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Dickson's Method for Generating Pythagorean Triples Revisited

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Abstract. The Dickson's method for generating Pythagorean triples states that the integers $a = r + s$, $b = r + t$, $c = r + s + t$ form a *Pythagorean triple* (a, b, c) on condition that $r^2 = 2st$, where r, s, t are positive integers. This paper presents a new simple proof of this method.

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1. Introduction

A triple (a, b, c) of positive integers is called a *Pythagorean triple* in case that $a^2 + b^2 = c^2$. There are several methods for generating *Pythagorean triples*. In this paper we investigate the Dickson's method, [1], that states that the integers $a = r + s$, $b = r + t$, $c = r + s + t$ form a *Pythagorean triple* (a, b, c) on condition that $r^2 = 2st$, where r, s, t are positive integers. The paper presents a new simple proof of this method.

2. Dickson's Method for Generating Pythagorean Triples

Theorem 1. *Given positive integers k, q, p, where $k > q$ and let $c = k + p$, $b = p + q$, $a = k$. Then $a^2 + b^2 = c^2$ if and only if $q^2 = 2p(k - q)$.*

Proof. We will show a bijection between triples (k, q, p) and (a, b, c). The bijection is depicted in Figures 1a and 1b (there are two examples for $k = 8$, $q = 4$, $p = 2$, $a = k = 8$, $b = p + q = 6$, $c = k + p = 10$ and $k = 8$, $p = 9$, $q = 6$, $a = k = 8$, $b = p + q = 15$, $c = k + p = 17$).

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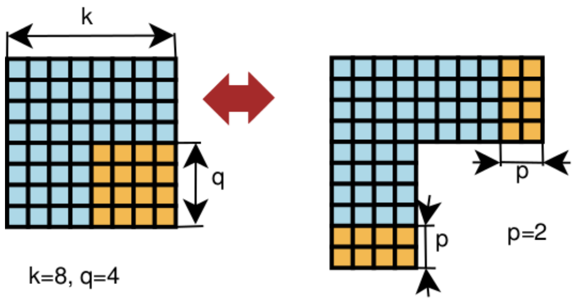
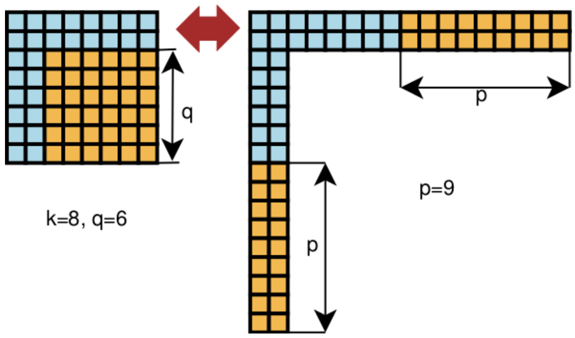


(a) $q = 4$ and $p = 2$



(b) $q = 6$ and $p = 9$

Figure 1: Examples of bijections between triples with $k = 8$ varying q and p.

The bijection consists in transforming the grid of $k \times k$ squares into an object that uniquely determines a *Pythagorean triple* (a, b, c). The crucial observation is that q^2 is divisible by $2(k-q)$ and hence the squares from the grid of q^2 squares can be equally separated in order to build up the L-shaped object on the right side of a figure; that object represents a *Pythagorean triple* $(k, q + p, k + p) = (a, b, c)$. To see this just note the L-shaped object is composed of k^2 squares and adding $(p + q)^2$ squares produces $(k + p)^2$ squares. On the other hand, given an L-shaped object composed of k^2 squares, then k, q, p are uniquely determined.
Also note that (b, a, c) and (a, b, c) will yield different values of (k, p, q).

References

[1] L.E. Dickson. History of the theory of numbers, vol. 2: Diophantine analysis. *Carnegie Institution of Washington, Publication No. 256*, 2:169, 1920.