



Compact Soft Multi Spaces

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Abstract. In this work, first we recall the concepts of soft multiset and soft multi topology. Then we will examine the concept of soft multi function which is defined between two soft multi class. Finally we will introduce soft multi compactness on soft multi topological space and give basic definitions and theorems about it.

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1. Introduction

Classical mathematical methods are not enough to solve the problems of daily life and also are not enough to meet the new requirements. Therefore, some theories such as Fuzzy set theory [17], Rough set theory [9], Soft set theory [8] and Multiset (or Bag) theory [16] have been developed to solve these problems.

Applications of these theories have been many areas of mathematics. Shabir and Naz [11] defined the soft topological space and studied the concepts of soft open set, soft multi interior point, soft neighborhood of a point, soft separation axioms, and subspace of a soft topological space. Aygunoglu and Aygun [2] introduced the soft continuity of soft mapping, soft product topology and studied soft compactness and generalized Tychono theorem to the soft topological space. Min [7] gave some results on soft topological spaces. Zorlutuna et al. [18] also investigated soft interior point and soft neighborhood. There are some other studies on the structure of soft topological spaces [3, 15]. Maji et al. [6] also initiated the more generalized concept of fuzzy soft sets which is a combination of fuzzy set and soft set. Tanay and Kandemir introduced topological structure of fuzzy soft set in [12] and gave an introductory theoretical base to carry further study on this concept. Following this study, some others [1, 5, 10, 14] studied on the concept of fuzzy soft topological spaces.

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The concept of soft multisets which is combining soft sets and multisets can be used to solve some real life problems. Also this concept can be used in many areas, such as data storage, computer science, information science, medicine, engineering, etc. The concept of soft multisets was introduced in [13]. Moreover, in [13] soft multi topology and its some properties was given. Also [13] soft multi connectedness was given.

In this work we will defined soft multi function between two soft multiset. After we will introduce soft multi compactness on soft multi topological space and will give basic definitions and theorems of soft multi compactness.

2. Preliminaries

2.1. Soft Set, Multiset and Soft Multiset

In this section, we present the basic definitions of soft set, multiset and soft multiset which may be found in earlier studies [4, 8, 13].

Definition 1 (Soft set). Let U be an initial universe set and E be set of parameters. Let $P(U)$ denotes the power set of U and $A \subseteq U$. A pair (F, A) is called a soft set over U , where F is a mapping given by $F : A \rightarrow P(U)$.

Definition 2 (Multiset). An mset M drawn from the set X is represented by a function Count M or C_M defined as $C_M : X \rightarrow N$ where N represented the set of non negative integers. The word "multiset" is often shortened to "mset".

Let M be an mset from X with x appearing n times in M . It is denoted by $x \in^n M$. $M = \{k_1/x_1, k_2/x_2, \dots, k_n/x_n\}$ where M is an mset with x_1 appearing k_1 times, x_2 appearing k_2 times and so on.

Definition 3. Let M be an mset drawn from a set X . The support set of M denoted by M^* is a subset of X and $M^* = \{x \in X : C_M(x) > 0\}$. i.e., M^* is an ordinary set and it is also called root set.

The power set of an mset is the support set of the power mset and is denoted by $P^*(M)$.

Example 1. Let $M = \{2/x, 3/y\}$ be an mset. Then $M^* = \{x, y\}$ is the support set of M and

$$P^*(M) = \{ \{2/x, 1/y\}, \{2/x, 2/y\}, \{1/x, 1/y\}, \{1/x, 2/y\}, \{1/x, 3/y\}, \{2/x\}, \{1/x\}, \{3/y\}, \{2/y\}, \{1/y\} \}$$

is the support set of $P(M)$.

Definition 4 (Soft multiset). Let U be an universal multiset, E be set of parameters and $A \subseteq E$. Then a pair (F, A) is called a soft multiset where F is a mapping given by $F : A \rightarrow P^*(U)$. For $\forall e \in A$, multiset $F(e)$ represent by count function $C_{F(e)} : U^* \rightarrow N$ where N represents the set of non negative integers.

Example 2. Let multiset and the parameter set be $U = \{1/x, 5/y, 3/z, 4/w\}$ and $E = \{p, q, r\}$. Define a mapping $F : E \rightarrow P^*(U)$ as follows:

$$\begin{aligned} F(p) &= \{1/x, 2/y, 3/z\}, \\ F(q) &= \{4/w\} \text{ and} \\ F(r) &= \{3/y, 1/z, 2/w\}. \end{aligned}$$

Then (F, A) is a soft multiset where for $\forall e \in A$, $F(e)$ multiset represent by count function $C_{F(e)} : U^* \rightarrow N$, which are defined as follows:

$$\begin{aligned} C_{F(p)}(x) &= 1, & C_{F(p)}(y) &= 2, & C_{F(p)}(z) &= 3, & C_{F(p)}(w) &= 0, \\ C_{F(q)}(x) &= 0, & C_{F(q)}(y) &= 0, & C_{F(q)}(z) &= 0, & C_{F(q)}(w) &= 4, \\ C_{F(r)}(x) &= 0, & C_{F(r)}(y) &= 3, & C_{F(r)}(z) &= 1, & C_{F(r)}(w) &= 2. \end{aligned}$$

Then $(F, A) = \{F(p), F(q), F(r)\} = \{\{1/x, 2/y, 3/z\}, \{4/w\}, \{3/y, 1/z, 2/w\}\}$.

Definition 5. For two soft multisets (F, A) and (G, B) over U , we say that (F, A) is a soft submultiset of (G, B) if

- i. $A \subseteq B$
- ii. $C_{F(e)}(x) \leq C_{G(e)}(x), \forall x \in U^*, \forall e \in A$

We write $(F, A) \tilde{\subseteq} (G, B)$. In addition to (F, A) is a whole soft submultiset of (G, B) if $C_{F(e)}(x) = C_{G(e)}(x), \forall x \in U^*, \forall e \in A$.

Definition 6. Let (F, A) and (G, B) be two soft multisets over U .

Equal $(F, A) = (G, B) \Leftrightarrow (F, A) \tilde{\subseteq} (G, B)$ and $(F, A) \tilde{\supseteq} (G, B)$.

Union $(H, C) = (F, A) \tilde{\cup} (G, B)$ where $C = A \cup B$ and $C_{H(e)}(x) = \max\{C_{F(e)}(x), C_{G(e)}(x)\}, \forall e \in A \cup B, \forall x \in U^*$.

Intersection $(H, C) = (F, A) \tilde{\cap} (G, B)$ where $C = A \cap B$ and $C_{H(e)}(x) = \min\{C_{F(e)}(x), C_{G(e)}(x)\}, \forall e \in A \cap B, \forall x \in U^*$. We write $(F, A) \tilde{\cap} (G, B)$.

Difference $(H, E) = (F, E) \setminus (G, E)$ where $C_{H(e)}(x) = \max\{C_{F(e)}(x) - C_{G(e)}(x), 0\}, \forall x \in U^*$.

Null A soft multiset (F, A) is said to be a NULL soft multiset denoted by Φ if for all $e \in A, F(e) = \emptyset$.

Complement The complement of a soft multiset (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where $F^c : A \rightarrow P^*(U)$ is a mapping given by $F^c(e) = U \setminus F(e)$ for all $e \in A$ where $C_{F^c(e)}(x) = C_U(x) - C_{F(e)}(x), \forall x \in U^*$.

Definition 7. Let (F, E) be a soft multiset over U and $a \in U^*$. We say that $a \in (F, E)$ read as a belongs to the soft multiset (F, E) whenever $a \in F(e)$ for all $e \in E$.

Note that for any $a \in U, a \notin (F, E)$, if $a \notin F(e)$ for some $e \in E$.

Let (F, E) be soft multiset over U . If for all $e \in E$ and $a \in U^*$, $C_{F(e)}(a) = n$ ($n \geq 1$) then we will write $a \in F(e)$ instead of $a \in^n F(e)$.

Definition 8. Let V be a non-empty submultiset of U , then \tilde{V} denotes the soft multiset (V, E) over U for which $V(e) = V$, for all $e \in E$.

In particular, (U, E) will be denoted by \tilde{U} .

Definition 9. Let $a \in U^*$, then (a, E) denotes the soft multiset over U for which $a(e) = \{a\}$, for all $e \in E$.

Definition 10. Let (F, E) be a soft multiset over U and V be a non-empty submultiset of U . Then the sub soft multiset of (F, E) over V denoted by $({}^V F, E)$, is defined as follows

$${}^V F(e) = V \cap F(e), \text{ for all } e \in E \text{ where } C_{{}^V F(e)}(x) = \min\{C_V(x), C_{F(e)}(x)\}, \forall x \in U^*$$

In other words $({}^V F, E) = \tilde{V} \tilde{\cap} (F, E)$.

2.2. Soft Multi Topology

In this section, we recall soft multi topology which given in [13].

Definition 11. Let X be universal multiset and E be set of parameters. Then the collection of all soft multisets over X with parameters from E is called a soft multi class and is denoted as X_E .

Definition 12. Let $\tau \subseteq X_E$, then τ is said to be a soft multi topology on X if the following conditions hold.

- i. Φ, \tilde{X} belong to τ .
- ii. The union of any number of soft multisets in τ belongs to τ .
- iii. The intersection of any two soft multisets in τ belongs to τ .

τ is called a soft multi topology over X and the binary (X_E, τ) is called a soft multi topological space over X .

The members of τ are said to be soft multi open sets in X .

A soft multiset (F, E) over X is said to be a soft multi closed set in X , if its complement $(F, E)^c$ belongs to τ .

Example 3. Let $X = \{2/x, 3/y, 4/z, 5/w\}$, $E = \{p, q\}$ and $\tau = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ where $(F_1, E), (F_2, E), (F_3, E)$ are soft multisets over X , defined as follows

$$\begin{aligned} F_1(p) &= \{1/x, 2/y, 3/z\}, & F_1(q) &= \{4/w\} \\ F_2(p) &= X, & F_2(q) &= \{1/x, 3/y, 4/z, 5/w\} \\ F_3(p) &= \{2/x, 3/y, 3/z, 1/w\}, & F_3(q) &= \{1/x, 4/w\}. \end{aligned}$$

Then τ defines a soft multi topology on X and hence (X_E, τ) is a soft multi topological space over X .

Definition 13. Let (X_E, τ_1) and (X_E, τ_2) be soft multi topological spaces. Then, the following hold.

- If $\tau_2 \supset \tau_1$, then T_2 is soft multi finer than τ_1 .
- If $\tau_2 \supset \tau_1$, then τ_2 is soft multi strictly finer than τ_1 .
- If either $\tau_2 \supseteq \tau_1$ or $\tau_2 \subseteq \tau_1$, then τ_1 is comparable with τ_2 .

Definition 14. Let X be universal multiset, E be the set of parameters.

- let τ be the collection of all soft multisets which can be defined over X . Then τ is called the soft multi discrete topology on X and (X_E, τ) is said to be a soft multi discrete space over X .
- $\tau = \{\Phi, \tilde{X}\}$ is called the soft multi indiscrete topology on X and (X_E, τ) is said to be a soft indiscrete space over X .

Definition 15. Let (X_E, τ) be a soft multi topological space over X and Y be a non-empty subset of X . Then

$$\tau_Y = \{(^Y F, E) : (F, E) \in \tau\}$$

is said to be the soft multi topology on Y and (Y_E, τ_Y) is called a soft multi subspace of (X_E, τ) .

We can easily verify that τ_Y is, in fact, a soft multi topology on Y .

3. Soft Multi Function

In this section, we defined soft multi function and examined its basic theorems.

Definition 16. Let X_E and Y_K be two soft multi class. Let $\varphi : X^* \rightarrow Y^*$ and $\psi : E \rightarrow K$ be two functions. Then the pair (φ, ψ) is called a soft multi function and denoted by $f = (\varphi, \psi) : X_E \rightarrow Y_K$ is defined as follows:

Let (F, E) be a soft multiset in X_E . Then the image of (F, E) under soft multi function f is soft multiset in Y_K defined by $f(F, E)$, where for $k \in \psi(E) \subseteq K$ and $y \in Y^*$,

$$C_{f(F,E)(k)}(y) = \begin{cases} \sup_{e \in \psi^{-1}(k) \cap E, x \in \varphi^{-1}(y)} C_{F(e)}(x) & , \text{if } \psi^{-1}(k) \neq \emptyset, \varphi^{-1}(y) \neq \emptyset; \\ 0 & , \text{otherwise.} \end{cases}$$

Let (G, K) be a soft multiset in Y_K . Then the inverse image of (G, K) under soft multi function f is soft multiset in X_E defined by $f^{-1}(G, K)$, where for $e \in \psi^{-1}(K) \subseteq E$ and $x \in X^*$,

$$C_{f^{-1}(G,K)(e)}(x) = C_{G(\psi(e))}(\varphi(x)).$$

Example 4. Let $X = \{2/a, 3/b, 4/c, 5/d\}$, $Y = \{5/x, 4/y, 3/z, 2/w\}$, $E = \{e_1, e_2, e_3, e_4\}$, $K = \{k_1, k_2, k_3\}$ and X_E, Y_K , classes of soft multisets. Let $\varphi : X^* \rightarrow Y^*$ and $\psi : E \rightarrow K$ be two function defined as

$$\begin{aligned} \varphi(a) &= z, & \varphi(b) &= y, & \varphi(c) &= y, & \varphi(d) &= x, \\ \psi(e_1) &= k_1, & \psi(e_2) &= k_3, & \psi(e_3) &= k_2, & \psi(e_4) &= k_1. \end{aligned}$$

Choose two soft multisets in X_E and Y_K , respectively, as

$$\begin{aligned} (F, A) &= \{e_1 = \{1/a, 2/b, 1/d\}, e_3 = \{3/b, 2/c, 1/d\}, e_4 = \{2/a, 5/d\}\}, \\ (G, B) &= \{k_1 = \{4/x, 2/w\}, k_2 = \{1/x, 1/y, 2/z, 2/w\}\} \end{aligned}$$

Then soft multiset image of (F, A) under $f : X_E \rightarrow Y_K$ is obtained as

$$\begin{aligned} C_{f(F,A)(k_1)}(x) &= \begin{cases} \sup_{e \in \psi^{-1}(k_1) \cap A, a \in \varphi^{-1}(x)} C_{F(e)}(a) & , \text{ if } \psi^{-1}(k_1) \neq \emptyset, \varphi^{-1}(x) \neq \emptyset; \\ 0 & , \text{ otherwise.} \end{cases} \\ &= \begin{cases} \sup_{e \in \{e_1, e_4\}, a \in \{d\}} C_{F(e)}(a) & , \text{ if } \psi^{-1}(k_1) \neq \emptyset, \varphi^{-1}(x) \neq \emptyset; \\ 0 & , \text{ otherwise.} \end{cases} \\ &= \sup \{C_{F(e_1)}(d), C_{F(e_4)}(d)\} \\ &= 5 \end{aligned}$$

$$C_{f(F,A)(k_1)}(y) = \sup \{C_{F(e_1)}(b), C_{F(e_4)}(b), C_{F(e_1)}(c), C_{F(e_4)}(c)\} = 2,$$

$$C_{f(F,A)(k_1)}(z) = \sup \{C_{F(e_1)}(a), C_{F(e_4)}(a)\} = 2,$$

$$C_{f(F,A)(k_1)}(w) = 0 \quad (\text{since } \varphi^{-1}(w) = \emptyset),$$

$$\begin{aligned} C_{f(F,A)(k_2)}(x) &= \begin{cases} \sup_{e \in \psi^{-1}(k_2) \cap A, a \in \varphi^{-1}(x)} C_{F(e)}(a) & , \text{ if } \psi^{-1}(k_2) \neq \emptyset, \varphi^{-1}(x) \neq \emptyset; \\ 0 & , \text{ otherwise.} \end{cases} \\ &= \begin{cases} \sup_{e \in \{e_3\}, a \in \{d\}} C_{F(e)}(a) & , \text{ if } \psi^{-1}(k_2) \neq \emptyset, \varphi^{-1}(x) \neq \emptyset; \\ 0 & , \text{ otherwise.} \end{cases} \\ &= \sup \{C_{F(e_3)}(d)\} \\ &= 1, \end{aligned}$$

$$C_{f(F,A)(k_2)}(y) = \sup \{C_{F(e_3)}(b), C_{F(e_3)}(c)\} = 3,$$

$$C_{f(F,A)(k_2)}(z) = \sup \{C_{F(e_3)}(a)\} = 0,$$

$$C_{f(F,A)(k_2)}(w) = 0 \quad (\text{since } \varphi^{-1}(w) = \emptyset).$$

Consequently, we have

$$(f(F,A), B) = \{k_1 = \{5/x, 2/y, 2/z\}, k_2 = \{1/x, 3/y\}\}.$$

Soft multiset inverse image of (G, B) under $f : X_E \rightarrow Y_K$ is obtained as

$$\begin{aligned} C_{f^{-1}(G,B)(e_3)}(a) &= C_{G(\psi(e_3))}(\varphi(a)) = C_{G(k_2)}(z) = 2, \\ C_{f^{-1}(G,B)(e_3)}(b) &= C_{G(\psi(e_3))}(\varphi(b)) = C_{G(k_2)}(y) = 1, \\ C_{f^{-1}(G,B)(e_3)}(c) &= C_{G(\psi(e_3))}(\varphi(c)) = C_{G(k_2)}(y) = 1, \\ C_{f^{-1}(G,B)(e_3)}(d) &= C_{G(\psi(e_3))}(\varphi(d)) = C_{G(k_2)}(x) = 1, \\ C_{f^{-1}(G,B)(e_4)}(a) &= C_{G(\psi(e_4))}(\varphi(a)) = C_{G(k_1)}(z) = 0, \\ C_{f^{-1}(G,B)(e_4)}(b) &= C_{G(\psi(e_4))}(\varphi(b)) = C_{G(k_1)}(y) = 0, \\ C_{f^{-1}(G,B)(e_4)}(c) &= C_{G(\psi(e_4))}(\varphi(c)) = C_{G(k_1)}(y) = 0, \\ C_{f^{-1}(G,B)(e_4)}(d) &= C_{G(\psi(e_4))}(\varphi(d)) = C_{G(k_1)}(x) = 4. \end{aligned}$$

Consequently, we have

$$(f^{-1}(G, B), D) = \{e_3 = \{2/a, 1/b, 1/c, 1/d\}, e_4 = \{4/d\}\}.$$

Theorem 1. Let $f : X_E \rightarrow Y_K$ be a soft multi function, $(F, A), (F_i, A)$ soft multisets in X_E and $(G, B), (G_i, B)$ soft multisets in Y_K .

- (1) $f(\Phi) = \Phi, f(\tilde{X}) \tilde{\subseteq} \tilde{Y}$,
- (2) $f^{-1}(\Phi) = \Phi, f^{-1}(\tilde{Y}) = \tilde{X}$,
- (3) $f((F_1, A_1) \tilde{\cup} (F_2, A_2)) = f(F_1, A_1) \tilde{\cup} f(F_2, A_2)$.
In general, $f(\tilde{\cup}_{i \in I} (F_i, A_i)) = \tilde{\cup}_{i \in I} f(F_i, A_i)$,
- (4) $f^{-1}((G_1, B) \tilde{\cup} (G_2, B)) = f^{-1}(G_1, B) \tilde{\cup} f^{-1}(G_2, B)$.
In general, $f^{-1}(\tilde{\cup}_{i \in I} (G_i, B)) = \tilde{\cup}_{i \in I} f^{-1}(G_i, B)$,
- (5) $f((F_1, A) \tilde{\cap} (F_2, A)) \tilde{\subseteq} f(F_1, A) \tilde{\cap} f(F_2, A)$.
In general, $f(\tilde{\cap}_{i \in I} (F_i, A)) \tilde{\subseteq} \tilde{\cap}_{i \in I} f(F_i, A)$,
- (6) $f^{-1}((G_1, B) \tilde{\cap} (G_2, B)) = f^{-1}(G_1, B) \tilde{\cap} f^{-1}(G_2, B)$.
In general, $f^{-1}(\tilde{\cap}_{i \in I} (G_i, B)) = \tilde{\cap}_{i \in I} f^{-1}(G_i, B)$,
- (7) If $(F_1, A) \tilde{\subseteq} (F_2, A)$, then $f(F_1, A) \tilde{\subseteq} f(F_2, A)$,
- (8) If $(G_1, B) \tilde{\subseteq} (G_2, B)$, then $f^{-1}(G_1, B) \tilde{\subseteq} f^{-1}(G_2, B)$.

Proof. By using Definition 16 , we only prove (3) – (8)

- (3) Suppose that $(F, A) = (F_1, A_1) \check{\cup} (F_2, A_2)$. We we should show that $f(F, A) = f(F_1, A_1) \check{\cup} f(F_2, A_2)$. Then for $k \in K$ and $y \in Y^*$,

$$\begin{aligned} C_{f(F,A)(k)}(y) &= \sup_{e \in \psi^{-1}(k) \cap A, x \in \varphi^{-1}(y)} C_{F(e)}(x) \\ &= \sup_{e \in \psi^{-1}(k) \cap A, x \in \varphi^{-1}(y)} \max\{C_{F_1(e)}(x), C_{F_2(e)}(x)\} \\ &= \max\left\{ \sup_{e \in \psi^{-1}(k) \cap A, x \in \varphi^{-1}(y)} C_{F_1(e)}(x), \sup_{e \in \psi^{-1}(k) \cap A, x \in \varphi^{-1}(y)} C_{F_2(e)}(x) \right\} \\ &= \max\{C_{f(F_1,A_1)(k)}(y), C_{f(F_2,A_2)(k)}(y)\}. \end{aligned}$$

Therefore $f(F, A) = f(F_1, A_1) \check{\cup} f(F_2, A_2)$. Hence $f((F_1, A_1) \check{\cup} (F_2, A_2)) = f(F_1, A_1) \check{\cup} f(F_2, A_2)$. Similarly, $f(\check{\cup}_{i \in I} (F_i, A_i)) = \check{\cup}_{i \in I} f(F_i, A_i)$.

- (4) Suppose that $(G, B) = (G_1, B) \check{\cup} (G_2, B)$. We we should show that $f^{-1}(G, B) = f^{-1}(G_1, B) \check{\cup} f^{-1}(G_2, B)$. Then for $e \in E$ and $x \in X^*$,

$$\begin{aligned} C_{f^{-1}(G,B)(e)}(x) &= C_{G(\psi(e))}(\varphi(x)) \\ &= \max\{C_{G_1(\psi(e))}(\varphi(x)), C_{G_2(\psi(e))}(\varphi(x))\} \\ &= \max\{C_{f^{-1}(G_1,B_1)(e)}(x), C_{f^{-1}(G_2,B_2)(e)}(x)\}. \end{aligned}$$

Therefore $f^{-1}((G_1, B) \check{\cup} (G_2, B)) = f^{-1}(G_1, B) \check{\cup} f^{-1}(G_2, B)$. Similarly, $f^{-1}(\check{\cup}_{i \in I} (G_i, B)) = \check{\cup}_{i \in I} f^{-1}(G_i, B)$.

- (5) Suppose that $(F, A) = (F_1, A_1) \check{\cap} (F_2, A_2)$. Then for $k \in K$ and $y \in Y^*$,

$$\begin{aligned} C_{f(F,A)(k)}(y) &= \sup_{e \in \psi^{-1}(k) \cap (A_1 \cap A_2), x \in \varphi^{-1}(y)} C_{F(e)}(x) \\ &= \sup_{e \in \psi^{-1}(k) \cap (A_1 \cap A_2), x \in \varphi^{-1}(y)} \min\{C_{F_1(e)}(x), C_{F_2(e)}(x)\} \\ &= \min\left\{ \sup_{e \in \psi^{-1}(k) \cap (A_1 \cap A_2), x \in \varphi^{-1}(y)} C_{F_1(e)}(x), \sup_{e \in \psi^{-1}(k) \cap (A_1 \cap A_2), x \in \varphi^{-1}(y)} C_{F_2(e)}(x) \right\} \\ &\leq \min\left\{ \sup_{e \in \psi^{-1}(k) \cap A_1, x \in \varphi^{-1}(y)} C_{F_1(e)}(x), \sup_{e \in \psi^{-1}(k) \cap A_2, x \in \varphi^{-1}(y)} C_{F_2(e)}(x) \right\} \\ &= \min\{C_{f(F_1,A_1)(k)}(y), C_{f(F_2,A_2)(k)}(y)\}. \end{aligned}$$

Therefore $f((F_1, A_1) \check{\cap} (F_2, A_2)) \check{\subseteq} f(F_1, A_1) \check{\cap} f(F_2, A_2)$. Similarly, $f(\check{\cup}_{i \in I} (F_i, A_i)) \check{\subseteq} \check{\cup}_{i \in I} f(F_i, A_i)$.

- (6) Suppose that $(G, B) = (G_1, B) \check{\cap} (G_2, B)$. We we should show that $f^{-1}(G, B) = f^{-1}(G_1, B) \check{\cap} f^{-1}(G_2, B)$. Then for $e \in E$ and $x \in X^*$,

$$\begin{aligned} C_{f^{-1}(G,B)(e)}(x) &= C_{G(\psi(e))}(\varphi(x)) \\ &= \min\{C_{G_1(\psi(e))}(\varphi(x)), C_{G_2(\psi(e))}(\varphi(x))\} \\ &= \min\{C_{f^{-1}(G_1,B_1)(e)}(x), C_{f^{-1}(G_2,B_2)(e)}(x)\}. \end{aligned}$$

Therefore $f^{-1}((G_1, B) \check{\cap} (G_2, B)) = f^{-1}(G_1, B) \check{\cap} f^{-1}(G_2, B)$. Similarly, $f^{-1}(\check{\cap}_{i \in I} (G_i, B)) = \check{\cap}_{i \in I} f^{-1}(G_i, B)$.

(7) Suppose that $(F_1, A_1) \tilde{\subseteq} (F_2, A_2)$. Then for all $e \in E$ and $x \in X^*$, $C_{F_1(e)}(x) \leq C_{F_2(e)}(x)$. Hence for $k \in K$ and $y \in Y^*$,

$$\begin{aligned} C_{f(F_1, A_1)(k)}(y) &= \sup_{e \in \psi^{-1}(k) \cap A_1, x \in \varphi^{-1}(y)} C_{F_1(e)}(x) \\ &\leq \sup_{e \in \psi^{-1}(k) \cap A_2, x \in \varphi^{-1}(y)} C_{F_2(e)}(x) \\ &= C_{f(F_2, A_2)(k)}(y). \end{aligned}$$

This show that $f(F_1, A_1) \tilde{\subseteq} f(F_2, A_2)$.

(8) Suppose that $(G_1, B_1) \tilde{\subseteq} (G_2, B_2)$. Then for all $k \in K$ and $y \in Y^*$, $C_{G_1(k)}(y) \leq C_{G_2(k)}(y)$. Hence for $e \in E$ and $x \in X^*$,

$$\begin{aligned} C_{f^{-1}(G_1, B_1)(e)}(x) &= C_{G_1(\psi(e))}(\varphi(x)) \\ &\leq C_{G_2(\psi(e))}(\varphi(x)) \\ &= C_{f^{-1}(G_2, B_2)(e)}(x) \end{aligned}$$

where $\psi(e) \in K$ and $\varphi(x) \in Y^*$. This show that $f^{-1}(G_1, B_1) \tilde{\subseteq} f^{-1}(G_2, B_2)$.

□

4. Compact Soft Multi Spaces

In this section, we introduced soft multi compactness on soft multi topological space and give basic definitions and theorems about it.

Definition 17. Let (X_E, τ) and (Y_K, σ) be two soft multi topological spaces.

- (1) A soft multi function $f : (X_E, \tau) \rightarrow (Y_K, \sigma)$ is called soft multi continuous if for all $(G, B) \in \sigma$, $f^{-1}(G, B) \in \tau$.
- (2) A soft multi function $f : (X_E, \tau) \rightarrow (Y_K, \sigma)$ is called soft multi open if for all $(F, A) \in \tau$, $f(F, A) \in \sigma$.

Definition 18. A family Ψ of soft multisets is a cover of a soft multiset (F, A) if

$$(F, A) \subseteq \cup \{ (F_i, A) : (F_i, A) \in \Psi, i \in I \}.$$

It is a soft multi open cover if each member of Ψ is a soft multi open set. A subcover of Ψ is a subfamily of Ψ which is also a cover.

Definition 19. Let (X_E, τ) be soft multi topological space and $(F, A) \in FS(X, E)$. Soft multiset (F, A) is called compact if each soft multi open cover of (F, A) has a finite subcover. Also soft multi topological space (X_E, τ) is called compact if each soft multi open cover of \tilde{X} has a finite subcover.

Example 5. A soft multi topological space (X_E, τ) is compact if X is finite.

Example 6. Let (X_E, τ) and (Y_K, σ) be two soft multi topological spaces and $\tau \subset \sigma$. Then, soft multi topological space (X_E, τ) is compact if (Y_K, σ) is compact.

Proposition 1. Let (G, B) be a whole soft multi closed set in soft multi compact space (X_E, τ) . Then (G, B) is also compact.

Proof. Let (F_i, A) be any open covering of (G, B) . Then

$$\tilde{X} \subseteq (\cup_{i \in I} (F_i, A)) \cup (G, B)^c;$$

that is, (F_i, A) together with soft multi open set $(G, B)^c$ is a open covering of \tilde{X} . Therefore there exists a finite subcovering $(F_1, A), (F_2, A), \dots, (F_n, A), (G, B)^c$. So

$$\tilde{X} \subseteq (F_1, A) \cup (F_2, A) \cup \dots \cup (F_n, A) \cup (G, B)^c.$$

Therefore

$$(G, B) \subseteq (F_1, A) \cup (F_2, A) \cup \dots \cup (F_n, A) \cup (G, B)^c$$

which clearly implies

$$(G, B) \subseteq (F_1, A) \cup (F_2, A) \cup \dots \cup (F_n, A)$$

since $(G, B) \cap (G, B)^c = \Phi$. Hence (G, B) has a finite subcovering and so is compact. □

Definition 20 ([13]). Let (X_E, τ) be a soft multi topological space over X and $x, y \in X$ such that $x \neq y$. If there exist soft multi open sets (F, A) and (G, A) such that $x \in (F, A), y \in (G, A)$ and $(F, A) \tilde{\cap} (G, A) = \Phi$, then (X_E, τ) is called a soft multi Hausdorff space.

Proposition 2. Let (G, B) be a whole soft multi compact set in soft multi Hausdorff space (X_E, τ) . Then (G, B) is closed.

Proof. Let $x \in (G, B)^c$. For each $y \in (G, B)$, we have $x \neq y$, so there are disjoint soft multi open sets (F_y, A) and (H_y, A) so that $x \in (F_y, A)$ and $y \in (H_y, A)$. Then $\{(H_y, A) : y \in (G, B)\}$ is an soft multi open cover of (G, B) Let $\{(H_{y_1}, A), (H_{y_2}, A), \dots, (H_{y_n}, A)\}$ be a finite subcover. Then $\cap_{i=1}^n (F_{y_i}, A)$ is an open set containing x and contained in $(G, B)^c$. Thus $(G, B)^c$ is soft multi open and (G, B) is closed. □

Theorem 2. Let (X_E, τ) and (Y_K, σ) be soft multi topological spaces and $f : (X_E, \tau) \rightarrow (Y_K, \sigma)$ continuous and onto soft multi function. If (X_E, τ) is soft multi compact, then (Y_K, σ) is soft multi compact.

Proof. We will use Theorem 1. Let (F_i, A) be any open covering of \tilde{Y} ; i.e., $\tilde{Y} \subseteq \cup_{i \in I} (F_i, A)$. Then $f^{-1}(\tilde{Y}) \subseteq f^{-1}(\cup_{i \in I} (F_i, A))$; and $\tilde{X} \subseteq \cup_{i \in I} f^{-1}(F_i, A)$. So $f^{-1}((F_i, A))$ is an open covering of \tilde{X} . As (X_E, τ) is compact, there are $1, 2, \dots, n$ in I such that

$$\tilde{X} \subseteq f^{-1}(F_1, A) \cup f^{-1}(F_2, A) \cup \dots \cup f^{-1}(F_n, A).$$

Since (φ, ψ) is surjective, we have

$$\begin{aligned}\tilde{Y} &= f(\tilde{X}) \\ &\subseteq f(f^{-1}((F_1, A)) \cup \dots \cup f^{-1}((F_n, A))) \\ &= f(f^{-1}((F_1, A))) \cup \dots \cup f(f^{-1}((F_n, A))) \\ &= (F_1, A) \cup (F_2, A) \cup \dots \cup (F_n, A).\end{aligned}$$

So we have $\tilde{Y} \subseteq (F_1, A) \cup (F_2, A) \cup \dots \cup (F_n, A)$; i.e., \tilde{Y} is covered by a finite number of (F_i, A) .

Hence (Y_K, σ) is compact. \square

Definition 21. Let (X_E, τ) and (Y_K, σ) be two soft multi topological spaces. A soft multi function $f : (X_E, \tau) \rightarrow (Y_K, \sigma)$ is called soft multi closed if $f((F, A))$ is soft multi closed set in (Y_K, σ) , for all soft multi closed set (F, A) in (X_E, τ) .

Theorem 3. Let (X_E, τ) be a soft multi topological space and (Y_K, σ) be a soft multi Hausdorff space. Soft multi function f is closed if soft multi function $f : (X_E, \tau) \rightarrow (Y_K, \sigma)$ is continuous.

Proof. Let (G, B) be any soft multi closed set in (X_E, τ) . By Proposition 1 we have (G, B) is compact. Since soft multi function (φ, ψ) is continuous, soft multiset $f((G, B))$ is compact in (Y_K, σ) . As (Y_K, σ) is soft multi Hausdorff space, soft multiset $f((G, B))$ is closed. Then soft multi function f is closed. \square

Definition 22. A family Ψ of whole soft multisets has the finite intersection property if the intersection of the members of each finite subfamily of Ψ is not the null soft multiset.

Theorem 4. A soft multi topological space is compact if and only if each family of whole soft multi closed sets with the finite intersection property has a nonnull intersection.

Proof. \Rightarrow : Let Ψ be any family of whole soft multi closed subset such that $\bigcap \{(F_i, A) : (F_i, A) \in \Psi, i \in I\} = \Phi$. Consider $\Omega = \{(F_i, A)^c : (F_i, A) \in \Psi, i \in I\}$. So Ω is a soft multi open cover of \tilde{X} . As soft multi topological space is compact, there exists a finite subcovering $(F_1, A)^c, (F_2, A)^c, \dots, (F_n, A)^c$. Then $\bigcap_{i=1}^n (F_i, A) = \tilde{X} - \bigcup_{i=1}^n (F_i, A)^c = \tilde{X} - \tilde{X} = \Phi$. Thence Ψ cannot have finite intersection property.

\Leftarrow : Assume that a soft multi topological space is not compact. Then any soft multi open cover of \tilde{X} has not a finite subcover. Let $\{(F_i, A) : i \in I\}$ be soft multi open cover of \tilde{X} . So $\bigcup_{i=1}^n (F_i, A) \neq \tilde{X}$. Therefore $\bigcap_{i=1}^n (F_i, A)^c \neq \Phi$. Thus, $\{(F_i, A)^c : i = 1, \dots, n\}$ have finite intersection property. By using hypothesis, $\bigcap (F_i, A)^c \neq \Phi$ and so $\bigcup (F_i, A) \neq \tilde{X}$. This is a contradiction. Thus soft multi topological space is compact. \square

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