

New Type of Strongly Continuous Functions In topological Spaces Via $\delta - \beta$ -Open Sets

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Abstract. In this paper we introduce and investigate a new class of strong continuous functions called strongly $\theta - \delta - \beta$ -continuous functions by using two new strong forms of $\delta - \beta$ -open sets called $\delta - \beta$ -regular sets and $\delta - \beta_\theta$ -open sets. This class is a generalization of both strongly θ -e-continuous functions and strongly $\theta - \beta$ -continuous functions. Several new characterizations and fundamental properties concerning strongly $\theta - \delta - \beta$ -continuous functions are obtained. Furthermore, the relationships between strongly $\theta - \delta - \beta$ -continuous functions and other well-known types of strong continuity are also discussed.

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1. Introduction

The notion of continuity is an important concept in general topology as well as all branches of mathematics and quantum physics of course its strong forms are important, too. Recently, strong continuity of functions in topological spaces has been introduced and investigated by many mathematicians and quantum physicists. Furthermore, generalized open and closed sets are, as well-known, the most important notions in both pure and applied mathematics. The notion of continuity by involving these notions is the subject-matter of topology which has penetrated in the whole body of science. In the course of time, mathematicians realized that it is very useful to generalize the notions of open and closed sets and accordingly the notion of continuity. In 1980, Noiri [37] introduced the notion of strong θ -continuity which is stronger than δ -continuity [37]. Some properties of strongly θ -continuous functions are studied by Long and Herrington [32]. Recently, five generalizations of strong θ -continuity are obtained

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- a) $\delta - \beta - Cl(A) = A \cup Int(Cl(\delta - Int(A)))$;
- b) $\delta - \beta - Int(A) = A \cap Cl(Int(\delta - \beta - Cl(A)))$;
- c) A is $\delta - \beta$ -closed if and only if $A = \delta - \beta - Cl(A)$;
- d) $\delta - \beta - Cl(A)$ is $\delta - \beta$ -closed;
- e) $X \setminus \delta - \beta - Cl(A) = \delta - \beta - Int(X \setminus A)$;
- f) $x \in \delta - \beta - Cl(A)$ if $A \cap U \neq \phi$ for every $\delta - \beta$ -open set U containing x .

Remark 3. From above definitions we have the following diagram in which the converses of implications need not be true, see the examples in [8, 9, 28].

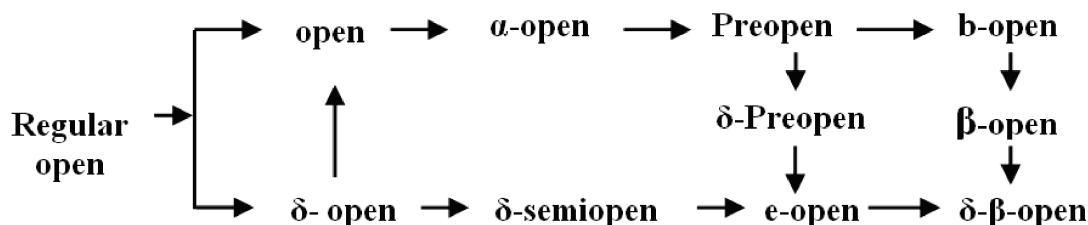


Figure 1: The relationships among some well-known generalized open sets in topological spaces

3. Strong Forms of $\delta - \beta$ -Open Sets

In this section we introduce two new strong forms of $\delta - \beta$ -open sets, called $\delta - \beta$ -regular sets and $\delta - \beta_\theta$ -open sets. By using these sets we introduce a several characterizations of $\delta - \beta$ -open sets and their properties.

Definition 1. A subset A of a topological space X is $\delta - \beta$ -regular if it is $\delta - \beta$ -open and $\delta - \beta$ -closed. The family of all $\delta - \beta$ -regular subsets of X containing a point $x \in X$ is denoted by $\delta - \beta R(X, x)$, the family of all $\delta - \beta$ -regular sets in X denoted by $\delta - \beta R(X, T)$.

Definition 2. A point x of X is called a $\delta - \beta_\theta$ -cluster point of A if $\delta - \beta - Cl(U) \cap A \neq \phi$ for every $U \in \delta - \beta \Sigma(X, x)$. The set of all $\delta - \beta_\theta$ -cluster points of A is called $\delta - \beta_\theta$ -closure of A and is denoted by $\delta - \beta - Cl_\theta(A)$. A subset A is said to be $\delta - \beta_\theta$ -closed if $A = \delta - \beta - Cl_\theta(A)$. The complement of a $\delta - \beta_\theta$ -closed set is said to be $\delta - \beta_\theta$ -open.

Remark 4. The union of two $\delta - \beta_\theta$ -closed sets is not necessarily $\delta - \beta_\theta$ -closed as Shown by the following example:

Example 1. Let $X = \{1, 2, 3\}$, Define a topology $T = \{\phi, X, \{1\}, \{2\}, \{1, 2\}\}$ on X . The subsets $\{1\}$ and $\{2\}$ are $\delta - \beta_\theta$ -closed in (X, T) but $\{1, 2\}$ is not $\delta - \beta_\theta$ -closed.

Remark 5. It can be easily shown that $\delta - \beta$ -regular $\Rightarrow \delta - \beta_\theta$ -open $\Rightarrow \delta - \beta$ -open. But the converses are not necessarily true as shown by the following examples:

Example 2. Let $X = \{1, 2, 3\}$, Define a topology $T = \{\phi, X, \{1\}, \{2\}, \{1, 2\}\}$ on X . The subsets $\{1, 2\}$ is $\delta - \beta_\theta$ -open in X but not $\delta - \beta$ -regular.

Example 3. Let $X = \{1, 2, 3, 4, 5\}$, Define a topology $T = \{\phi, X, \{1\}, \{3\}, \{1, 3\}, \{3, 4\}, \{1, 3, 4\}\}$ on X . then The subsets $\{1\}$ is $\delta - \beta$ -open in X but not $\delta - \beta_\theta$ -open.

The following interesting results will play an important role in the sequel.

Theorem 1. The following properties hold for a subset A of a topological space (X, T) :

- a) $A \in \delta - \beta\Sigma(X, T)$ if and only if $\delta - \beta\text{-Cl}(A) \in \delta - \beta R(X, T)$;
- b) $A \in \delta - \beta C(X, T)$ if and only if $\delta - \beta\text{-Int}(A) \in \delta - \beta R(X, T)$.

Proof. a). (Necessity) Let $A \in \delta - \beta\Sigma(X, T)$, Then we have $A \subset \text{Cl}(\text{Int}(\delta - \text{Cl}(A)))$ and hence $\delta - \beta - \text{Cl}(A) \subset \delta - \beta - \text{Cl}[\text{Cl}(\text{Int}(\delta - \text{Cl}(A)))] = \text{Cl}(\text{Int}(\delta - \text{Cl}(A)))$. Since $A \subset \delta - \beta - \text{Cl}(A)$, we have $\delta - \beta - \text{Cl}(A) \subset \text{Cl}(\text{Int}(\delta - \text{Cl}(\delta - \beta - \text{Cl}(A))))$. This shows that $\delta - \beta\text{-Cl}(A)$ is a $\delta - \beta$ -open set. On the other hand, $\delta - \beta\text{-Cl}(A)$ is always an $\delta - \beta$ -closed set. Therefore $\delta - \beta - \text{Cl}(A) \in \delta - \beta R(X, T)$.

(Sufficiency). Let $\delta - \beta - \text{Cl}(A) \in \delta - \beta R(X, T)$. Then we have

$$A \subset \delta - \beta - \text{Cl}(A) \subset \text{Cl}(\text{Int}(\delta - \text{Cl}(\delta - \beta - \text{Cl}(A)))) \subset \text{Cl}(\text{Int}(\delta - \text{Cl}(\delta - \text{Cl}(A)))) = \text{Cl}(\text{Int}(\delta - \text{Cl}(A))).$$

Hence we have $A \subset \text{Cl}(\text{Int}(\delta - \text{Cl}(A)))$, There for $A \in \delta - \beta\Sigma(X, T)$.

b). This Proof is follows from a) and Lemma (1). □

Theorem 2. For a subset A of a topological space X ; the following are equivalent:

- a) $A \in \delta - \beta R(X, T)$;
- b) $A = \delta - \beta - \text{Cl}(\delta - \beta - \text{Int}(A))$;
- c) $A = \delta - \beta - \text{Int}(\delta - \beta - \text{Cl}(A))$.

Proof. The proofs of the implications $(a) \Rightarrow (b)$ and $(a) \Rightarrow (c)$ are obvious thus omitted.

$(b) \Rightarrow (a)$ since $\delta - \beta\text{-Cl}(A)$ is $\delta - \beta$ -closed, then by Theorem (1) we have

$\delta - \beta\text{-Int}(\delta - \beta - \text{Cl}(A)) \in \delta - \beta R(X, T)$ and $A \in \delta - \beta R(X, T)$.

$(c) \Rightarrow (a)$ Since $\delta - \beta\text{-Int}(A)$ is $\delta - \beta$ -open, then by Theorem (1) we have

$\delta - \beta - \text{Cl}(\delta - \beta - \text{Int}(A)) \in \delta - \beta R(X, T)$ and $A \in \delta - \beta R(X, T)$. □

Theorem 3. For each subset A of a topological space (X, T) , we have:

$$\delta - \beta - \text{Cl}_\theta(A) = \bigcap \{V : A \subset V \text{ and } V \text{ is } \delta - \beta_\theta\text{-closed}\} = \bigcap \{V : A \subset V \text{ and } V \in \delta - \beta R(X, T)\}.$$

Proof. We prove only the first equality since the other is similarly proved. First, suppose that $x \notin \delta - \beta - Cl_{\theta}(A)$. Then there exists $V \in \delta - \beta - \Sigma(X, x)$ such that $\delta - \beta - Cl(V) \cap A = \phi$. By Theorem 1, $X \setminus \delta - \beta - Cl(V)$ is $\delta - \beta$ -regular and hence $X \setminus \delta - \beta - Cl(V)$ is an $\delta - \beta_{\theta}$ -closed set containing A and $x \notin X \setminus \delta - \beta - Cl(V)$. Therefore, we have $x \notin \bigcap \{V : A \subset V \text{ and } V \text{ is } \delta - \beta_{\theta} - \text{closed}\}$. Conversely, suppose that $x \notin \bigcap \{V : A \subset V \text{ and } V \text{ is } \delta - \beta_{\theta} - \text{closed}\}$. There exists an $\delta - \beta_{\theta}$ -closed set V such that $A \subset V$ and $x \notin V$. There exists $U \in \delta - \beta - \Sigma(X, T)$ such that $x \in U \subset \delta - \beta - Cl(U) \subset X \setminus V$. Therefore, we have $\delta - \beta - Cl(U) \cap A \subset \delta - \beta - Cl(U) \cap V = \phi$. This shows that $x \notin \delta - \beta - Cl_{\theta}(A)$. \square

Theorem 4. Let A and B be any two subsets of a topological space (X, T) . Then the following properties hold:

- $x \in \delta - \beta - Cl_{\theta}(A)$ if and only if $U \cap A \neq \phi$; for each $U \in \delta - \beta R(X, x)$,
- If $A \subset B$; then $\delta - \beta - Cl_{\theta}(A) \subset \delta - \beta - Cl_{\theta}(B)$,
- $\delta - \beta - Cl_{\theta}(\delta - \beta - Cl_{\theta}(A)) = \delta - \beta - Cl_{\theta}(A)$,
- If A_{λ} is $\delta - \beta_{\theta}$ -closed in X for each $\lambda \in \Delta$; then $\bigcap_{\lambda \in \Delta} A_{\lambda}$ is $\delta - \beta_{\theta}$ -closed in X .

Proof. The proofs of properties (a) and (b) are obvious, thus omitted. (c) Generally we have $\delta - \beta - Cl_{\theta}(\delta - \beta - Cl_{\theta}(A)) \supset \delta - \beta - Cl_{\theta}(A)$. Suppose that $x \notin \delta - \beta - Cl_{\theta}(A)$. There exists $U \in \delta - \beta R(X, x)$ such that $U \cap A = \phi$. Since $U \in \delta - \beta R(X, T)$; we have $\delta - \beta - Cl_{\theta}(A) \cap U = \phi$. This shows that $x \notin \delta - \beta - Cl_{\theta}(\delta - \beta - Cl_{\theta}(A))$. Therefore, we obtain $\delta - \beta - Cl_{\theta}(\delta - \beta - Cl_{\theta}(A)) \subset \delta - \beta - Cl_{\theta}(A)$. (d) Let A_{λ} is $\delta - \beta_{\theta}$ -closed in X for each $\lambda \in \Delta$. For each $\lambda \in \Delta$. $A_{\lambda} = \delta - \beta - Cl_{\theta}(A_{\lambda})$. Hence $\delta - \beta - Cl_{\theta}(\bigcap_{\lambda \in \Delta} A_{\lambda}) \subset \bigcap_{\lambda \in \Delta} \delta - \beta - Cl_{\theta}(A_{\lambda}) = \bigcap_{\lambda \in \Delta} A_{\lambda} \subset \delta - \beta - Cl_{\theta}(\bigcap_{\lambda \in \Delta} A_{\lambda})$. Therefore, we obtain: $\delta - \beta - Cl_{\theta}(\bigcap_{\lambda \in \Delta} A_{\lambda}) = \bigcap_{\lambda \in \Delta} A_{\lambda}$. This shows that $\bigcap_{\lambda \in \Delta} A_{\lambda}$ is $\delta - \beta_{\theta}$ -closed in X . \square

Corollary 1. Let A and A_{λ} ($\lambda \in \Delta$) be any subsets of topological space (X, T) . Then the following properties hold:

- A is $\delta - \beta_{\theta}$ -open in X if and only if for each $x \in A$ there exists $U \in \delta - \beta R(X, x)$ Such that $x \in U \subset A$,
- $\delta - \beta - Cl_{\theta}(A)$ is $\delta - \beta_{\theta}$ -closed and $\delta - \beta - Int_{\theta}(A)$ is $\delta - \beta_{\theta}$ -open,
- If A_{λ} is $\delta - \beta_{\theta}$ -open in X for each $\lambda \in \Delta$, then $\bigcup_{\lambda \in \Delta} A_{\lambda}$ is $\delta - \beta_{\theta}$ -open in X .

Theorem 5. For a subset A of a space X , the following properties hold:

- If $A \in \delta - \beta \Sigma(X, T)$, Then $\delta - \beta Cl(A) = \delta - \beta - Cl_{\theta}(A)$,
- $A \in \delta - \beta R(X, T)$ if and only if A is $\delta - \beta_{\theta}$ -open and $\delta - \beta_{\theta}$ -closed.

Proof. (a) Generally we have $\delta - \beta - Cl(B) \subset \delta - \beta - Cl_\theta(B)$ for every subset B of X. Let $A \in \delta - \beta - \Sigma(X, T)$. Suppose that $x \notin \delta - \beta - Cl(A)$. Then there exists $U \in \delta - \beta - \Sigma(X, x)$ such that $U \cap A = \phi$. Since $A \in \delta - \beta - \Sigma(X, T)$, we have $\delta - \beta - Cl(U) \cap A = \phi$. This shows that $x \notin \delta - \beta - Cl_\theta(A)$. Therefore, we obtain $\delta - \beta - Cl_\theta(A) \subset \delta - \beta - Cl(A)$. Hence $\delta - \beta - Cl(A) = \delta - \beta - Cl_\theta(A)$.

(b) Let $A \in \delta - \beta - R(X, T)$. Then $A \in \delta - \beta - \Sigma(X, T)$ and by (a), $A = \delta - \beta - Cl(A) = \delta - \beta - Cl_\theta(A)$. Therefore, A is $\delta - \beta_\theta$ -closed. Since $X \setminus A \in \delta - \beta - R(X, T)$, by the argument above. We have $X \setminus A$ is $\delta - \beta_\theta$ -closed and hence A is $\delta - \beta_\theta$ -open. The converse is obvious \square

4. Characterizations of Strongly $\delta - \beta$ -Continuous Functions

In this section, we introduce some characterizations and basic properties concerning strongly $\theta - \delta - \beta$ -continuous functions.

Definition 3. A function $f : (X, T) \rightarrow (Y, T^*)$ is said to be strongly $\theta - \delta - \beta$ -continuous (briefly, st. $\theta - \delta - \beta.c.$) if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists an $\delta - \beta$ -open set U of X containing x, such that $f(\delta - \beta - Cl(U)) \subset V$.

Theorem 6. For a function $f : (X, T) \rightarrow (Y, T^*)$, the following are equivalent:

- a) f is strongly $\theta - \delta - \beta$ -continuous,
- b) For each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in \delta - \beta - R(X, x)$ such that $f(U) \subset V$,
- c) $f^{-1}(V)$ is $\delta - \beta_\theta$ -open in X for each open set V of Y,
- d) $f^{-1}(F)$ is $\delta - \beta_\theta$ -closed in X for each closed set F of Y,
- e) $f(\delta - \beta - Cl_\theta(A)) \subset Cl(f(A))$ for each subset A of X,
- f) $\delta - \beta - Cl_\theta(f^{-1}(B)) \subset f^{-1}(Cl(B))$ for each subset B of Y.

Proof. (a) \Rightarrow (b). It follows directly from Theorem (1).

(b) \Rightarrow (c). Let V be any open set of Y and $x \in f^{-1}(V)$. There exists $U \in \delta - \beta - R(X, x)$ such that $f(U) \subset V$. Therefore, we have $x \in U \subset f^{-1}(V)$. Hence by Corollary (1)(a), $f^{-1}(V)$ is $\delta - \beta_\theta$ -open in X.

(c) \Rightarrow (d). This is obvious thus omitted.

(d) \Rightarrow (e). Let A be any open set of X. since $Cl(f(A))$ is closed in Y, by (d) $f^{-1}(Cl(f(A)))$ is $\delta - \beta_\theta$ -closed and we have,

$$\delta - \beta - Cl_\theta(A) \subset \delta - \beta - Cl_\theta(f^{-1}(f(A))) \subset \delta - \beta - Cl_\theta(f^{-1}(Cl(f(A)))) = f^{-1}(Cl(f(A))).$$

There for, we obtain $f(\delta - \beta - Cl_\theta(A)) \subset Cl(f(A))$.

(e) \Rightarrow (f). Let B be any subset of Y. By (e), we obtain

$$f(\delta - \beta - Cl_\theta(f^{-1}(B))) \subset Cl(f(f^{-1}(B))) \subset Cl(B) \text{ and hence } \delta - \beta - Cl_\theta(f^{-1}(B)) \subset f^{-1}(Cl(B)).$$

(f) \Rightarrow (a). Let $x \in X$ and V be any open set of Y containing $f(x)$. Since $Y \setminus V$ is closed in Y, we

have $\delta - \beta - Cl_{\theta}(f^{-1}(Y \setminus V)) \subset f^{-1}(Cl(Y \setminus V)) = f^{-1}(Y \setminus V)$. Therefore, $f^{-1}(Y \setminus V)$ is $\delta - \beta_{\theta}$ -closed in X and $f^{-1}(V)$ is an $\delta - \beta_{\theta}$ -open set containing x . There exists $U \in \delta - \beta\Sigma(X, x)$ such that $\delta - \beta - Cl(U) \subset f^{-1}(V)$ and hence $f(\delta - \beta - Cl(U)) \subset V$. This shows that f is strongly $\theta - \delta - \beta$ -continuous. \square

Definition 4 ([9, 27]). A function $f : (X, T) \rightarrow (Y, T^*)$ is said to be $\delta - \beta$ -continuous if $f^{-1}(A)$ is $\delta - \beta$ -open in X for every $A \in T^*$.

Theorem 7. Let Y be a regular space. Then $f : (X, T) \rightarrow (Y, T^*)$ is strongly $\theta - \delta - \beta$ -continuous if and only if f is $\delta - \beta$ -continuous.

Proof. Let $x \in X$ and V an open set of Y containing $f(x)$. Since Y is regular, there exists an open set H such that $f(x) \in H \subset Cl(H) \subset V$. If f is $\delta - \beta$ -continuous, there exists $U \in \delta - \beta\Sigma(X, x)$ such that $f(U) \subset H$. We shall show that $f(\delta - \beta - Cl(U)) \subset Cl(H)$. Suppose that $y \notin Cl(H)$. There exists an open set W containing y such that $W \cap H = \phi$. Since f is $\delta - \beta$ -continuous, Then $f^{-1}(W) \in \delta - \beta\Sigma(X, T)$ and $f^{-1}(W) \cap U = \phi$. and hence $f^{-1}(W) \cap \delta - \beta - Cl(U) = \phi$. Therefore, we obtain $W \cap f(\delta - \beta - Cl(U)) = \phi$ and $y \notin f(\delta - \beta - Cl(U))$. Consequently, we have $f(\delta - \beta - Cl(U)) \subset Cl(H) \subset V$. The converse is obvious. \square

Definition 5. A space X is said to be $\delta - \beta$ -regular if for each closed set $F \subset X$ and each point $x \in X \setminus F$, there exist disjoint $\delta - \beta$ -open sets U and V such that $x \in U$ and $F \subset V$.

Lemma 2. For a space X the following are equivalent:

- X is $\delta - \beta$ -regular;
- For each point $x \in X$ and for each open set U of X containing x , there exists $V \in \delta - \beta\Sigma(X, T)$ such that $x \in V \subset \delta - \beta - Cl(V) \subset U$,
- For each subset A of X and each closed set F such that $A \cap F = \phi$. There exist disjoint $U, V \in \delta - \beta\Sigma(X, T)$ such that $A \cap U \neq \phi$. and $F \subset V$,
- For each closed set F of X , $F = \bigcap \{ \delta - \beta - Cl(V) : F \subset V, V \in \delta - \beta\Sigma(X, T) \}$.

Theorem 8. A continuous function $f : (X, T) \rightarrow (Y, T^*)$ is strongly $\theta - \delta - \beta$ -continuous if and only if X is $\delta - \beta$ -regular.

Proof. (Necessity). Let $f : X \rightarrow X$ be the identity function. Then f is continuous and strongly $\theta - \delta - \beta$ -continuous by our hypothesis. For any open set U of X and any point $x \in U$, we have $f(x) = x \in U$ and there exists $V \in \delta - \beta\Sigma(X, x)$ such that $f(\delta - \beta - Cl(V)) \subset U$. Therefore, we have $x \in V \subset \delta - \beta - Cl(V) \subset U$. It follows from Lemma 2 that X is $\delta - \beta$ -regular. (Sufficiency). Suppose that $f : X \rightarrow Y$ is continuous and X is $\delta - \beta$ -regular. For any $x \in X$ and open set V containing $f(x)$, $f^{-1}(V)$ is an open set containing x . Since X is $\delta - \beta$ -regular, then there exists $U \in \delta - \beta\Sigma(X, T)$ such that $x \in U \subset \delta - \beta - Cl(U) \subset f^{-1}(V)$. Therefore, we have $f(\delta - \beta - Cl(U)) \subset V$. This shows that f is strongly $\theta - \delta - \beta$ -continuous. \square

Theorem 9. Let $f : X \rightarrow Y$ be a function and $g : X \rightarrow X \times Y$ be the graph function of f . Then, the following properties are hold:

- a) If g is strongly $\theta - \delta - \beta$ -continuous, then f is strongly $\theta - \delta - \beta$ -continuous. And X is $\delta - \beta$ -regular.
- b) If f is strongly $\theta - \delta - \beta$ -continuous, and X is $\delta - \beta$ -regular, then g is strongly $\theta - \delta - \beta$ -continuous.

Proof. (a). Suppose that g is strongly $\theta - \delta - \beta$ -continuous. First, we show that f is strongly $\theta - \delta - \beta$ -continuous. Let $x \in X$ and V an open set of Y containing $f(x)$. Then $X \times V$ is an open set of $X \times Y$ containing $g(x)$. Since g is strongly $\theta - \delta - \beta$ -continuous, then there exists $U \in \delta - \beta\Sigma(X, x)$ such that $g(\delta - \beta - Cl(U)) \subset X \times V$. Therefore, we obtain, $f(\delta - \beta - Cl(U)) \subset V$. Next, we show that X is $\delta - \beta$ -regular. Let U be any open set of X and $x \in U$. Since $g(x) \in U \times Y$ and $U \times Y$ is an open in $X \times Y$, Then there exists $W \in \delta - \beta\Sigma(X, x)$ such that $g(\delta - \beta - Cl(W)) \subset U \times Y$. Therefore we obtain $x \in W \subset \delta - \beta - Cl(W) \subset U$ and hence X is $\delta - \beta$ -regular.

(b). Let $x \in X$ and H an open set of $X \times Y$ containing $g(x)$. There exists open sets $U_1 \subset X$ and $V \subset Y$ such that $g(x) = (x, f(x)) \in U_1 \times V \subset H$. since f is strongly $\theta - \delta - \beta$ -continuous, There exists $U_2 \in \delta - \beta\Sigma(X, x)$ such that $f(\delta - \beta - Cl(U_2)) \subset V$. since X is $\delta - \beta$ -regular and $U_1 \cap U_2 \in \delta - \beta\Sigma(X, x)$, so there exists $U \in \delta - \beta\Sigma(X, x)$ such that $x \in U \subset \delta - \beta - Cl(U) \subset U_1 \cap U_2$. Therefore we obtain $g(\delta - \beta - Cl(U)) \subset U_1 \times f(\delta - \beta - Cl(U_2)) \subset U_1 \times V \subset H$. This show that g is strongly $\theta - \delta - \beta$ -continuous. \square

5. Comparisons and Examples

In this section, we investigate the relationships between strongly $\theta - \delta - \beta$ -continuous function and other well-known types of strong continuity.

Definition 6. A function $f : (X, T) \rightarrow (Y, T^*)$ is said to be:

- a) Strongly θ -continuous [37] if for each $x \in X$ and each open set V of Y Containing $f(x)$, there exists an open set U of X containing x such that $f(Cl(U)) \subset V$;
- b) Strongly θ -semicontinuous [29] if for each $x \in X$ and each open set V of Y Containing $f(x)$, there exists a semi-open set U of X containing x such that $f(sCl(U)) \subset V$;
- c) Strongly θ -precontinuous [38] if for each $x \in X$ and each open set V of Y Containing $f(x)$, there exists a preopen set U of X containing x such that $f(pCl(U)) \subset V$;
- d) Strongly $\theta - \beta$ -continuous [39] if for each $x \in X$ and each open set V of Y Containing $f(x)$, there exists a semi-preopen set U of X containing x such that, $f(\beta Cl(U)) \subset V$;
- e) Strongly θ -b-continuous [42] if for each $x \in X$ and each open set V of Y Containing $f(x)$, there exists a b-open set U of X containing x such that $f(bCl(U)) \subset V$;

- f) Strongly b -continuous [10] if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in BO(X, x)$ such that $f(U) \subset V$;
- g) Strongly θ - e -continuous [41] if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists an e -open set U of X containing x such that $f(e - Cl(U)) \subset V$.

Remark 6. From Definitions 3 and 6 we have the following diagram. However the converses are not true in general by Examples (4.4, 4.5, 4.6, 4.7, 4.8) of [42] and (4.2, 4.3, 4.4, 4.5) [41] and the following examples.

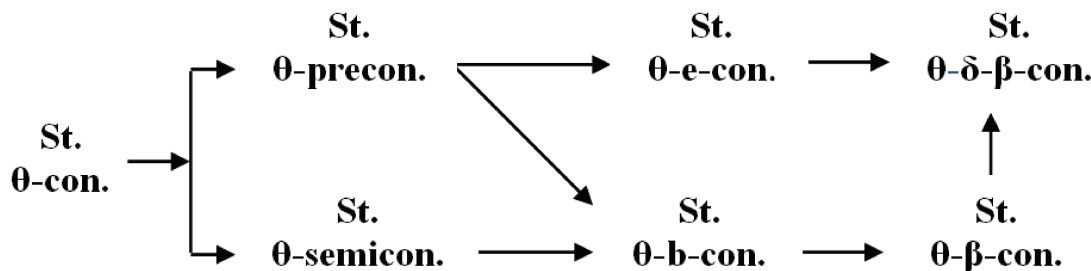


Figure 2: The relationships between strongly $\hat{\gamma}\hat{\alpha}\hat{\beta}$ -continuous functions and other known types of strong continuity

Example 4. Let $X = \{1, 2, 3, 4, 5\}$, Define a topology $T = \{\phi, X, \{1\}, \{3\}, \{1, 3\}, \{3, 4\}, \{1, 3, 4\}\}$ on X and a topology $T^* = \{\phi, X, \{4\}\}$ on Y . Then the identity function $f : X \rightarrow Y$ is strongly $\theta - \delta - \beta$ -continuous but not strongly θ - b -continuous.

Example 5. Let $X = \{1, 2, 3, 4, 5\}$, Define a topology $T = \{\phi, X, \{1\}, \{3\}, \{1, 3\}, \{3, 4\}, \{1, 3, 4\}\}$ on X and a topology $T^* = \{\phi, X, \{2, 3, 4\}\}$ on Y . Then the identity function $f : X \rightarrow Y$ is strongly $\theta - b$ -continuous but not strongly θ - e -continuous.

Example 6. Let T be the usual topology for R and $T^* = \{[0, 1] \cup (1, 2) \cap Q\}$ where Q denotes the set of rational numbers. Then the identity function $f : (R, T) \rightarrow (R, T^*)$ is strongly $\theta - \delta - \beta$ -continuous but neither strongly θ -precontinuous nor strongly θ -semicontinuous.

Example 7. Let $X = \{1, 2, 3, 4\}$, Define $T = \{\phi, X, \{1\}, \{3\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}, \{1, 3, 4\}\}$ on X and a topology $T^* = \{\phi, X, \{2, 4\}\}$ on Y . Then the identity function $f : (X, T) \rightarrow (X, T^*)$ is strongly $\theta - \delta - \beta$ -continuous but neither strongly θ - e -continuous nor strongly $\theta - \beta$ -continuous.

Recall that a space X is said to be submaximal [45] if each dense subset of X is open in X . It is shown in [45] that a space X is submaximal if and only if every preopen set of X is open. A space X is said to be extremally disconnected [3] if the closure of each open set of X is open. Note that an extremally disconnected space is exactly the space where every semiopen set is α -open

Theorem 10. Let X be a submaximal extremally disconnected space. Then the following properties are equivalent for a function $f : X \rightarrow Y$.

- a) f is strongly θ -continuous;
- b) f is strongly θ -semicontinuous;
- c) f is strongly θ -precontinuous;
- d) f is strongly θ -b-continuous;
- e) f is strongly θ -e-continuous;
- f) f is strongly $\theta - \beta$ -continuous;
- g) f is strongly $\theta - \delta - \beta$ -continuous.

Proof. it follows from the fact that if X is submaximal extremally disconnected, then open set, preopen set, semiopen set, b-open set, e-open set, semipreopen set and $\theta - \beta$ -open set are equivalent. \square

Theorem 11. Let $f : (X, T) \rightarrow (Y, T^*)$ and $g : (Y, T^*) \rightarrow (Z, T^{**})$ be a functions. If f is strongly $\theta - \delta - \beta$ -continuous. And g is continuous, then the composition function $g \circ f : (X, T) \rightarrow (Z, T^{**})$ is strongly $\theta - \delta - \beta$ -continuous.

Proof. This proof follows directly from Theorem 6. \square

6. Strongly $\theta - \delta - \beta$ -continuous Functions and Separation Axioms

Definition 7 ([7, 28]). A space X is said to be $\delta - \beta - T_2$ if for each pair of distinct points x and y in X , there exist $U \in \delta - \beta \Sigma(X, x)$ and $V \in \delta - \beta \Sigma(X, y)$ such that $U \cap V = \phi$.

Lemma 3. A space X is $\delta - \beta - T_2$ if and only if for each pair of distinct points x and y in X , there exist $U \in \delta - \beta \Sigma(X, x)$ and $V \in \delta - \beta \Sigma(X, y)$ such that $\delta - \beta - Cl(U) \cap \delta - \beta - Cl(V) = \phi$.

Theorem 12. If a function $f : (X, T) \rightarrow (Y, T^*)$ is (st. $\theta - \delta - \beta$.c.) injection and Y is T_0 , then X is $\delta - \beta - T_2$.

Proof. For any distinct points x and y of X , by hypothesis $f(x) \neq f(y)$ and there exists either an open set V containing $f(x)$ not containing $f(y)$ or an open set H containing $f(y)$ not containing $f(x)$. If the first case holds, then there exists $U \in \delta - \beta \Sigma(X, x)$ such that $f(\delta - \beta - Cl(U)) \subset V$. Thus, we obtain $f(y) \notin f(\delta - \beta - Cl(U))$ and hence $X \setminus \delta - \beta - Cl(U) \in \delta - \beta \Sigma(X, y)$. If the second case holds, then we obtain a similar result. Thus, X is $\delta - \beta - T_2$. \square

Theorem 13. If $f : (X, T) \rightarrow (Y, T^*)$ (st. $\theta - \delta - \beta$.c.)function and Y is Hausdorff, then the subset $A = \{(x, y) : f(x) = f(y)\}$ is $\delta - \beta_\theta$ -closed in $X \times X$.

Proof. It is clear that $f(x) \neq f(y)$ for each $(x, y) \notin A$. Since Y is Hausdorff, there exist open sets V and H of Y containing $f(x)$ and $f(y)$, respectively, such that $V \cap H = \emptyset$. Since f is (st. $\theta - \delta - \beta.c.$). There exist $U \in \delta - \beta \Sigma(X, x)$ and $W \in \delta - \beta \Sigma(X, y)$ such that $f(\delta - \beta - Cl(U)) \subset V$ and $f(\delta - \beta - Cl(W)) \subset H$. Set $D = f(\delta - \beta - Cl(U)) \times f(\delta - \beta - Cl(W))$. It follows that $(x, y) \in D \in \delta - \beta R(X \times X)$ and $D \cap A = \emptyset$. This means $\delta - \beta - Cl_\theta(A) \subset A$ and thus, A is $\delta - \beta_\theta$ -closed in $X \times X$.

Recall that for a function $f : X \rightarrow Y$, the subset $\{(x, f(x)) : x \in X\}$ of $X \times Y$ is called the graph of f and is denoted by $G(f)$. \square

Definition 8. The graph $G(f)$ of a function $f : X \rightarrow Y$ is said to be strongly $\delta - \beta$ -closed if for each $(x, y) \in (X \times Y) \setminus G(f)$; there exist $U \in \delta - \beta \Sigma(X, x)$ and an open set V in Y containing y such that $(\delta - \beta - Cl(U) \times V) \cap G(f) = \emptyset$.

Lemma 4. The graph $G(f)$ of a function $f : X \rightarrow Y$ is strongly $\delta - \beta$ -closed if and only if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exist $U \in \delta - \beta \Sigma(X, x)$ and an open set V in Y containing y such that $f(\delta - \beta - Cl(U)) \cap V = \emptyset$.

Theorem 14. If $f : X \rightarrow Y$ is (st. $\theta - \delta - \beta.c.$) and Y is Hausdorff, then $G(f)$ is strongly $\delta - \beta$ -closed in $X \times Y$.

Proof. It is clear that $f(x) \neq y$ for each $(x, y) \in (X \times Y) \setminus G(f)$. Since Y is Hausdorff, there exist open sets V and H in Y containing $f(x)$ and y , respectively, such that $V \cap H = \emptyset$. Since f is (st. $\theta - \delta - \beta.c.$), There exist $U \in \delta - \beta \Sigma(X, x)$ such that $f(\delta - \beta - Cl(U)) \subset V$. Thus, $f(\delta - \beta - Cl(U)) \cap H = \emptyset$ and then by Lemma (4), $G(f)$ is strongly $\delta - \beta$ -closed in $X \times Y$. \square

7. Covering Properties

Definition 9. A space X is said to be:

- $\delta - \beta$ -closed if every cover of X by $\delta - \beta$ -open sets has a finite subcover whose preclosures cover X ,
- Countably $\delta - \beta$ -closed if every countable cover of X by $\delta - \beta$ -open sets has a Finite subcover whose preclosures cover X .

A subset K of a space X is said to be $\delta - \beta$ -closed relative to X if for every cover $\{V_\lambda : \lambda \in \Delta\}$ of K by $\delta - \beta$ -open sets of X , there exists a finite subset Δ_0 of Δ such that $K \subset \bigcup \{\delta - \beta - Cl(V_\lambda) : \lambda \in \Delta_0\}$.

Theorem 15. If $f : X \rightarrow Y$ is (st. $\theta - \delta - \beta.c.$) and K is $\delta - \beta$ -closed relative to X , then, $f(K)$ is a compact set of Y .

Proof. Suppose that $f : X \rightarrow Y$ is (st. $\theta - \delta - \beta.c.$) and K is $\delta - \beta$ -closed relative to X , Let $\{V_\lambda : \lambda \in \Delta\}$ be a cover of $f(K)$ by open sets of Y . For each point $x \in K$, there exists $\lambda(x) \in \Delta$

such that $f(x) \in V_{\lambda(x)}$. Since f is (st. $\theta - \delta - \beta.c.$), there exists $U_x \in \delta - \beta\Sigma(X, x)$ such that $f(\delta - \beta - Cl(U_x)) \subset V_{\lambda(x)}$. The family $\{U_x : x \in K\}$ is a cover of K by $\delta - \beta$ -open sets of X and hence there exists a finite subset K_0 of K such that $K \subset \bigcup_{x \in K_0} \delta - \beta - Cl(U_x)$. Therefore, we obtain $f(K) \subset \bigcup_{x \in K_0} V_{\lambda(x)}$. This shows that $f(K)$ is compact. \square

Corollary 2. Let $f : X \rightarrow Y$ is (st. $\theta - \delta - \beta.c.$) surjection. Then the following Properties hold:

- a) If X is $\delta - \beta$ -closed, then Y is compact,
- b) If X is countably $\delta - \beta$ -closed, then Y is countably compact.

Theorem 16. If a function $f : X \rightarrow Y$ has a strongly $\delta - \beta$ -closed graph, then $f(K)$ is closed in Y for each subset K which is $\delta - \beta$ -closed relative to X .

Proof. Let K be $\delta - \beta$ -closed relative to X and $y \in Y \setminus f(K)$. Then for each $x \in K$ we have $(x, y) \notin G(f)$ and by Lemma 4 there exist $U_x \in \delta - \beta\Sigma(X, x)$ and an open set V_x of Y containing y such that $f(\delta - \beta - Cl(U_x)) \cap V_x = \phi$. The family $\{U_x : x \in K\}$ is a cover of K by $\delta - \beta$ -open sets of X . Since K is $\delta - \beta$ -closed relative to X , there exists a finite subset K_0 of K such that $K \subset \bigcup \{\delta - \beta - Cl(U_x) : x \in K_0\}$.

Put $V = \bigcap \{V_x : x \in K_0\}$. Then V is an open set containing y and $f(K) \cap V \subset [\bigcup_{x \in K_0} f(\delta - \beta - Cl(U_x))] \cap V \subset \bigcup_{x \in K_0} [f(\delta - \beta - Cl(U_x)) \cap V_x] = \phi$. Therefore, we have $y \notin Cl(f(K))$ and hence $f(K)$ is closed in Y . \square

Theorem 17. Let X be a submaximal extremally disconnected space. If a function $f : X \rightarrow Y$ has a strongly $\delta - \beta$ -closed graph, then $f^{-1}(K)$ is θ -closed in X for each compact set K of Y .

Proof. Let K be a compact set of Y and $x \notin f^{-1}(K)$. Then for each $y \in K$ we have $(x, y) \notin G(f)$ and by Lemma 4 there exist $U_y \in \delta - \beta\Sigma(X, x)$ and an open set V_y of Y containing y such that $f(\delta - \beta - Cl(U_y)) \cap V_y = \phi$. The family $\{V_y : y \in K\}$ is an open cover of K and there exists a finite subset K_0 of K such that $K \subset \bigcup_{y \in K_0} V_y$. Since X is submaximal extremally disconnected, therefor each U_y is open in X and $\delta - \beta - Cl(U_y) = Cl(U_y)$. Set $U = \bigcap_{y \in K_0} U_y$, then U is an open set containing x and $f(Cl(U)) \cap K \subset \bigcup_{y \in K_0} [f(Cl(U)) \cap V_y] \subset \bigcup_{y \in K_0} [f(\delta - \beta - Cl(U_y)) \cap V_y] = \phi$. so, we have $Cl(U) \cap f^{-1}(K) = \phi$ and hence $x \notin Cl_{\theta}(f^{-1}(K))$. This shows that $f^{-1}(K)$ is θ -closed in X . \square

Corollary 3. Let X be a submaximal extremally disconnected space and Y be a compact Hausdorff space. For a function $f : X \rightarrow Y$, the following properties are equivalent:

- a) f is (st. $\theta - \delta - \beta.c.$);
- b) $G(f)$ is strongly $\delta - \beta$ -closed in $X \times Y$;
- c) f is strongly θ -continuous;
- d) f is continuous;

e) f is $\delta - \beta$ -continuous.

Proof. (a) \Rightarrow (b). It follows directly from Theorem 14.

(b) \Rightarrow (c). It follows directly from Theorem 17. (c) \Rightarrow (d) \Rightarrow (e). These are clear.

(e) \Rightarrow (a). Since Y is regular, it follows from Theorem 7. \square

8. Conclusion

Topology as a field of mathematics is concerned with all questions directly or indirectly related to continuity. Therefore, generalization of continuity is one of the most important subjects in topology. One of the most important subjects in studying topology and physics is continuity, has been researched and investigated by many mathematicians and quantum physicists. [4–6, 17, 22, 40, 43, 44] from the different points of views. Relation of topology and physics have been appeared in [17, 18, 26, 47], El-Naschie in [13, 17, 26] have indicate that topology plays a significant role in quantum physics, high energy physics and superstring theory. One can observe the influence made in the realms of applied research by general topological spaces, properties and structures. In digital topology, information systems, particle physic [30], computational topology for geometric design and molecular design [35], Furthermore, Rosen and Peters [46] have used topology in computer-aided geometric design and engineering design. Also since El-Naschie has shown that the notion of fuzzy topology have very important applications in quantum particle physics especially in related to both string theory and ε^∞ theory. Thus we study a new class of strong continuity which may have very important applications in quantum particle physics, theoretical Physics, particularly in connections with string theory and ε^∞ theory [12, 14–16, 19–21, 23–25, 34, 47]. Also the fuzzy topological version of the concepts and results introduced in this paper are very important.

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References

- [1] D. Andrijevic. Semipreopen sets. *Matematicki Vesnik*, 38(1):24–32, 1986.
- [2] D. Andrijevic. On b-open sets. *Matematicki Vesnik*, 48:59–64, 1996.
- [3] N. Bourbaki. *Elements of Mathematics: General Topology*. Hermann, 1966.
- [4] M. Caldas, S. Jafari, T. Noiri, and R. K. Saraf. Weak and strong forms of α -irresolute maps. *Chaos, Solitons and Fractals*, 24(1):223–228, 2005.
- [5] M. Caldas, S. Jafari, T. Noiri, and M. Simões. A new generalization of contra-continuity via levine's g-closed sets. *Chaos, Solitons and Fractals*, 32(4):1597–1603, 2007.

- [6] E. Ekici. On almost π gp-continuous functions. *Chaos, Solitons and Fractals*, 32(5):1935–1944, 2007.
- [7] E. Ekici. New forms of contra-continuity. *Carpathian Journal of Mathematics*, 24(1):37–45, 2008.
- [8] E. Ekici. On e-open sets, dp^* -sets and dpe^* -sets and decompositions of continuity. *Arabian Journal for Science and Engineering*, 33(24):269–282, 2008.
- [9] E. Ekici. On e^* -open sets and $(d,s)^*$ -sets. *Mathematica Moravica*, 13(1):29–36, 2009.
- [10] A. A. El-Atik. *A study of some types of mappings on topological spaces*. PhD thesis, Masters Thesis, Faculty of Science, Tanta University, Tanta, Egypt, 1997.
- [11] M. E. A. EL-Monsef, S. N. EL-Deeb, and R. A. Mahmoud. β -open sets and β -continuous mapping. *Bulletin of the Faculty of Science Assiut University*, 12(1):77–90, 1983.
- [12] M. S. EL-Naschie. Fredholm operators and the wave-particle duality in cantor space. *Chaos, Solitons and Fractals*, 9(6):975–978, 1998.
- [13] M. S. EL-Naschie. On the uncertainty of cantor space geometry and the two-slit experiment. *Chaos, Solitons and Fractals*, 9(3):517–529, 1998.
- [14] M. S. EL-Naschie. Superstrings, knots, and noncommutative geometry in space. *International journal of theoretical physics*, 37(12):2935–2951, 1998.
- [15] M. S. EL-Naschie. On a class of general theories for high energy particle physics. *Chaos, Solitons and Fractals*, 14(4):649–668, 2002.
- [16] M. S. EL-Naschie. Complex vacuum fluctuation as a chaotic limit set of any kleinian group transformation and the mass spectrum of high energy particle physics via spontaneous self-organization. *Chaos, Solitons and Fractals*, 17(4):631–638, 2003.
- [17] M. S. EL-Naschie. Quantum gravity, clifford algebras, fuzzy set theory and the fundamental constants of nature. *Chaos, Solitons and Fractals*, 20(3):437–450, 2004.
- [18] M. S. EL-Naschie. The symplectic vacuum, exotic quasi particles and gravitational instanton. *Chaos, Solitons and Fractals*, 22(1):1–11, 2004.
- [19] M. S. EL-Naschie. A few hints and some theorems about witten’s m-theory and t-duality. *Chaos, Solitons and Fractals*, 25(3):545–548, 2005.
- [20] M. S. EL-Naschie. On a fuzzy kähler-like manifold which is consistent with the two slit experiment. *International Journal of Nonlinear Sciences and Numerical Simulation*, 6(2):95–98, 2005.
- [21] M. S. EL-Naschie. Elementary prerequisites for e-infinity:(recommended background readings in nonlinear dynamics, geometry and topology). *Chaos, Solitons and Fractals*, 30(3):579–605, 2006.

- [22] M. S. EL-Naschie. Topics in the mathematical physics of e-infinity theory. *Chaos, Solitons and Fractals*, 30(3):656–663, 2006.
- [23] M. S. EL-Naschie. On the topological ground state of e-infinity spacetime and the super string connection. *Chaos, Solitons and Fractals*, 32(2):468–470, 2007.
- [24] M. S. EL-Naschie. Probability set particles. *International Journal of Nonlinear Sciences and Numerical Simulation*, 8(1):117–120, 2007.
- [25] M. S. EL-Naschie. Su(5) grand unification in a transfinite form. *Chaos, Solitons and Fractals*, 32(2):370–374, 2007.
- [26] M. S. EL-Naschie, O. E. Rossler, and G. Oed. information and diffusion in quantum physics. *Chaos, Solitons and Fractals*, 7(5):Special Issue, 1996.
- [27] E. Hatir and T. Noiri. Decompositions of continuity and complete continuity. *Acta Mathematica Hungarica*, 113(4):281–287, 2006.
- [28] E. Hatir and T. Noiri. On $\delta - \beta$ -continuous functions. *Chaos, Solitons and Fractals*, 42(1):205–211, 2009.
- [29] S. Jafari and T. Noiri. On strongly θ -semi-continuous functions. *Indian Journal of Pure and Applied Mathematics*, 29(1):1195–1201, 1998.
- [30] G. Landi. An introduction to noncommutative spaces and their geometry. *Lecture notes in physics*, New York, Springer, 1997.
- [31] N. Levine. Semi-open sets and semi-continuity in topological spaces. *American Mathematical Monthly*, 70(1):36–41, 1963.
- [32] P. E. Long and L. L. Herrington. Strongly θ -continuous functions. *Journal of the Korean Mathematical Society*, 18(1):21–28, 1981/82.
- [33] A. S. Mashhour, M. E. Abd-ELMonsef, and S. N. EL-Deeb. On pre-continuous and weak pre continuous mappings. *Proceedings of the Mathematical and Physical Society of Egypt*, 53:47–53, 1982.
- [34] R. D. Mauldin and S. C. Williams. Random recursive constructions: asymptotic geometric and topological properties. *Transactions of the American Mathematical Society*, 295(1):325–346, 1986.
- [35] E. L. F. Moore and T. J. Peters. Computational topology for geometric design and molecular design. *Mathematics for Industry: Challenges and Frontiers*. SIAM, pages 125–137, 2005.
- [36] O. Njastad. On some classes of nearly open sets. *Pacific Journal of Mathematics*, 15(3):961–970, 1965.

- [37] T. Noiri. On δ -continuous functions. *Journal of the Korean Mathematical Society*, 16(2):161–166, 1979/80.
- [38] T. Noiri. Strongly θ -precontinuous functions. *Acta Mathematica Hungarica*, 90(4):307–316, 2001.
- [39] T. Noiri and V. Popa. Strongly θ - β -continuous functions. *Journal of pure mathematics*, 19:31–39, 2002.
- [40] T. Noiri and V. Popa. Faintly m -continuous functions. *Chaos, Solitons and Fractals*, 19(5):1147–1159, 2004.
- [41] M. Özkoç and G. Aslim. On strongly θ - e -continuous functions. *Bulletin of the Korean Mathematical Society*, 47(5):1025–1036, 2010.
- [42] J. H. Park. Strongly θ - b -continuous functions. *Acta Mathematica Hungarica*, 110(4):347–359, 2006.
- [43] J. H. Park, S. W. Bae, and Y. B. Park. Almost strongly θ -precontinuous functions. *Chaos, Solitons and Fractals*, 28(1):32–41, 2006.
- [44] J. H. Park and J. K. Park. On π gp-continuous functions in topological spaces. *Chaos, Solitons and Fractals*, 20(3):467–477, 2004.
- [45] I. L. Reilly and M. K. Vamanamurthy. On some questions concerning preopen sets. *Kyungpook Mathematical Journal*, 30(1):87–93, 1990.
- [46] D. W. Rosen and T. J. Peters. The role of topology in engineering design research. *Research in Engineering Design*, 8(2):81–98, 1996.
- [47] K. Svozil. Quantum field theory on fractal space-time: a new regularisation method. *Journal of Physics A: Mathematical and General*, 20(12):3861–3875, 1987.
- [48] N. V. Velicko. H-closed topological spaces. *American Mathematical Society Translations*, 78:103–118, 1968.