



Robust Control System for Spacecraft Motion Trajectory

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Abstract. In aerospace field the economic realization of a spacecraft is one of the main objectives which should be accomplished by conceiving the optimal propulsion system and best control programs. This article focuses on the implementation of uncertain control system theory and development of a robust control system of Spacecraft Motion Trajectory (SMT). The proposed strategy involves the nonlinear mathematical model of SMT expressed in the central field, which is linearized by the Taylor expansion, and model reference Adaptive Control Approach (ACA) with the second Lyapunov method to offer a high rate and unfailing performance in the functioning. The efficiencies of the linearization procedure and the control approach are theoretically investigated through some realistic simulations and tests under MATLAB.

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1. Introduction

Most modern controlled processes are complex, multi-dimensional nonlinear systems and have as a potential application the conception of flying objects for the aerospace industry [18]. Sometimes, for these objects the first problem is related to the development of a mathematical model because there is a little information about the data that describe the object behavior or there is usually a change in the structure and/or parameters of the mathematical model during the spacecraft missions. The second problem is related to the control functions.

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In literature, the adaptive control approaches has been widely developed in aerospace applications on the base of a large panel of methods. The authors developed in [16] a method based on the self-learning to accomplish a precise mission of the spacecrafts. The developed adaptive controllers through these methods used the principles of characteristics frequency stabilization and the reference model. In [10], an attitude adaptive control of satellites in elliptic orbits is proposed. The paper presents a novel non certainty-equivalent adaptive control system for the satellites pitch control. The adaptive law is based on the attractive manifold design using filtered signals for the synthesis. The developed work in [11] is devoted to the adaptive control of a flexible spacecraft in noisy environment. A new control law for a large angle rotational maneuver of the spacecraft is derived. The control system includes a state predictor to generate the estimates of the unknown parameters to be used in the feedback law. The controller is synthesized using only the pitch angle and its first derivative. A new design of a simple adaptive system for the rotational maneuver and the vibration suppression of an orbiting spacecraft with flexible appendages is proposed in [14]. The control output variable is chosen as the linear combination of the pitch angle and rate. A simple adaptive control law is derived for the pitch angle control and the elastic mode stabilization. The adaptation rule requires only four adjustable parameters and the structure of the control system does not depend on the order of the truncated spacecraft model. The authors in [21] described a hybrid control scheme for a flexible spacecraft. To obtain good static and dynamic characteristics, the hybrid control scheme is put forward with a variable structure and an intelligent adaptive control method.

A flying spacecraft formation controller is designed in [3] using a sliding mode control (SMC) scheme with an adaptive gain. The main benefit of the SMC is the robust stability of the closed-loop system. To improve the performance of the SMC, an adaptive controller based on neural networks is used to compensate for the effects of the modeling error, the external disturbance, and the non-linearity terms. An adaptive filter with a periodic gain is proposed in [13]. The filter is applicable in the case of discrete-time linear periodic systems and non-linear systems with a periodic nominal trajectory. The problem of adaptive control for a flying satellite formation under a thrust misalignment is treated in [15]. An ACA combined with a back stepping technique is developed by using a Lyapunov control approach for the relative position tracking problem of a satellite formation in the case of a misalignment uncertainty and disturbances. An approach based on an adaptive asymptotic tracking of spacecraft attitude motion with inertia matrix identification is developed in [8]. Moreover, the problem of a spacecraft trajectory tracking is addressed in [8] using an adaptive feedback control. The control law, which has the form of a sixth-order dynamic compensator, does not require the knowledge of the inertia or the center of mass of the spacecraft. The authors in [20] described the problem of adaptive spacecraft attitude control with actuator uncertainties. An ACA is developed when the spin axis directions and/or the gains of the flywheel actuators are uncertain. A smooth projection algorithm is applied to keep the estimates inside a singularity free region and to avoid a bursting. The authors in [5] focused on the angular rate control using the retrospective cost adaptive control of a spacecraft attitude. Two problems are treated in this paper: the first one is to bring the body to a specified attitude in the attitude control case; the second one seeks to bring the spacecraft to a spin about a specified body axis. In [19], an

adaptive output feedback tracking control of multiple spacecrafts is presented. The control law is designed to provide a filtered velocity measurement and a desired adaptive compensation with a semi-global and asymptotic position tracking. The proposed control law is simulated in the case of two and three spacecrafts and the results showed the semi-global and asymptotic tracking of the relative position errors.

Demand for high performance in systems with nonlinear behavior and model uncertainties is one of most challenging area in control theory [2]. An adaptive robust controller represents a systematic way to design a controller for such requirement, and it combines adaptive and robust control approaches to preserve the advantages of the both methods while overcoming their drawbacks. In literature for spacecraft the approach that has been developed in [9] was implied to combine the adaptive robust controller with dynamic backstepping method. In this method the robust controller is used as the main controller for trajectory tracking, and adaptive controller tries to decrease the uncertainty and helps to reduce tracking error especially at steady state.

In this article we developed an adaptive control strategy for robust control system of SMT. This approach involves the reference model which includes the information on the desired dynamics of the spacecraft trajectory. Robustness in SMT model is determined by the uncertain parameters.

We chose the control law which is necessary to describe the purpose of control and we selected tunable parameters of the mathematical model and the purpose of adaptation. After that, we selected the adaptation algorithm based on the gradient type. The algorithm uses the error between the output of the considered dynamic system including the reactive acceleration and the reference model. Finally, we synthesized the adaptive controller taking into account the random disturbances. As a result, we obtained the system which works as follows: the spacecraft is moving according to the desired optimal trajectory, in the case of deviations from the desired trajectory it triggers the adaptive controller which returns the system to the original optimal trajectory.

The primary contributions of this paper can be stated within the following points:

- 1) The linearization of the mathematical model of SMT, which is proposed in [7]. The linearization of this nonlinear time-varying model is achieved by a Taylor expansion, which allows to obtain a satisfactory linear time-invariant reference model to be used in the adaptive control.
- 2) The design of an adaptive control system of SMT based on the proposed linearized reference model and the Lyapunov theory. In our knowledge, the use of adaptive control within the SMT model is new and leads to obtain a satisfactory control of the spacecraft trajectory.

This paper is organized as follows. Section II presents the mathematical model of SMT which is linearized by the Taylor expansion. Section III describes the synthesis of the proposed model reference ACA. Section IV is devoted to the theoretical simulations and tests under MATLAB of the states evolution of the nonlinear mathematical model of the reactive acceleration, the linearized model and the adaptive controller. We end the paper in section V with some conclusions and future works.

2. Robust Mathematical Model of SMT

In this section, the mathematical model of robust control system of SMT is firstly presented [7] and secondly linearized to be used later in the development of the adaptive control strategy. In this paper, the mathematical model of SMT in a central field is considered. The model is built for the initial elliptical orbit and expresses a transition from an elliptical to parabolic orbit (for example) in the interplanetary flight and is obtained by using the Pontryagin Maximum Principle [7].

2.1. Optimal Mathematical Model of SMT

Let consider the mathematical model of SMT which is described by the nonlinear differential equations system based on the boundary conditions [7]:

$$\begin{cases} \dot{p}(t) = 2(p(t))^{(3/2)}(Q(t)+1)^{-1}a_\phi \\ \dot{Q}(t) = (-p(t))^{(-3/2)}(Q(t)+1)^2L(t) + 2\sqrt{p(t)}a_\phi \\ \dot{L}(t) = (p(t))^{(-3/2)}(Q(t)+1)^2Q(t) + \sqrt{p(t)}(Q(t)+1)^{-1}L(t)a_\phi + \sqrt{p(t)}a_r \end{cases} \quad (1)$$

where $p(t) \in \mathfrak{R}$ denotes the parameter of the osculating ellipse and $\{Q(t), L(t)\} \in \mathfrak{R}$ denote the phase coordinates. The accelerations $\{a_r, a_\phi\} \in I(\mathfrak{R})$ define the components of the reactive acceleration in the spherical coordinates and $u(t)$ is the command signal and robust character of the system:

$$a_r = a_r \pm \Delta a_r, \quad a_\phi = a_\phi \pm \Delta a_\phi, \quad \{a_r, a_\phi\} \in I(\mathfrak{R}), \quad a_r, a_\phi \in \mathfrak{R}$$

where $I(\mathfrak{R})$ - set of uncertain interval parameters with real bounders, $\Delta a_r, \Delta a_\phi$ - parameter deviations from nominal values due to external perturbations.

The initial conditions $\{p_0, Q_0, L_0\}$ that determine the optimal trajectory can be written in terms of the initial conditions of the eccentricity ε_0 and true anomaly ν_0 such as:

$$p_0 = 1 - \varepsilon_0^2; \quad Q_0 = \varepsilon_0 \cos \nu_0; \quad L_0 = \varepsilon_0 \sin \nu_0 \quad (2)$$

For more clarity, the Table I summarizes the main parameters of SMT mathematical model (1).

2.2. Linearization of the SMT Mathematical Model

The mathematical model of SMT is described by the nonlinear differential equations system (1). The first step to design an ACA for the spacecraft model states is the linearization of the system (1) using the Taylor series expansion, as detailed in [6, 17].

To work with the uncertain system with interval-set parameters the results were obtained by the authors according to which the uncertain parameters can be the key point with the use of auxiliary parameters [4]:

$$z = \{z \in \mathfrak{R} : z \in \{-1, 1\}\},$$

and defining q key parameters $d_q \in \mathfrak{R}$ interval value $d \in I(\mathfrak{R})$, as:

$$d_q \in d,$$

$$d_q = \begin{cases} d_q, & z = 1 \\ \frac{1}{d_q}, & z = -1 \end{cases}$$

In accordance with this method there can be used uncertain parameters $\{a_r, a_\varphi\} \in I(\mathfrak{R})$ as the point parameters with values $a_\varphi = 0.94, a_r = 0.11$.

The mathematical model can be expanded in a Taylor series around the operating point. Since the deviations from the operating point are small, the expansion takes only the term with the degree 1. The resulting equations after expansion are subtracted from the equilibrium equations which lead to obtain a linearized equations around the operating point.

Let consider the first equation of the system (1):

$$\dot{p}(t) = 2(p(t))^{(3/2)}(Q(t) + 1)^{-1} a_\phi \tag{3}$$

The equation (3) can be re-written in the following way:

$$\dot{p}(t) - \frac{2(p(t))^{(3/2)} \cdot a_\phi}{Q(t) + 1} = 0 \tag{4}$$

The equation (4) can be defined as a function $F(\dot{p}, p, Q, t)$:

$$F(\dot{p}, p, Q, t) = \dot{p}(t) - \frac{2(p(t))^{(3/2)} \cdot a_\phi}{Q(t) + 1} = 0 \tag{5}$$

To linearize the equation (3), the function $F(\dot{p}, p, Q, t)$ is decomposed in Taylor series around of the operating point $(0, p_0, Q_0)$, where $p_0 = 0.75$ and $Q_0 = 0.49$ (see Table I).

Table 1: SMT model paramters

Parameters	Descriptions	Values
p_0	Initial parameter of the osculating ellipse	0.75
Q_0	Initial phase coordinate	0.49
L_0	Initial phase coordinate	0.1
a_ϕ	component of reactive acceleration in the spherical coordinates	0.94
a_r	component of reactive acceleration in the spherical coordinates	0.11
ε_0	Initial condition of the eccentricity	0.5
ν_0	Initial condition of the true anomaly	0.205
ρ	Atmospheric density	1.2

Then, the exclusion of the terms of the second and higher orders in this expansion leads to the following equation:

$$F(\dot{p}, p, Q, t) = F_0 + \frac{\partial F}{\partial \dot{p}} \Delta \dot{p}(t) + \frac{\partial F}{\partial p} \Delta p(t) + \frac{\partial F}{\partial Q} \Delta Q(t) \tag{6}$$

By using the equation (6) and the parameter $a_\phi = 0.94$ (see Table I), it is possible to find the value of the function F_0 and the partial derivative terms $\frac{\partial F}{\partial p}$, $\frac{\partial F}{\partial p}$ and $\frac{\partial F}{\partial Q}$:

$$\begin{cases} F_0 = 0 \\ \left. \frac{\partial F}{\partial p} \right|_0 = 1 \\ \left. \frac{\partial F}{\partial p} \right|_0 = -\frac{3}{2} \cdot \frac{2p_0^{(3/2)-1} \cdot a_\phi}{Q_0+1} = -1.63 \\ \left. \frac{\partial F}{\partial Q} \right|_0 = \frac{2p_0^{(3/2)} \cdot a_\phi}{(1+Q_0)^2} = 0.53 \end{cases} \quad (7)$$

Then, equation (6) can be written in deviations from the operating point $(0, p_0, Q_0)$:

$$\Delta \dot{p}(t) = 1.63 \Delta p(t) - 0.53 \Delta Q(t) \quad (8)$$

The equation (8) represents the linearized form of the first nonlinear equation (3) of the mathematical model (1). In the same way, we applied the linearization method to the second and third motion equations of SMT model (1).

Finally, we obtained the following linear mathematical model (Σ') of the nonlinear system (1):

$$\begin{cases} \Delta \dot{p}(t) = 1.63 \Delta p(t) - 0.53 \Delta Q(t) \\ \Delta \dot{Q}(t) = -0.46 \Delta Q(t) - 3.42 \Delta L(t) + 1.37 \Delta p(t) \\ \Delta \dot{L}(t) = 0.55 \Delta L(t) + 5.63 \Delta Q(t) - 2.13 \Delta p(t) \end{cases} \quad (9)$$

The linearized mathematical model (Σ') of SMT will be used below in the design of the adaptive control method.

3. Adaptive Control Approach Design for SMT Model

3.1. General Overview of the ACA Block Diagram

Let first present an overview of the ACA design for SMT model based on the synthesis of a model reference adaptive controller [12]. In fact, among the various types of adaptive system configuration, model reference adaptive techniques are important since they lead to relatively easy-to-implement systems with a high speed of adaptation which can be used in a variety of situations. The block diagram of the adaptive control system is shown in Fig. 1. The upper, middle, and lower portions of the diagram correspond to the three main components of the approach. The upper portion of the diagram depicts the reference model, which includes the information of the desired dynamics of the spacecraft trajectory $x_M(t)$. The lower portion of the diagram is composed of the adjustable system with the control law $u(t)$ from the adaptive controller. The middle portion of the diagram depicts the adaptation law. When the spacecraft deviates from its trajectory, the ACA acts on the spacecraft control system to correct this deviation and follow the optimal reference trajectory. In other words, when the states of

the controlled plant $y(t)$ are different from the reference values $x_M(t)$, then the error signal $e(t) = x_M(t) - y(t)$ is provided as an input to the adaptation law, which contains the adaptation algorithm. In this paper, the adaptation is developed using the Lyapunov theory.

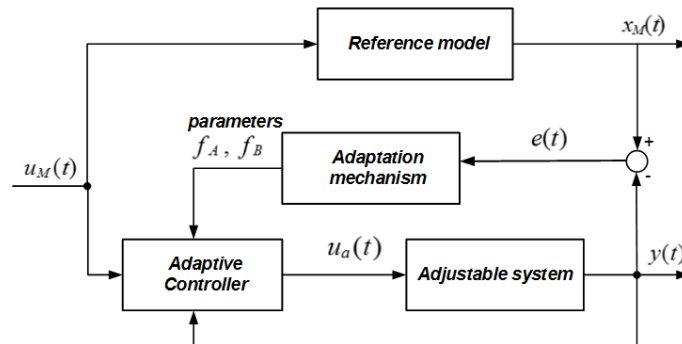


Figure 1: Block diagram of the model reference adaptive control of SMT

When the spacecraft deviates from its trajectory, the ACA acts on the spacecraft control system to correct this deviation and follow the optimal reference trajectory. In other words, when the states of the adjustable system $y(t)$ are different from the reference values $x_M(t)$, then the error signal $e(t)$ is provided as an input to the adaptation law block, which contains the adaptation algorithm based Lyapunov theory.

3.2. Model-Reference Adaptive Control Algorithm Design

The linearized model of SMT (9) can be written in the following matrix representation:

$$\begin{cases} \Delta \dot{p}(t) = 1.63 \Delta p(t) - 0.53 \Delta Q(t) + b_1 u(t) \\ \Delta \dot{Q}(t) = -0.46 \Delta Q(t) - 3.42 \Delta L(t) + 1.37 \Delta p(t) + b_2 u(t) \\ \Delta \dot{L}(t) = 0.55 \Delta L(t) + 5.63 \Delta Q(t) - 2.13 \Delta p(t) + b_3 u(t) \end{cases} \quad (10)$$

Around the equilibrium point, we consider $x_1(t) = \Delta p(t) \approx p(t)$, $x_2(t) = \Delta Q(t) \approx Q(t)$, $x_3(t) = \Delta L(t) \approx L(t)$ as the state variables of the mathematical model (10), then we can re-write the linear system such as:

$$\begin{bmatrix} \Delta \dot{p}(t) \\ \Delta \dot{Q}(t) \\ \Delta \dot{L}(t) \end{bmatrix} = \begin{bmatrix} 1.63 & -0.53 & 0 \\ 1.37 & -0.46 & -3.42 \\ -2.13 & 5.63 & 0.55 \end{bmatrix} \begin{bmatrix} \Delta p(t) \\ \Delta Q(t) \\ \Delta L(t) \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} u(t) \quad (11)$$

Then, the mathematical model (11) can be written as a general state-space representation of a linear system:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (12)$$

where $x(t) = [x_1(t) \ x_2(t) \ x_3(t)]^T$ is the state vector of the model, A is a state matrix representing the linearized inter-relationships between the states $p(t)$, $Q(t)$ and $L(t)$, B is the input

matrix representing the linearized effects of the control parameters and $u(t)$ is the control law vector.

The adjustable system of SMT shown in Figure 1 can be written according to the form of the linear system (12) but due to the changing of the system (1) properties and the operating point, its matrices A and B are poorly known and their values are time-dependent. These drawbacks can be addressed by the application of a model reference adaptive control law [12] that results in a time-dependent feedback control matrix $f(t)$. Therefore, we consider in the upper portion of the diagram in Fig. 1 the reference model that represents the desired closed-loop dynamics of the system. This reference model should correspond to the optimal trajectory model of SMT (11) and is established by the selection of its fundamental matrices A_M and B_M :

$$\dot{x}_M(t) = A_M x_M(t) + B_M u(t) \tag{13}$$

where $x_M(t) = [p_M(t)Q_M(t)L_M(t)]^T$ is the state vector of the reference model and the fundamental matrices A_M and B_M are equal to the state matrix A and the input matrix B of the mathematical model (11), which leads to obtain:

$$A_M = \begin{bmatrix} 1.63 & -0.53 & 0 \\ 1.37 & -0.46 & -3.42 \\ -2.13 & 5.63 & 0.55 \end{bmatrix}, \quad B_M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{14}$$

The lower portion of the diagram in Fig. 1 represents the mathematical model of the adjustable system which is given by the following state-space representation:

$$\dot{y}(t) = A_S(e, t)y(t) + B_S(e, t)u_a(t) \tag{15}$$

where $y(t) = [p_s(t)Q_s(t)L_s(t)]^T$ is the state vector of the adjustable system, $A_S(e, t) \in \mathfrak{R}^{3 \times 3}$ and $B_S(e, t) \in \mathfrak{R}^{3 \times 3}$ are time-varying matrices whose terms depend on the state generalized error vector $[e_1(t) \ e_2(t) \ e_3(t)]^T$ and

$$u_a(t) = f_y(t) \cdot y(t) + f_M(t) \cdot u_M(t)$$

is the control law where $f_y(t)$ and $f_M(t)$ represent two state variable feedback control matrices which will be tuned later based on the second method of Lyapunov [1].

For tuning of $f_y(t), f_M(t)$ the input signal of the adjustable system (15) is $u_M(t)$. The error vector $e(t)$ in the middle portion of the diagram is calculated as the discrepancy between the states of the reference model and the active response of the adjustable system with input signal $u_M(t)$:

$$e(t) = x_M(t) - y(t) \tag{16}$$

The error vector $e(t)$ becomes as an input information to the adaptation block where an adaptation law is implemented by using the Lyapunov theory [1], resulting in an asymptotic adaptation where the three states control errors will converge to zero regardless of initial conditions. Firstly, we differentiate the state generalized error vector in (16) to obtain its derivative:

$$\dot{e}(t) = A_M e(t) + [A_M - A_S(e, t)]y(t) + [B_M - B_S(e, t)]u(t) \tag{17}$$

After, we define a Lyapunov function V , which includes the generalized error $e(t)$. The chosen function takes the following form [1]:

$$\begin{aligned}
 V = & e^T(t) \cdot P \cdot e(t) \\
 & + tr \left\{ [A_M - A_S(e, t)]^T \cdot f_A^{-1} \cdot [A_M - A_S(e, t)] \right\} \\
 & + tr \left\{ [B_M - B_S(e, t)]^T \cdot f_B^{-1} \cdot [B_M - B_S(e, t)] \right\}
 \end{aligned} \tag{18}$$

According to the considered Lyapunov function, V , and the second Lyapunov method, the matrix P should be positive definite and skew-symmetric, $P = P^T > 0$, and f_A and f_B are positive parameters of the adaptive mechanism, which will be defined later.

When differentiating (18), the derivative of the Lyapunov function can be written such as:

$$\begin{aligned}
 \dot{V} = & e^T(t) (A_M^T \cdot P + P \cdot A) e(t) \\
 & + 2tr \{ [A_M - A_S(e, t)]^T \dots [P \cdot e(t) \cdot y^T(t) - f_A^{-1} \cdot \dot{A}_S(e, t)] \} \\
 & + 2tr \{ [B_M - B_S(e, t)]^T (P \cdot e(t) \cdot u^T(t) - f_B^{-1} \cdot \dot{B}_S(e, t)) \}
 \end{aligned} \tag{19}$$

The first term in (19) is negative definite for all $e(t) \neq 0$ and the second and third terms are identically equal to zero if one chooses the following adaptation laws [12]:

$$\dot{A}_S(e, t) = f_A \cdot (P \cdot e(t)) \cdot y^T(t) \tag{20}$$

$$\dot{B}_S(e, t) = f_B \cdot (P \cdot e(t)) \cdot u(t) \tag{21}$$

When integrating (20) and (21), one obtains:

$$A_S(e, t) = \int_0^t f_A \cdot (P \cdot e(\tau)) \cdot y^T(\tau) d\tau + A_S(0) \tag{22}$$

$$B_S(e, t) = \int_0^t f_B \cdot (P \cdot e(\tau)) \cdot u(\tau) d\tau + B_S(0) \tag{23}$$

One notes that f_A and f_B are used to establish a functional relationship between $A_S(e, t)$, $B_S(e, t)$ and the values of the error vector $e(t)$ in the interval $0 \leq \tau \leq t$ [12]. Then, the problem of model reference adaptive control system can be seen as a regulation problem for the spacecraft system displaced from its equilibrium state (defined by $A_M = A_S$, $B_M = B_S$) using the adjusted parameters f_A and f_B (see Figure 1). Therefore the adaptive control design problem can be specified from the following proposition:

Proposition 1. *Given an unknown initial differences $[A_M - A_S(t)]$, $[B_M - B_S(t)]$ between the parameters of the reference model and those of the adjustable system at $t = t_0$ and a known initial state generalized error vector $e(t_0) = x(t_0) - y(t_0)$, find an adaptation law independent of the initial conditions which achieves an asymptotic adaptation characterized by:*

$$\lim_{t \rightarrow \infty} e(t) = 0 \tag{24}$$

$$\lim_{t \rightarrow \infty} A_S(t) = A_M \tag{25}$$

$$\lim_{t \rightarrow \infty} B_S(t) = B_M \tag{26}$$

Finally, the control law $u_a(t)$ for the adjustable system is established using the conditions (24)-(26) and the adjusted parameters f_A and f_B as follows [12]:

$$\begin{aligned}
 u_a(t) &= f_y(t) \cdot y(t) + f_M(t) \cdot u_M(t) \\
 \frac{d}{dt} f_y(t) &= f_A \cdot B_S^T \cdot P \cdot e(t) \cdot (y(t))^T \\
 \frac{d}{dt} f_M(t) &= f_B \cdot B_S^T \cdot P \cdot e(t) \cdot u_M(t)
 \end{aligned}
 \tag{27}$$

4. Simulation Results

This section aims to illustrate the performance and accuracy of the designed model reference ACA via some numerical simulations carried out under MATLAB. The performed simulations concerned the linearization of SMT model and the adaptive control system design.

4.1. Simulation Results of SMT Model Linearization

We carried out some realistic simulations of the states evolution of the SMT nonlinear mathematical model (1) and its linearized version (9). The obtained results in Figures 2 (a) and (b) show the evolution of the osculating ellipse $p(t)$ and the phase coordinates $\{Q(t), L(t)\}$ before and after linearization, respectively. The initial conditions presented in Table I.

In accordance with Figure 2, the dynamics of the linearized mathematical model (Figure 2b) corresponds to the dynamics of the nonlinear mathematical model (Figure 2a), then we can use the linearized mathematical model (9) for solving the problem of the adaptive control synthesis.

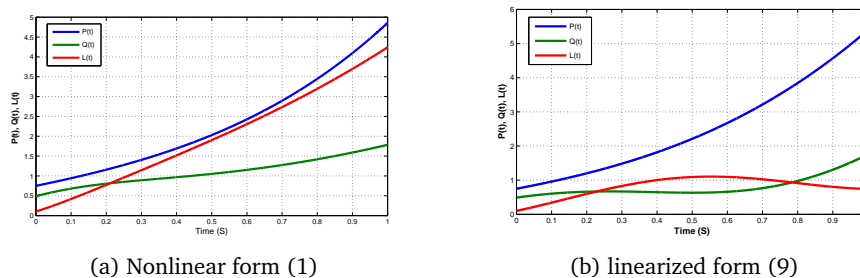


Figure 2: States evolution of the SMT model

4.2. Adaptive Control Algorithm Implementation under MATLAB

To implement the ACA under MATLAB, similarly as done in [15], we proposed the following adaptive control simulation results based on the equations (13), (15), (16), (20) and (21).

Algorithm 1. The adaptation algorithm steps implementation under MATLAB (tuning of the parameters f_A, f_B)

- 1) Take the initial conditions and parameters of the SMT mathematical model (see Table I).
- 2) Selection of the parameters f_A, f_B .
- 3) Calculation of the values of output signal of the reference model (13).
- 4) Calculation of the values of output signal of the adjustable system (15) with input signal $u_M(t)$.
- 5) Calculation of the elements of the state generalized error vector $e(t)$ by using (16).
- 6) Calculate the elements of the matrix $\dot{A}_S(t)$ and $\dot{B}_S(t)$ from (20) and (21).
- 7) Plot the results of the adjustable system and the reference model.

4.3. Simulation Results of the Adaptation Algorithm

The SMT linearized reference model and adjustable system are simulated based on the equations (13), (15), (16), (20), (21) and algorithm 1. Matrices of reference model and adjustable system, $A_M, B_M, A_S(t), B_S(t)$, are selected in accordance with matrices A_M, B_M (14). But the initial conditions of the adjustable system and the reference model are chosen different such as:

$$\begin{bmatrix} y_1(0) & y_2(0) & y_3(0) \end{bmatrix}^T = \begin{bmatrix} p_S(0) & Q_S(0) & L_S(0) \end{bmatrix}^T = \begin{bmatrix} 0.1 & 0.2 & 0.01 \end{bmatrix}^T$$

$$\begin{bmatrix} x_{M1}(0) & x_{M2}(0) & x_{M3}(0) \end{bmatrix}^T = \begin{bmatrix} p_M(0) & Q_M(0) & L_M(0) \end{bmatrix}^T = \begin{bmatrix} 0.75 & 0.489 & 0.102 \end{bmatrix}^T$$

Input signal for reference model is chosen as $u_M(t) = 1$, matrix P - unit matrix (positive definite and skew-symmetric). The implementation of the adaptation algorithm 1 were obtained the parameters for adaptive controller $f_A = 1000, f_B = 1000$.

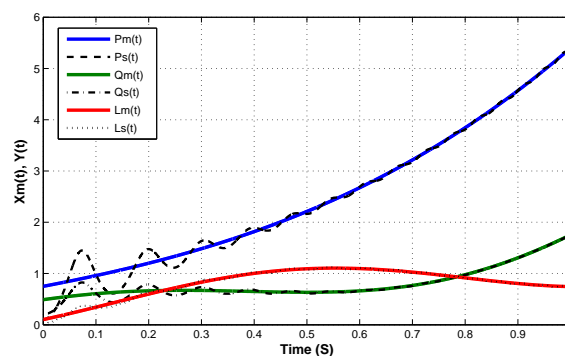


Figure 3: States evolution of the adjustable system (15) using the adaptation algorithm and the reference model (13)

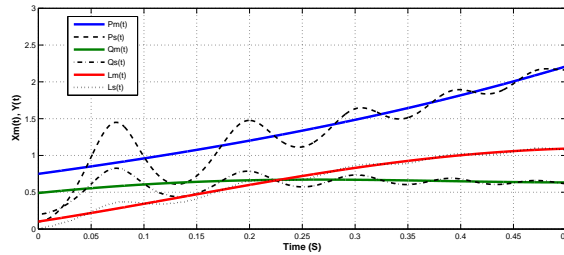


Figure 4: Zoom on the states evolution of the adjustable system (15)

The obtained states evolution of the adjustable system (15) and the reference model (13) are compared for 1 second in Figure 3. This figure shows the states $p_S(t), Q_S(t), L_S(t)$ of the adaptive SMT scheme adapting itself with the reference $p_M(t), Q_M(t), L_M(t)$. The first interval, 0 to 0.5 seconds, of Figure 3 shows the adapted states $p_S(t), Q_S(t), L_S(t)$ oscillate and adjust to converge to the desired states $p_M(t), Q_M(t), L_M(t)$. The second interval, 0.5 to 1 sec, shows convergence settling smoothly to the desired reference. The obtained results prove that the adapted states of the SMT model can match with the desired reference model with the aid of the Lyapunov stability criterion which is applied in feedback for tuning the parameters to make the error between the reference and the plant tends to zero. For more clarity of the convergence rate, we took a zoom in Figure 4 of the first interval of the states evolution in Figure 3.

The generalized error vector of control $e(t) = [e_1(t) \ e_2(t) \ e_3(t)]^T$ between the reference model and the adjustable dynamic model of SMT is calculated from the equation (15) and is presented in Figure 5. It is clear that these errors converge to zero which proves the efficiency of the proposed adaptation algorithm.

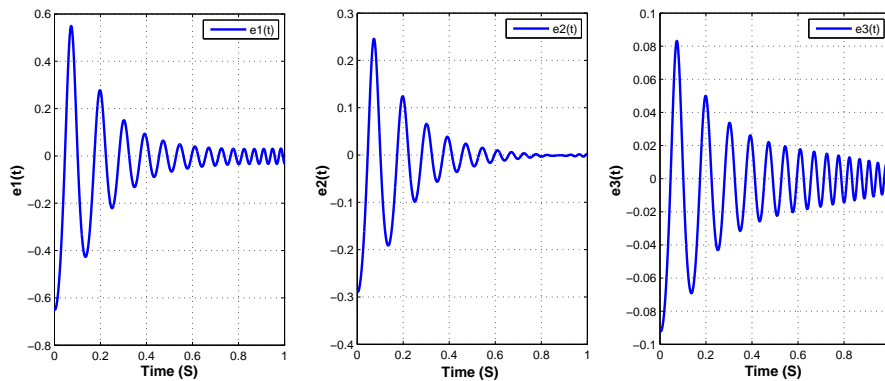


Figure 5: The generalized error vector of control $e(t)$

As a result of the functioning of adaptation algorithm the parameters of adaptation mechanism $f_A = 1000, f_B = 1000$ chosen in such a way that the state errors of control converge to zero.

4.4. The Adaptive Control Algorithm

Algorithm 2. The adaptive control algorithm

- 1) Take the initial conditions and parameters of the SMT mathematical model (see Table I).
- 2) Calculation of the values of output signal of the reference model (13).
- 3) Calculation of the values of output signal of the adjustable system (15) with adaptive control law (27), where f_A, f_B are the parameters from algorithm.
- 4) Plot the results of the adjustable system (15) with adaptive control (27) and the reference model (13).

4.5. Simulation Results of the Adaptive Control Algorithm

The proposed ACA for the SMT linearized model is simulated based on the algorithm 2 (Figure 6). The matrices and initial conditions of the adjustable system and the reference model are chosen according to algorithm 1.

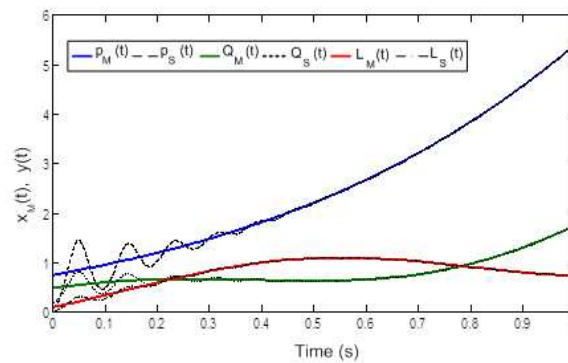


Figure 6: States evolution of the adjustable system (15) using the adaptive control (27) and the reference model (13)

Results of modeling (Figure 6) are identical to the results presented in Figure 3.

5. Conclusion

This paper presents the design of a model reference adaptive control system for a new form of linearized robust SMT model in the interplanetary flight. The proposed adaptive strategy involves the nonlinear mathematical model of SMT in the central field, which is linearized by the Taylor expansion, and the second Lyapunov method. The adaptive control system, which used the deviation and feedback principles, is composed of the main feedback control loop and the adaptation loop. In conclusion, the spacecraft is moving according to the desired optimal trajectory, in the case of deviations from the desired trajectory it triggers the adaptive

controller which returns the system to the original optimal trajectory. The obtained simulation results of the adaptive control strategy prove the efficiency and performance of the proposed adaptive control algorithm. The state errors of control converge to zero over a short period of 0.5 seconds.

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References

- [1] B R Andrievsky and A L Fradkov. Selected chapters of the control theory with the examples on the language matlab. pages 308–310. Science, 2000.
- [2] M Ataollahi and H D Taghirad. Adaptive robust controller design for non-minimum phase systems. *The proceedings of TOK'11, Izmir, Turkey*, 2011.
- [3] J Bae and Y Kim. Adaptive controller design for spacecraft formation flying using sliding mode controller and neural networks. *Journal of the Franklin Institute*, 349(2):578–603, 2012.
- [4] N A Bobylev. About positive definiteness of interval families of symmetric matrices. *Automation and Remote Control*, 61(8):4–9, 2000.
- [5] G Cruz, M D A Amato, and S D Bernstein. Retrospective cost adaptive control of spacecraft attitude. *AIAA Guidance, Navigation, and Control Conference*, 2012.
- [6] F Golnaraghi, C Benjamin, and C B Kuo. *Automatic Control Systems*. 9th, Willey, 2009.
- [7] G L Grodzovskii, N Ivanov, and V V Tokarev. *Mechanics of space flight (optimization problem)*. Science, Moscow, 1975.
- [8] A Jasim, V T Coppola, and S D Bernstein. Adaptive asymptotic tracking of spacecraft attitude. *Journal of Guidance, Control, and Dynamics*, 21(5):684–691, 1998.
- [9] Y Jiang, Q Hu, and G Ma. Adaptive backstepping fault-tolerant control for flexible spacecraft with unknown bounded disturbances and actuator failures. *ISA Transactions*, 49(1):57–69, 2010.
- [10] W L Keum and N S Sahjendra. Attractive manifold-based adaptive solar attitude control of satellites in elliptic orbits. *Acta Astronautica*, 68(1-2):185–196, 2011.
- [11] W L Keum and N S Sahjendra. L adaptive control of flexible spacecraft despite disturbances. *Acta Astronautica*, 80(11):24–35, 2012.
- [12] Y Landau. *Adaptive control - The Model Reference approach*. New York, 1979.

- [13] J Li, H Wu, and Y Chen. An adaptive filter with periodic gain, and its application in autonomous satellite navigation. *Control Engineering Practice*, 4(12):1727–1734, 1996.
- [14] Z-B Li, Z-L Wang, and J-F Li. A hybrid control scheme of adaptive and variable structure for flexible spacecraft. *Aerospace Science and Technology*, 8(5):423–430, 2004.
- [15] H Lim and H Bang. Adaptive control for satellite formation flying under thrust misalignment. *Acta Astronautica*, 65(1-2):112–122, 2009.
- [16] V Y Rutkowski, S D Zemlaykov, V M Suhanov, and V M Glumov. Some methods and adaptive the coordinate-parametric control objects for the aviation and space applications. *Gyroscopy and Navigation*, 2(3):118, 2006.
- [17] S P Sharyi. *A finite-interval analysis*. XYZ Institute of Computational Technologies, Novosibirsk: SORAN, 2013.
- [18] V P Sienkiewicz. Modern society, space and cosmic worldview. *International Scientific Public Conference - Cosmic outlook - new thinking XXI century*, 3:504, 2003.
- [19] H Wongl, V Kapila, and A Sparks. Adaptive output feedback tracking control of multiple spacecraft. *Proceedings of the American control conference*, 2001.
- [20] H Yoon and P Triostas. Adaptive spacecraft attitude tracking control with actuator uncertainties. *Journal of the Astronautically Sciences*, 56(2):251–268, 2008.
- [21] Y Zeng, A D Araujo, and S N Singh. Output feedback variable structure adaptive control of a flexible spacecraft. *Acta Astronautica*, 44(1):11–22, 1999.