EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
Vol. 9, No. 4, 2016, 402-418
ISSN 1307-5543 – www.ejpam.com

On Regular Multiplicative Hyperrings

Reza Ameri*, Ali Kordi

School of Mathematics, Statistics and Computer Science, College of Sciences, University of Tehran, P.O. Box: 14155-6455, Tehran, Iran

---

Abstract. We introduce and study regular multiplicative hyperrings, as a generalization of classical rings. Also, we use the fundamental relation γ^* on a given regular multiplicative hyperring R and prove that the fundamental ring R/γ^* of R is a regular ring. Finally, we investigate the algebraic properties of $M(R)$, the regular hyperideal of R, generated by all elements of R such that its generated hyperideal is regular.

2010 Mathematics Subject Classifications: 20N20

Key Words and Phrases: Multiplicative hyperring, Regular hyperring, Fundamental relation, Hyperideal, Regular element, Idempotent

---

1. Introduction and Primary

The theory of hyperstructures has been introduced by Marty in 1934 during the 8^{th} Congress of the Scandinavian Mathematicians [19]. Marty introduced hypergroups as a generalization of groups. He published some notes on hypergroups, using them in different contexts as algebraic functions, rational fractions, non-commutative groups and then many researchers have been worked on this new field of modern algebra and developed it. It was later observed that the theory of hyperstructures has many applications in both pure and applied sciences; for example, semi-hypergroups are the simplest algebraic hyperstructures that possess the properties of closure and associativity. The theory of hyperstructures has been widely reviewed [8, 9, 12, 19, 31].

In [9] Corsini and Leoreanu-Fotea have collected numerous applications of algebraic hyperstructures, especially those from the last fifteen years to the following subjects: geometry, hypergraphs, binary relations, lattices, fuzzy sets and rough sets, automata, cryptography, codes, median algebras, relation algebras, artificial intelligence, and probabilities. The hyperrings were introduced and studied by Krasner [18], Nakasis [21], Massouros [19] and especially studied by Davvaz and Leoreanu-Fotea [13], Zahedi and Ameri [33], Ameri and

*Corresponding author.

Email addresses: rameri@ut.ac.ir (R. Ameri), ali.kordi@gmail.com (A. Kordi)

Norouzi [1, 2]. The study on hyperrings in [31] ends with an outline of applications in chemistry and physics, analyzing several special kinds of hyperstructures: e-hyperstructures and transposition hypergroups. The theory of suitable modified hyperstructures can serve as a mathematical background in the field of quantum communication systems. A well-known type of a hyperring, called the Krasner hyperring [18]. Krasner hyperrings are essentially hyperrings, with approximately modified axioms in which addition is a hyperoperation, while the multiplication is an operation. Then, this concept has been studied by a variety of authors. Some principal notions of hyperring theory can be found in [12, 13, 20, 30, 32]. The another type of hyperrings was introduced by Rota in 1982 which the multiplication is a hyperoperation, while the addition is an operation, and it is called it a multiplicative hyperring (for more details see [26–29]) which was subsequently investigated by Olson and Ward [22] and many others. De Salvo [14] introduced hyperrings in which the additions and the multiplications are hyperoperations. Moreover, there exists other types of hyperrings that both the addition and multiplication are hyperoperations and instead associativity, commutativity and distributivity satisfy in weak associativity, weak commutativity and weak distributivity, which is called H_v-hyperrings, this type of hyperrings can be seen in [31, 32]. Also, there are other types of hyperrings which were completely studied in [12]. These hyperrings are studied by Rahnamai Barghi [25]. Procesi and Rota in [23] have studied ring of fractions in Krasner hyperrings and also they conceptualized in [24] the notion of primeness of hyperideal in a multiplicative hyperring, and in [10], Dasgupta extended the prime and primary hyperideals in multiplicative hyperrings. Asokkumar and Velrajan [4, 5] have studied Von Neumann regularity in Krasner hyperrings.

A special equivalence relations which is called fundamental relations play important roles in the the theory of algebraic hyperstructures. The fundamental relations are one of the most important and interesting concepts in algebraic hyperstructures that ordinary algebraic structures are derived from algebraic hyperstructures by them. The fundamental relation β^* on hypergroups was defined by Koskas [17], mainly studied by Corsini [19], Freni [15, 16], Vougiouklis [32] (for more details about hyperrings and fundamental relations on hyperrings see [1, 2, 11, 12, 30, 32]). In this paper we consider the classes of multiplicative hyperring as a hyperstructures $(R, +, .)$, where $(R, +)$ is an abelian group, $(R, +)$ is a semihypergroup and the hyperoperation "." is distributive with respect to the operation "$+$", i.e. $a.(b+c) \subseteq a.b + a.c$. The purpose of this paper is the study study regular multiplicative hyperrings. In this regards we study the properties of regular multiplicative hyperring R and obtain some results. We will proceed to use the fundamental relation γ^* on R and prove that the fundamental ring R/γ^* of R is regular whenever R is regular. Also, we show that this process make a functor from the category of regular multiplicative hyperrings to the category of regular rings. Finally, the notion of regular hyperideal $M(R)$, consisting of the elements of R such that the generated hyperideal by these elements are regular hyperideals, are introduced and its basic properties are investigated.

2. Regular Multiplicative Hyperring

Recall that a *hyperoperation* "." on nonempty set H is a mapping of $H \times H$ into the family of all nonempty subsets of H. Let "." be a hyperoperation on H. Then, $(H, .)$ is called a *hypergroupoid*. we can extend the hyperoperation on H to subsets of H as follows. For $A, B \subseteq H$ and $h \in H$, then

$$AB = \cup_{a \in A, b \in B} ab, \ Ah = A\{h\}, hB = \{h\}B.$$

A *semihypergroup* is a hypergroupoid $(H, .)$, which is associative, that is $(a.b).c = a.(b.c)$ or fall $a, b, c \in H$. A *hypergroup* is a semihypergroup $(H, .)$, that satisfies the *reproduction axioms*, that is $a.H = H = H.a$ for all $a \in H$.

A non-empty set R with two hyperoperations $+$ and . is said to be a *hyperring* if $(R, +)$ is a *canonical hypergroup*, $(R, .)$ is a semihypergroup with $r.0 = 0.r = 0$ for all $r \in R$ (0 as a bilaterally absorbing element) and the hyperoperation . is distributive with respect to $+$, i.e., for every $a, b, c \in R$; $a(b+c) = ab + ac$ and $(a+b)c = ac + bc$.

A *multiplicative hyperring* is an additive commutative group $(R, +)$ endowed with a hyperoperation "." which satisfies the following conditions:

(1) $\forall a, b, c \in R : a(bc) = (ab)c$;

(2) $\forall a, b, c \in R : (a+b)c \subseteq ac + bc, a(b+c) \subseteq ab + ac$;

(3) $\forall a, b \in R : (-a)b = a(-b) = -(ab)$.

If in (2) we have equalities instead of inclusions, then we say that the multiplicative hyperring is *strongly distributive*.

Definition 1. *Let R be a multiplicative hyperring. Then*

(i) An element $e \in R$ is said to be a left(resp. right)identity if $a \in e.a$ (resp. $a \in a.e$) for $a \in R$. An element e is called an identity element if it is both left and right identity element.

(ii) An element $e \in R$ is said to be a left(resp. right) scalar identity if $a = e.a (resp., \ a = a.e)$ for $a \in R$. An element e is called an scalar identity element if it is both left and right scalar identity element.

(iii) An element a is called a left (right) invertible (with respect to e), if there exists $x \in R$, such that $e \in xa (e \in ax)$ and a is called invertible if it is both a left and a right invertible.

A multiplicative hyperring R is called a left (right) invertible if every element of R has a left (right) invertible and R is called invertible if it is both a left and a right invertible. Denote the set of all invertible elements in R by $U(R)$ (with respect to the identity e by $U_e(R)$).

Definition 2. *Let R be a multiplicative hyperring. We called $a \in R$ is regular if there exists $x \in R$ such that $a \in axa$. So, we can define that R is regular multiplicative hyperring, if all of elements in R are regular elements. The set of all regular elements in R is denoted by $V(R)$.*

Example 1. *Let $(R,+,.)$ be the regular commutative ring with an unitary element. For every subset $A \in P^*(R) = P(R) - \{\emptyset\}$, $|A| \geq 2$, and $1 \in A$, define a multiplicative hyperring $(R_A, +, \circ)$, where $R_A = R$ and for all $x, y \in R_A$, $x \circ y = \{xay | a \in A\}$. Then $(R_A, +, \circ)$ is a regular multiplicative hyperring. Since, for all $a \in R$, there exists $r \in R$ such that $a = ara$. Now, by setting $x = r$ we have, $a \circ x \circ a = \{asx | s \in A\} \circ a = \{asxta | s, t \in A\} = \{axast | s, t \in A\} = \{ast | s, t \in A\}$, since $1 \in A$, we have $a \in a \circ x \circ a$. Hence $(R_A, +, \circ)$ is regular.*

Example 2. *Let $(R,+,.)$ be a non-zero regular ring and for all $a, b \in R$, define a hyperoperation $a \circ b = \{a.b, 2a.b, 3a.b, \ldots\}$. Then $(R, +, \circ)$ is a regular multiplicative hyperring, which is not strongly distributive. Also, for all $a \in R$, there exists $r \in R$ such that $a = ara$. Now by setting $x = r$ we have*

$$a \circ x \circ a = \{ar, 2ar, \ldots, nar, \ldots\} \circ a = \{ara, 2ara, \ldots, nara, \ldots\}.$$

Then $a \in a \circ x \circ a$.

2.1. Applications of the γ^*-Relation in Regular Multiplicative Hyperrings

Let $(R,+,.)$ be a hyperring. We define the relation γ as follows:

$a\gamma b$ if and only if $\{a, b\} \subseteq U$ where U is a finite sum of finite products of elements of R, i.e.,

$$a\gamma b \Leftrightarrow \exists z_1, \ldots, z_n \in R \text{ such that } \{a, b\} \subseteq \sum_{j\in J} \prod_{i \in I_j} z_i;\ I_j, J \subseteq \{1, \ldots, n\}.$$

We denote the transitive closure of γ by γ^*. The relation γ^* as the smallest equivalence relation on a multiplicative hyperring $(R,+,.)$ such that the quotient R/γ^*, the set of all equivalence classes, is a fundamental ring. Let $\mathcal{U}$ be the set of all finite sums of products of elements of R we can rewrite the definition of γ^* on R as follows:

$a\gamma^* b \Leftrightarrow \exists z_1, \ldots, z_{n+1} \in R$ with $z_1 = a, z_{n+1} = b$ and $u_1, \ldots, u_n \in \mathcal{U}$ such that $\{z_i, z_{i+1}\} \subseteq u_i$ for $i \in \{1, \ldots, n\}$.

Suppose that $\gamma^*(a)$ is the equivalence class containing $a \in R$. Then, both the sum $\oplus$ and the product $\odot$ in R/γ^* are defined as follows:$\gamma^*(a) \oplus \gamma^*(b) = \gamma^*(c)$ for all $c \in \gamma^*(a) + \gamma^*(b)$ and $\gamma^*(a) \odot \gamma^*(b) = \gamma^*(d)$ for all $d \in \gamma^*(a).\gamma^*(b)$. Then R/γ^* is a ring, which is called fundamental ring of R (see also [31]).

Theorem 1. *Let R be a regular multiplicative hyperring. Then R/γ^* is regular ring.*

Proof. Assume that $x \in R/\gamma^*$. Thus there is a $r \in R$ such that $x = \gamma^*(r)$. Since R is a regular hyperring, then there exists $r' \in R$ such that $r \in rr'r$. So

$$\gamma^*(r) = \gamma^*(rr'r) = \gamma^*(r) \odot \gamma^*(r') \odot \gamma^*(r).$$

Therefore R/γ^* is a regular ring. □

Remark 1. *The converse of Theorem 1 is not valid. For example let $(R,+,.)$ be a non regular ring. Consider $(R,+,.)$ as a hyperring under operators "+" and ".". Clearly $R/\gamma^* \cong R$.*

Definition 3. *A multiplicative hyperring R is said multiplicatively n-complete (n-MC) if for all $x_1, \ldots, x_n \in R$ we have*

$$\gamma(\prod_{i=1}^{n} x_i) = \prod_{i=1}^{n} x_i.$$

Also [7], we called that a hyperring R is n-complete if for all $(k_1, \ldots, k_n) \in \mathbb{N}^n$ and for all $(x_{1j}, \ldots, x_{ik_i}) \in R^{k_i}$ we have

$$\gamma(\sum_{i=1}^{n}(\prod_{j=1}^{k_i} x_{ij})) = \sum_{i=1}^{n}(\prod_{j=1}^{k_i} x_{ij}).$$

Theorem 2. *Let R be a 3-MC multiplicative hyperring. If R/γ^* is a regular ring then R is a regular multiplicative hyperring.*

Proof. Assume that $a \in R$. Then there exists $r \in R$ such that

$$\gamma^*(a) \odot \gamma^*(r) \odot \gamma^*(a) = \gamma^*(a).$$

Then

$$\gamma^*(a) = \gamma^*(ara).$$

Thus $a \in \gamma^*(ara) = ara$, since R is 3-MC and hence $a \in ara$. □

Proposition 1. *([25]) Let R be a commutative ring and $a \in V(R)$. Then there is a unique $x \in R$ with $axa = a$ and $xax = x$.*

Obviously, if R is a regular commutative multiplicative hyperring, then for each $a \in R$ there exists a unique element $\gamma^*(x)$ in R/γ^* such that $\gamma^*(axa) = \gamma^*(a)$ and $\gamma^*(xax) = \gamma^*(x)$.

Theorem 3. *If R is a commutative 3-MC multiplicative hyperring and $a \in R$ be a regular element of R. Then there exists $x \in R$ such that $a \in axa$ and $x \in xax$.*

Proof. Since R is a commutative multiplicative hyperring then it is easy to check that R/γ^* is commutative ring. As $a \in R$ is a regular element then $\gamma^*(a)$ is a regular element of R/γ^*, by Theorem 1, then, by Proposition 1, there is a unique $\gamma^*(x) \in R/\gamma^*$ for $x \in R$ such that

$$\gamma^*(a) \odot \gamma^*(x) \odot \gamma^*(a) = \gamma^*(a)$$
$$\gamma^*(x) \odot \gamma^*(a) \odot \gamma^*(x) = \gamma^*(x).$$

Then $\gamma^*(axa) = \gamma^*(a)$ and $\gamma^*(xax) = \gamma^*(x)$. Therefore, $a \in axa$ and $x \in xax$, since R is 3-MC. □

Definition 4. *Let R be a multiplicative hyperring. Then we called that $M_n(R)$, as the set of all hypermatrices of R. Also we called that for all $\mathscr{A} = (A_{ij})_{n\times n}$, $\mathscr{B} = (B_{ij})_{n\times n} \in P^*(M_n(R))$, $\mathscr{A} \subseteq \mathscr{B}$ if and only if $A_{ij} \subseteq B_{ij}$.*

Remark 2. *Let R be a multiplicative hyperring with a scalar identity 1 and $M_n(R)$ denotes the set of all $n \times n$ matrices with entries in R. It is easy to verify that $M_n(R)$ is a non-commutative multiplicative hyperring with unitary element under usual matrix operations. Let $A = (a_{ij})_{n\times n}$ be a matrix, where $a_{rs} = ab$ for some $1 \leq r, s \leq n$ and in other positions $a_{ij} = 0$. Then $A = BC$, where $B = (b_{ij})_{n\times n}$ such that $b_{rs} = a$ and $C = (c_{ij})$ such that $c_{ss} = b$ and in other entries of B, C we have b_{ij} and $c_{ij} = 0$.*

Recall that R has a *zero absorbing property* if for all $a \in R$, $0 \circ a = a \circ 0 = \{0\}$.

Theorem 4. *Let $(R, +, .)$ be a multiplicative hyperring such that it has zero absorbing property. Then*

$$M_n(R)/\gamma^* \cong M_n(R/\gamma^*).$$

Proof. Consider the projection homomorphism $\phi : M_n(R) \to M_n(R/\gamma^*)$ defined by $\phi((a_{ij})_{n\times n}) = (\gamma^*(a_{ij}))_{n\times n}$ for all $a_{ij} \in R$ and $1 \leq i, j \leq n$. We denote the equivalence relation associated with ϕ by ρ. That is,

$$\begin{aligned}(a_{ij})_{n\times n}\rho(b_{ij})_{n\times n} \Longleftrightarrow & (\gamma^*(a_{ij}))_{n\times n} \\ & =(\gamma^*(b_{ij}))_{n\times n}, \\ & \quad \forall a_{ij}, b_{ij} \in R, \quad 1 \leq i, j \leq n.\end{aligned}$$

In fact, $\rho = ker(\phi)$. Since ϕ is an epimorphism, we have

$$M_n(R)/\rho = M_n(R)/ker(\phi) \cong M_n(R/\gamma^*).$$

We know that $M_n(R/\gamma^*)$ is a ring, and so $M_n(R)/\rho$ is a ring. Thus $\gamma^* \subseteq \rho$, since γ^* is the smallest equivalence relation on $M_n(R)$ such that $M_n(R)/\gamma^*$ is a ring.

Let $(a'_{ij})_{n\times n}\rho(a_{ij})_{n\times n}$ for all $a_{ij} \in R$ and $1 \leq i, j \leq n$. Hence

$$(\gamma^*(a'_{ij}))_{n\times n} = (\gamma^*(a_{ij}))_{n\times n} \Longleftrightarrow \gamma^*(a'_{ij}) = \gamma^*(a_{ij}), \quad \forall a_{ij} \in R.$$

Then we conclude that

$$a_{ij}, a'_{ij} \in \sum_{s=1}^{m} \prod_{t=1}^{k_s} x_{st}$$

for some $(x_{s1}, \ldots, x_{sk_s}) \in R^{k_s}$ and $1 \leq i, j \leq n$. Then we have

$$(a_{ij})_{n\times n}, (a'_{ij})_{n\times n} \in (\sum_{s=1}^{m} \prod_{t=1}^{k_s} x_{st}^{ij})_{n\times n} = \sum_{s=1}^{m} (\prod_{t=1}^{k_s} x_{st}^{ij})_{n\times n} = \sum_{s=1}^{m} \sum_{i=1,j=1}^{n} A^{ij},$$

where $A^{ij} = (b_{pq})_{n\times n}$, such that

$$b_{pq} = \begin{cases} \prod_{t=1}^{k_s} x_{st}^{ij} & \text{if } p = i, q = j, \\ 0 & \text{otherwise} \end{cases}$$

now by the Remark 2, we have $A^{ij} = B^{ij}(B^{jj})^{k_s-1}$ for $s = 1, \dots m$, where $B^{ij} = (c_{uv})_{n\times n}$, where

$$c_{uv} = \begin{cases} x^{ij}_{st} & \text{if } u = s, v = t, \\ 0 & \text{otherwise} \end{cases}$$

so we have

$$\{(a_{ij})_{n\times n}, (a'_{ij})_{n\times n}\} \in \sum_{s=1}^{m} \sum_{i=1,j=1}^{n} A^{ij} = \sum_{s=1}^{m} \sum_{i=1,j=1}^{n} B^{ij}(B^{jj})^{k_s-1},$$

i.e., $(a_{ij})_{n\times n}\gamma(a'_{ij})_{n\times n}$. Hence $(a_{ij})_{n\times n}\gamma^*(a'_{ij})_{n\times n}$. Consequently,

$$(a'_{ij})_{n\times n} \in \gamma^*((a_{ij})_{n\times n})$$

and therefore $\rho \subseteq \gamma^*$. Then $\gamma^* = \rho$ and so $M_n(R)/\gamma^* \cong M_n(R/\gamma^*)$. □

Theorem 5. *Let R be a multiplicative hyperring. R is n-complete (n-MC) if and only if $M_n(R)$ is n-complete (n-MC).*

Proof. (⇒) Assume that R is n-complete and $T = (a_{ij})_{n\times n} \in \gamma(\sum_{s=1}^{n}(\prod_{t=1}^{k_s}(x_{st})_{n\times n}))$, then

$$\{(a_{ij})_{n\times n}, \sum_{s=1}^{n}(\prod_{t=1}^{k_s}(x_{st})_{n\times n})\} \subseteq \sum_{z=1}^{n}(\prod_{\ell=1}^{w_z}(y_{uv})^{z\ell}_{n\times n}).$$

Now, for convenience, let

$$A = (A_{ij})_{n\times n} = \sum_{s=1}^{n}(\prod_{t=1}^{k_s}(x_{st})_{n\times n})$$

and

$$B = (B_{ij})_{n\times n} = \sum_{z=1}^{n}(\prod_{\ell=1}^{w_z}(y_{uv})^{z\ell}_{n\times n}),$$

then $a_{ij} \in B_{ij}, A_{ij} \subseteq B_{ij}$, so $\gamma(a_{ij}) = \gamma(B_{ij}), \gamma(A_{ij}) = \gamma(B_{ij})$. Since R is n-complete, then $a_{ij} \in \gamma(B_{ij}) = \gamma(A_{ij}) = A_{ij}$, i.e., $a_{ij} \in A_{ij}$ for all $1 \leq i, j \leq n$. Hence $(a_{ij})_{n\times n} \in A$, and so $\gamma(\sum_{s=1}^{n}(\prod_{t=1}^{k_s}(x_{st})_{n\times n})) \subseteq A$.

(⇐) Suppose that $M_n(R)$ is n-complete. Let $x \in \gamma(\sum_{i=1}^{n}(\prod_{j=1}^{k_i} x_{ij}))$. Thus

$$\{x, \sum_{i=1}^{n}(\prod_{j=1}^{k_i} x_{ij})\} \subseteq \sum_{s=1}^{n}(\prod_{t=1}^{k_s} y_{st}).$$

Since $M_n(R)$ is n-complete then $x \in \sum_{i=1}^{n}(\prod_{j=1}^{k_i} x_{ij})$, i.e., R is n-complete. □

Proposition 2. *([6]) Let R be a strongly distributive multiplicative hyperring and $a \in R$. If there exists $x \in R$ and $c \in axa - a$ such that c is regular, then a is regular.*

In 1950, Brown and McCoy [3] proved that, R is a regular ring if and only if $M_n(R)$ is regular. Now we can extend it by the following Theorem:

Theorem 6. *Let R be a strongly distributive multiplicative hyperring. R is regular if and only if $M_n(R)$ is regular multiplicative hyperring.*

Proof. ($\Rightarrow$) First of all we'll prove it for $n = 2$, and then we will extend it to arbitrary n. Assume that

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2(R),$$

since b is regular there exists $b' \in R$ such that $b \in bb'b$. If we set, $X = \begin{pmatrix} 0 & 0 \\ b' & 0 \end{pmatrix} \in M_2(R)$ and denote $B = AXA - A$ then by an easy calculation we can see that

$$B' = \begin{pmatrix} bb'a - a & bb'b - b \\ db'a - c & db'b - d \end{pmatrix} \subseteq B.$$

Now, we can consider $B^* = \begin{pmatrix} s & 0 \\ t & u \end{pmatrix} \in B'$, where $s \in bb'a - a, t \in db'a - c, u \in db'b - d$. Since s, u are regular there exist $s', u' \in R$ such that $s \in ss's$, $u \in uu'u$. If $L = \begin{pmatrix} s' & 0 \\ 0 & u' \end{pmatrix}$, then by simple calculation we can show that

$$C = B^*LB^* - B^* \supseteq \begin{pmatrix} ss's - s & 0 \\ -t + ts's + uu't & uu'u - u \end{pmatrix} = C'.$$

Let $C^* = \begin{pmatrix} 0 & 0 \\ m & 0 \end{pmatrix} \in C'$, where $m \in -t + ts's + uu't$. Since m is regular there exists $m' \in R$, such that $m \in mm'm$. Finally, if $K = \begin{pmatrix} 0 & m' \\ 0 & 0 \end{pmatrix}$ we can see that $0 \in C^*KC^* - C^*$. So, C^* is regular, then by the Proposition 2, B^* is regular and hence A is regular. Therefore, for $n = 2$, if R is regular then $M_2(R)$ is regular.

Since $M_2(M_2(R)) \cong M_4(R)$, then $M_4(R)$ is regular. Thus by continuing this process we can show that for any positive integer k, $M_{2^k}(R)$ is regular. Now, assume that n is an arbitrary positive integer, choose k such that $2^k \geq n$. If $A \in M_n(R)$, let A_1 be the matrix of $M_{2^k}(R)$ with A in the upper left-hand corner and zeros elsewhere. Assume that $A_1 \in M_{2^k}(R)$, is regular, then there exists an element $T = \begin{pmatrix} B & C \\ D & E \end{pmatrix}$ of $M_{2^k}(R)$ such that $A_1 \in A_1TA_1$. However, this implies that $A \in ABA$ and hence A is regular.

($\Leftarrow$) Assume that $M_n(R)$ is regular and a is an arbitrary element of R, then

$$A = \begin{pmatrix} a & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}$$

is in $M_n(R)$. Since $M_n(R)$ is regular, then there exists

$$B = \begin{pmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nn} \end{pmatrix} \in M_n(R)$$

such that $A \in ABA$, this means that $a \in ab_{11}a$, i.e., a is regular. □

Theorem 7. *Let $(R, +, .)$ be a commutative multiplicative hyperring with zero absorbing property. Then*

$$R[x]/\gamma^* \cong (R/\gamma^*)[x].$$

Proof. Consider the map $\phi : R[x] \to (R/\gamma^*)[x]$ defined by $\phi(\sum_{i=1}^{n} a_i x^i) = \sum_{i=1}^{n} \gamma^*(a_i)x^i$. By [13, Theorem 5.6.5], ϕ is a projection homomorphism. We denote the equivalence relation associated with ϕ by ρ. That is,

$$(\sum_{i=1}^{n} a_i x^i)\rho(\sum_{i=1}^{n} b_i x^i) \Longleftrightarrow \sum_{i=1}^{n} \gamma^*(a_i)x^i = \sum_{i=1}^{n} \gamma^*(b_i)x^i, \quad \forall a_i, b_i \in R.$$

Since ϕ is epimorphism we have

$$R[x]/\rho = R[x]/ker(\phi) \cong (R/\gamma^*)[x].$$

We know that $(R/\gamma^*)[x]$ is a ring, then $R[x]/\rho$ is a ring. Since γ^* is the smallest equivalence relation on $R[x]$ such that $R[x]/\gamma^*$ is a ring, then $\gamma^* \subseteq \rho$. Now, let $(\sum_{i=1}^{n} a_i x^i)\rho(\sum_{i=1}^{n} b_i x^i)$ for all $b_i \in R$. Hence

$$\sum_{i=1}^{n} \gamma^*(a_i)x^i = \sum_{i=1}^{n} \gamma^*(b_i)x^i \Longleftrightarrow \gamma^*(a_i) = \gamma^*(b_i), \quad \forall a_i \in R.$$

Thus, $\{a_i, b_i\} \subseteq \prod_{s=1}^{m} t_s$. Therefore,

$$\{\sum_{i=1}^{n} a_i x^i, \sum_{i=1}^{n} b_i x^i\} \subseteq \sum_{i=1}^{n} (\prod_{s=1}^{m} t_s)_i x^i.$$

This means that $(\sum_{i=1}^{n} a_i x^i)\gamma(\sum_{i=1}^{n} b_i x^i)$. Therefore $(\sum_{i=1}^{n} a_i x^i)\gamma^*(\sum_{i=1}^{n} b_i x^i)$. Hence $\rho \subseteq \gamma^*$, i.e., $\rho = \gamma^*$, and so $R[x]/\gamma^* \cong (R/\gamma^*)[x]$. □

Remark 3. *If R is a commutative multiplicative hyperring with zero absorbing property and $R[x]$ is regular, then R is regular. But the converse is not true.*

2.2. Some Properties of Regular Multiplicative Hyperrings

Definition 5. *Let R be a multiplicative hyperring. A subset A of R is idempotent if $A \subseteq A^2$. The set of all idempotent elements of R is denoted by $Idem(R)$.*

Definition 6. *We say that I is a hyperideal of multiplicative hyperring $(R, +, .)$ if it satisfies the following conditions:*

(1) $I - I \subseteq I$,

(2) $\forall x \in I,\ r \in R,\ xr \cup rx \subseteq I$.

Definition 7. *Let R be a multiplicative hyperring. The element $a \in R$ is nilpotent, if there exists an n such that $a^n = \{0\}$. Denote the set of all nilpotent elements of R by $nil(R)$.*

Definition 8. *Let R be a multiplicative hyperring and $x \in R$. Then a left(right) annihilator of x is $Ann(x) = \{r \in R | rx = 0\}$ ($Ann(x) = \{r \in R | xr = 0\}$). For a non-empty subset B of a multiplicative hyperring R, the annihilator of B is $Ann(B) = \cap\{Ann(x) | x \in B\}$.*

Theorem 8. *Let R be a commutative multiplicative hyperring with a scalar identity 1, and $a \in R$. Then we have the following statements:*

(1) If $a \in ara$ for $r \in R$, then $ar \in Idem(R)$.

(2) $V(R) \cap nil(R) = \{0\}$.

Proof. (1) This is clear.

(2) Suppose $a \in V(R) \cap nil(R)$. Then there is a $r \in R$ and $n \in \mathbb{N}$ such that $a \in ara$ and $a^n = \{0\}$. Thus we have,

$$a \in ara \subseteq (ara)r(ara) = a^4r^3 = (ara)a^2r^2 \subseteq \ldots \subseteq a^n r' = \{0\}r' = \{0\},$$

because $0.r' = (1-1).r' \subseteq 1.r' - 1.r' = \{r'\} - \{r'\} = \{0\}$. Therefore, $a = 0$. □

Theorem 9. *Suppose that R is a commutative multiplicative hyperring with a scalar identity 1. Then we have the following statements:*

(1) If for some $u \in U(R)$ and $a \in R$, $a \in aua$, then $a \in ve$ for some $v \in U(R)$ and $e \in Idem(R)$.

(2) If $\zeta = ue$ for some $u \in U(R)$ and $e \in Idem(R)$, then for some $v \in U(R)$, $\zeta \subseteq \zeta v \zeta$.

(3) If for $a \in R$, there exists $b \in R$ such that $ab = 0$, with $a + b \in U(R)$, then a is regular.

(4) If $\eta = ue$ for some $u \in U(R)$ and $e \in Idem(R)$ and $|\eta^2| = 1$, then there exists $\ell \in \eta$ such that $\ell b = 0$ for $b \in R$ with $\ell + b \in U(R)$.

Proof. (1) Assume that $a \in aua$ for some $u \in U(R)$. Since u is invertible then there is a $v \in U(R)$ such that $1 \in uv$. Let $e = au$, thus we have

$$e^2 = (au)^2 = (au)(au) = (aua)u \supseteq au = e,$$

then $e \in Idem(R)$. Hence $ve = v(au) = a(vu) \supseteq a.1 \ni a$.

(2) Since $u \in U(R)$, then there exists $v \in U(R)$ such that $1 \in vu$. Thus we have:

$$e = 1.e \subseteq (vu)e = v(ue) = v\zeta \Rightarrow e \subseteq e^2 \subseteq ev\zeta \Rightarrow \zeta = ue \subseteq uev\zeta = \zeta v\zeta,$$

i.e., $\zeta \subseteq \zeta v\zeta$ for some $v \in U(R)$.

(3) Let $u = a + b$. Then $au = a(a + b) \subseteq a^2 + ab = a^2$. Since u is invertible, so there is a $v \in U(R)$ such that $1 \in uv$. Therefore, we have:

$$a \in a.1 \subseteq a(uv) = (au)v \subseteq a^2 v = ava.$$

Hence a is regular.

(4) Assume that $\eta = ue$ for some $u \in U(R)$ and $e \in Idem(R)$. Let $\tau = u(1 - e)$. Then $\eta\tau = ue(u(1 - e)) \subseteq ueu1 - ueue = ueu - ueue \subseteq ueue - ueue = \eta^2 - \eta^2 = 0$, i.e., for all $\ell \in \eta, b \in \tau$, $\ell b = 0$. Now, we need to show that $\eta + b \cap U(R) \neq \emptyset$. We have

$$1 \in u.1.u \subseteq u(e + (1 - e))u \subseteq (ue + u(1 - e))u = (\eta + b)u,$$

i.e., $1 \in (\eta + b)u$, so there exist $b \in \tau, \ell \in \eta$ such that $1 \in b + \ell$. Therefore $\ell + b \in U(R)$. □

Definition 9. *Let R be a multiplicative hyperring. Then R is said to be π-regular if for all $a \in R$ there are $r \in R$ and an integer $n \geq 1$, such that $a^n \subseteq a^n r a^n$.*

Clearly, a regular multiplicative hyperring is π-regular multiplicative hyperring. Also, if $a \in R$ is π-regular multiplicative hyperring then for an $n \geq 1$, a^n is regular.

Theorem 10. *Let R be a commutative multiplicative hyperring with a scalar identity* 1*. Then for $a \in R$, the following hold:*

(1) a is a π-regular if and only if for some $n \geq 1$, a^n is regular.

(2) If $a^n \subseteq a^n r a^n$ for $r \in R$ and $n \geq 1$, then $a^n r \subseteq Idem(R)$.

(3) If $a^n = ue$ for some $u \in U(R)$ and $e \in Idem(R)$ and $|a^{2n}| = 1$ then there exists $\ell \in a^n$ such that ℓ is π-regular.

Proof. It's straightforward by Theorem 9. □

For each hyperideal I of multiplicative hyperring R, letting $O_I = \{a \in I : a \in aI\}$. Then O_I is called a *pure part* of I. An hyperideal I is called a *pure hyperideal* if $I = O_I$.

Theorem 11. *Let R be a commutative multiplicative hyperring with a scalar identity* 1 *and $a \in R$ and also for $r \in R$ there exists $n \in \mathbb{N}$ such that $|ar^n| = 1$. If hyperideal $< a >$ is pure hyperideal, then $R =< a > +Ann(a)$.*

Proof. Suppose that $< a >$ is pure hyperideal, then there exists $x = sa + \sum_{i=1}^{n} a \cdot x_i \subseteq < a >$, such that $a \in ax$, where $s \in \mathbb{N}$, $r, x_i \in R$. So, there exists $\ell \in x$ such that $a \in a\ell$. Thus we have $a(1-\ell) \subseteq a - a\ell \subseteq a - a\ell^2 \subseteq \ldots \subseteq a - a\ell^n = 0$, i.e., $a(1-\ell) = 0$, which implies that $1 - \ell \in Ann(a)$. Therefore $1 = \ell + (1-\ell) \in < a > +Ann(a)$. Hence $R = < a > +Ann(a)$. □

Theorem 12. *Let R be a multiplicative hyperring with a scalar identity* 1 *and M be a maximal hyperideal of R. Also, for* $a \in M$ *and* $r \in R$ *there exists* $s \in \mathbb{N}$*, such that* $|ar^s| = 1$*. Then we have the following statements:*

(1) if $a \in V(R)$ *then for* $a \in M$, $a \in O_M$*,*

(2) $a \in O_M$ *for* $a \in M$ *if and only if* $Ann(a)$ *is not contained in* M*.*

Proof. (1) Since $a \in V(R)$, then for some $r \in R$, $a \in ara$. Now, for a maximal hyperideal M such that $a \in M$, we have $a \in ara = a(ra) \subseteq aM$. Hence $a \in O_M$.

(2): ($\Rightarrow$) Suppose that $a \in O_M$ and $Ann(a) \subseteq M$. Then there is a $m \in M$ such that $a \in am$. Thus $a(1-m) \subseteq a - am \subseteq a - am^2 \subseteq \ldots \subseteq a - am^s = 0$, i.e., $a(1-m) = 0$. Therefore, $1 - m \in Ann(a) \subseteq M$, i.e., $1 - m \in M$ and it's contradiction. Hence $Ann(a) \nsubseteq M$.

($\Leftarrow$) Assume that $Ann(a)$ is not contained in M. Then $R = M + Ann(a)$. So, there exist $m \in M$, $x \in Ann(a)$ such that $1 = m + x$, then $a \in a.1 = a(m + x) \subseteq am + ax = am$, i.e., $a \in am$. Hence $a \in O_M$. □

3. Properties of $M(R)$

Definition 10. *Let R be a multiplicative hyperring. Denote by* $M(R)$ *the set of all elements in R such that the generated hyperideals by each of these elements are regular. Clearly,* $M(R)$ *is a regular hyperring.*

Lemma 1. *If a and b are two regular elements in a commutative multiplicative hyperring R. Then the following statements hold:*

(1) $\gamma^*(ab)$ *is regular in* R/γ^**,*

(2) Moreover, if $|ab| = 1$*, then so is* ab*.*

Proof. (1) Since a, b are regular in R, then there exist $r_1, r_2 \in R$, such that $a \in ar_1a$, $b \in br_2b$. So

$$\gamma^*(ab) = \gamma^*(ab) \odot \gamma^*(r_1r_2) \odot \gamma^*(ab).$$

Hence $\gamma^*(ab)$ is regular in R/γ^*.

(2) By definition of regular element, there exist $r_1, r_2 \in R$, such that $a \in ar_1a$ and $b \in br_2b$. Since $|ab| = 1$, we have $ab \in (ar_1a)(br_2b) = (ab)r_1r_2(ab)$. □

Theorem 13. *Let R be a strongly distributive multiplicative hyperring and* $a \in R$*. Thus we have the following statements:*

(1) If there is a regular element c in $a^2 - a$, then a and $1 - a$ are regular.

(2) If a and $1 - a$ are regular in R such that $|a(1 - a)| = 1$, then so is $a(1 - a)$.

Proof. (1) It immediately follows by Proposition 2.
(2) By Lemma 1(2) it is clear. □

Theorem 14. *If R be a strongly distributive multiplicative hyperring, then a right hyperideal I in the hyperring $M(R)$ is a right hyperideal in R.*

Proof. Suppose $a \in I$, $r \in R$, then $ar \subseteq M(R)$, hence for some element $r' \in R$, $ar \subseteq arr'ar$. But $rr'ar \subseteq M(R)$, so $ar \subseteq I$. Thus I is a right hyperideal in R. □

Theorem 15. *Let R be a strongly distributive multiplicative hyperring. Then $M(R)$ is a hyperideal of R.*

Proof. Let $z \in M(R)$ and $r \in R$. Since $< zr > \subseteq < z >$ and $< rz > \subseteq < z >$, we have $zr \subseteq M(R)$ and $rz \subseteq M(R)$, thus $zr \cup rz \subseteq M(R)$. Now, assume that $t_1, t_2 \in M(R)$. We need to prove that all of elements in $< t_1 - t_2 >$ is regular. For achieving to it, let $a \in < t_1 - t_2 >$. Then for some $u \in < t_1 >$ and $v \in < t_2 >$ we have $a = u - v$. As $< t_1 >$ is regular then there exists $r \in R$ such that $u \in uru$. Thus by distributive property of R we have

$$\begin{aligned}
ara =&(u - v)r(u - v) - u + v \\
=&uru - urv - vru + vrv - u + v \\
\subseteq&uru - uru + v - urv - vru + vrv \\
=&u(r - r)u + v - urv - vru + vrv \\
=&u0u + v - urv - vru + vrv \\
=&u(v - v)u + v - urv - vru + vrv \\
=&uvu - uvu + v - urv - vru + vrv,
\end{aligned}$$

since the right side is in $< t_2 >$, then $ara - a \subseteq < t_2 >$, i.e., $ara - a$ is regular and by Proposition 2, a is regular. Hence $< t_1 - t_2 >$ is a regular hyperideal and $t_1 - t_2 \in M(R)$, i.e., $M(R) - M(R) \subseteq M(R)$. □

Let $(R, +, .)$ be a multiplicative hyperring and I be a hyperideal of it. We consider the usual addition of cosets and the multiplication defined as follows:

$$(a + I) * (b + I) = \{c + I | c \in a.b\}.$$

On the set $R/I = \{r + I | r \in R\}$ of all cosets of I. Then $(R/I, +, *)$ is a multiplicative hyperring. here here here

Proposition 3. *([19]) If $(R, +, .)$ is a strongly distributive multiplicative hyperring and I is a hyperideal of R, then $(R/I, +, *)$ is ring.*

Corollary 1. *If* $(R, +, .)$ *is a strongly distributive multiplicative hyperring, then* $(R/M(R), +, *)$ *is ring.*

Theorem 16. *If* $(R, +, .)$ *is a strongly distributive multiplicative hyperring, then* $M(R/M(R)) = \{0\}$.

Proof. Assume that $a + M(R)$ denote the residue class modulo $M(R)$ which contains the element a of R. If $b + M(R) \in M(R/M(R))$ and $a \in< b >$, then $a + M(R) \in< b + M(R) >$. Since $< b + M(R) >$ is a regular ideal in $R/M(R)$, then $a + M(R)$ is regular. Thus, for some $x+M(R) \in R/M(R)$, $a+M(R) = (a+M(R))*(x+M(R))*(a+M(R))$ or $a+M(R) = axa+M(R)$, i.e., $axa - a \subseteq M(R)$. Therefore $axa - a$ is regular element of strongly distributive multiplicative hyperring R and by Proposition 2, a is regular. Thus $< b >$ is regular hyperideal and hence $b \in M(R)$. So, $b + M(R) = 0_{R/M(R)}$. □

Theorem 17. *Let* B *be a hyperideal of a strongly distributive multiplicative hyperring* R*. Then* $M(B) = B \cap M(R)$.

Proof. Assume that B is a hyperideal of R and let b be a element of B such that generated a regular hyperideal $< b >^*$ in the strongly distributive multiplicative hyperring B. Suppose $< b >$ be the hyperideal in R generated by the element b, and let

$$c = nb + rb + bs + \Sigma r_i b s_i, \text{ where } n \in \mathbb{Z}, r, s, r_i, s_i \in R$$

be any element of $< b >$. Since b is regular in B we have for some $x \in B$, $b \in bxb$. Hence

$$\begin{aligned} c \subseteq & nb + r(bxb) + (bxb)s + \Sigma r_i(bxb)s_i \\ = & nb + r(bxb) + (bxb)s + \Sigma r_i(bxbxb)s_i \\ = & nb + r(bxb) + (bxb)s + \Sigma(r_i bx)b(xbs_i), \end{aligned}$$

and thus $c \subseteq< b >^*$. As $< b >$ coincides with $< b >^*$, then $< b >$ is regular. Hence, for $b \in M(B)$ we have $b \in B \cap M(R)$, i.e., $M(B) \subseteq B \cap M(R)$.

Conversely, let $b \in B \cap M(R)$, then $b \in B$, $b \in M(R)$. As b is regular in R, then there exists $r \in R$ such that $b \in brb$ and since B is a hyperideal and $b \in B$ we have $rb \subseteq B$ then $b \in brb \subseteq b(rbr)b$, where $rbr \subseteq B$. It shows that b is regular in B. Therefore $B \cap M(R)$ is regular hyperideal in the strongly distributive multiplicative hyperring B. Thus $B \cap M(R) \subseteq M(B)$. This completes the proof. □

Theorem 18. *If* R *is a multiplicative hyperring and it has zero absorbing property, then* $M(R) \cap Ann(M(R)) = \{0\}$.

Proof. For all $a \in M(R) \cap Ann(M(R))$ we have $a \in M(R)$, $a \in Ann(M(R))$. As $a \in M(R)$ thus there exists $r \in R$ such that $a \in ara$. Since $ra \subseteq M(R)$, we have $a \in a(ra) = \{0\}$. Therefore $a = 0$. □

Proposition 4. *([19]) Every strongly distributive hyperring* $(R, +, .)$ *with a scalar identity* 1 *is a ring.*

Theorem 19. *Let R be a multiplicative hyperring and I be a strongly distributive hyperideal of R such that it has a scalar identity e and for all $x \in R$, $|xe| = 1$, then $R = I + Ann(I)$.*

Proof. Let x be an arbitrary element of R, then $xe \cup ex \subseteq I$. So $(xe)e = e(xe)$, i.e., $xe = exe$ and similarly $ex = exe$. Thus $xe = ex$ and e is in center of R. As, by Proposition 4, $ex \in I$ and for all $y \in I$, $y(x-ex) = ye(x-ex) = y(ex-e^2x) = y(ex-ex) = y0 = 0$, i.e., $x-ex \in Ann(I)$. Thus we have,

$$x = ex + (x - ex).$$

Hence for all $x \in R$ as a sum of elements $ex \in I$ and $x - ex \in I$ of $Ann(I)$. This completes the proof. □

The following example show that under Theorem 19, R is not a ring:

Example 3. *Let $R = \mathbb{Z} \bigoplus \mathbb{Z}$ and define*

$$(a,b) \circ (c,d) = \begin{cases} (ac, \mathbb{Z}) & bd \neq 0 \\ (ac, 0) & bd = 0 \end{cases}$$

Then $(R,+,\circ)$ is a multiplicative hyperring such that is not strongly distributive, because by considering $a = (1,1)$, $b = (0,2)$, $c = (0,-2)$, we have $a \circ (b+c) = (1,1) \circ (0,0) = (0,0)$ but $a \circ b + a \circ c = (0,\mathbb{Z}) + (0,\mathbb{Z}) = (0, \mathbb{Z}+\mathbb{Z})$. Now, let $I = (\mathbb{Z}, 0)$ be a strongly distributive hyperideal of R and $e = (1,0)$ is scalar identity element of I such that for all $r \in R$ we have $|xe| = 1$. It is clear that $Ann(I) = (0,\mathbb{Z})$ and $R = I + Ann(I)$ but $(R,+,\circ)$ is not a ring.

Corollary 2. *([7]) If an ideal I in a ring R has a unit element e, then $R = I + Ann(I)$.*

ACKNOWLEDGEMENTS The first author partially has been supported by "Algebraic Hyperstructure Excellence, Tarbiat Modares University, Tehran, Iran" and "Research Center in Algebraic Hyperstructures and Fuzzy Mathematics, University of Mazandaran, Babolsar, Iran".

References

[1] R. Ameri and M. Norouzi. *New fundamental relation of hyperrings*, European Journal of Combinatorics, 34, 884–891. 2013.

[2] R. Ameri and M. Norouzi. *Prime and primary hyperideals in Krasner*, European Journal of Combinatorics, 34, 379–390. 2013.

[3] R. Ameri and M.M. Zahedi. *Hyperalgebraic systems*, Italian Journal of Pure and Applied Mathematics, 6, 21-32. 1999.

[4] A. Asokkumar and M. Velrajan. *Characterizations of regular hyperrings*, Italian Journal of Pure and Applied Mathematics, 22, 115-124. 2007.

[5] A. Asokkumar and M. Velrajan. *Hyperring of matrices over a regular hyperring*, Italian Journal of Pure and Applied Mathematics, 23, 13-120. 2008.

[6] A. Asokkumar and M. Velrajan. *A radical property of hyperrings*, Italian Journal of Pure and Applied Mathematics, 29, 301-308. 2012.

[7] B. Brown and N. H. McCoy. *The maximal regular ideal of a ring*, Proceedings of the American Mathematical Society, 1, 165-171. 1950.

[8] P. Corsini. *Prolegomena of hypergroup theory*, Second edition, Aviani Editore, 1993.

[9] P. Corsini and V. Leoreanu. *Applications of hyperstructures theory*, Advances in Mathematics, Kluwer Academic Publishers, 2003.

[10] U. Dasgupta. *Prime and primary hyperideals of a multiplicative hyperring*, Analele Stiintifice Ale Uniersitatii Al. I Cuza din Iasi (S. N.) Matematica, LVIII(1): 19-36. 2012.

[11] B. Davvaz and S. Mirvakili. *On α-relation and transitive condition of α*, Communications in Algebra, 36(5), 1695-1703. 2008.

[12] B. Davvaz and T. Vougiouklis. *Commutative rings obtained from hyperrings (H_v-rings) with α^*-relations*, Communications in Algebra, 35, 3307-3320. 2007.

[13] B. Davvaz and V. Leoreanu-Fotea. *Hyperring theory and applications*, International Academic Press, USA, 2007.

[14] M. De Salvo and G. Lo Faro. *On the n^*-complete hypergroups*, Discrete Mathematics, 208/209, 177-188. 1990.

[15] D. Freni. *A new characterization of the derived hypergroup via strongly regular equivalences*, Communications in Algebra, 30(8), 3977-3989. 2002.

[16] D. Freni. *Strongly transitive geometric spaces: Applications to hypergroups and semigroups theory*, Communications in Algebra, 32, 969-988. 2004.

[17] M. Koskas. *Groupoids, demi-groupes et hypergroupes*, J. Math. Pures Appl., 49, 155-192. 1970.

[18] M. Krasner. *A class of hyperrings and hyperfields*, International Journal of Mathematics and Mathematical Sciences, 2, 307-312. 1983.

[19] C. G. Massouros. *On the theory of hyperrings and hyperfields*, Algebra i Logika, 24, 728-742. 1985.

[20] J. Mittas. *Hypergroups canoniques*, Mathemaica Balkanica, 2, 165-179. 1972.

[21] A. Nakassis. *Expository and Survey Article Recent Result in hyperring and Hyperfield Theory*, International Journal of Mathematics and Mathematical Sciences, 11(2), 209- 220. 1988.

[22] D. M. Olson and V.K. Ward. *A note on multiplicative hyperrings*, Italian Journal of Pure and Applied Mathematics, 1, 77-84. 1997.

[23] R. Procesi-Ciampi and R. Rota. *The hyperring spectrum*, Riv. Mat. Pura Appl., 1, 71-80. 1987.

[24] R. Procesi and R. Rota. *On some classes of hyperstructures*, Discrete Mathemaatics, 208/209, 485-497. 1999.

[25] A. R. Barghi. *A class of hyperrings*, Journal of Discrete Mathematical Sciences and Cryptography, 6, 227-233. 2003.

[26] R. Rota. *Strongly distributive multiplicative hyperrings*, Journal of Geometry, 39, 130-138. 1990.

[27] R. Rota. *Sugli iperanelli moltiplicativi*, Rend. Di Mat., Series VII, 4(2), 711-724. 1982.

[28] R. Rota. *Congruenze sugli iperanelli moltiplicativi*, Rend. Di Mat., Series VII, 1(3), 17-31. 1983.

[29] R. Rota. *Sulla categoria degli iperanelli moltiplicativi*, Rend. Di Mat., Series VII, 1(4), 75-84. 1984.

[30] S. Spartalis and T. Vougiouklis. *The fundamental relations on H_v-rings*, Rivista Mat. Pura ed Appl., 13, 7-20. 1994.

[31] T. Vougiouklis. *Hyperstructures and Their Representations*, 115 Hadronic Press, Inc., Palm Harber, USA, 1994.

[32] T. Vougiouklis. *The fundamental relation in hyperrings, The general hyperfield*, in: Proceedings of the Fourth International Congress on Algebraic Hyperstructures and Applications, AHA, 1990, World Scientific, 203-211. 1991.

[33] M. M. Zahedi and R. Ameri. *On the prime, primary and maximal subhypermodules*, Italian Journal of Pure and Applied Mathematics, 5, 61-80. 1999.