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Rough spaces on covering based rough sets

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Abstract. In this paper, we discuss open (closed) sets, rough interiors, rough closures, continuous mappings, open (closed) mappings and homeomorphism mappings of covering based rough topology of Akduman, Özcelik and Özel. We also construct the nano topology on a given covering based rough set.

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1. Introduction

The rough set theory was introduced by Pawlak in [2]. It deals with impression, vagueness, and uncertainty in data analysis and information systems. The rough set theory offers ways to find the deciding factors or core from data. The classical rough set theory (Pawlak version) is based on the equivalence relations. Pawlak defined an approximation space as an ordered pair $< U, \mathcal{R} >$ where U is a non-empty set and $\mathcal{R}$ is an equivalence relation defined on U. Then for any $X \in P(U)$, he presented definitions for lower and upper approximations. Next, he determined rough sets by establishing rough equivalence relation.

Later in 1988, Pomykala [3] defined corresponding operations on Pawlak rough sets. A year later, Bryniarski [1] extended classical rough sets given by Pawlak to covering based rough sets. Moreover, he restricted coverings by some conditions to make operations of

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Pawlak rough sets, well defined.

• Let U be a non-empty set and C be a family of non-empty subsets of U such that $\cup C = U$. Then C is called a **covering** of U. Obviously, a partition is indeed a covering of U, so that a covering is an extension of a partition.
• Bryniarski also defined the lower and upper approximations and the boundary region in a similar way as Pawlak. But, instead of using elementary sets (classes), He used the elements of coverings. Let $X \in P(U)$. The ordered pair $(\underline{X}, \overline{X})$ is then the covering based rough set of X. He also introduced a restricted condition for a covering called an **approximation condition**. Since we extend the Pawlak operations to the covering based rough sets, these operations may not be well defined.

Definition 1. *(Bryniarski,* 1989*) Let U be a set with at least two elements and C be its covering.*
• *C satisfies the approximation condition $\iff$ for all $A, B \subset C$ such that $A \subset B$, there exists $X \subset U$ with $A = \underline{X}$ and $B = \overline{X}$.*
• *C is minimal $\iff$ for all $K \in C$, we have $\cup C \setminus \{K\} \neq U$.*
• *C is representable $\iff$ for all $K \in C$ there exists $x \in K$ such that for all $L \in C$ and $L \neq K$, we have $x \notin L$. This element $x \in K$ is called a representative of K.*
• *C is a well representable if every $K \in C$ has at least two distinct representatives.*
• *C is called an exact $\iff$ for all $A \subset C, A = \{K \in C, K \subset \cup A\}$.*

Theorem 1. *[1] Let C be a covering, then the following conditions are equivalent:*

(i) C is minimal;

(ii) C is representable;

(iii) C is exact.

Theorem 2. *[1] For a covering C of the universe U, the following conditions are equivalent:*

(i) C is well representable;

(ii) C satisfies the approximation condition.

We can say that an ordered pair (U, C) is a **covering approximation space** if C satisfies one condition of Theorem 2.

Let (U, C) be a covering approximation space and $X, Y \in P(U)$. Then the operations $\vee$, $\wedge$ and complement on the rough sets are defined in the following:

(i) $X \vee Y = (\underline{X} \vee \underline{Y}, \overline{X} \vee \overline{Y})$;

(ii) $X \wedge Y = (\underline{X} \wedge \underline{Y}, \overline{X} \wedge \overline{Y})$;

(iii) $X^c = (C \setminus \overline{X}, C \setminus \underline{X})$.

Note that the union and intersection are about classes, not elements in the classes.

A rough set X is **exact** if $\underline{X} = \overline{X}$. The family of all rough sets of elements belonging to $P(U)$ is called a **rough set of the first order**. This family is a distributive lattice with operations $\vee$ and $\wedge$. Also, it satisfies De Morgan laws with operations $\vee$ and $\wedge$ and complement. In addition, the set of all rough exact sets with operations $\vee, \wedge$ and complement forms a Boolean algebra [1].

Proposition 1. *[1] If (U, C) is a covering approximation space and A and B are subsets of U, then*

(i) $\underline{A} \subseteq A \subseteq \overline{A}$;

(ii) $\underline{\emptyset} = \overline{\emptyset} = \emptyset$;

(iii) $\underline{U} = \overline{U} = U$;

(iv) $\overline{A \vee B} = \overline{A} \vee \overline{B}$;

(v) $\overline{A \wedge B} \subseteq \overline{A} \wedge \overline{B}$;

(vi) $\underline{A \wedge B} = \underline{A} \wedge \underline{B}$.

(vii) $\underline{A} \vee \underline{B} \subseteq \underline{A \vee B}$;

(viii) If $A \subseteq B$, *then* $\underline{A} \subseteq \underline{B}$ *and* $\overline{A} \subseteq \overline{B}$;

(ix) $\underline{A^c} = (\overline{A})^c$, *and* $\overline{A^c} = (\underline{A})^c$;

(x) $\overline{\overline{A}} = \underline{(\overline{A})} = \overline{A}$;

(xi) $\underline{\underline{A}} = \overline{(\underline{A})} = \underline{A}$.

Definition 2. *[5] Let (U, C) be a covering approximation space and $X = (\underline{X}, \overline{X})$ be a covering based rough set. The collection τ consisting of rough subsets of $X = (\underline{X}, \overline{X})$ is called a* **covering based rough topology** *on $X = (\underline{X}, \overline{X})$ if the following conditions are satisfied:*

(i) $\emptyset, X = (\underline{X}, \overline{X}) \in \tau$;

(ii) τ *is closed under a finite intersection;*

(iii) τ *is closed under an arbitrary union.*

In 2015, Akduman, Özcelik and Özel [4] constructed a topology on classical rough sets. They called it a rough topological space. Also, they defined a topology on a given covering based rough set.

2. Main results

To define open and closed sets on a covering based rough topology, we need the next result.

Theorem 3. *[1] For every covering approximation space (U, C), there exists a one-to-one mapping of the set of all pairs (A, B) such that $A \subset B \subset C$ onto the set of all rough sets of the first order defined as $(A, B) = X \iff A = \underline{X}$ and $B = \overline{X}$, where $X \subseteq U$.*

Let $Y \subseteq U$ be a rough open set in a covering based rough space (X, τ), then there exist $A, B \subset U$ such that $A \subset B \subset C$ with $\underline{Y} = A$, $\overline{Y} = B$, $A \subseteq \underline{X}$ and $B \subseteq \overline{X}$.

To define closed sets in X, we define the rough complement of any rough open set with respect to the rough set X. If $Y = (\underline{Y}, \overline{Y})$ is a rough open set in X, then $Y^c = (\underline{X} \backslash \overline{Y}, \overline{X} \backslash \underline{Y})$ is a closed set in the covering based rough space (X, τ) such that $\underline{X} \backslash \overline{Y} \subseteq \underline{X}$ and $\overline{X} \backslash \underline{Y} \subseteq \overline{X}$.

Similarly as the classical rough topology, we can define strong and weak elements for any covering based rough topology. Then we can apply a neighborhood for strong (resp. weak) element and strong (resp. weak) interior and strong (resp. weak) exterior point in similar way.

However, Bryniarski [1] defined an exact element of a rough set as a nonempty subset of the set of representatives of some class $K \in C$ or the whole class K.

Definition 3. *[1] Let $< U, C >$ be an approximation space. An element of a rough set of the first order is defined as follows: for all $X, Y \in P(U)$, $Y \in X_c \iff Y \neq \emptyset, Y_C \subset X_C$ and there exists $K \in C$ such that $\overline{Y} = \{K\}$.*

We are ready to introduce a rough exact neighborhood of an exact element. Let Y be an exact element in the covering based rough space X and A be a covering based rough open set in X. Then A is a rough exact neighborhood of Y if $Y_C \subseteq A$, that is, the exact element Y is included in A roughly.

Definition 4. *Let (X, τ) be a covering based rough space, where $X \subseteq U$. For $A \subseteq X$,*

(i) **the covering based rough interior** *of the set A is defined as the union of all covering based rough open subsets contained in A. It is denoted by $cint(A)$ or A°;*

(ii) **the covering based rough closure** *of the set A is defined as the intersection of all covering based rough closed subsets containing A. It is denoted by $ccl(A)$;*

(iii) **the exact interior point** *of a subset A is an element $Y \in X_C$ having a rough exact neighborhood V such that $Y \in V \subseteq A$;*

(iv) **the exact exterior point** *of a subset A is an element $Y \in X_C$ having a rough exact neighborhood V such that $Y \in V \subseteq A^c$.*

Example 1. *Let* $U = \{a, b, c, d, e\}$ *and* $C = \{\{a, b, e\}, \{c, d, e\}\}$*. Clearly,* $\bigcup C = U$ *and the covering* C *is well representable, so it satisfies an approximation condition. Assume that* $X = \{a, e, d\}$*, then*

$$\underline{X} = \emptyset,$$

and

$$\overline{X} = \{\{a, b, e\}\{c, d, e\}\}.$$

Define $\tau_C = \{X = (\underline{X}, \overline{X}), = (\emptyset, \emptyset), \{a\}, \{d\}\}$*, then* τ_C *is a covering based rough space, where the approximations of each rough open set are as follows:*

$$\underline{\{a\}} = \emptyset, \overline{\{a\}} = \{\{a, b, e\}\},$$

and

$$\underline{\{d\}} = \emptyset, \overline{\{d\}} = \{\{c, d, e\}\}.$$

Exact elements of this covering based rough topology are $\{a\}, \{d\}, \{e\}, \{a, e\}, \{a, d\}$, $\{e, d\}, \{a, e, d\}$*, which satisfy the condition in Definition 3.*

Proposition 2. *Let* (U, C) *be a covering approximation space and* $X \subseteq U$ *be such that* (X, τ) *is a covering based rough topology. Suppose* A *and* B *are subsets of* X*.*

(i) If $A \subseteq B$*, then* $A^\circ \subseteq B^\circ$*;*

(ii) If $A \subseteq B$*, then* $ccl(A) \subseteq ccl(B)$*;*

(iii) $(A \wedge B)^\circ = A^\circ \wedge B^\circ$*;*

(iv) $A^\circ \vee B^\circ \subseteq (A \vee B)^\circ$*;*

(v) $ccl(A \vee B) = ccl(A) \vee ccl(B)$*;*

(vi) $ccl(A) \wedge ccl(B) \subseteq ccl(A \wedge B)$*.*

Proof.

(i) We know that $A^\circ \subseteq A$ and $B^\circ \subseteq B$. But $A \subseteq B$, so

$$\underline{A^\circ} \subseteq \underline{A} \subseteq \underline{B},$$

$$\overline{A^\circ} \subseteq \overline{A} \subseteq \overline{B}.$$

Thus, $A^\circ \subseteq B$. By definition of interior of B as the biggest rough open set contained in B, $A^\circ \subseteq B^\circ$.

(ii) We have $A \subseteq ccl(A)$ and $B \subseteq ccl(B)$. Hence

$$\underline{A} \subseteq \underline{ccl(A)}, \overline{A} \subseteq \overline{ccl(A)},$$

$$\underline{B} \subseteq \underline{ccl(B)}, \overline{B} \subseteq \overline{ccl(B)}.$$

As $A \subseteq B$, by definition of closure of A, we get

$$\underline{ccl(A)} \subseteq \underline{ccl(B)}, \overline{ccl(A)} \subseteq \overline{ccl(B)}.$$

Hence $ccl(A) \subseteq ccl(B)$.

Rest items are similar to the classical case.

Subspaces

The inclusion operation on covering based rough sets X and Y is defined by

$$Y \subseteq_C X \iff \underline{Y} \subseteq \underline{X},$$

and

$$\overline{Y} \subseteq \overline{X}.$$

Let (U, C) be a covering approximation space. Suppose that Y and X are elements of the power set of U such that $Y \subseteq_C X$. Our aim is to define topological subspaces of a covering based rough topology. Following [1], the set of all rough sets of the first order with operations $\vee$ and $\wedge$ is a distributive lattice, where

$$\vee_i (V_i \wedge Y) = (\vee_i V_i) \wedge Y,$$

$$\wedge_i (V_i \vee Y) = (\wedge_i V_i) \vee Y.$$

That is, V_i and Y are rough sets. Then

$$\vee_i (\underline{V_i \wedge Y}) = \underline{\vee_i (V_i)} \wedge \underline{Y},$$

$$\wedge_i (\underline{V_i \vee Y}) = \underline{\wedge_i (V_i)} \vee \underline{Y},$$

$$\vee_i (\overline{V_i \wedge Y}) = \overline{\vee_i (V_i)} \wedge \overline{Y},$$

and

$$\wedge_i (\overline{V_i \vee Y}) = \overline{\wedge_i (V_i)} \vee \overline{Y}.$$

Therefore, we can define covering based rough subspaces.

Definition 5. *Suppose (U, C) be a covering approximation space. Let (X, τ) be a covering based rough topology with the rough set $Y \subseteq X$. Define $\tau' = \{V \wedge Y : V \in \tau\}$. Then (Y, τ') is a* **covering based rough subspace** *of (X, τ).*

The intersection of Y with all covering rough open sets in τ induces a topology space on Y.

Exact base and exact subbase of covering based rough space

A base for a covering based rough space (X, τ) is a collection of covering based rough open sets, that are, subsets of X satisfying the following fact: for every rough open set $Y_C = (\underline{Y}, \overline{Y})$ and for every exact element $Y \in Y_C$, there exists a basic open set $B = (\underline{B}, \overline{B})$ of that collection such that $Y \in B \subseteq Y_C$, i.e., $\underline{B} \subseteq \underline{Y}$ and $\overline{B} \subseteq \overline{Y}$.

Now, we will give a result related to a characterization of the base in the covering based rough topology on rough sets.

Proposition 3. *Let $\mathcal{B} = \{(A, B); A \subseteq B, B \in C, A \subseteq \underline{X} \wedge B \subseteq \overline{X}\}$ be a collection of covering based rough open subsets of X. Then $\mathcal{B}$ satisfies the following two conditions:*

(i) For every $Y \in X_C$, there exists $(A, B) \in \mathcal{B}$ such that $Y \in (A, B)$;

(ii) For any $(A_1, B_1), (A_2, B_2) \in \mathcal{B}$ and every element $Y \in ((A_1, B_1) \wedge (A_2, B_2))$, there exists $(A, B) \in \mathcal{B}$ such that $Y \in (A, B) \subseteq ((A_1, B_1) \wedge (A_2, B_2))$.

Therefore $\mathcal{B}$ is an exact base for the covering based rough space (X, τ) .

Proof. We need the two following conditions:

(i) Let $Y \in X_C \implies Y \neq \emptyset \wedge Y_C \subseteq X_C \wedge \exists K \in C$ such that $\overline{Y} = K$. We have $\underline{Y} \subseteq \overline{Y} = K \subseteq C$. So that $Y_C = (\underline{Y}, \overline{Y}) \in \mathcal{B}$, with $\underline{Y} \subseteq \underline{X} \wedge \overline{Y} \subseteq \overline{X}$, which is the desired result.

(ii) Let $Y \in G_C = (\underline{G}, \overline{G}) = ((A_1, B_1) \wedge (A_2, B_2))$, where $(A_1, B_1), (A_2, B_2) \in \mathcal{B}$. This implies that $Y \neq \phi, Y_C \subseteq G_C$ and there exists $B \in C$ such that $\overline{Y} = B$. Then $Y_C = (\underline{Y}, \overline{Y})$ with $\underline{Y} \subseteq \overline{Y} = B$. Hence $Y_C \in \mathcal{B}$ and $Y \in Y_C \subseteq ((A_1, B_1) \wedge (A_2, B_2))$.

So $\mathcal{B}$ is an exact base for the covering based rough topology on the covering based rough set X.

A family $\mathcal{F} \subseteq \tau$ is called **an exact subbase** for a topological space (X, τ) if the family of all finite intersections of members of $\mathcal{F}$ forms a base for (X, τ).

Definition 6. *Let $\mathcal{F}$ be a family of covering based rough open subsets of X_C. Then $\mathcal{F}$ is called* **an exact covering based rough subbase** *if the family $\{Y_{1C} \wedge Y_{2C} \wedge \cdots \wedge Y_{nC} : n \in \mathbb{N}, Y_{iC} \in \mathcal{F}, \forall i \in \{1, 2, \ldots, n\}\}$ forms a base for (X_C, τ).*

Remark 1. *Let $\mathcal{F}$ be an exact covering based rough subbase. Then it generates a covering based rough topology on X.*

Now, we introduce separation axioms with respect to exact elements for a covering based rough space (X, τ).
The space (X, τ) is called **an exact covering based rough $T_\circ$-space** if for every exact distinct elements $Y, Y' \in X$, there is a covering based rough open set A of X such that $Y \notin A \ni Y'$ or $Y' \notin A \ni Y$.
The space (X, τ) is called **an exact covering based rough T_1-space** if for every exact

distinct elements $Y, Y' \in X$, there are two covering based rough open sets A and B of X such that $Y \notin A \ni Y'$ and $Y' \notin B \ni Y$.
The space (X, τ) is called **an exact covering based rough T_2-space** if for every exact distinct elements $Y, Y' \in X$, there are two disjoint covering based rough open sets A and B of X such that $Y \notin A \ni Y'$ and $Y' \notin B \ni Y$.

Note that, there is no exact rough T_1-space. Also, there is no covering based rough T_1-space. This is true only in the case that the rough set X is a singleton and this singleton is a class, since exact elements are subset of each other.

Continuous, closed, open and homeomorphism mappings

Here, we will define covering based rough continuous maps between two covering based rough topological spaces.

Definition 7. *Let (U, C) be a covering based approximation space and $f : U \to U$ be a map. Assume that X, Y are covering based rough topological spaces over U. Then the restriction of f from X to Y is* **a covering based rough continuous map***, if the pre image of every $V \subseteq Y$, which is a covering based rough open (resp. closed) set in Y, gives a subset of X being a covering based rough open (resp. closed) in X.*

Definition 8. *Let (U, C) and (V, C') be a covering based approximation spaces such that $X \subseteq U$ and $Y \subseteq V$. Consider the two rough topological spaces (X, τ), (Y, τ'). A mapping f from X to Y is called a* **covering based rough continuous** *if $f^{-1}(B) \in \tau$ for any covering based rough open (resp. closed) set $B \subseteq Y$. That is, if the inverse image of any subset B of Y being covering based rough open (resp. closed) subset of Y is a subset of X such that $f^{-1}(B) \in \tau$.*

Now, we define the concept of a covering based rough closed (resp. open) map. A covering based rough continuous map $f : X \to Y$ is called a **covering based rough closed (resp. open)**, if for every covering based rough closed (resp. open) set $A \subset X$, the image $f(A)$ is a covering based rough closed (resp. open) in Y. A covering based rough map, which is both covering based rough closed and covering based rough open, is called a **covering based rough clopen map**.

A covering based rough continuous map $f : X \to Y$ is called **a homeomorphism** if f maps X onto Y in a one-to-one way and the inverse map f^{-1} of Y to X is a covering based rough continuous map. We say that two covering based rough topological spaces X and Y are homeomorphic if there exists a homeomorphism of X onto Y. Also, a covering based rough homeomorphism is always an isomorphism, but the converse may be not true.

If there is a covering based rough homeomorphism from X into Y, then there is one-to-one corresponding between lower (resp. upper) approximations. Then we can say there exist isomorphisms f_l and f_u such that $f_l : \underline{X} \to \underline{Y}$ and $f_u : \overline{X} \to \overline{Y}$.

If there are lower and upper isomorphisms $f_l : \underline{X} \to \underline{Y}$ and $f_u : \overline{X} \to \overline{Y}$, then the pair $f = (f_l, f_u)$ is not always a homomorphism from X to Y.

The Product of covering based rough topologies

Let U and V be non-empty universes and let C and D be a covering of U and V such that C and D satisfy the approximation condition. Let (X, τ_C) and (Y, τ_D) be covering based rough topologies. Consider the product cartesian of C and D as follows:

$$C \times D = \{l \times k : l \in C, k \in D\}.$$

Let x, y be two representative elements of l and $x^{'}, y^{'}$ be two representative elements of k. Then $(x, x^{'}), (y, y^{'}), (x, y^{'}), (y, x^{'})$ are representative elements of $l \times k$. So that, $C \times D$ satisfies the approximation condition.

In addition, assume that for every $A_C \times A_D, B_C \times B_D \subseteq C \times D$ such that $A_C \times A_D \subseteq B_C \times B_D$, there exists a set $X \times Y \subseteq U \times V$ with $A_C \times A_D = \underline{X \times Y}$ and $B_C \times B_D = \overline{X \times Y}$. Now, suppose that $X \times Y \subseteq U \times V$. Then define

(i) $\underline{X \times Y} = \{l \times k \in C \times D : l \times k \subseteq X \times Y\}$;

(ii) $\overline{X \times Y} = \{l \times k \in X \times Y : l \times k \wedge X \times Y \neq \phi\}$;

(iii) $BN(X \times Y) = \overline{X \times Y} - \underline{X \times Y}$.

Let $A, B \subseteq U \times V$. We can define the union, the intersection and the complement of two sets in $U \times V$ as follows:

(i) $A \vee B = (\underline{A} \vee \underline{B}, \overline{A} \vee \overline{B})$;

(ii) $A \wedge B = (\underline{A} \wedge \underline{B}, \overline{A} \wedge \overline{B})$;

(iii) $A^{'} = (C \times D \setminus \overline{A}, C \times D \setminus \underline{A})$.

Definition 9. *Let $X \subseteq U$ and $Y \subseteq V$ such that (X, τ_C) and (Y, τ_D) are covering based rough topological spaces on the coverings C and D, respectively. Then the set $\{A \times B : A \in \tau_C, B \in \tau_D\}$ is a base for the product covering based rough topology $X \times Y$ on $C \times D$.*

Now, let W be a covering based rough open set in the product topology $\tau_{X \times Y}$. Consider the pair $(X_1, X_2) \in W$, where X_1, X_2 are exact elements in X and Y, respectively. So there exist $W_X \in \tau_X$ and $W_Y \in \tau_Y$ such that $X_1 \in W_X$ and $X_2 \in W_Y$. This implies that $\underline{X_1} \neq \phi \neq \underline{X_2}$ and $X_1 \subseteq_C W_C, X_2 \subseteq_D W_Y$. Then $\exists K_C \in C, K_D \in D$ with $\overline{X_1} = \{K_C\}$ and $\overline{X_2} = \{K_D\}$. Hence $(X_1, X_2) \neq \emptyset$ and $X_1 \times X_2 \subseteq_{C \times D} W$. There exists $K_C \times K_D \in C \times D$ with $\overline{W} = \{K_C \times K_D\}$. We can then write $(X_1, X_2) \in W_X \times W_Y \subseteq W$, where $W_X \times W_Y$ is a covering based rough basic open set.

Remark 2. *Consider the product covering based rough topology $X \times Y$ on $C \times D$. Let $A \subset X$ and $B \subset Y$. Then*

(i) $cint(A \times B) = cint(A) \times cint(B)$;

(ii) $ccl(A \times B) = ccl(A) \times ccl(B)$.

Properties of Covering Based Rough Topology

Let (U, C) be a covering approximation space. Suppose that X and Y are rough subsets of U such that $Y \subseteq X$. Consider the covering based rough topology (X, τ). We generalize the definition of density from classical rough topology to covering based rough topology.

Definition 10. *The rough subset Y of X is* **dense** *in the covering based rough topology (X, τ), if the upper approximation of Y is equal to the upper approximation of X, i.e., $\overline{X} = \overline{Y}$.*

Generating Covering Based Rough Topology

Let X be a subset of the universe U such that the covering based rough set of X is $X_C = (\underline{X}, \overline{X})$. Let $\mathcal{O}$ be a family of subsets of X_C that satisfies the base conditions.

Definition 11. *The set τ consisting of all possible unions of members of $\mathcal{O}$ together with $\emptyset_C = (\emptyset, \emptyset)$ is called a* **generating covering based rough topology** *and we write $\tau =< \mathcal{O} >$, where $\mathcal{O}$ is the base of τ.*

Now, let $\tau_1 =< \mathcal{O}_1 >$ and $\tau_2 =< \mathcal{O}_2 >$ be two generating covering based rough topologies on the rough set X_C. Then we say that τ_1 is coarser than τ_2 or τ_2 is finer than τ_1 and write $\tau_1 \subseteq \tau_2$ if and only if for every $A_C \in \tau_1$ and for every $Y \in A_C$, there exists $B \in \mathcal{O}_2$ such that $Y \in B \subseteq A_C$. In addition, we write $\tau_1 = \tau_2$ if and only if for every $A_C \in \tau_1$, $B_C \in \tau_2$, $Y \in A_C$ and $Y^{'} \in B_C$, there exist $K \in \mathcal{O}_1$ and $K^{'} \in \mathcal{O}_2$ such that $Y \in K^{'} \subseteq A_C$ and $Y^{'} \in K \subseteq B_C$.

Covering Based Nano Space

In this section, we extend the definition of classical nano topology to include a rough set X determined by a covering C.
Let (U, C) be a covering based approximation space. Similarly, we define the covering based rough set $X \subseteq U$ as $X = (\underline{X}, \overline{X}, BN(X))$. If $Y \subseteq X$, then $\underline{Y} \subseteq \underline{X}, \overline{Y} \subseteq \overline{X}$ and $BN(Y) \subseteq BN(X)$. Now, we are constructing a nano topology on the covering based rough topology.

Definition 12. *Let (U, C) be an approximation space and τ be the family of all rough subsets of $X = (\underline{X}, \overline{X}, BN(X))$ satisfying the following conditions:*

(i) $X, \emptyset \in \tau$;

(ii) It is closed under finite intersection;

(iii) It is closed under arbitrary union.

Then τ is called a **covering based nano topology** *on a given rough set.*

In covering based approximation space, we also find that the nano topology coincides with the rough topology. The reasons are similar with the ones in a classical approximation space.

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Competing Interests

The authors declare that they have no competing interests.

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