



Soft semi local functions in soft ideal topological spaces

F. A. Gharib^{1,*}, A. M. Abd El-latif^{1,2}

¹ Faculty of Arts and Science, Northern Border University, Rafha, P. O. Box, 840, K. S. A.

² Mathematics Department, Faculty of Education, Ain Shams University, Roxy, 11341, Cairo, Egypt

Abstract. In this paper, we define a soft semi local function $(F, E)^{*}_s(\tilde{I}, \tau)$ by using semi open soft sets in a soft ideal topological space (X, τ, E, \tilde{I}) . This concept is discussed with a view to find new soft topologies from the original one, called $*_s$ -soft topology. Some properties and characterizations of soft semi local function are explored. Finally, the notion of soft semi compatibility of soft ideals with soft topologies is introduced and some equivalent conditions concerning this topic are established here.

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1. Introduction

The notion of ideal topological spaces can be found in some classical texts of Kuratowski [16, 17] and Vaidyanathaswamy [29]. Some early applications of ideal topological spaces can be found in various branches of mathematics, like measure theory by Scheinberg [27]. In 1990 Jankovic and Hamlett [6] wrote a paper in which they, among their results, included many other results in this area using modern notation, and logically and systematically arranging them. This paper rekindled the interest in this topic, resulting in many generalizations of the ideal topological space and many generalizations of the notion of open sets, like in papers of Jafari and Rajesh [5] and Manoharan and Thangavelu [22]. In 1966, Velicko [30] introduced the notions of θ -open and θ -closed sets, and also a θ -closure, examining H-closed spaces in terms of an arbitrary filter base.

*Corresponding author.

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Email addresses: fatouhalmg@yahoo.com (F. A. Gharib),
alaa_8560@yahoo.com, alaa.ali@nbu.edu.sa (A. M. Abd El-latif)

In [2], Al-Omari and Noiri introduced the local closure function as a generalization of the θ -closure and the local function in an ideal topological space. They proved some basic properties for the local closure function, and also introduced two new topologies obtained from the original one using the local closure function. Abd El Monsef et al. [1] introduced semi local function in 1992 and defined a topology τ^{*s} , which investigated recently in 2010 in [15]. In 2012, S. Mistry and S. Modak [20] defined pre local function.

The concept of soft sets was first introduced by Molodtsov [24] in 1999 as a general mathematical tool for dealing with uncertain objects. After presentation of the operations of soft sets [21], the properties and applications of soft set theory have been studied increasingly [3, 18, 23, 26]. Recently, in 2011, Shabir and Naz [28] initiated the study of soft topological spaces. The notion of soft ideal was initiated for the first time by Kandil et al. [10]. They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft ideal topological spaces (X, τ, E, \tilde{I}) . Applications to various fields were further investigated by Kandil et al. [8, 9, 11–14].

In this paper, we will introduce and study two different notions via soft ideals namely, soft semi local function and soft semi compatibility of τ with \tilde{I} and investigate their relationships with other types of similar operators.

2. Preliminaries

In this section, we will present the basic definitions and results of soft set theory which will be needed in the sequel.

Definition 1. [24] Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E . A pair (F, A) denoted by F_A is called a soft set over X , where F is a mapping given by $F : A \rightarrow P(X)$ i.e $F_A = \{(e, F(e)) : e \in A \subseteq E, F : A \rightarrow P(X)\}$. The family of all these soft sets denoted by $SS(X)_A$.

Definition 2. [28] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E , then it is called a soft topology on X if:

- (1) $\tilde{X}, \tilde{\phi} \in \tau$, where $\tilde{\phi}(e) = \phi$ and $\tilde{X}(e) = X, \forall e \in E$,
- (2) the union of any number of soft sets in τ belongs to τ ,
- (3) the intersection of any two soft sets in τ belongs to τ .

The triplet (X, τ, E) is called a soft topological space over X . A soft set (F, A) over X is said to be closed soft set in X , if its relative complement $(F, A)'$ is an open soft set. We denote the set of all open soft sets over X by $OS(X, \tau, E)$, or $OS(X)$ and the set of all closed soft sets by $CS(X, \tau, E)$, or $CS(X)$.

Definition 3. [28] Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. The soft closure of (F, E) , denoted by $cl(F, E)$ is the intersection of all closed soft super sets of (F, E) .

Definition 4. [31] Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. The soft interior of (G, E) , denoted by $int(G, E)$ is the union of all open soft subsets of (G, E) .

Definition 5. [31] The soft set $(F, E) \in SS(X)_E$ is called a soft point in \tilde{X} if there exist $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e') = \phi$ for each $e' \in E - \{e\}$, and the soft point (F, E) is denoted by x_e . We denote the set of all soft point of the universal set X by ε .

Definition 6. [31] The soft point x_e is said to be belonging to the soft set (G, A) , denoted by $x_e \tilde{\in} (G, A)$, if for the element $e \in A$, $F(e) \subseteq G(e)$.

Definition 7. [25] Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. Define $\tau_{(F,E)} = \{(G, E) \tilde{\cap} (F, E) : (G, E) \in \tau\}$, which is a soft topology on (F, E) . This soft topology is called a soft relative topology of τ on (F, E) , and $[(F, E), \tau_{(F,E)}]$ is called a soft subspace of (X, τ, E) .

Definition 8. [4, 7] A soft set (F, E) of a soft topological space (X, τ, E) is called semi open soft, if $F_E \tilde{\subseteq} cl(int(F_E))$ (resp., semi closed soft, if $int(cl(F_E)) \tilde{\subseteq} F_E$). The set of all semi open soft sets is denoted by $SOS(X)$ and the set of all semi closed soft sets is denoted by $SCS(X)$. Also, The semi soft closure of (F, E) , denoted by $scl(F, E)$, is defined by the intersection of all semi closed soft sets containing (F, E) .

Definition 9. [6]. A non-empty collection I of subsets of a set X is called an ideal on X , if it is closed under finite unions and subsets.

3. Soft semi local functions and generated a new soft topology

In this section, we will introduce a soft semi local function $(F, E)^{*}_s(\tilde{I}, \tau)$ by using semi open soft sets in a soft ideal topological space (X, τ, E, \tilde{I}) . This concept is discussed with a view to find new soft topologies from the original one, called $*_s$ -soft topology. Some properties and characterizations of soft semi local function will be studied.

Definition 10. [10] Let \tilde{I} be a non-null collection of soft sets over a universe X with the same set of parameters E . Then, $\tilde{I} \subseteq SS(X)_E$ is called a soft ideal on X if it is closed under finite soft unions and soft subsets.

Definition 11. [10] Let (X, τ, E) be a soft topological space and \tilde{I} be a soft ideal over X with the same set of parameters E . Then,

$$(F, E)^*(\tilde{I}, \tau) ((F, E)^*(\tilde{I}) \text{ or } (F, E)^*) = \tilde{\cup} \{x_e \in \varepsilon : O_{x_e} \tilde{\cap} (F, E) \tilde{\notin} \tilde{I} \forall O_{x_e} \in \tau\}$$

is called the soft local function of (F, E) with respect to \tilde{I} and τ , where O_{x_e} is an open soft set containing x_e .

Definition 12. Let (X, τ, E) be a soft topological space and \tilde{I} be a soft ideal over X with the same set of parameters E . Then,

$$(F, E)^{*s}(\tilde{I}, \tau) ((F, E)^{*s}(\tilde{I}) \text{ or } (F, E)^{*s}) = \tilde{U}\{x_e \in \varepsilon : O_{x_e} \tilde{\cap}(F, E) \tilde{\not\subseteq} \tilde{I} \forall O_{x_e} \in \text{SOS}(X)\}$$

is called the soft semi local function of (F, E) with respect to \tilde{I} and τ , where O_{x_e} is a semi open soft set containing x_e .

Theorem 1. Let \tilde{I} be a soft ideal with the same set of parameters E on a soft topological space (X, τ, E) and $(F, E) \in \text{SS}(X)_E$. Then, $(F, E)^{*s} \tilde{\subseteq} (F, E)^*$.

Proof. Let $x_e \tilde{\in} (F, E)^{*s}$. Then, $O_{x_e} \tilde{\cap}(F, E) \not\subseteq \tilde{I} \forall O_{x_e} \in \text{SOS}(X)$. Since $\tau \subseteq \text{SOS}(X)$ and $O_{x_e} \tilde{\cap}(F, E) \not\subseteq \tilde{I}$, $O_{x_e} \tilde{\cap}(F, E) \not\subseteq \tilde{I} \forall O_{x_e} \in \tau$ from Definition 10. Hence, $x_e \tilde{\in} (F, E)^*$.

Remarks 1. The converse of the above theorem is not true in general as will shown in the following example.

Example 1. Suppose that there are three cars in the universe X given by $X = \{c_1, c_2, c_3\}$. Let $E = \{e_1, e_2\}$ be the set of decision parameters which are stands for "expensive" and "color" respectively.

Let $(F_1, E), (F_2, E), (F_3, E)$ be soft sets over the common universe X , which describe the composition of the cars, where

$$\begin{aligned} F_1(e_1) &= \{c_1\}, & F_1(e_2) &= \{c_1\}. \\ F_2(e_1) &= \{c_2\}, & F_2(e_2) &= \{c_2\}. \\ F_3(e_1) &= \{c_1, c_2\}, & F_3(e_2) &= \{c_1, c_2\}. \end{aligned}$$

Then, $\tau = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$ defines a soft topology on X . Let $\tilde{I} = \{\tilde{\phi}\}$ and (G, E) be a soft set defined by $G(e_1) = \{c_2, c_3\}$, $G(e_2) = \{c_2\}$. Hence, $(G, E)^{*s} = \text{scl}(G, E) = \text{scl}(G, E)^{*s} = (G, E)$ and $(G, E)^* = \text{cl}(G, E) = \text{cl}(G, E)^* = (H, E)$, where $H(e_1) = \{c_2, c_3\}$, $H(e_2) = \{c_2, c_3\}$.

Remarks 2. The collection of all semi open soft sets of a soft topological space (X, τ, E) fails to form a soft ideal on X as will shown in the following example.

Example 2. Suppose that there are two houses in the universe X given by $X = \{h_1, h_2\}$. Let $E = \{e_1, e_2\}$ be the set of decision parameters which are stands for "wooden" and "position" respectively.

Let $(F_1, E), (F_2, E)$ be soft sets over the common universe X , which describe the composition of the houses, where

$$\begin{aligned} F_1(e_1) &= \{h_1\}, & F_1(e_2) &= \{h_2\}. \\ F_2(e_1) &= \{h_2\}, & F_2(e_2) &= \{h_1\}. \end{aligned}$$

Then, $\tau = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E)\}$ defines a soft topology on X . Also, $\text{SOS}(X) = \{\tilde{\phi}, (F_1, E), (F_2, E), \tilde{X}\}$ is not soft ideal, because it is not closed under soft subset.

Remarks 3. (1) If $(F, E) \in \tilde{I}$, then $(F, E)^{*s} = \tilde{\phi}$.

(2) If $\tilde{I} = \text{SS}(X)_E$, then $(F, E)^{*s} = \tilde{\phi} = (F, E)^*$.

(3) If $\tilde{I} = \{\tilde{\phi}\}$, then $(F, E)^{*s} = \text{scl}(F, E) \neq \text{cl}(F, E)$.

(4) Neither $(F, E)^{*s} \tilde{\subseteq} (F, E)$ nor $(F, E) \tilde{\subseteq} (F, E)^{*s}$ in general.

(5) If $\tau = SOS(X)$, then $(F, E)^{*s} = (F, E)^*$.

Theorem 2. Let \tilde{I} and \tilde{J} be any two soft ideals with the same set of parameters E on a soft topological space (X, τ, E) . Let $(F, E), (G, E) \in SS(X)_E$. Then,

(1) $(\tilde{\phi})^{*s} = \tilde{\phi}$,

(2) If $(F, E) \tilde{\subseteq} (G, E)$, then $(F, E)^{*s} \tilde{\subseteq} (G, E)^{*s}$,

(3) If $\tilde{I} \subseteq \tilde{J}$, then $(F, E)^{*s}(\tilde{J}) \tilde{\subseteq} (F, E)^{*s}(\tilde{I})$,

(4) $(F, E)^{*s} = scl(F, E)^{*s} \tilde{\subseteq} scl(F, E)$, where scl is the semi soft closure w.r.t. τ , $(F, E)^{*s}$ is semi closed soft set.

(5) $((F, E)^{*s})^{*s} \tilde{\subseteq} (F, E)^{*s}$,

(6) $((F, E) \tilde{\cup} (G, E))^{*s} = (F, E)^{*s} \tilde{\cup} (G, E)^{*s}$,

(7) $\tilde{\cup}_j (F, E)^{*s} = (\tilde{\cup}_j (F, E))^{*s}$,

(8) $((F, E) \tilde{\cap} (G, E))^{*s} \subseteq (F, E)^{*s} \tilde{\cap} (G, E)^{*s}$,

(9) $(F, E)^{*s} - (G, E)^{*s} = ((F, E) - (G, E))^{*s} - (G, E)^{*s} \tilde{\subseteq} ((F, E) - (G, E))^{*s}$,

(10) If $(I, E) \in \tilde{I}$, then $((F, E) - (I, E))^{*s} = (F, E)^{*s} = ((F, E) \tilde{\cup} (I, E))^{*s}$.

Proof.

(1) Obvious from Definition 12.

(2) Assume that $x_e \tilde{\in} (F, E)^{*s}$ and $x_e \tilde{\notin} (G, E)^{*s}$. Then, there exist $O_{x_e} \in SOS(X)$ such that $O_{x_e} \tilde{\cap} (G, E) \in \tilde{I}$. Since $(F, E) \tilde{\subseteq} (G, E)$, $O_{x_e} \tilde{\cap} (F, E) \tilde{\subseteq} O_{x_e} \tilde{\cap} (G, E)$ and $O_{x_e} \tilde{\cap} (F, E) \in \tilde{I}$. Hence, $x_e \tilde{\in} (F, E)^{*s}$, which is a contradiction. Thus, $x_e \tilde{\in} (G, E)^{*s}$ and so $(F, E)^{*s} \tilde{\subseteq} (G, E)^{*s}$.

(3) Let $x_e \tilde{\in} (F, E)^{*s}(\tilde{J})$. Then, $O_{x_e} \tilde{\cap} (F, E) \notin \tilde{J} \forall O_{x_e} \in SOS(X)$. Since $\tilde{I} \subseteq \tilde{J}$. Then, $O_{x_e} \tilde{\cap} (F, E) \notin \tilde{I} \forall O_{x_e} \in SOS(X)$. Hence, $x_e \tilde{\in} (F, E)^{*s}(\tilde{I})$. Thus, $(F, E)^{*s}(\tilde{J}) \tilde{\subseteq} (F, E)^{*s}(\tilde{I})$.

(4) We first prove that $(F, E)^{*s} = scl(F, E)^{*s}$. Let $x_e \tilde{\in} scl(F, E)^{*s}$. Then, $O_{x_e} \tilde{\cap} (F, E)^{*s} \neq \tilde{\phi}$ for every $O_{x_e} \in SOS(X)$. Therefore, there exists a soft point $y_{e'}$ such that $y_{e'} \tilde{\in} O_{x_e} \tilde{\cap} (F, E)^{*s}$ and so $y_{e'} \tilde{\in} (F, E)^{*s}$. Hence, $O_{y_{e'}} \tilde{\cap} (F, E) \notin \tilde{I}$. Thus, $x_e \tilde{\in} (F, E)^{*s}$. This shows that, $scl(F, E)^{*s} \tilde{\subseteq} (F, E)^{*s}$ but we have $(F, E)^{*s} \tilde{\subseteq} scl(F, E)^{*s}$.

Now, we prove that $scl(F, E)^{*s} = (F, E)^{*s} \tilde{\subseteq} scl(F, E)$. Assume that, $x_e \tilde{\notin} scl(F, E)$. Then, there exists $O_{x_e} \in SOS(X)$ such that $O_{x_e} \tilde{\cap} (F, E) = \tilde{\phi} \in \tilde{I}$. Hence, $x_e \tilde{\notin} (F, E)^{*s}$. Thus, $(F, E)^{*s} \tilde{\subseteq} scl(F, E)$.

(5) Since $(F, E)^{*s} \tilde{\subseteq} scl(F, E)$ from (4). Replace (F, E) with $(F, E)^{*s}$, we get $((F, E)^{*s})^{*s} \tilde{\subseteq} scl(F, E)^{*s} = (F, E)^{*s}$ from (4).

- (6) Let $x_e \in \tilde{I} \cap ((F, E) \tilde{\cup} (G, E))^*_{s}$. Then, $O_{x_e} \tilde{\cap} ((F, E) \tilde{\cup} (G, E)) = (O_{x_e} \tilde{\cap} (F, E)) \tilde{\cup} (O_{x_e} \tilde{\cap} (G, E)) \notin \tilde{I} \forall O_{x_e} \in SOS(X)$. Hence, either $O_{x_e} \tilde{\cap} (F, E) \notin \tilde{I}$ or $O_{x_e} \tilde{\cap} (G, E) \notin \tilde{I}$ from Definition 10 $\forall O_{x_e} \in SOS(X)$. This means that, either $x_e \notin (F, E)^*_{s}$ or $x_e \notin (G, E)^*_{s}$. Thus, $x_e \notin (F, E)^*_{s} \tilde{\cup} (G, E)^*_{s}$. It follows that, $((F, E) \tilde{\cup} (G, E))^*_{s} \subseteq (F, E)^*_{s} \tilde{\cup} (G, E)^*_{s}$. For the reverse inclusion, since $(F, E), (G, E) \subseteq ((F, E) \tilde{\cup} (G, E))$. Then, $(F, E)^*_{s} \subseteq ((F, E) \tilde{\cup} (G, E))^*_{s}$ and $(G, E)^*_{s} \subseteq ((F, E) \tilde{\cup} (G, E))^*_{s}$ from (2). Hence, $(F, E)^*_{s} \tilde{\cup} (G, E)^*_{s} \subseteq ((F, E) \tilde{\cup} (G, E))^*_{s}$ and it implies that $((F, E) \tilde{\cup} (G, E))^*_{s} = (F, E)^*_{s} \tilde{\cup} (G, E)^*_{s}$.
- (7) Obvious from (6).
- (8) Since $((F, E) \tilde{\cap} (G, E)) \subseteq (F, E), (G, E)$. Then, $((F, E) \tilde{\cap} (G, E))^*_{s} \subseteq (F, E)^*_{s}$ and $((F, E) \tilde{\cap} (G, E))^*_{s} \subseteq (G, E)^*_{s}$ from (2). Hence, $((F, E) \tilde{\cap} (G, E))^*_{s} \subseteq (F, E)^*_{s} \tilde{\cap} (G, E)^*_{s}$.
- (9) We first prove that $(F, E)^*_{s} - (G, E)^*_{s} = ((F, E) - (G, E))^*_{s} - (G, E)^*_{s}$. Since $(F, E) - (G, E) \subseteq (F, E)$. Then, $((F, E) - (G, E))^*_{s} \subseteq (F, E)^*_{s}$. Hence, $((F, E) - (G, E))^*_{s} - (G, E)^*_{s} \subseteq (F, E)^*_{s} - (G, E)^*_{s}$. For the reverse inclusion, since $(F, E) = [(F, E) - (G, E)] \tilde{\cup} [(F, E) \tilde{\cap} (G, E)]$. Then, $(F, E)^*_{s} = [((F, E) - (G, E)) \tilde{\cup} ((F, E) \tilde{\cap} (G, E))]^*_{s}$. It follows, $(F, E)^*_{s} = [((F, E) - (G, E)) \tilde{\cup} ((F, E) \tilde{\cap} (G, E))]^*_{s} = [(F, E) - (G, E)]^*_{s} \tilde{\cup} [(F, E) \tilde{\cap} (G, E)]^*_{s} \subseteq [(F, E) - (G, E)]^*_{s} \tilde{\cup} (G, E)^*_{s}$ from (6). Hence, $(F, E)^*_{s} - (G, E)^*_{s} \subseteq [(F, E) - (G, E)]^*_{s} \tilde{\cup} (G, E)^*_{s} - (G, E)^*_{s} = [((F, E) - (G, E)) \tilde{\cup} (G, E)]^*_{s} \tilde{\cap} (G, E)^*_{s} = [(F, E) - (G, E)]^*_{s} - (G, E)^*_{s}$. Thus, $(F, E)^*_{s} - (G, E)^*_{s} = ((F, E) - (G, E))^*_{s} - (G, E)^*_{s}$. Now, if $x_e \in ((F, E) - (G, E))^*_{s} - (G, E)^*_{s}$. Then, $x_e \in ((F, E) - (G, E))^*_{s}$ and consequently $(F, E)^*_{s} - (G, E)^*_{s} = ((F, E) - (G, E))^*_{s} - (G, E)^*_{s} \subseteq ((F, E) - (G, E))^*_{s} - (G, E)^*_{s}$.
- (10) We first prove that $(F, E)^*_{s} = ((F, E) - (I, E))^*_{s}$. Since $((F, E) - (I, E)) \subseteq (F, E)$, $((F, E) - (I, E))^*_{s} \subseteq (F, E)^*_{s}$ from (2). For the reverse inclusion, let $x_e \in (F, E)^*_{s}$. Then, there exists $O_{x_e} \in SOS(X)$ such that $O_{x_e} \tilde{\cap} ((F, E) - (I, E)) \in \tilde{I}$. Since $(I, E) \in \tilde{I}$. Then, $(I, E) \tilde{\cup} (O_{x_e} \tilde{\cap} ((F, E) - (I, E))) \in \tilde{I}$. Hence, $(I, E) \tilde{\cup} (O_{x_e} \tilde{\cap} (F, E)) \in \tilde{I}$. Thus, $O_{x_e} \tilde{\cap} (F, E) \in \tilde{I}$ for some $O_{x_e} \in SOS(X)$. It follows that, $x_e \in (F, E)^*_{s}$. This means that, $(F, E)^*_{s} \subseteq ((F, E) - (I, E))^*_{s}$. Now, we prove that $((F, E) \tilde{\cup} (I, E))^*_{s} = (F, E)^*_{s}$. From (2), $((F, E) \tilde{\cup} (I, E))^*_{s} = (F, E)^*_{s} \tilde{\cup} (I, E)^*_{s} = (F, E)^*_{s} \tilde{\cup} \phi = (F, E)^*_{s}$ from Remark 3 (1). This completes the proof.

Corollary 1. Let \tilde{I} be a soft ideal with the same set of parameters E on a soft topological space (X, τ, E) . Let $(F, E), (G, E) \in SS(X)_E$. Then,

- (1) $(F, E)^*_{s}$ is semi closed soft set.
- (2) $((F, E)^*_{s})^*_{s} \subseteq (F, E)^*$,
- (3) $((F, E)^*_{s})^* \subseteq (F, E)^*$,
- (4) $((F, E)^*)^*_{s} \subseteq (F, E)^*$,

- (5) $scl(F, E)^{*s} \subseteq cl(F, E)$,
- (6) $(F, E)^{*s} \subseteq (F, E)^* \subseteq cl(F, E)$,
- (7) If $(I, E) \in \tilde{I}$, then $[\tilde{X} - (I, E)]^{*s} = \tilde{X}^{*s}$.

Proof. It follows from Theorem 1 and Theorem 2.

Theorem 3. Let \tilde{I} and \tilde{J} be two soft ideals with the same set of parameters E on a soft topological space (X, τ, E) and $(F, E) \in SS(X)_E$. Then, $(F, E)^{*s}(\tilde{I} \cap \tilde{J}) = (F, E)^*(\tilde{I}) \tilde{\cup} (F, E)^*(\tilde{J})$.

Proof. Since $\tilde{I} \cap \tilde{J} \subseteq \tilde{I}$ and $\tilde{I} \cap \tilde{J} \subseteq \tilde{J}$, $(F, E)^*(\tilde{I}), (F, E)^*(\tilde{J}) \subseteq (F, E)^{*s}(\tilde{I} \cap \tilde{J})$ from Theorem 2 (3). Therefore, $(F, E)^*(\tilde{I}) \tilde{\cup} (F, E)^*(\tilde{J}) \subseteq (F, E)^{*s}(\tilde{I} \cap \tilde{J})$. For the reverse inclusion, let $x_e \in (F, E)^{*s}(\tilde{I} \cap \tilde{J})$. Then, $O_{x_e} \tilde{\cap} (F, E) \notin (\tilde{I} \cap \tilde{J}) \forall O_{x_e} \in SOS(X)$ and so $O_{x_e} \tilde{\cap} (F, E) \notin \tilde{I}$ or $O_{x_e} \tilde{\cap} (F, E) \notin \tilde{J}$. It follows, $x_e \in (F, E)^{*s}(\tilde{I})$ or $x_e \in (F, E)^{*s}(\tilde{J})$ and consequently $x_e \in (F, E)^{*s}(\tilde{I}) \tilde{\cup} (F, E)^{*s}(\tilde{J})$. Thus, $(F, E)^{*s}(\tilde{I} \cap \tilde{J}) \subseteq (F, E)^*(\tilde{I}) \tilde{\cup} (F, E)^*(\tilde{J})$. This completes the proof.

Theorem 4. Let (X, τ, E) be a soft topological space and \tilde{I} be a soft ideal over X with the same set of parameters E . Then the operator $cl^{*s} : SS(X)_E \rightarrow SS(X)_E$ defined by:

$$cl^{*s}(F, E) = (F, E) \tilde{\cup} (F, E)^{*s}. \tag{1}$$

is a soft closure operator.

Proof. $cl^{*s}(\tilde{\phi}) = \tilde{\phi} \tilde{\cup} (\tilde{\phi})^{*s} = \tilde{\phi} \tilde{\cup} \tilde{\phi} = \tilde{\phi}$ from Theorem 1 (1), and obviously $(F, E) \subseteq cl^{*s}(F, E) \forall (F, E) \in SS(X)_E$. Now, $cl^{*s}((F, E) \tilde{\cup} (G, E)) = ((F, E) \tilde{\cup} (G, E)) \tilde{\cup} ((F, E) \tilde{\cup} (G, E))^{*s} = ((F, E) \tilde{\cup} (G, E)) \tilde{\cup} ((F, E)^{*s} \tilde{\cup} (G, E)^{*s}) = ((F, E) \tilde{\cup} (F, E)^{*s}) \tilde{\cup} ((G, E) \tilde{\cup} (G, E)^{*s}) = cl^{*s}(F, E) \tilde{\cup} cl^{*s}(G, E)$ from Theorem 2 (6). Also, for any $(F, E) \in SS(X)_E$, $cl^{*s}(cl^{*s}(F, E)) = cl^{*s}((F, E) \tilde{\cup} (F, E)^{*s}) = ((F, E) \tilde{\cup} (F, E)^{*s}) \tilde{\cup} ((F, E) \tilde{\cup} (F, E)^{*s})^{*s} = ((F, E) \tilde{\cup} (F, E)^{*s}) \tilde{\cup} ((F, E)^{*s} \tilde{\cup} ((F, E)^{*s})^{*s}) \subseteq ((F, E) \tilde{\cup} (F, E)^{*s}) \tilde{\cup} ((F, E)^{*s} \tilde{\cup} ((F, E)^{*s})) = cl^{*s}(F, E)$ from Theorem 2 (5).

Definition 13. Let (X, τ, E) be a soft topological space, \tilde{I} be a soft ideal over X with the same set of parameters E and $cl^{*s} : SS(X)_E \rightarrow SS(X)_E$ be the soft closure operator. Then there exists a unique soft topology over X with the same set of parameters E , finer than τ , called the $*\text{-soft topology}$, denoted by $\tau^{*s}(\tilde{I})$ or τ^{*s} , given by

$$\tau^{*s}(\tilde{I}) = \{(F, E) \in SS(X)_E : cl^{*s}(F, E)' = (F, E)'\}. \tag{2}$$

Example 3. (1) If $\tilde{I} = \{\tilde{\phi}\}$, then $(F, E)^{*s}(\tilde{I}, \tau) = scl(F, E) \forall (F, E) \in SS(X)_E$. Hence, $cl^{*s}(F, E) = scl(F, E)$.

(2) If $\tilde{I} = SS(X)_E$, then $(F, E)^{*s}(\tilde{I}, \tau) = \tilde{\phi} \forall (F, E) \in SS(X)_E$. Hence, $cl^{*s}(F, E) = (F, E)$ and $\tau^{*s} = SS(X)_E$ (the soft discrete topology).

Theorem 5. Let \tilde{I} and \tilde{J} be any two soft ideals with the same set of parameters E on a soft topological space (X, τ, E) . If $\tilde{I} \subseteq \tilde{J}$, then $\tau^{*s}(\tilde{I}) \subseteq \tau^{*s}(\tilde{J})$.

Proof. Let (F, E) be a $\tau^{*s}(\tilde{I})$ -closed soft set. Since $\tilde{I} \subseteq \tilde{J}$, $(F, E)^{*s}(\tilde{J}, \tau) \subseteq (F, E)^{*s}(\tilde{I}, \tau)$. Therefore, $cl_{\tilde{J}}^{*s}(F, E) \subseteq cl_{\tilde{I}}^{*s}(F, E)$. Hence, $(F, E) \subseteq cl_{\tilde{J}}^{*s}(F, E) \subseteq cl_{\tilde{I}}^{*s}(F, E) = (F, E)$ and consequently (F, E) is $\tau^{*s}(\tilde{J})$ -closed soft.

Proposition 1. *Let (X, τ, E) be a soft topological space and \tilde{I} be a soft ideal over X with the same set of parameters E . Then,*

$$\beta(\tilde{I}, \tau) = \{(F, E) - (G, E) : (F, E) \in SOS(X), (G, E) \in \tilde{I}\}$$

*is a soft basis for the soft topology $\tau^{*s}(\tilde{I})$.*

Proof. Since $\tilde{X} \in \tau, \tilde{\phi} \in \tilde{I}$. Then, $\tilde{X} - \tilde{\phi} \in \beta$. Hence, $\tilde{X} \in \beta$ and $\bigcup_{j \in J} ((F_j, E) - (G_j, E)) = \tilde{X}$. Also, let $((F_1, E) - (G_1, E)), ((F_2, E) - (G_2, E)) \in \beta$ such that $x_e \in ((F_1, E) - (G_1, E)) \tilde{\cap} ((F_2, E) - (G_2, E))$. Then $x_e \in ((F_1, E) - (G_1, E)) \tilde{\cap} ((F_2, E) - (G_2, E)) = ((F_1, E) \tilde{\cap} (F_2, E)) - ((G_1, E) \tilde{\cap} (G_2, E)) \in \beta(\tilde{I}, \tau)$. Thus, β is a soft basis of τ^{*s} .

Corollary 2. *Let (X, τ, E) be a soft topological space and \tilde{I} be a soft ideal over X with the same set of parameters E . Then, $\tau \subseteq \beta(\tilde{I}, \tau) \subseteq \tau^*(\tilde{I}) \subseteq \tau^{*s}(\tilde{I})$.*

Proof. It is obvious from Theorem 2(3) and Proposition 1.

4. Soft semi-compatibility of τ with \tilde{I}

In this section, we will introduce the notion of soft semi compatibility of soft ideals with soft topologies and some equivalent conditions concerning this topic will be investigated here.

Definition 14. *Let (X, τ, E) be a soft topological space and \tilde{I} be a soft ideal over X with the same set of parameters E . We say that τ is semi compatible with \tilde{I} , denoted by $\tau \sim^s \tilde{I}$, if the following holds for each $(F, E) \in SS(X)_E$:*

if for each soft point x_e and $x_e \in (F, E)$ there exists $O_{x_e} \in SOS(X)$ such that $O_{x_e} \tilde{\cap} (F, E) \in \tilde{I}$, then $(F, E) \in \tilde{I}$.

Theorem 6. *Let (X, τ, E) be a soft topological space, \tilde{I} be a soft ideal over X with the same set of parameters E and $\tau \sim^s \tilde{I}$. Then, the following are equivalent for each $(F, E) \in SS(X)_E$:*

- (1) *If $(F, E) \tilde{\cap} (F, E)^{*s} = \tilde{\phi}$, then $(F, E)^{*s} = \tilde{\phi}$.*
- (2) *$((F, E) - (F, E)^{*s})^* = \tilde{\phi}$.*
- (3) *$((F, E) \tilde{\cap} (F, E)^{*s})^* = (F, E)^{*s}$.*

Proof.

(1) \implies (2) Let $(F, E) \in SS(X)_E$. Since $((F, E) - (F, E)^{*s}) \tilde{\cap} ((F, E) - (F, E)^{*s})^* = [((F, E) - (F, E)^{*s}) \tilde{\cap} (F, E)^{*s}] \tilde{\cap} [((F, E) - (F, E)^{*s}) \tilde{\cap} (\tilde{X} - (F, E)^{*s})^*] = \tilde{\phi} \tilde{\cap} [((F, E) - (F, E)^{*s}) \tilde{\cap} (\tilde{X} - (F, E)^{*s})^*] = \tilde{\phi}$, $((F, E) - (F, E)^{*s})^* = \tilde{\phi}$ by (1).

(2) \implies (3) Let $(F, E) \in SS(X)_E$. Since $(F, E) = ((F, E) - ((F, E) \tilde{\cap} (F, E)^{*s})) \tilde{\cup} ((F, E) \tilde{\cap} (F, E)^{*s})$,
 $(F, E)^{*s} = [(F, E) - ((F, E) \tilde{\cap} (F, E)^{*s}) \tilde{\cup} ((F, E) \tilde{\cap} (F, E)^{*s})]^{*s} = [(F, E) - ((F, E) \tilde{\cap} (F, E)^{*s})]^{*s}$
 $\tilde{\cup} [(F, E) \tilde{\cap} (F, E)^{*s}]^{*s} = [\tilde{\phi} \tilde{\cup} ((F, E) - (F, E)^{*s})]^{*s} \tilde{\cup} [(F, E) \tilde{\cap} (F, E)^{*s}]^{*s} = [(F, E) \tilde{\cap} (F, E)^{*s}]^{*s}$
 from (2).

(3) \implies (1) Let $(F, E) \in SS(X)_E$ and $(F, E) \tilde{\cap} (F, E)^{*s} = \tilde{\phi}$. Then, $(F, E)^{*s} = [(F, E) \tilde{\cap} (F, E)^{*s}]^{*s} =$
 $(\tilde{\phi})^{*s} = \tilde{\phi}$.

Corollary 3. Let (X, τ, E) be a soft topological space, \tilde{I} be a soft ideal over X with the same set of parameters E , $(F, E) \in SS(X)_E$ and $\tau \sim^s \tilde{I}$. Then, $((F, E)^{*s})^{*s} = (F, E)^{*s}$.

Proof. Let $(F, E) \in SS(X)_E$. Since $(F, E)^{*s} = ((F, E) \tilde{\cap} (F, E)^{*s})^{*s} \tilde{\subseteq} (F, E)^{*s}$ from Theorem 6. But, we have $((F, E)^{*s})^{*s} \tilde{\subseteq} (F, E)^{*s}$ from Theorem 2 (5). Thus, $((F, E)^{*s})^{*s} = (F, E)^{*s}$.

Theorem 7. Let (X, τ, E) be a soft topological space and \tilde{I} be a soft ideal over X with the same set of parameters E . Then, the following are equivalent:

- (1) $\tau \sim^s \tilde{I}$.
- (2) If $(F, E) \in SS(X)_E$ has a cover of semi open soft sets each of whose soft intersection with (F, E) is in \tilde{I} , then $(F, E) \in \tilde{I}$.
- (3) For every $(F, E) \in SS(X)_E$ such that $(F, E) \tilde{\cap} (F, E)^{*s} = \tilde{\phi}$ implies $(F, E) \in \tilde{I}$.
- (4) For every $(F, E) \in SS(X)_E$, $(F, E) - (F, E)^{*s} \in \tilde{I}$.
- (5) For every τ^{*s} -closed soft subset (F, E) , $(F, E) - (F, E)^{*s} \in \tilde{I}$.
- (6) For every $(F, E) \in SS(X)_E$, if (F, E) contains no non-null soft set (G, E) with $(G, E) \tilde{\subseteq} (G, E)^{*s}$, then $(F, E) \in \tilde{I}$.

Proof.

(1) \implies (2) Obvious from Definition 14.

(2) \implies (3) Let $(F, E) \in SS(X)_E$ such that $(F, E) \tilde{\cap} (F, E)^{*s} = \tilde{\phi}$ and $x_e \tilde{\in} (F, E)$. Then, $x_e \notin (F, E)^{*s}$ and then $O_{x_e} \tilde{\cap} (F, E) \in \tilde{I}$ for some $O_{x_e} \in SOS(X)$. Therefore, $(F, E) \tilde{\subseteq} \tilde{\cup} \{O_{x_e} : x_e \tilde{\in} (F, E) \text{ and } O_{x_e} \in SOS(X)\}$. By (2), $(F, E) \in \tilde{I}$.

(3) \implies (4) Let $(F, E) \in SS(X)_E$. Since $((F, E) - (F, E)^{*s}) \tilde{\cap} ((F, E) - (F, E)^{*s})^{*s} = ((F, E) \tilde{\cap} (F, E)^{*s})^{*s}$
 $\tilde{\cap} ((F, E) - (F, E)^{*s})^{*s} \tilde{\subseteq} ((F, E) \tilde{\cap} (F, E)^{*s}) \tilde{\cap} (F, E)^{*s} = \tilde{\phi}$. Then, $(F, E) - (F, E)^{*s} \in \tilde{I}$
 by (3).

(4) \implies (5) Immediate.

- (5) \implies (1) Let $(F, E) \in SS(X)_E$ and assume that for every $x_e \in \tilde{E}(F, E)$ there exists $O_{x_e} \in SOS(X)$ such that $O_{x_e} \tilde{\cap} (F, E) \in \tilde{I}$. Then, $x_e \tilde{\in} (F, E)^{*s}$. Hence, $((F, E) \tilde{\cap} (F, E)^{*s}) = \tilde{\phi}$ and since $(F, E) \tilde{\cup} (F, E)^{*s}$ is τ^{*s} -closed soft, we have $((F, E) \tilde{\cup} (F, E)^{*s}) - ((F, E) \tilde{\cup} (F, E)^{*s})^{*s} \in \tilde{I}$ by (5). Hence, $((F, E) \tilde{\cup} (F, E)^{*s}) - ((F, E)^{*s} \tilde{\cup} ((F, E)^{*s})^{*s}) = ((F, E) \tilde{\cup} (F, E)^{*s}) - ((F, E)^{*s}) = (F, E) \in \tilde{I}$ by Theorem 2 (5,6). Therefore, $\tau \sim^s \tilde{I}$.
- (4) \implies (6) Let $(F, E) \in SS(X)_E$ such that (F, E) contains no non-null soft set (G, E) with $(G, E) \tilde{\subseteq} (G, E)^{*s}$. Since $(F, E) \tilde{\cap} (F, E)^{*s} \tilde{\subseteq} (F, E)^{*s} = ((F, E) \tilde{\cap} (F, E)^{*s})^{*s}$ from Theorem 6 (3). It follows, $(F, E) \tilde{\cap} (F, E)^{*s} \tilde{\subseteq} ((F, E) \tilde{\cap} (F, E)^{*s})^{*s}$. By assumption, $(F, E) \tilde{\cap} (F, E)^{*s} = \tilde{\phi}$. Thus, $(F, E) = (F, E) - (F, E)^{*s} \in \tilde{I}$ by (4).
- (6) \implies (4) Let $(F, E) \in SS(X)_E$. Since $((F, E) - (F, E)^{*s}) \tilde{\cap} ((F, E) - (F, E)^{*s})^{*s} = \tilde{\phi}$ and $(F, E) - (F, E)^{*s}$ contains no non-null soft set (G, E) with $(G, E) \tilde{\subseteq} (G, E)^{*s}$. Hence, $(F, E) - (F, E)^{*s} \in \tilde{I}$ by (6).

Theorem 8. *If (X, τ, E) is a soft topological space, \tilde{I} be a soft ideal over X with the same set of parameters E and semi compatible with τ . Then, a soft set is τ^{*s} -closed if and only if it is the union of a τ -semi closed soft set and a soft set in \tilde{I} .*

Proof.

Let (F, E) be a τ^{*s} -closed soft set. Then, $cl^{*s}(F, E) = (F, E) = (F, E) \tilde{\cup} (F, E)^{*s} = (F, E)$. It follows, $(F, E)^{*s} \tilde{\subseteq} (F, E)$. Hence, $(F, E) = ((F, E) - (F, E)^{*s}) \tilde{\cup} (F, E)^{*s}$, where $(F, E) - (F, E)^{*s} \in \tilde{I}$ from Theorem 7 and $(F, E)^{*s}$ is τ -semi closed soft from Corollary 1. Conversely, let $(F, E) = (G, E) \tilde{\cup} (I, E)$, for some τ -semi closed soft set (G, E) and $(I, E) \in \tilde{I}$. Then, $(F, E)^{*s} = ((G, E) - (I, E))^{*s} = (G, E)^{*s} \tilde{\subseteq} scl(G, E) = (G, E) \tilde{\subseteq} (F, E)$ from Theorem 2 (4,12). It follows, $(F, E) \tilde{\cup} (F, E)^{*s} = (F, E)$. Hence, $cl^{*s}(F, E) = (F, E)$. Therefore, (F, E) is τ^{*s} -closed soft.

Definition 15. [19] *A soft topological space (X, τ, E) is called a soft semi compact space if each semi open soft cover of \tilde{X} has a finite soft subcover.*

Theorem 9. [19] *Each semi closed soft subspace of a soft semi compact topological space is a soft semi compact.*

Theorem 10. *Let (X, τ, E) be a soft semi compact space in which every soft subset of \tilde{X} is a semi closed soft and \tilde{I} be a soft ideal over X with the same set of parameters E . Then, $\tau \sim^s \tilde{I}$.*

Proof. Let (F, E) be a soft subset of \tilde{X} . Assume that, for each $x_e \in \tilde{E}(F, E)$, there exists a semi open soft set O_{x_e} such that $O_{x_e} \tilde{\cap} (F, E) \in \tilde{I}$. Then, $(F, E) \tilde{\subseteq} \tilde{\cup} \{O_{x_e} : x_e \in \tilde{E}(F, E) \text{ and } O_{x_e} \in SOS(X)\}$. It follows, the family $\tilde{\cup} \{O_{x_e} : x_e \in \tilde{E}(F, E)\}$ is a semi open soft cover of (F, E) . By assumption, (F, E) is a semi closed soft set. By Theorem 9, (F, E) is a soft semi compact. Hence, there exist a finite number of soft points, say, $x_{1e_1}, x_{2e_2}, \dots, x_{ne_n}$ in (F, E) such that $(F, E) \tilde{\subseteq} \tilde{\cup}_{i=1}^n O_{x_{ie_i}}$ and hence $(F, E) = (F, E) \tilde{\cap} [\tilde{\cup}_{i=1}^n O_{x_{ie_i}}] = \tilde{\cup}_{i=1}^n [(F, E) \tilde{\cap} O_{x_{ie_i}}]$. Since $O_{x_{ie_i}} \tilde{\cap} (F, E) \in \tilde{I}$ for each i , $(F, E) \in \tilde{I}$. Therefore, $\tau \sim^s \tilde{I}$.

5. Conclusion

In this paper, we will introduce and study two different notions via soft ideals namely, soft semi local function and soft semi compatibility of τ with \tilde{I} and investigate their relationships with other types of similar operators. Some properties and characterizations of soft semi local function are explored. In future, we will introduce the notions of soft θ -open and soft θ -closed sets and soft θ -closure. Also, we introduce the notion of soft local closure functions as a generalization of the soft θ -closure and the soft (semi) local function in a soft ideal topological space and the future research will be undertaken in this direction.

Conflict of Interest

We declare that, there is no conflict of interest regarding the publication of this manuscript.

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