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Neutrosophic Set Theory Applied to UP-Algebras†

Metawee Songsaeng[1], Aiyared Iampan[1,*]

[1] *Department of Mathematics, School of Science, University of Phayao, Phayao 56000, Thailand*

Abstract. The notions of neutrosophic UP-subalgebras, neutrosophic near UP-filters, neutrosophic UP-filters, neutrosophic UP-ideals, and neutrosophic strongly UP-ideals of UP-algebras are introduced, and several properties are investigated. Conditions for neutrosophic sets to be neutrosophic UP-subalgebras, neutrosophic near UP-filters, neutrosophic UP-filters, neutrosophic UP-ideals, and neutrosophic strongly UP-ideals of UP-algebras are provided. Relations between neutrosophic UP-subalgebras (resp., neutrosophic near UP-filters, neutrosophic UP-filters, neutrosophic UP-ideals, neutrosophic strongly UP-ideals) and their level subsets are considered.

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Key Words and Phrases: UP-algebra, neutrosophic UP-subalgebra, neutrosophic near UP-filter, neutrosophic UP-filter, neutrosophic UP-ideal, neutrosophic strongly UP-ideal

1. Introduction

Among many algebraic structures, algebras of logic form important class of algebras. Examples of these are BCK-algebras [7], BCI-algebras [8], BCH-algebras [4], KU-algebras [18], SU-algebras [13] UP-algebras [5] and so on. They are strongly connected with logic. For example, BCI-algebras were introduced by Iséki [8] in 1966 have connections with BCI-logic being the BCI-system in combinatory logic which has application in the language of functional programming. BCK and BCI-algebras are two classes of logical algebras. They were introduced by Imai and Iséki [7, 8] in 1966 and have been extensively investigated by many researchers. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. The above-mentioned section has been derived from [12].

The branch of the logical algebra, UP-algebras were introduced by Iampan [5]. Later Somjanta et al. [23] studied fuzzy UP-subalgebras, fuzzy UP-ideals and fuzzy UP-filters of UP-algebras. Guntasow et al. [3] introduced and studied fuzzy translations of a fuzzy set in UP-algebras. Kesorn et al. [14] studied intuitionistic fuzzy sets in UP-algebras. Kaijae et al. [11] introduced and investigated anti-fuzzy UP-ideals and anti-fuzzy UP-subalgebras.

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*Corresponding author.
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Email addresses: metawee.faith@gmail.com (M. Songsaeng), aiyared.ia@up.ac.th (A. Iampan)

Tanamoon et al. [26] introduced and studied Q-fuzzy sets in UP-algebras. Sripaeng et al. [25] studied anti Q-fuzzy UP-ideals and anti Q-fuzzy UP-subalgebras of UP-algebras. Dokkhamdang et al. [2] studied Generalized fuzzy sets in UP-algebras. Songsaeng and Iampan [24] studied $\mathcal{N}$-fuzzy UP-algebras and their level subsets.

The notion of neutrosophic sets was introduced by Smarandache [22] in 1999. Wang et al. [28] introduced the notion of interval neutrosophic sets in 2005. The notion of neutrosophic $\mathcal{N}$-structures and their applications in semigroups was introduced by Khan et al. [15] in 2017. Jun et al. [9] applied the notion of neutrosophic $\mathcal{N}$-structures to BCK/BCI-algebras in 2017. Khan et al. [15] discussed neutrosophic $\mathcal{N}$-structures and their applications in semigroups in 2017. Jun et al. [10] studied neutrosophic positive implicative $\mathcal{N}$-ideals in BCK-algebras in 2018. Kim et al. [16] studied generalizations of neutrosophic subalgebras in BCK/BCI-algebras based on neutrosophic points in 2018. Rangsuk et al. [19] introduced the notions of (special) neutrosophic $\mathcal{N}$-UP-subalgebras, (special) neutrosophic $\mathcal{N}$-near UP-filters, (special) neutrosophic $\mathcal{N}$-UP-filters, (special) neutrosophic $\mathcal{N}$-UP-ideals, and (special) neutrosophic $\mathcal{N}$-strongly UP-ideals of UP-algebras in 2019.

In this paper, the notions of neutrosophic UP-subalgebras, neutrosophic near UP-filters, neutrosophic UP-filters, neutrosophic UP-ideals, and neutrosophic strongly UP-ideals of UP-algebras are introduced, and several properties are investigated. Conditions for neutrosophic sets to be neutrosophic UP-subalgebras, neutrosophic near UP-filters, neutrosophic UP-filters, neutrosophic UP-ideals, and neutrosophic strongly UP-ideals of UP-algebras are provided. Relations between neutrosophic UP-subalgebras (resp., neutrosophic near UP-filters, neutrosophic UP-filters, neutrosophic UP-ideals, neutrosophic strongly UP-ideals) and their level subsets are considered.

2. Basic results on UP-algebras

Before we begin our study, we will give the definition and useful properties of UP-algebras.

Definition 1. *[5] An algebra $X = (X, \cdot, 0)$ of type $(2, 0)$ is called a UP-algebra where X is a nonempty set, $\cdot$ is a binary operation on X, and 0 is a fixed element of X (i.e., a nullary operation) if it satisfies the following axioms:*

(UP-1) $(\forall x, y, z \in X)((y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0)$,

(UP-2) $(\forall x \in X)(0 \cdot x = x)$,

(UP-3) $(\forall x \in X)(x \cdot 0 = 0)$, *and*

(UP-4) $(\forall x, y \in X)(x \cdot y = 0, y \cdot x = 0 \Rightarrow x = y)$.

From [5], we know that the notion of UP-algebras is a generalization of KU-algebras (see [18]).

Example 1. *[21] Let X be a universal set and let $\Omega \in \mathcal{P}(X)$ where $\mathcal{P}(X)$ means the power set of X. Let $\mathcal{P}_\Omega(X) = \{A \in \mathcal{P}(X) \mid \Omega \subseteq A\}$. Define a binary operation $\cdot$ on $\mathcal{P}_\Omega(X)$ by* *putting* $A \cdot B = B \cap (A^C \cup \Omega)$ *for all* $A, B \in \mathcal{P}_\Omega(X)$ *where* A^C *means the complement of a subset* A*. Then* $(\mathcal{P}_\Omega(X), \cdot, \Omega)$ *is a UP-algebra and we shall call it the generalized power UP-algebra of type 1 with respect to* Ω*. Let* $\mathcal{P}^\Omega(X) = \{A \in \mathcal{P}(X) \mid A \subseteq \Omega\}$*. Define a binary operation* $*$ *on* $\mathcal{P}^\Omega(X)$ *by putting* $A * B = B \cup (A^C \cap \Omega)$ *for all* $A, B \in \mathcal{P}^\Omega(X)$*. Then* $(\mathcal{P}^\Omega(X), *, \Omega)$ *is a UP-algebra and we shall call it the generalized power UP-algebra of type 2 with respect to* Ω*. In particular,* $(\mathcal{P}(X), \cdot, \emptyset)$ *is a UP-algebra and we shall call it the power UP-algebra of type 1, and* $(\mathcal{P}(X), *, X)$ *is a UP-algebra and we shall call it the power UP-algebra of type 2.*

Example 2. *[2] Let* $\mathbb{N}$ *be the set of all natural numbers with two binary operations* $\circ$ *and* $\bullet$ *defined by*

$$(\forall x, y \in \mathbb{N}) \left(x \circ y = \begin{cases} y & \text{if } x < y, \\ 0 & \text{otherwise} \end{cases} \right)$$

and

$$(\forall x, y \in \mathbb{N}) \left(x \bullet y = \begin{cases} y & \text{if } x > y \text{ or } x = 0, \\ 0 & \text{otherwise} \end{cases} \right).$$

Then $(\mathbb{N}, \circ, 0)$ *and* $(\mathbb{N}, \bullet, 0)$ *are UP-algebras.*

Example 3. *[17] Let* $X = \{0, 1, 2, 3, 4, 5\}$ *be a set with a binary operation* $\cdot$ *defined by the following Cayley table:*

$\cdot$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	0	0	2	3	2	5
2	0	1	0	3	1	5
3	0	1	2	0	4	5
4	0	0	0	3	0	5
5	0	0	2	0	2	0

Then $(X, \cdot, 0)$ *is a UP-algebra.*

For more examples of UP-algebras, see [1, 6, 20, 21].

In a UP-algebra $X = (X, \cdot, 0)$, the following assertions are valid (see [5, 6]).

$$(\forall x \in X)(x \cdot x = 0), \tag{2.1}$$

$$(\forall x, y, z \in X)(x \cdot y = 0, y \cdot z = 0 \Rightarrow x \cdot z = 0), \tag{2.2}$$

$$(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow (z \cdot x) \cdot (z \cdot y) = 0), \tag{2.3}$$

$$(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow (y \cdot z) \cdot (x \cdot z) = 0), \tag{2.4}$$

$$(\forall x, y \in X)(x \cdot (y \cdot x) = 0), \tag{2.5}$$

$$(\forall x, y \in X)((y \cdot x) \cdot x = 0 \Leftrightarrow x = y \cdot x), \tag{2.6}$$

$$(\forall x, y \in X)(x \cdot (y \cdot y) = 0), \tag{2.7}$$

$$(\forall a, x, y, z \in X)((x \cdot (y \cdot z)) \cdot (x \cdot ((a \cdot y) \cdot (a \cdot z))) = 0), \tag{2.8}$$

$$(\forall a, x, y, z \in X)((((a \cdot x) \cdot (a \cdot y)) \cdot z) \cdot ((x \cdot y) \cdot z) = 0), \tag{2.9}$$

$$(\forall x, y, z \in X)(((x \cdot y) \cdot z) \cdot (y \cdot z) = 0), \tag{2.10}$$

$$(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow x \cdot (z \cdot y) = 0), \tag{2.11}$$

$$(\forall x, y, z \in X)(((x \cdot y) \cdot z) \cdot (x \cdot (y \cdot z)) = 0), \text{ and} \tag{2.12}$$

$$(\forall a, x, y, z \in X)(((x \cdot y) \cdot z) \cdot (y \cdot (a \cdot z)) = 0). \tag{2.13}$$

On a UP-algebra $X = (X, \cdot, 0)$, we define a binary relation $\leq$ on X [5] as follows:

$$(\forall x, y \in X)(x \leq y \Leftrightarrow x \cdot y = 0).$$

Definition 2. *[3, 5, 23] A nonempty subset S of a UP-algebra $(X, \cdot, 0)$ is called*

(1) a UP-subalgebra of X if $(\forall x, y \in S)(x \cdot y \in S)$.

(2) a near UP-filter of X if

(i) the constant 0 of X is in S, and

(ii) $(\forall x, y \in X)(y \in S \Rightarrow x \cdot y \in S)$.

(3) a UP-filter of X if

(i) the constant 0 of X is in S, and

(ii) $(\forall x, y \in X)(x \cdot y \in S, x \in S \Rightarrow y \in S)$.

(4) a UP-ideal of X if

(i) the constant 0 of X is in S, and

(ii) $(\forall x, y, z \in X)(x \cdot (y \cdot z) \in S, y \in S \Rightarrow x \cdot z \in S)$.

(5) a strongly UP-ideal of X if

(i) the constant 0 of X is in S, and

(ii) $(\forall x, y, z \in X)((z \cdot y) \cdot (z \cdot x) \in S, y \in S \Rightarrow x \in S)$.

Guntasow et al. [3] proved that the notion of UP-subalgebras is a generalization of near UP-filters, the notion of near UP-filters is a generalization of UP-filters, the notion of UP-filters is a generalization of UP-ideals, and the notion of UP-ideals is a generalization of strongly UP-ideals. Moreover, they also proved that a UP-algebra X is the only one strongly UP-ideal of itself.

3. NSs in UP-algebras

In 1965, Zadeh [29] introduced the notion of fuzzy sets as the following definition.

A *fuzzy set* (briefly, FS) in a nonempty set X (or a fuzzy subset of X) is an arbitrary function $f : X \to [0, 1]$ where $[0, 1]$ is the unit segment of the real line, and the fuzzy set $\overline{f}$ defined by $\overline{f}(x) = 1 - f(x)$ for all $x \in X$ is said to be the *complement* of f in X.

In 1999, Smarandache [22] introduced the notion of neutrosophic sets as the following definition.

A *neutrosophic set* (briefly, NS) in a nonempty set X is a structure of the form:

$$\Lambda = \{(x, \lambda_T(x), \lambda_I(x), \lambda_F(x)) \mid x \in X\} \tag{3.1}$$

where $\lambda_T : X \to [0, 1]$ is a *truth membership function*, $\lambda_I : X \to [0, 1]$ is an *indeterminate membership function*, and $\lambda_F : X \to [0, 1]$ is a *false membership function*.

For our convenience, we will denote a NS as $\Lambda = (X, \lambda_T, \lambda_I, \lambda_F) = (X, \lambda_{T,I,F}) = \{(x, \lambda_T(x), \lambda_I(x), \lambda_F(x)) \mid x \in X\}$.

Definition 3. *[22] Let Λ be a NS in a nonempty set X. The NS $\overline{\Lambda} = (X, \overline{\lambda}_{T,I,F})$ in X defined by*

$$(\forall x \in X)\begin{pmatrix}\overline{\lambda}_T(x) = 1 - \lambda_T(x)\\ \overline{\lambda}_I(x) = 1 - \lambda_I(x)\\ \overline{\lambda}_F(x) = 1 - \lambda_F(x)\end{pmatrix} \tag{3.2}$$

is called the complement of Λ in X.

Remark 1. *For all NS Λ in a nonempty set X, we have $\Lambda = \overline{\overline{\Lambda}}$.*

Lemma 1. *[27] Let $a, b, c \in \mathbb{R}$. Then the following statements hold:*

(1) $a - \min\{b, c\} = \max\{a - b, a - c\}$, *and*

(2) $a - \max\{b, c\} = \min\{a - b, a - c\}$.

The following lemma is easily proved.

Lemma 2. *Let f be a fuzzy set in a nonempty set X. Then the following statements hold:*

(1) $(\forall x, y, z \in X)(\overline{f}(x) \geq \min\{\overline{f}(y), \overline{f}(z)\} \Leftrightarrow f(x) \leq \max\{f(y), f(z)\})$,

(2) $(\forall x, y, z \in X)(\overline{f}(x) \leq \min\{\overline{f}(y), \overline{f}(z)\} \Leftrightarrow f(x) \geq \max\{f(y), f(z)\})$,

(3) $(\forall x, y, z \in X)(\overline{f}(x) \geq \max\{\overline{f}(y), \overline{f}(z)\} \Leftrightarrow f(x) \leq \min\{f(y), f(z)\})$, *and*

(4) $(\forall x, y, z \in X)(\overline{f}(x) \leq \max\{\overline{f}(y), \overline{f}(z)\} \Leftrightarrow f(x) \geq \min\{f(y), f(z)\})$.

In what follows, let X denote a UP-algebra $(X, \cdot, 0)$ unless otherwise specified.

Now, we introduce the notions of neutrosophic UP-subalgebras, neutrosophic near UP-filters, neutrosophic UP-filters, neutrosophic UP-ideals, and neutrosophic strongly UP-ideals of UP-algebras, provide the necessary examples, investigate their properties, and prove their generalizations.

Definition 4. *A NS Λ in X is called a neutrosophic UP-subalgebra of X if it satisfies the following conditions:*

$$(\forall x, y \in X)(\lambda_T(x \cdot y) \geq \min\{\lambda_T(x), \lambda_T(y)\}), \tag{3.3}$$

$$(\forall x, y \in X)(\lambda_I(x \cdot y) \leq \max\{\lambda_I(x), \lambda_I(y)\}), \tag{3.4}$$

$$(\forall x, y \in X)(\lambda_F(x \cdot y) \geq \min\{\lambda_F(x), \lambda_F(y)\}). \tag{3.5}$$

Example 4. *Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:*

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	2	4
2	0	0	0	2	4
3	0	0	0	0	4
4	0	1	2	3	0

We define a NS Λ in X as follows:

$$\lambda_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.9 & 0.7 & 0.5 & 0.3 & 0.3 \end{pmatrix}, \lambda_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 0.8 & 0.4 & 0.2 & 0.4 \end{pmatrix}, \lambda_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 0.6 & 0.8 & 0.3 & 0.2 \end{pmatrix}.$$

Hence, Λ is a neutrosophic UP-subalgebra of X.

Definition 5. *A NS Λ in X is called a neutrosophic near UP-filter of X if it satisfies the following conditions:*

$$(\forall x \in X)(\lambda_T(0) \geq \lambda_T(x)), \tag{3.6}$$

$$(\forall x \in X)(\lambda_I(0) \leq \lambda_I(x)), \tag{3.7}$$

$$(\forall x \in X)(\lambda_F(0) \geq \lambda_F(x)), \tag{3.8}$$

$$(\forall x, y \in X)(\lambda_T(x \cdot y) \geq \lambda_T(y)), \tag{3.9}$$

$$(\forall x, y \in X)(\lambda_I(x \cdot y) \leq \lambda_I(y)), \tag{3.10}$$

$$(\forall x, y \in X)(\lambda_F(x \cdot y) \geq \lambda_F(y)). \tag{3.11}$$

Example 5. *Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:*

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	1	2	4
2	0	0	0	1	4
3	0	0	0	0	4
4	0	1	2	3	0

We define a NS Λ in X as follows:

$$\lambda_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 0.7 & 0.5 & 0.4 & 0.8 \end{pmatrix}, \lambda_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.1 & 0.2 & 0.3 & 0.7 & 0.6 \end{pmatrix}, \lambda_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.9 & 0.8 & 0.4 & 0.3 & 0.5 \end{pmatrix}.$$

Hence, Λ is a neutrosophic near UP-filter of X.

Definition 6. *A NS Λ in X is called a neutrosophic UP-filter of X if it satisfies the following conditions:* (3.6), (3.7), (3.8), *and*

$$(\forall x, y \in X)(\lambda_T(y) \geq \min\{\lambda_T(x \cdot y), \lambda_T(x)\}), \tag{3.12}$$

$$(\forall x, y \in X)(\lambda_I(y) \leq \max\{\lambda_I(x \cdot y), \lambda_I(x)\}), \tag{3.13}$$

$$(\forall x, y \in X)(\lambda_F(y) \geq \min\{\lambda_F(x \cdot y), \lambda_F(x)\}). \tag{3.14}$$

Example 6. *Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:*

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	0	0	3	3
3	0	1	2	0	3
4	0	1	2	0	0

We define a NS Λ in X as follows:

$$\lambda_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.9 & 0.4 & 0.3 & 0.1 & 0.1 \end{pmatrix}, \lambda_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.2 & 0.3 & 0.7 & 0.8 & 0.8 \end{pmatrix}, \lambda_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0.8 & 0.7 & 0.4 & 0.3 & 0.3 \end{pmatrix}.$$

Hence, Λ is a neutrosophic UP-filter of X.

Definition 7. *A NS Λ in X is called a neutrosophic UP-ideal of X if it satisfies the following conditions:* (3.6), (3.7), (3.8), *and*

$$(\forall x, y, z \in X)(\lambda_T(x \cdot z) \geq \min\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\}), \tag{3.15}$$

$$(\forall x, y, z \in X)(\lambda_I(x \cdot z) \leq \max\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\}), \tag{3.16}$$

$$(\forall x, y, z \in X)(\lambda_F(x \cdot z) \geq \min\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}). \tag{3.17}$$

Example 7. *Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:*

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	0	0	2	4
3	0	0	0	0	4
4	0	1	2	3	0

We define a NS Λ in X as follows:

$$\lambda_T = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 0.7 & 0.6 & 0.6 & 0.4 \end{pmatrix}, \lambda_I = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 0.3 & 0.5 & 0.5 & 0.7 \end{pmatrix}, \lambda_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 0.8 & 0.7 & 0.7 & 0.5 \end{pmatrix}.$$

Hence, Λ is a neutrosophic UP-ideal of X.

Definition 8. *A NS Λ in X is called a neutrosophic strongly UP-ideal of X if it satisfies the following conditions:* (3.6), (3.7), (3.8), *and*

$$(\forall x, y, z \in X)(\lambda_T(x) \geq \min\{\lambda_T((z \cdot y) \cdot (z \cdot x)), \lambda_T(y)\}), \tag{3.18}$$

$$(\forall x, y, z \in X)(\lambda_I(x) \leq \max\{\lambda_I((z \cdot y) \cdot (z \cdot x)), \lambda_I(y)\}), \tag{3.19}$$

$$(\forall x, y, z \in X)(\lambda_F(x) \geq \min\{\lambda_F((z \cdot y) \cdot (z \cdot x)), \lambda_F(y)\}). \tag{3.20}$$

Example 8. *Let $X = \{0, 1, 2, 3, 4\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:*

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	1	0	2	4
3	0	1	0	0	4
4	0	1	0	3	0

We define a NS Λ in X as follows:

$$(\forall x \in X) \begin{pmatrix} \lambda_T(x) = 1 \\ \lambda_I(x) = 0.2 \\ \lambda_F(x) = 0.8 \end{pmatrix}.$$

Hence, Λ is a neutrosophic strongly UP-ideal of X.

Definition 9. *A NS Λ in X is said to be constant if Λ is a constant function from X to $[0, 1]^3$. That is, λ_T, λ_I, and λ_F are constant functions from X to $[0, 1]$.*

Theorem 1. *Every neutrosophic UP-subalgebra of X satisfies the conditions* (3.6), (3.7), *and* (3.8).

Proof. Assume that Λ is a neutrosophic UP-subalgebra of X. Then for all $x \in X$,

$$\lambda_T(0) = \lambda_T(x \cdot x) \geq \min\{\lambda_T(x), \lambda_T(x)\} = \lambda_T(x), \quad \text{(2.1) and (3.3)}$$

$$\lambda_I(0) = \lambda_I(x \cdot x) \leq \max\{\lambda_I(x), \lambda_I(x)\} = \lambda_I(x), \quad \text{(2.1) and (3.4)}$$

$$\lambda_F(0) = \lambda_F(x \cdot x) \geq \min\{\lambda_F(x), \lambda_F(x)\} = \lambda_F(x). \quad \text{(2.1) and (3.5)}$$

Hence, Λ satisfies the conditions (3.6), (3.7), and (3.8).

Theorem 2. *A NS Λ in X is constant if and only if it is a neutrosophic strongly UP-ideal of X.*

Proof. Assume that Λ is constant. Then for all $x \in X$, $\lambda_T(x) = \lambda_T(0), \lambda_I(x) = \lambda_I(0)$, and $\lambda_F(x) = \lambda_F(0)$ and so $\lambda_T(0) \geq \lambda_T(x), \lambda_I(0) \leq \lambda_I(x)$, and $\lambda_F(0) \geq \lambda_F(x)$. Next, for all $x, y, z \in X$,

$$\begin{aligned}
\lambda_T(x) = \lambda_T(0) = \min\{\lambda_T(0), \lambda_T(0)\} = \min\{\lambda_T((z \cdot y) \cdot (z \cdot x)), \lambda_T(y)\},\\
\lambda_I(x) = \lambda_I(0) = \max\{\lambda_I(0), \lambda_I(0)\} = \max\{\lambda_I((z \cdot y) \cdot (z \cdot x)), \lambda_I(y)\},\\
\lambda_F(x) = \lambda_F(0) = \min\{\lambda_F(0), \lambda_F(0)\} = \min\{\lambda_F((z \cdot y) \cdot (z \cdot x)), \lambda_F(y)\}.
\end{aligned}$$

Hence, Λ is a neutrosophic strongly UP-ideal of X.

Conversely, assume that Λ is a neutrosophic strongly UP-ideal of X. For any $x \in X$, we have

$$\begin{aligned}
\lambda_T(x) &\geq \min\{\lambda_T((x \cdot 0) \cdot (x \cdot x)), \lambda_T(0)\} && (3.18)\\
&= \min\{\lambda_T(0 \cdot (x \cdot x)), \lambda_T(0)\} && \text{(UP-3)}\\
&= \min\{\lambda_T(x \cdot x), \lambda_T(0)\} && \text{(UP-2)}\\
&= \min\{\lambda_T(0), \lambda_T(0)\} && (2.1)\\
&= \lambda_T(0),\\
\lambda_I(x) &\leq \max\{\lambda_I((x \cdot 0) \cdot (x \cdot x)), \lambda_I(0)\} && (3.19)\\
&= \max\{\lambda_I(0 \cdot (x \cdot x)), \lambda_I(0)\} && \text{(UP-3)}\\
&= \max\{\lambda_I(x \cdot x), \lambda_I(0)\} && \text{(UP-2)}\\
&= \max\{\lambda_I(0), \lambda_I(0)\} && (2.1)\\
&= \lambda_I(0),\\
\lambda_F(x) &\geq \min\{\lambda_F((x \cdot 0) \cdot (x \cdot x)), \lambda_F(0)\} && (3.20)\\
&= \min\{\lambda_F(0 \cdot (x \cdot x)), \lambda_F(0)\} && \text{(UP-3)}\\
&= \min\{\lambda_F(x \cdot x), \lambda_F(0)\} && \text{(UP-2)}\\
&= \min\{\lambda_F(0), \lambda_F(0)\} && (2.1)\\
&= \lambda_F(0).
\end{aligned}$$

Thus $\lambda_T(x) = \lambda_T(0), \lambda_I(x) = \lambda_I(0)$, and $\lambda_F(x) = \lambda_F(0)$ for all $x \in X$. Hence, Λ is constant.

Theorem 3. *Every neutrosophic strongly UP-ideal of X is a neutrosophic UP-ideal.*

Proof. Assume that Λ is a neutrosophic strong UP-ideal of X. Then Λ satisfies the conditions (3.6), (3.7), and (3.8). By Theorem 2, we have Λ is constant. Then for all $x \in X$, $\lambda_T(x) = \lambda_T(0), \lambda_I(x) = \lambda_I(0)$, and $\lambda_F(x) = \lambda_F(0)$. Thus

$$\lambda_T(x \cdot z) = \min\{\lambda_T((z \cdot y) \cdot (z \cdot (x \cdot z))), \lambda_T(y)\} \tag{3.18}$$

$$
\begin{aligned}
&= \min\{\lambda_T((z\cdot y)\cdot 0), \lambda_T(y)\} && (2.5)\\
&= \min\{\lambda_T(0), \lambda_T(y)\} && \text{(UP-3)}\\
&= \lambda_T(y) && (3.6)\\
&\geq \min\{\lambda_T(x\cdot(y\cdot z)), \lambda_T(y)\},\\
\lambda_I(x\cdot z) &= \max\{\lambda_I((z\cdot y)\cdot(z\cdot(x\cdot z))), \lambda_I(y)\} && (3.19)\\
&= \max\{\lambda_I((z\cdot y)\cdot 0), \lambda_I(y)\} && (2.5)\\
&= \max\{\lambda_I(0), \lambda_I(y)\} && \text{(UP-3)}\\
&= \lambda_I(y) && (3.7)\\
&\leq \max\{\lambda_I(x\cdot(y\cdot z)), \lambda_I(y)\},\\
\lambda_F(x\cdot z) &= \min\{\lambda_F((z\cdot y)\cdot(z\cdot(x\cdot z))), \lambda_F(y)\} && (3.20)\\
&= \min\{\lambda_F((z\cdot y)\cdot 0), \lambda_F(y)\} && (2.5)\\
&= \min\{\lambda_F(0), \lambda_F(y)\} && \text{(UP-3)}\\
&= \lambda_F(y) && (3.8)\\
&\geq \min\{\lambda_F(x\cdot(y\cdot z)), \lambda_F(y)\}.
\end{aligned}
$$

Hence, Λ is a neutrosophic UP-ideal of X.

The following example show that the converse of Theorem 3 is not true.

Example 9. *From Example 7, we have Λ is a neutrosophic UP-ideal of X. Since Λ is not constant, it follows from Theorem 2 that it is not a neutrosophic strongly UP-ideal of X.*

Theorem 4. *Every neutrosophic UP-ideal of X is a neutrosophic UP-filter.*

Proof. Assume that Λ is a neutrosophic UP-ideal of X. Then Λ satisfies the conditions (3.6), (3.7), and (3.8). Next, let $x, y \in X$. Then

$$
\begin{aligned}
\lambda_T(y) &= \lambda_T(0\cdot y) && \text{(UP-2)}\\
&\geq \min\{\lambda_T(0\cdot(x\cdot y)), \lambda_T(x)\} && (3.15)\\
&= \min\{\lambda_T(x\cdot y), \lambda_T(x)\}, && \text{(UP-2)}\\
\lambda_I(y) &= \lambda_I(0\cdot y) && \text{(UP-2)}\\
&\leq \max\{\lambda_I(0\cdot(x\cdot y)), \lambda_I(x)\} && (3.16)\\
&= \max\{\lambda_I(x\cdot y), \lambda_I(x)\}, && \text{(UP-2)}\\
\lambda_F(y) &= \lambda_F(0\cdot y) && \text{(UP-2)}\\
&\geq \min\{\lambda_F(0\cdot(x\cdot y)), \lambda_F(x)\} && (3.17)\\
&= \min\{\lambda_F(x\cdot y), \lambda_F(x)\}. && \text{(UP-2)}
\end{aligned}
$$

Hence, Λ is a neutrosophic UP-filter of X.

The following example show that the converse of Theorem 4 is not true.

Example 10. *From Example 6, we have* Λ *is a neutrosophic UP-filter of* X*. Since* $\lambda_F(3 \cdot 4) = 0.3 < 0.4 = \min\{\lambda_F(3 \cdot (2 \cdot 4)), \lambda_F(2)\}$*, we have* Λ *is not a neutrosophic UP-ideal of* X*.*

Theorem 5. *Every neutrosophic UP-filter of* X *is a neutrosophic near UP-filter.*

Proof. Assume that Λ is a neutrosophic UP-filter. Then Λ satisfies the conditions (3.6), (3.7), and (3.8). Next, let $x, y \in X$. Then

$$\begin{aligned}
\lambda_T(x \cdot y) &\geq \min\{\lambda_T(y \cdot (x \cdot y)), \lambda_T(y)\} && (3.12)\\
&= \min\{\lambda_T(0), \lambda_T(y)\} && (2.5)\\
&= \lambda_T(y), && (3.6)\\
\lambda_I(x \cdot y) &\leq \max\{\lambda_I(y \cdot (x \cdot y)), \lambda_I(y)\} && (3.13)\\
&= \max\{\lambda_I(0), \lambda_I(y)\} && (2.5)\\
&= \lambda_I(y), && (3.7)\\
\lambda_F(x \cdot y) &\geq \min\{\lambda_F(y \cdot (x \cdot y)), \lambda_F(y)\} && (3.14)\\
&= \min\{\lambda_F(0), \lambda_F(y)\} && (2.5)\\
&= \lambda_F(y). && (3.8)
\end{aligned}$$

Hence, Λ is a neutrosophic near UP-filter of X.

The following example show that the converse of Theorem 5 is not true.

Example 11. *From Example 5, we have* Λ *is a neutrosophic near UP-filter of* X*. Since* $\lambda_I(3) = 0.7 > 0.3 = \max\{\lambda_I(2 \cdot 3), \lambda_I(2)\}$*, we have* Λ *is not a neutrosophic UP-filter of* X*.*

Theorem 6. *Every neutrosophic near UP-filter of* X *is a neutrosophic UP-subalgebra.*

Proof. Assume that Λ is a neutrosophic near UP-filter of X. Then for all $x, y \in X$

$$\begin{aligned}
\lambda_T(x \cdot y) \geq \lambda_T(y) &\geq \min\{\lambda_T(x), \lambda_T(y)\}, && (3.9)\\
\lambda_I(x \cdot y) \leq \lambda_I(y) &\leq \max\{\lambda_I(x), \lambda_I(y)\}, && (3.10)\\
\lambda_F(x \cdot y) \geq \lambda_F(y) &\geq \min\{\lambda_F(x), \lambda_F(y)\}. && (3.11)
\end{aligned}$$

Hence, Λ is a neutrosophic UP-subalgebra of X.

The following example show that the converse of Theorem 6 is not true.

Example 12. *From Example 4, we have* Λ *is a neutrosophic UP-subalgebra of* X*. Since* $\lambda_I(2 \cdot 3) = 0.4 > 0.2 = \lambda_I(3)$*, we have* Λ *is not a neutrosophic near UP-filter of* X*.*

By Theorems 3, 4, 5, and 6 and Examples 9, 10, 11, and 12, we have that the notion of neutrosophic UP-subalgebras is a generalization of neutrosophic near UP-filters, the notion of neutrosophic near UP-filters is a generalization of neutrosophic UP-filters, the notion of neutrosophic UP-filters is a generalization of neutrosophic UP-ideals, and the notion of neutrosophic UP-ideals is a generalization of neutrosophic strongly UP-ideals. Moreover, by Theorem 2, we obtain that neutrosophic strongly UP-ideals and constant neutrosophic set coincide.

Theorem 7. *If* Λ *is a neutrosophic UP-subalgebra of* X *satisfying the following condition:*

$$(\forall x, y \in X)\left(x \cdot y \neq 0 \Rightarrow \begin{cases} \lambda_T(x) \geq \lambda_T(y) \\ \lambda_I(x) \leq \lambda_I(y) \\ \lambda_F(x) \geq \lambda_F(y) \end{cases}\right), \tag{3.21}$$

then Λ *is a neutrosophic near UP-filter of* X*.*

Proof. Assume that Λ is a neutrosophic UP-subalgebra of X satisfying the condition (3.21). By Theorem 1, we have Λ satisfies the conditions (3.6), (3.7), and (3.8). Next, let $x, y \in X$.

Case 1: $x \cdot y = 0$. Then

$$\lambda_T(x \cdot y) = \lambda_T(0) \geq \lambda_T(y), \tag{3.6}$$
$$\lambda_I(x \cdot y) = \lambda_I(0) \leq \lambda_I(y), \tag{3.7}$$
$$\lambda_F(x \cdot y) = \lambda_F(0) \geq \lambda_F(y). \tag{3.8}$$

Case 2: $x \cdot y \neq 0$. Then

$$\lambda_T(x \cdot y) \geq \min\{\lambda_T(x), \lambda_T(y)\} = \lambda_T(y), \quad \text{(3.3) and (3.21) for } \lambda_T$$
$$\lambda_I(x \cdot y) \leq \max\{\lambda_I(x), \lambda_I(y)\} = \lambda_I(y), \quad \text{(3.4) and (3.21) for } \lambda_I$$
$$\lambda_F(x \cdot y) \geq \min\{\lambda_F(x), \lambda_F(y)\} = \lambda_F(y). \quad \text{(3.5) and (3.21) for } \lambda_F$$

Hence, Λ is a neutrosophic near UP-filter of X.

Theorem 8. *If* Λ *is a neutrosophic near UP-filter of* X *satisfying the following condition:*

$$\lambda_T = \lambda_I = \lambda_F, \tag{3.22}$$

then Λ *is a neutrosophic UP-filter of* X*.*

Proof. Assume that Λ is a neutrosophic near UP-filter of X satisfying the condition (3.22). Then Λ satisfies the conditions (3.6), (3.7), and (3.8). Next, let $x, y \in X$. Then

$$\begin{aligned}
\min\{\lambda_T(x \cdot y), \lambda_T(x)\} &= \min\{\lambda_I(x \cdot y), \lambda_T(x)\} && (3.22)\\
&\leq \min\{\lambda_I(y), \lambda_T(x)\} && (3.10)\\
&= \min\{\lambda_T(y), \lambda_T(x)\} && (3.22)\\
&\leq \lambda_T(y),\\
\max\{\lambda_I(x \cdot y), \lambda_I(x)\} &= \max\{\lambda_T(x \cdot y), \lambda_I(x)\} && (3.22)\\
&\geq \max\{\lambda_T(y), \lambda_I(x)\} && (3.9)\\
&= \max\{\lambda_I(y), \lambda_I(x)\} && (3.22)\\
&\geq \lambda_I(y),\\
\min\{\lambda_F(x \cdot y), \lambda_F(x)\} &= \min\{\lambda_I(x \cdot y), \lambda_F(x)\} && (3.22)
\end{aligned}$$

$$
\begin{aligned}
&\leq \min\{\lambda_I(y), \lambda_F(x)\} && (3.10)\\
&= \min\{\lambda_F(y), \lambda_F(x)\} && (3.22)\\
&\leq \lambda_F(y).
\end{aligned}
$$

Hence, Λ is a neutrosophic UP-filter of X.

Theorem 9. *If Λ is a neutrosophic UP-filter of X satisfying the following condition:*

$$(\forall x, y, z \in X)\begin{pmatrix}\lambda_T(y \cdot (x \cdot z)) = \lambda_T(x \cdot (y \cdot z))\\ \lambda_I(y \cdot (x \cdot z)) = \lambda_I(x \cdot (y \cdot z))\\ \lambda_F(y \cdot (x \cdot z)) = \lambda_F(x \cdot (y \cdot z))\end{pmatrix}, \tag{3.23}$$

then Λ is a neutrosophic UP-ideal of X.

Proof. Assume that Λ is a neutrosophic UP-filter of X satisfying the condition (3.23). Then Λ satisfies the conditions (3.6), (3.7), and (3.8). Next, let $x, y, z \in X$. Then

$$
\begin{aligned}
\lambda_T(x \cdot z) &\geq \min\{\lambda_T(y \cdot (x \cdot z)), \lambda_T(y)\} && (3.12)\\
&= \min\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\}, && (3.23) \text{ for } \lambda_T\\
\lambda_I(x \cdot z) &\leq \max\{\lambda_I(y \cdot (x \cdot z)), \lambda_I(y)\} && (3.13)\\
&= \max\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\}, && (3.23) \text{ for } \lambda_I\\
\lambda_F(x \cdot z) &\geq \min\{\lambda_F(y \cdot (x \cdot z)), \lambda_F(y)\} && (3.14)\\
&= \min\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}. && (3.23) \text{ for } \lambda_F
\end{aligned}
$$

Hence, Λ is a neutrosophic UP-ideal of X.

Theorem 10. *If Λ is a NS in X satisfying the following condition:*

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases}\lambda_T(z) \geq \min\{\lambda_T(x), \lambda_T(y)\}\\ \lambda_I(z) \leq \max\{\lambda_I(x), \lambda_I(y)\}\\ \lambda_F(z) \geq \min\{\lambda_F(x), \lambda_F(y)\}\end{cases}\right), \tag{3.24}$$

then Λ is a neutrosophic UP-subalgebra of X.

Proof. Assume that Λ is a NS in X satisfying the condition (3.24). Let $x, y \in X$. By (2.1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \leq x \cdot y$. It follows from (3.24) that

$$
\begin{aligned}
\lambda_T(x \cdot y) &\geq \min\{\lambda_T(x), \lambda_T(y)\},\\
\lambda_I(x \cdot y) &\leq \max\{\lambda_I(x), \lambda_I(y)\},\\
\lambda_F(x \cdot y) &\geq \min\{\lambda_F(x), \lambda_F(y)\}.
\end{aligned}
$$

Hence, Λ is a neutrosophic UP-subalgebra of X.

Theorem 11. *If Λ is a NS in X satisfying the following condition:*

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} \lambda_T(z) \geq \lambda_T(y) \\ \lambda_I(z) \leq \lambda_I(y) \\ \lambda_F(z) \geq \lambda_F(y) \end{cases}\right), \tag{3.25}$$

then Λ is a neutrosophic near UP-filter of X.

Proof. Assume that Λ is a NS in X satisfying the condition (3.25). Let $x \in X$. By (UP-2) and (2.1), we have $0 \cdot (x \cdot x) = 0$, that is, $0 \leq x \cdot x$. It follows from (3.25) that $\lambda_T(0) \geq \lambda_T(x), \lambda_I(0) \leq \lambda_I(x)$, and $\lambda_F(0) \geq \lambda_F(x)$. Next, let $x, y \in X$. By (2.1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \leq x \cdot y$. It follows from (3.25) that $\lambda_T(x \cdot y) \geq \lambda_T(y), \lambda_I(x \cdot y) \leq \lambda_I(y)$, and $\lambda_F(x \cdot y) \geq \lambda_F(y)$. Hence, Λ is a neutrosophic near UP-filter of X.

Theorem 12. *If Λ is a NS in X satisfying the following condition:*

$$(\forall x, y, z \in X)\left(z \leq x \cdot y \Rightarrow \begin{cases} \lambda_T(y) \geq \min\{\lambda_T(z), \lambda_T(x)\} \\ \lambda_I(y) \leq \max\{\lambda_I(z), \lambda_I(x)\} \\ \lambda_F(y) \geq \min\{\lambda_F(z), \lambda_F(x)\} \end{cases}\right), \tag{3.26}$$

then Λ is a neutrosophic UP-filter of X.

Proof. Assume that Λ is a NS in X satisfying the condition (3.26). Let $x \in X$. By (UP-3), we have $x \cdot (x \cdot 0) = 0$, that is, $x \leq x \cdot 0$. It follows from (3.26) that

$$\begin{aligned} \lambda_T(0) &\geq \min\{\lambda_T(x), \lambda_T(x)\} = \lambda_T(x), \\ \lambda_I(0) &\leq \max\{\lambda_I(x), \lambda_I(x)\} = \lambda_I(x), \\ \lambda_F(0) &\geq \min\{\lambda_F(x), \lambda_F(x)\} = \lambda_F(x). \end{aligned}$$

Next, let $x, y \in X$. By (2.1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \leq x \cdot y$. It follows from (3.26) that

$$\begin{aligned} \lambda_T(y) &\geq \min\{\lambda_T(x \cdot y), \lambda_T(x)\}, \\ \lambda_I(y) &\leq \max\{\lambda_I(x \cdot y), \lambda_I(x)\}, \\ \lambda_F(y) &\geq \min\{\lambda_F(x \cdot y), \lambda_F(x)\}. \end{aligned}$$

Hence, Λ is a neutrosophic UP-filter of X.

Theorem 13. *If Λ is a NS in X satisfying the following condition:*

$$(\forall a, x, y, z \in X)\left(a \leq x \cdot (y \cdot z) \Rightarrow \begin{cases} \lambda_T(x \cdot z) \geq \min\{\lambda_T(a), \lambda_T(y)\} \\ \lambda_I(x \cdot z) \leq \max\{\lambda_I(a), \lambda_I(y)\} \\ \lambda_F(x \cdot z) \geq \min\{\lambda_F(a), \lambda_F(y)\} \end{cases}\right), \tag{3.27}$$

then Λ is a neutrosophic UP-ideal of X.

Proof. Assume that Λ is a NS in X satisfying the condition (3.27). Let $x \in X$. By (UP-3), we have $x \cdot (0 \cdot (x \cdot 0) = 0$, that is, $x \leq 0 \cdot (x \cdot 0)$. It follows from (3.27) that

$$\lambda_T(0) = \lambda_T(0 \cdot 0) \geq \min\{\lambda_T(x), \lambda_T(x)\} = \lambda_T(x), \qquad \text{(UP-2)}$$
$$\lambda_I(0) = \lambda_I(0 \cdot 0) \leq \max\{\lambda_I(x), \lambda_I(x)\} = \lambda_I(x), \qquad \text{(UP-2)}$$
$$\lambda_F(0) = \lambda_F(0 \cdot 0) \geq \min\{\lambda_F(x), \lambda_F(x)\} = \lambda_F(x). \qquad \text{(UP-2)}$$

Next, let $x, y, z \in X$. By (2.1), we have $(x \cdot (y \cdot z)) \cdot (x \cdot (y \cdot z)) = 0$, that is, $x \cdot (y \cdot z) \leq x \cdot (y \cdot z)$. It follows from (3.27) that

$$\lambda_T(x \cdot z) \geq \min\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\},$$
$$\lambda_I(x \cdot z) \leq \max\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\},$$
$$\lambda_F(x \cdot z) \geq \min\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}.$$

Hence, Λ is a neutrosophic UP-ideal of X.

For any fixed numbers $\alpha^+, \alpha^-, \beta^+, \beta^-, \gamma^+, \gamma^- \in [0,1]$ such that $\alpha^+ > \alpha^-, \beta^+ > \beta^-, \gamma^+ > \gamma^-$ and a nonempty subset G of X, a NS $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}] = (X, \lambda^G_T[^{\alpha^+}_{\alpha^-}], \lambda^G_I[^{\beta^-}_{\beta^+}], \lambda^G_F[^{\gamma^+}_{\gamma^-}])$ in X where $\lambda^G_T[^{\alpha^+}_{\alpha^-}], \lambda^G_I[^{\beta^-}_{\beta^+}]$, and $\lambda^G_F[^{\gamma^+}_{\gamma^-}]$ are functions on X which are given as follows:

$$\lambda^G_T[^{\alpha^+}_{\alpha^-}](x) = \begin{cases} \alpha^+ & \text{if } x \in G, \\ \alpha^- & \text{otherwise,} \end{cases}$$

$$\lambda^G_I[^{\beta^-}_{\beta^+}](x) = \begin{cases} \beta^- & \text{if } x \in G, \\ \beta^+ & \text{otherwise,} \end{cases}$$

$$\lambda^G_F[^{\gamma^+}_{\gamma^-}](x) = \begin{cases} \gamma^+ & \text{if } x \in G, \\ \gamma^- & \text{otherwise.} \end{cases}$$

Lemma 3. *If the constant* 0 *of* X *is in a nonempty subset* G *of* X*, then a NS* $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ *in* X *satisfies the conditions* (3.6), (3.7), *and* (3.8).

Proof. If $0 \in G$, then $\lambda^G_T[^{\alpha^+}_{\alpha^-}](0) = \alpha^+, \lambda^G_I[^{\beta^-}_{\beta^+}](0) = \beta^-, \lambda^G_F[^{\gamma^+}_{\gamma^-}](0) = \gamma^+$. Thus

$$(\forall x \in X) \begin{pmatrix} \lambda^G_T[^{\alpha^+}_{\alpha^-}](0) = \alpha^+ \geq \lambda^G_T[^{\alpha^+}_{\alpha^-}](x) \\ \lambda^G_I[^{\beta^-}_{\beta^+}](0) = \beta^- \leq \lambda^G_I[^{\beta^-}_{\beta^+}](x) \\ \lambda^G_F[^{\gamma^+}_{\gamma^-}](0) = \gamma^+ \geq \lambda^G_F[^{\gamma^+}_{\gamma^-}](x) \end{pmatrix}.$$

Hence, $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ satisfies the conditions (3.6), (3.7), and (3.8).

Lemma 4. *If a NS* $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ *in* X *satisfies the condition* (3.6) *(resp.,* (3.7), (3.8)*), then the constant* 0 *of* X *is in a nonempty subset* G *of* X.

Proof. Assume that the NS $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ in X satisfies the condition (3.6). Then $\lambda^G_T[^{\alpha^+}_{\alpha^-}](0) \geq \lambda^G_T[^{\alpha^+}_{\alpha^-}](x)$ for all $x \in X$. Since G is nonempty, there exists $g \in G$. Thus $\lambda^G_T[^{\alpha^+}_{\alpha^-}](g) = \alpha^+$ and so $\lambda^G_T[^{\alpha^+}_{\alpha^-}](0) \geq \lambda^G_T[^{\alpha^+}_{\alpha^-}](g) = \alpha^+ \geq \lambda^G_T[^{\alpha^+}_{\alpha^-}](0)$, that is, $\lambda^G_T[^{\alpha^+}_{\alpha^-}](0) = \alpha^+$. Hence, $0 \in G$.

Theorem 14. *A NS $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ in X is a neutrosophic UP-subalgebra of X if and only if a nonempty subset G of X is a UP-subalgebra of X.*

Proof. Assume that $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a neutrosophic UP-subalgebra of X. Let $x, y \in G$. Then $\lambda^G_T[^{\alpha^+}_{\alpha^-}](x) = \alpha^+ = \lambda^G_T[^{\alpha^+}_{\alpha^-}](y)$. Thus

$$\lambda^G_T[^{\alpha^+}_{\alpha^-}](x \cdot y) \geq \min\{\lambda^G_T[^{\alpha^+}_{\alpha^-}](x), \lambda^G_T[^{\alpha^+}_{\alpha^-}](y)\} = \alpha^+ \geq \lambda^G_T[^{\alpha^+}_{\alpha^-}](x \cdot y) \tag{3.3}$$

and so $\lambda^G_T[^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^+$. Thus $x \cdot y \in G$. Hence, G is a UP-subalgebra of X.

Conversely, assume that G is a UP-subalgebra of X. Let $x, y \in X$.

Case 1: $x, y \in G$. Then

$$\begin{aligned}
\lambda^G_T[^{\alpha^+}_{\alpha^-}](x) &= \alpha^+ = \lambda^G_T[^{\alpha^+}_{\alpha^-}](y), \\
\lambda^G_I[^{\beta^-}_{\beta^+}](x) &= \beta^- = \lambda^G_I[^{\beta^-}_{\beta^+}](y), \\
\lambda^G_F[^{\gamma^+}_{\gamma^-}](x) &= \gamma^+ = \lambda^G_F[^{\gamma^+}_{\gamma^-}](y).
\end{aligned}$$

Thus

$$\begin{aligned}
\min\{\lambda^G_T[^{\alpha^+}_{\alpha^-}](x), \lambda^G_T[^{\alpha^+}_{\alpha^-}](y)\} &= \alpha^+, \\
\max\{\lambda^G_I[^{\beta^-}_{\beta^+}](x), \lambda^G_I[^{\beta^-}_{\beta^+}](y)\} &= \beta^-, \\
\min\{\lambda^G_F[^{\gamma^+}_{\gamma^-}](x), \lambda^G_F[^{\gamma^+}_{\gamma^-}](y)\} &= \gamma^+.
\end{aligned}$$

Since G is a UP-subalgebra of X, we have $x \cdot y \in G$ and so $\lambda^G_T[^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^+$, $\lambda^G_I[^{\beta^-}_{\beta^+}](x \cdot y) = \beta^-$, and $\lambda^G_F[^{\gamma^+}_{\gamma^-}](x \cdot y) = \gamma^+$. Hence,

$$\begin{aligned}
\lambda^G_T[^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^+ &\geq \alpha^+ = \min\{\lambda^G_T[^{\alpha^+}_{\alpha^-}](x), \lambda^G_T[^{\alpha^+}_{\alpha^-}](y)\}, \\
\lambda^G_I[^{\beta^-}_{\beta^+}](x \cdot y) = \beta^- &\leq \beta^- = \max\{\lambda^G_I[^{\beta^-}_{\beta^+}](x), \lambda^G_I[^{\beta^-}_{\beta^+}](y)\}, \\
\lambda^G_F[^{\gamma^+}_{\gamma^-}](x \cdot y) = \gamma^+ &\geq \gamma^+ = \min\{\lambda^G_F[^{\gamma^+}_{\gamma^-}](x), \lambda^G_F[^{\gamma^+}_{\gamma^-}](y)\}.
\end{aligned}$$

Case 2: $x \notin G$ or $y \notin G$. Then

$$\begin{aligned}
\lambda^G_T[^{\alpha^-}_{\alpha^-}](x) &= \alpha^- \text{ or } \lambda^G_T[^{\alpha^+}_{\alpha^-}](y) = \alpha^-, \\
\lambda^G_I[^{\beta^-}_{\beta^+}](x) &= \beta^+ \text{ or } \lambda^G_I[^{\beta^-}_{\beta^+}](y) = \beta^+, \\
\lambda^G_F[^{\gamma^+}_{\gamma^-}](x) &= \gamma^- \text{ or } \lambda^G_F[^{\gamma^+}_{\gamma^-}](y) = \gamma^-.
\end{aligned}$$

Thus

$$\min\{\lambda_T^G[^{\alpha^+}_{\alpha^-}](x), \lambda_T^G[^{\alpha^+}_{\alpha^-}](y)\} = \alpha^-,$$
$$\max\{\lambda_I^G[^{\beta^-}_{\beta^+}](x), \lambda_I^G[^{\beta^-}_{\beta^+}](y)\} = \beta^+,$$
$$\min\{\lambda_F^G[^{\gamma^+}_{\gamma^-}](x), \lambda_F^G[^{\gamma^+}_{\gamma^-}](y)\} = \gamma^-.$$

Therefore,

$$\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot y) \geq \alpha^- = \min\{\lambda_T^G[^{\alpha^+}_{\alpha^-}](x), \lambda_T^G[^{\alpha^+}_{\alpha^-}](y)\},$$
$$\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot y) \leq \beta^+ = \max\{\lambda_I^G[^{\beta^-}_{\beta^+}](x), \lambda_I^G[^{\beta^-}_{\beta^+}](y)\},$$
$$\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot y) \geq \gamma^- = \min\{\lambda_F^G[^{\gamma^+}_{\gamma^-}](x), \lambda_F^G[^{\gamma^+}_{\gamma^-}](y)\}.$$

Hence, $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a neutrosophic UP-subalgebra of X.

Theorem 15. *A NS $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ in X is a neutrosophic near UP-filter of X if and only if a nonempty subset G of X is a near UP-filter of X.*

Proof. Assume that $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is neutrosophic near UP-filter of X. Since $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ satisfies the condition (3.6), it follows from Lemma 4 that $0 \in G$. Next, let $x \in X$ and $y \in G$. Then $\lambda_T^G[^{\alpha^+}_{\alpha^-}](y) = \alpha^+$. Thus

$$\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot y) \geq \lambda_T^G[^{\alpha^+}_{\alpha^-}](y) = \alpha^+ \geq \lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot y) \tag{3.9}$$

and so $\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^+$. Thus $x \cdot y \in G$. Hence, G is a near UP-filter of X.

Conversely, assume that G is a near UP-filter of X. Since $0 \in G$, it follows from Lemma 3 that $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ satisfies the conditions (3.6), (3.7), and (3.8). Next, let $x, y \in X$.

Case 1: $y \in G$. Then $\lambda_T^G[^{\alpha^+}_{\alpha^-}](y) = \alpha^+, \lambda_I^G[^{\beta^-}_{\beta^+}](y) = \beta^-$, and $\lambda_F^G[^{\gamma^+}_{\gamma^-}](y) = \gamma^+$. Since G is a near UP-filter of X, we have $x \cdot y \in G$ and so $\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^+, \lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot y) = \beta^-$, and $\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot y) = \gamma^+$. Thus

$$\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^+ \geq \alpha^+ = \lambda_T^G[^{\alpha^+}_{\alpha^-}](y),$$
$$\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot y) = \beta^- \leq \beta^- = \lambda_I^G[^{\beta^-}_{\beta^+}](y),$$
$$\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot y) = \gamma^+ \geq \gamma^+ = \lambda_F^G[^{\gamma^+}_{\gamma^-}](y).$$

Case 2: $y \notin G$. Then $\lambda_T^G[^{\alpha^+}_{\alpha^-}](y) = \alpha^-, \lambda_I^G[^{\beta^-}_{\beta^+}](y) = \beta^+$, and $\lambda_F^G[^{\gamma^+}_{\gamma^-}](y) = \gamma^-$. Thus

$$\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot y) \geq \alpha^- = \lambda_T^G[^{\alpha^+}_{\alpha^-}](y),$$
$$\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot y) \leq \beta^+ = \lambda_I^G[^{\beta^-}_{\beta^+}](y),$$

$$\lambda_F^G[{}^{\gamma^+}_{\gamma^-}](x \cdot y) \geq \gamma^- = \lambda_F^G[{}^{\gamma^+}_{\gamma^-}](y).$$

Hence, $\Lambda^G[{}^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a neutrosophic near UP-filter of X.

Theorem 16. *A NS $\Lambda^G[{}^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ in X is a neutrosophic UP-filter of X if and only if a nonempty subset G of X is a UP-filter of X.*

Proof. Assume that $\Lambda^G[{}^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a neutrosophic UP-filter of X. Since $\Lambda^G[{}^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ satisfies the condition (3.6), it follows from Lemma 4 that $0 \in G$. Next, let $x, y \in X$ be such that $x \cdot y \in G$ and $x \in G$. Then $\lambda_T^G[{}^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^+ = \lambda_T^G[{}^{\alpha^+}_{\alpha^-}](x)$. Thus

$$\lambda_T^G[{}^{\alpha^+}_{\alpha^-}](y) \geq \min\{\lambda_T^G[{}^{\alpha^+}_{\alpha^-}](x \cdot y), \lambda_T^G[{}^{\alpha^+}_{\alpha^-}](x)\} = \alpha^+ \geq \lambda_T^G[{}^{\alpha^+}_{\alpha^-}](y) \tag{3.12}$$

and so $\lambda_T^G[{}^{\alpha^+}_{\alpha^-}](y) = \alpha^+$. Thus $y \in G$. Hence, G is a UP-filter of X.

Conversely, assume that G is a UP-filter of X. Since $0 \in G$, it follows from Lemma 3 that $\Lambda^G[{}^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ satisfies the conditions (3.6), (3.7), and (3.8). Next, let $x, y \in X$.

Case 1: $x \cdot y \in G$ and $x \in G$. Then

$$\lambda_T^G[{}^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^+ = \lambda_T^G[{}^{\alpha^+}_{\alpha^-}](x),$$
$$\lambda_I^G[{}^{\beta^-}_{\beta^+}](x \cdot y) = \beta^- = \lambda_I^G[{}^{\beta^-}_{\beta^+}](x),$$
$$\lambda_F^G[{}^{\gamma^+}_{\gamma^-}](x \cdot y) = \gamma^+ = \lambda_F^G[{}^{\gamma^+}_{\gamma^-}](x).$$

Since G is a UP-filter of X, we have $y \in G$ and so $\lambda_T^G[{}^{\alpha^+}_{\alpha^-}](y) = \alpha^+, \lambda_I^G[{}^{\beta^-}_{\beta^+}](y) = \beta^-$, and $\lambda_F^G[{}^{\gamma^+}_{\gamma^-}](y) = \gamma^+$. Thus

$$\lambda_T^G[{}^{\alpha^+}_{\alpha^-}](y) = \alpha^+ \geq \alpha^+ = \min\{\lambda_T^G[{}^{\alpha^+}_{\alpha^-}](x \cdot y), \lambda_T^G[{}^{\alpha^+}_{\alpha^-}](x)\},$$
$$\lambda_I^G[{}^{\beta^-}_{\beta^+}](y) = \beta^- \leq \beta^- = \max\{\lambda_I^G[{}^{\beta^-}_{\beta^+}](x \cdot y), \lambda_I^G[{}^{\beta^-}_{\beta^+}](x)\},$$
$$\lambda_F^G[{}^{\gamma^+}_{\gamma^-}](y) = \gamma^+ \geq \gamma^+ = \min\{\lambda_F^G[{}^{\gamma^+}_{\gamma^-}](x \cdot y), \lambda_F^G[{}^{\gamma^+}_{\gamma^-}](x)\}.$$

Case 2: $x \cdot y \notin G$ or $x \notin G$. Then

$$\lambda_T^G[{}^{\alpha^+}_{\alpha^-}](x \cdot y) = \alpha^- \text{ or } \lambda_T^G[{}^{\alpha^+}_{\alpha^-}](x) = \alpha^-,$$
$$\lambda_I^G[{}^{\beta^-}_{\beta^+}](x \cdot y) = \beta^+ \text{ or } \lambda_I^G[{}^{\beta^-}_{\beta^+}](x) = \beta^+,$$
$$\lambda_F^G[{}^{\gamma^+}_{\gamma^-}](x \cdot y) = \gamma^- \text{ or } \lambda_F^G[{}^{\gamma^+}_{\gamma^-}](x) = \gamma^-.$$

Thus

$$\min\{\lambda_T^G[{}^{\alpha^+}_{\alpha^-}](x \cdot y), \lambda_T^G[{}^{\alpha^+}_{\alpha^-}](x)\} = \alpha^-,$$
$$\max\{\lambda_I^G[{}^{\beta^-}_{\beta^+}](x \cdot y), \lambda_I^G[{}^{\beta^-}_{\beta^+}](x)\} = \beta^+,$$

$$\min\{\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot y), \lambda_F^G[^{\gamma^+}_{\gamma^-}](x)\} = \gamma^-.$$

Therefore,

$$\lambda_T^G[^{\alpha^+}_{\alpha^-}](y) \geq \alpha^- = \min\{\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot y), \lambda_T^G[^{\alpha^+}_{\alpha^-}](x)\},$$
$$\lambda_I^G[^{\beta^-}_{\beta^+}](y) \leq \beta^+ = \max\{\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot y), \lambda_I^G[^{\beta^-}_{\beta^+}](x)\},$$
$$\lambda_F^G[^{\gamma^+}_{\gamma^-}](y) \geq \gamma^- = \min\{\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot y), \lambda_F^G[^{\gamma^+}_{\gamma^-}](x)\}.$$

Hence, $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a neutrosophic UP-filter of X.

Theorem 17. *A NS $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ in X is a neutrosophic UP-ideal of X if and only if a nonempty subset G of X is a UP-ideal of X.*

Proof. Assume that $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a neutrosophic UP-ideal of X. Since $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ satisfies the condition (3.6), it follows from Lemma 4 that $0 \in G$. Next, let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in G$ and $y \in G$. Then $\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot (y \cdot z)) = \alpha^+ = \lambda_T^G[^{\alpha^+}_{\alpha^-}](y)$. Thus

$$\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot z) \geq \min\{\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot (y \cdot z)), \lambda_T^G[^{\alpha^+}_{\alpha^-}](y)\} = \alpha^+ \geq \lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot z) \tag{3.18}$$

and so $\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot z) = \alpha^+$. Thus $x \cdot z \in G$. Hence, G is a UP-ideal of X.

Conversely, assume that G is a UP-ideal of X. Since $0 \in G$, it follows from Lemma 3 that $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ satisfies the conditions (3.6), (3.7), and (3.8). Next, let $x, y, z \in X$.

Case 1: $x \cdot (y \cdot z) \in G$ and $y \in G$. Then

$$\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot (y \cdot z)) = \alpha^+ = \lambda_T^G[^{\alpha^+}_{\alpha^-}](y),$$
$$\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot (y \cdot z)) = \beta^- = \lambda_I^G[^{\beta^-}_{\beta^+}](y),$$
$$\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot (y \cdot z)) = \gamma^+ = \lambda_F^G[^{\gamma^+}_{\gamma^-}](y).$$

Thus

$$\min\{\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot (y \cdot z)), \lambda_T^G[^{\alpha^+}_{\alpha^-}](y)\} = \alpha^+,$$
$$\max\{\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot (y \cdot z)), \lambda_I^G[^{\beta^-}_{\beta^+}](y)\} = \beta^-,$$
$$\min\{\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot (y \cdot z)), \lambda_F^G[^{\gamma^+}_{\gamma^-}](y)\} = \gamma^+.$$

Since G is a UP-ideal of X, we have $x \cdot z \in G$ and so $\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot z) = \alpha^+$, $\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot z) = \beta^-$, and $\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot z) = \gamma^+$. Thus

$$\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot z) = \alpha^+ \geq \alpha^+ = \min\{\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot (y \cdot z)), \lambda_T^G[^{\alpha^+}_{\alpha^-}](y)\},$$
$$\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot z) = \beta^- \leq \beta^- = \max\{\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot (y \cdot z)), \lambda_I^G[^{\beta^-}_{\beta^+}](y)\},$$

$$\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot z) = \gamma^+ \geq \gamma^+ = \min\{\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot (y \cdot z)), \lambda_F^G[^{\gamma^+}_{\gamma^-}](y)\}.$$

Case 2: $x \cdot (y \cdot z) \notin G$ or $y \notin G$. Then

$$\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot (y \cdot z)) = \alpha^- \text{ or } \lambda_T^G[^{\alpha^+}_{\alpha^-}](y) = \alpha^-,$$
$$\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot (y \cdot z)) = \beta^+ \text{ or } \lambda_I^G[^{\beta^-}_{\beta^+}](y) = \beta^+,$$
$$\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot (y \cdot z)) = \gamma^- \text{ or } \lambda_F^G[^{\gamma^+}_{\gamma^-}](y) = \gamma^-.$$

Thus

$$\min\{\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot (y \cdot z)), \lambda_T^G[^{\alpha^+}_{\alpha^-}](y)\} = \alpha^-,$$
$$\max\{\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot (y \cdot z)), \lambda_I^G[^{\beta^-}_{\beta^+}](y)\} = \beta^+,$$
$$\min\{\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot (y \cdot z)), \lambda_F^G[^{\gamma^+}_{\gamma^-}](y)\} = \gamma^-.$$

Therefore,

$$\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot z) \geq \alpha^- = \min\{\lambda_T^G[^{\alpha^+}_{\alpha^-}](x \cdot (y \cdot z)), \lambda_T^G[^{\alpha^+}_{\alpha^-}](y)\},$$
$$\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot z) \leq \beta^+ = \max\{\lambda_I^G[^{\beta^-}_{\beta^+}](x \cdot (y \cdot z)), \lambda_I^G[^{\beta^-}_{\beta^+}](y)\},$$
$$\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot z) \geq \gamma^- = \min\{\lambda_F^G[^{\gamma^+}_{\gamma^-}](x \cdot (y \cdot z)), \lambda_F^G[^{\gamma^+}_{\gamma^-}](y)\}.$$

Hence, $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a neutrosophic UP-ideal of X.

Theorem 18. *A NS $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ in X is a neutrosophic strongly UP-ideal of X if and only if a nonempty subset G of X is a strongly UP-ideal of X.*

Proof. Assume that $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a neutrosophic strongly UP-ideal of X. By Theorem 2, we have $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is constant, that is, $\lambda_T^G[^{\alpha^+}_{\alpha^-}]$ is constant. Since G is nonempty, we have $\lambda_T^G[^{\alpha^+}_{\alpha^-}](x) = \alpha^+$ for all $x \in X$. Thus $G = X$. Hence, G is a strongly UP-ideal of X.

Conversely, assume that G is a strongly UP-ideal of X. Then $G = X$, so

$$(\forall x \in X)\begin{pmatrix} \lambda_T^G[^{\alpha^+}_{\alpha^-}](x) = \alpha^+ \\ \lambda_I^G[^{\beta^-}_{\beta^+}](x) = \beta^- \\ \lambda_F^G[^{\gamma^+}_{\gamma^-}](x) = \gamma^+ \end{pmatrix}.$$

Thus $\lambda_T^G[^{\alpha^+}_{\alpha^-}], \lambda_I^G[^{\beta^-}_{\beta^+}]$, and $\lambda_F^G[^{\gamma^+}_{\gamma^-}]$ are constant, that is, $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is constant. By Theorem 2, we have $\Lambda^G[^{\alpha^+,\beta^-,\gamma^+}_{\alpha^-,\beta^+,\gamma^-}]$ is a neutrosophic strongly UP-ideal of X.

4. Level subsets of a NS

In this section, we discuss the relationships between neutrosophic UP-subalgebras (resp., neutrosophic near UP-filters, neutrosophic UP-filters, neutrosophic UP-ideals, neutrosophic strongly UP-ideals) of UP-algebras and their level subsets.

Definition 10. *[23] Let f be a fuzzy set in A. For any $t \in [0,1]$, the sets*

$$U(f;t) = \{x \in X \mid f(x) \geq t\},$$
$$L(f;t) = \{x \in X \mid f(x) \leq t\},$$
$$E(f;t) = \{x \in X \mid f(x) = t\}$$

are called an upper t-level subset, a lower t-level subset, and an equal t-level subset of f, respectively.

Theorem 19. *A NS Λ in X is a neutrosophic UP-subalgebra of X if and only if for all $\alpha, \beta, \gamma \in [0,1]$, the sets $U(\lambda_T;\alpha), L(\lambda_I;\beta)$, and $U(\lambda_F;\gamma)$ are UP-subalgebras of X if $U(\lambda_T;\alpha), L(\lambda_I;\beta)$, and $U(\lambda_F;\gamma)$ are nonempty.*

Proof. Assume that Λ is a neutrosophic UP-subalgebra of X. Let $\alpha, \beta, \gamma \in [0,1]$ be such that $U(\lambda_T;\alpha), L(\lambda_I;\beta)$, and $U(\lambda_F;\gamma)$ are nonempty.

Let $x, y \in U(\lambda_T;\alpha)$. Then $\lambda_T(x) \geq \alpha$ and $\lambda_T(y) \geq \alpha$, so α is an lower bound of $\{\lambda_T(x), \lambda_T(y)\}$. By (3.3), we have $\lambda_T(x \cdot y) \geq \min\{\lambda_T(x), \lambda_T(y)\} \geq \alpha$. Thus $x \cdot y \in U(\lambda_T;\alpha)$.

Let $x, y \in L(\lambda_I;\beta)$. Then $\lambda_I(x) \leq \beta$ and $\lambda_I(y) \leq \beta$, so β is a upper bound of $\{\lambda_I(x), \lambda_I(y)\}$. By (3.4), we have $\lambda_I(x \cdot y) \leq \max\{\lambda_I(x), \lambda_I(y)\} \leq \beta$. Thus $x \cdot y \in L(\lambda_I;\beta)$.

Let $x, y \in U(\lambda_F;\gamma)$. Then $\lambda_F(x) \geq \gamma$ and $\lambda_F(y) \geq \gamma$, so γ is an lower bound of $\{\lambda_F(x), \lambda_F(y)\}$. By (3.5), we have $\lambda_F(x \cdot y) \geq \min\{\lambda_F(x), \lambda_F(y)\} \geq \gamma$. Thus $x \cdot y \in U(\lambda_F;\gamma)$.

Hence, $U(\lambda_T;\alpha), L(\lambda_I;\beta)$, and $U(\lambda_F;\gamma)$ are UP-subalgebras of X.

Conversely, assume that for all $\alpha, \beta, \gamma \in [0,1]$, the sets $U(\lambda_T;\alpha), L(\lambda_I;\beta)$, and $U(\lambda_F;\gamma)$ are UP-subalgebras of X if $U(\lambda_T;\alpha), L(\lambda_I;\beta)$, and $U(\lambda_F;\gamma)$ are nonempty.

Let $x, y \in X$. Then $\lambda_T(x), \lambda_T(y) \in [0,1]$. Choose $\alpha = \min\{\lambda_T(x), \lambda_T(y)\}$. Thus $\lambda_T(x) \geq \alpha$ and $\lambda_T(y) \geq \alpha$, so $x, y \in U(\lambda_T;\alpha) \neq \emptyset$. By assumption, we have $U(\lambda_T;\alpha)$ is a UP-subalgebra of X and so $x \cdot y \in U(\lambda_T;\alpha)$. Thus $\lambda_T(x \cdot y) \geq \alpha = \min\{\lambda_T(x), \lambda_T(y)\}$.

Let $x, y \in X$. Then $\lambda_I(x), \lambda_I(y) \in [0,1]$. Choose $\beta = \max\{\lambda_I(x), \lambda_I(y)\}$. Thus $\lambda_I(x) \leq \beta$ and $\lambda_I(y) \leq \beta$, so $x, y \in L(\lambda_I;\beta) \neq \emptyset$. By assumption, we have $L(\lambda_I;\beta)$ is a UP-subalgebra of X and so $x \cdot y \in L(\lambda_I;\beta)$. Thus $\lambda_I(x \cdot y) \leq \beta = \max\{\lambda_I(x), \lambda_I(y)\}$.

Let $x, y \in X$. Then $\lambda_F(x), \lambda_F(y) \in [0,1]$. Choose $\gamma = \min\{\lambda_F(x), \lambda_F(y)\}$. Thus $\lambda_F(x) \geq \gamma$ and $\lambda_F(y) \geq \gamma$, so $x, y \in U(\lambda_F;\gamma) \neq \emptyset$. By assumption, we have $U(\lambda_F;\gamma)$ is a UP-subalgebra of X and so $x \cdot y \in U(\lambda_F;\gamma)$. Thus $\lambda_F(x \cdot y) \geq \gamma = \min\{\lambda_F(x), \lambda_F(y)\}$.

Therefore, Λ is a neutrosophic UP-subalgebra of X.

Theorem 20. *A NS Λ in X is a neutrosophic near UP-filter of X if and only if for all $\alpha, \beta, \gamma \in [0,1]$, the sets $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are near UP-filters of X if $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are nonempty.*

Proof. Assume that Λ is a neutrosophic near UP-filter of X. Let $\alpha, \beta, \gamma \in [0,1]$ be such that $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are nonempty.

Let $x \in U(\lambda_T; \alpha)$. Then $\lambda_T(x) \geq \alpha$. By (3.6), we have $\lambda_T(0) \geq \lambda_T(x) \geq \alpha$. Thus $0 \in U(\lambda_T; \alpha)$. Next, let $x \in X$ and $y \in U(\lambda_T; \alpha)$. Then $\lambda_T(y) \geq \alpha$. By (3.9), we have $\lambda_T(x \cdot y) \geq \lambda_T(y) \geq \alpha$. Thus $x \cdot y \in U(\lambda_T; \alpha)$.

Let $x \in L(\lambda_I; \beta)$. Then $\lambda_I(x) \leq \beta$. By (3.7), we have $\lambda_I(0) \leq \lambda_I(x) \leq \beta$. Thus $0 \in L(\lambda_I; \beta)$. Next, let $x \in X$ and $y \in L(\lambda_I; \beta)$. Then $\lambda_I(y) \leq \beta$. By (3.10), we have $\lambda_I(x \cdot y) \leq \lambda_I(y) \leq \beta$. Thus $x \cdot y \in L(\lambda_I; \beta)$.

Let $x \in U(\lambda_F; \gamma)$. Then $\lambda_F(x) \geq \gamma$. By (3.8), we have $\lambda_F(0) \geq \lambda_F(x) \geq \gamma$. Thus $0 \in U(\lambda_F; \gamma)$. Next, let $x \in X$ and $y \in U(\lambda_F; \gamma)$. Then $\lambda_F(y) \geq \gamma$. By (3.11), we have $\lambda_F(x \cdot y) \geq \lambda_F(y) \geq \gamma$. Thus $x \cdot y \in U(\lambda_F; \gamma)$.

Hence, $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are near UP-filters of X.

Conversely, assume that for all $\alpha, \beta, \gamma \in [0,1]$, the sets $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are near UP-filters of X if $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are nonempty.

Let $x \in X$. Then $\lambda_T(x) \in [0,1]$. Choose $\alpha = \lambda_T(x)$. Thus $\lambda_T(x) \geq \alpha$, so $x \in U(\lambda_T; \alpha) \neq \emptyset$. By assumption, we have $U(\lambda_T; \alpha)$ is a near UP-filter of X and so $0 \in U(\lambda_T; \alpha)$. Thus $\lambda_T(0) \geq \alpha = \lambda_T(x)$. Next, let $x, y \in X$. Then $\lambda_T(y) \in [0,1]$. Choose $\alpha = \lambda_T(y)$. Thus $\lambda_T(y) \geq \alpha$, so $y \in U(\lambda_T; \alpha) \neq \emptyset$. By assumption, we have $U(\lambda_T; \alpha)$ is a near UP-filter of X and so $x \cdot y \in U(\lambda_T; \alpha)$. Thus $\lambda_T(x \cdot y) \geq \alpha = \lambda_T(y)$.

Let $x \in X$. Then $\lambda_I(x) \in [0,1]$. Choose $\beta = \lambda_I(x)$. Thus $\lambda_I(x) \leq \beta$, so $x \in L(\lambda_I; \beta) \neq \emptyset$. By assumption, we have $L(\lambda_I; \beta)$ is a near UP-filter of X and so $0 \in L(\lambda_I; \beta)$. Thus $\lambda_I(0) \leq \beta = \lambda_I(x)$. Next, let $x, y \in X$. Then $\lambda_I(y) \in [0,1]$. Choose $\beta = \lambda_I(y)$. Thus $\lambda_I(y) \leq \beta$, so $y \in L(\lambda_I; \beta) \neq \emptyset$. By assumption, we have $L(\lambda_I; \beta)$ is a near UP-filter of X and so $x \cdot y \in L(\lambda_I; \beta)$. Thus $\lambda_I(x \cdot y) \leq \beta = \lambda_I(y)$.

Let $x \in X$. Then $\lambda_F(x) \in [0,1]$. Choose $\gamma = \lambda_F(x)$. Thus $\lambda_F(x) \geq \gamma$, so $x \in U(\lambda_F; \gamma) \neq \emptyset$. By assumption, we have $U(\lambda_F; \gamma)$ is a near UP-filter of X and so $0 \in U(\lambda_F; \gamma)$. Thus $\lambda_F(0) \geq \gamma = \lambda_F(x)$. Next, let $x, y \in X$. Then $\lambda_F(y) \in [0,1]$. Choose $\gamma = \lambda_F(y)$. Thus $\lambda_F(y) \geq \gamma$, so $y \in U(\lambda_F; \gamma) \neq \emptyset$. By assumption, we have $L(\lambda_F; \gamma)$ is a near UP-filter of X and so $x \cdot y \in U(\lambda_F; \gamma)$. Thus $\lambda_F(x \cdot y) \geq \gamma = \lambda_F(y)$.

Therefore, Λ is a neutrosophic near UP-filter of X.

Theorem 21. *A NS Λ in X is a neutrosophic UP-filter of X if and only if for all $\alpha, \beta, \gamma \in [0,1]$, the sets $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are UP-filters of X if $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are nonempty.*

Proof. Assume that Λ is a neutrosophic UP-filter of X. Let $\alpha, \beta, \gamma \in [0,1]$ be such that $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are nonempty.

Let $x \in U(\lambda_T; \alpha)$. Then $\lambda_T(x) \geq \alpha$. By (3.6), we have $\lambda_T(0) \geq \lambda_T(x) \geq \alpha$. Thus $0 \in U(\lambda_T; \alpha)$. Next, let $x, y \in X$ be such that $x \cdot y \in U(\lambda_T; \alpha)$ and $x \in U(\lambda_T; \alpha)$. Then

$\lambda_T(x \cdot y) \geq \alpha$ and $\lambda_T(x) \geq \alpha$, so α is an lower bound of $\{\lambda_T(x \cdot y), \lambda_T(x)\}$. By (3.12), we have $\lambda_T(y) \geq \min\{\lambda_T(x \cdot y), \lambda_T(x)\} \geq \alpha$. Thus $y \in U(\lambda_T; \alpha)$.

Let $x \in L(\lambda_I; \beta)$. Then $\lambda_I(x) \leq \beta$. By (3.7), we have $\lambda_I(0) \leq \lambda_I(x) \leq \beta$. Thus $0 \in L(\lambda_I; \beta)$. Next, let $x, y \in X$ be such that $x \cdot y \in L(\lambda_I; \beta)$ and $x \in L(\lambda_I; \beta)$. Then $\lambda_I(x \cdot y) \leq \beta$ and $\lambda_I(x) \leq \beta$, so β is a upper bound of $\{\lambda_I(x \cdot y), \lambda_I(x)\}$. By (3.13), we have $\lambda_I(y) \leq \max\{\lambda_I(x \cdot y), \lambda_I(x)\} \leq \beta$ Thus $y \in L(\lambda_I; \beta)$.

Let $x \in U(\lambda_F; \gamma)$. Then $\lambda_F(x) \geq \gamma$. By (3.8), we have $\lambda_F(0) \geq \lambda_F(x) \geq \gamma$. Thus $0 \in U(\lambda_F; \gamma)$. Next, let $x, y \in X$ be such that $x \cdot y \in U(\lambda_F; \gamma)$ and $x \in U(\lambda_F; \gamma)$. Then $\lambda_F(x \cdot y) \geq \gamma$ and $\lambda_F(x) \geq \gamma$, so γ is an lower bound of $\{\lambda_F(x \cdot y), \lambda_F(x)\}$. By (3.14), we have $\lambda_F(y) \geq \min\{\lambda_F(x \cdot y), \lambda_F(x)\} \geq \gamma$. Thus $y \in U(\lambda_F; \gamma)$.

Hence, $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are UP-filters of X.

Conversely, assume that for all $\alpha, \beta, \gamma \in [0, 1]$, the sets $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are UP-filters of X if $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are nonempty.

Let $x \in X$. Then $\lambda_T(x) \in [0, 1]$. Choose $\alpha = \lambda_T(x)$. Thus $\lambda_T(x) \geq \alpha$, so $x \in U(\lambda_T; \alpha) \neq \emptyset$. By assumption, we have $U(\lambda_T; \alpha)$ is a UP-filter of X and so $0 \in U(\lambda_T; \alpha)$. Thus $\lambda_T(0) \geq \alpha = \lambda_T(x)$. Next, let $x, y \in X$. Then $\lambda_T(x \cdot y), \lambda_T(x) \in [0, 1]$. Choose $\alpha = \min\{\lambda_T(x \cdot y), \lambda_T(x)\}$. Thus $\lambda_T(x \cdot y) \geq \alpha$ and $\lambda_T(x) \geq \alpha$, so $x \cdot y, x \in U(\lambda_T; \alpha) \neq \emptyset$. By assumption, we have $U(\lambda_T; \alpha)$ is a UP-filter of X and so $y \in U(\lambda_T; \alpha)$. Thus $\lambda_T(y) \geq \alpha = \min\{\lambda_T(x \cdot y), \lambda_T(x)\}$.

Let $x \in X$. Then $\lambda_I(x) \in [0, 1]$. Choose $\beta = \lambda_I(x)$. Thus $\lambda_I(x) \leq \beta$, so $x \in L(\lambda_I; \beta) \neq \emptyset$. By assumption, we have $L(\lambda_I; \beta)$ is a UP-filter of X and so $0 \in L(\lambda_I; \beta)$. Thus $\lambda_I(0) \leq \beta = \lambda_I(x)$. Next, let $x, y \in X$. Then $\lambda_I(x \cdot y), \lambda_I(x) \in [0, 1]$. Choose $\beta = \max\{\lambda_I(x \cdot y), \lambda_I(x)\}$. Thus $\lambda_I(x \cdot y) \leq \beta$ and $\lambda_I(x) \leq \beta$, so $x \cdot y, x \in L(\lambda_I; \beta) \neq \emptyset$. By assumption, we have $L(\lambda_I; \beta)$ is a UP-filter of X and so $y \in L(\lambda_I; \beta)$. Thus $\lambda_I(y) \leq \beta = \max\{\lambda_I(x \cdot y), \lambda_I(x)\}$.

Let $x \in X$. Then $\lambda_F(x) \in [0, 1]$. Choose $\gamma = \lambda_F(x)$. Thus $\lambda_F(x) \geq \gamma$, so $x \in U(\lambda_F; \gamma) \neq \emptyset$. By assumption, we have $U(\lambda_F; \gamma)$ is a UP-filter of X and so $0 \in U(\lambda_F; \gamma)$. Thus $\lambda_F(0) \geq \gamma = \lambda_F(x)$. Next, let $x, y \in X$. Then $\lambda_F(x \cdot y), \lambda_F(x) \in [0, 1]$. Choose $\gamma = \min\{\lambda_F(x \cdot y), \lambda_F(x)\}$. Thus $\lambda_F(x \cdot y) \geq \gamma$ and $\lambda_F(x) \geq \gamma$, so $x \cdot y, x \in U(\lambda_F; \gamma) \neq \emptyset$. By assumption, we have $U(\lambda_F; \gamma)$ is a UP-filter of X and so $y \in U(\lambda_F; \gamma)$. Thus $\lambda_F(y) \geq \gamma = \min\{\lambda_F(x \cdot y), \lambda_F(x)\}$.

Therefore, Λ is a neutrosophic UP-filter of X.

Theorem 22. *A NS Λ in X is a neutrosophic UP-ideal of X if and only if for all $\alpha, \beta, \gamma \in [0, 1]$, the sets $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are UP-ideals of X if $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are nonempty.*

Proof. Assume that Λ is a neutrosophic UP-ideal of X. Let $\alpha, \beta, \gamma \in [0, 1]$ be such that $U(\lambda_T; \alpha), L(\lambda_I; \beta)$, and $U(\lambda_F; \gamma)$ are nonempty.

Let $x \in U(\lambda_T; \alpha)$. Then $\lambda_T(x) \geq \alpha$. By (3.6), we have $\lambda_T(0) \geq \lambda_T(x) \geq \alpha$. Thus $0 \in U(\lambda_T; \alpha)$. Next, let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in U(\lambda_T; \alpha)$ and $y \in U(\lambda_T; \alpha)$. Then $\lambda_T(x \cdot (y \cdot z)) \geq \alpha$ and $\lambda_T(y) \geq \alpha$, so α is an lower bound of $\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\}$. By (3.15), we have $\lambda_T(x \cdot z) \geq \min\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\} \geq \alpha$. Thus $x \cdot z \in U(\lambda_T; \alpha)$.

Let $x \in L(\lambda_I;\alpha)$. Then $\lambda_I(x) \leq \beta$. By (3.7), we have $\lambda_I(0) \leq \lambda_I(x) \leq \beta$. Thus $0 \in L(\lambda_I;\beta)$. Next, let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in L(\lambda_I;\beta)$ and $y \in L(\lambda_I;\beta)$. Then $\lambda_I(x \cdot (y \cdot z)) \leq \beta$ and $\lambda_I(y) \leq \beta$, so β is a upper bound of $\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\}$. By (3.16), we have $\lambda_I(x \cdot z) \leq \max\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\} \leq \beta$. Thus $x \cdot z \in L(\lambda_I;\beta)$.

Let $x \in U(\lambda_F;\gamma)$. Then $\lambda_F(x) \geq \gamma$. By (3.8), we have $\lambda_F(0) \geq \lambda_F(x) \geq \gamma$. Thus $0 \in U(\lambda_F;\gamma)$. Next, let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in U(\lambda_F;\gamma)$ and $y \in U(\lambda_F;\gamma)$. Then $\lambda_F(x \cdot (y \cdot z)) \geq \gamma$ and $\lambda_F(y) \geq \gamma$, so γ is an lower bound of $\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}$. By (3.17), we have $\lambda_F(x \cdot z) \geq \min\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\} \geq \gamma$. Thus $x \cdot z \in U(\lambda_F;\gamma)$.

Hence, $U(\lambda_T;\alpha), L(\lambda_I;\beta)$, and $U(\lambda_F;\gamma)$ are UP-ideals of X.

Conversely, assume that for all $\alpha, \beta, \gamma \in [0, 1]$, the sets $U(\lambda_T;\alpha), L(\lambda_I;\beta)$, and $U(\lambda_F;\gamma)$ are UP-ideals of X if $U(\lambda_T;\alpha), L(\lambda_I;\beta)$, and $U(\lambda_F;\gamma)$ are nonempty.

Let $x \in X$. Then $\lambda_T(x) \in [0, 1]$. Choose $\alpha = \lambda_T(x)$. Thus $\lambda_T(x) \geq \alpha$, so $x \in U(\lambda_T;\alpha) \neq \emptyset$. By assumption, we have $U(\lambda_T;\alpha)$ is a UP-ideal of X and so $0 \in U(\lambda_T;\alpha)$. Thus $\lambda_T(0) \geq \alpha = \lambda_T(x)$. Next, let $x, y, z \in X$. Then $\lambda_T(x \cdot (y \cdot z)), \lambda_T(y) \in [0, 1]$. Choose $\alpha = \min\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\}$. Thus $\lambda_T(x \cdot (y \cdot z)) \geq \alpha$ and $\lambda_T(y) \geq \alpha$, so $x \cdot (y \cdot z), y \in U(\lambda_T;\alpha) \neq \emptyset$. By assumption, we have $U(\lambda_T;\alpha)$ is a UP-ideal of X and so $x \cdot z \in U(\lambda_T;\alpha)$. Thus $\lambda_T(x \cdot z) \geq \alpha = \min\{\lambda_T(x \cdot (y \cdot z)), \lambda_T(y)\}$.

Let $x \in X$. Then $\lambda_I(x) \in [0, 1]$. Choose $\beta = \lambda_I(x)$. Thus $\lambda_I(x) \leq \beta$, so $x \in L(\lambda_I;\beta) \neq \emptyset$. By assumption, we have $L(\lambda_I;\beta)$ is a UP-ideal of X and so $0 \in L(\lambda_I;\beta)$. Thus $\lambda_I(0) \leq \beta = \lambda_I(x)$. Next, let $x, y, z \in X$. Then $\lambda_I(x \cdot (y \cdot z)), \lambda_I(y) \in [0, 1]$. Choose $\beta = \max\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\}$. Thus $\lambda_I(x \cdot (y \cdot z)) \leq \beta$ and $\lambda_I(y) \leq \beta$, so $x \cdot (y \cdot z), y \in L(\lambda_I;\beta) \neq \emptyset$. By assumption, we have $L(\lambda_I;\beta)$ is a UP-ideal of X and so $x \cdot z \in L(\lambda_I;\beta)$. Thus $\lambda_I(x \cdot z) \leq \beta = \max\{\lambda_I(x \cdot (y \cdot z)), \lambda_I(y)\}$.

Let $x \in X$. Then $\lambda_F(x) \in [0, 1]$. Choose $\gamma = \lambda_F(x)$. Thus $\lambda_F(x) \geq \gamma$, so $x \in U(\lambda_F;\gamma) \neq \emptyset$. By assumption, we have $U(\lambda_F;\gamma)$ is a UP-ideal of X and so $0 \in U(\lambda_F;\gamma)$. Thus $\lambda_F(0) \geq \gamma = \lambda_F(x)$. Next, let $x, y, z \in X$. Then $\lambda_F(x \cdot (y \cdot z)), \lambda_F(y) \in [0, 1]$. Choose $\gamma = \min\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}$. Thus $\lambda_F(x \cdot (y \cdot z)) \geq \gamma$ and $\lambda_F(y) \geq \gamma$, so $x \cdot (y \cdot z), y \in U(\lambda_F;\gamma) \neq \emptyset$. By assumption, we have $U(\lambda_F;\gamma)$ is a UP-ideal of X and so $x \cdot z \in U(\lambda_F;\gamma)$. Thus $\lambda_F(x \cdot z) \geq \gamma = \min\{\lambda_F(x \cdot (y \cdot z)), \lambda_F(y)\}$.

Therefore, Λ is a neutrosophic UP-ideal of X.

Theorem 23. *A NS Λ in X is a neutrosophic strongly UP-ideal of X if and only if the sets $E(\lambda_T;\lambda_T(0)), E(\lambda_I;\lambda_I(0))$, and $E(\lambda_F;\lambda_F(0))$ are strongly UP-ideals of X.*

Proof. Assume that Λ is a neutrosophic strongly UP-ideal of X. By Theorem 2, we have Λ is constant, that is, λ_T, λ_I, and λ_F are constant. Thus

$$(\forall x \in X)\begin{pmatrix}\lambda_T(x) = \lambda_T(0)\\ \lambda_I(x) = \lambda_I(0)\\ \lambda_F(x) = \lambda_F(0)\end{pmatrix}.$$

Hence, $E(\lambda_T;\lambda_T(0)) = X, E(\lambda_I;\lambda_I(0)) = X$, and $E(\lambda_F;\lambda_F(0)) = X$ and so $E(\lambda_T;\lambda_T(0))$, $E(\lambda_I;\lambda_I(0))$, and $E(\lambda_F;\lambda_F(0))$ are strongly UP-ideals of X.

Conversely, assume that $E(\lambda_T;\lambda_T(0)), E(\lambda_I;\lambda_I(0))$, and $E(\lambda_F;\lambda_F(0))$ are strongly UP-ideals of X. Then $E(\lambda_T;\lambda_T(0)) = X, E(\lambda_I;\lambda_I(0)) = X$, $E(\lambda_F;\lambda_F(0)) = X$ and so

$$(\forall x \in X)\begin{pmatrix}\lambda_T(x) = \lambda_T(0)\\ \lambda_I(x) = \lambda_I(0)\\ \lambda_F(x) = \lambda_F(0)\end{pmatrix}.$$

Thus λ_T, λ_I, and λ_F are constant, that is, Λ is constant. By Theorem 2, we have Λ is a neutrosophic strongly UP-ideal of X.

Definition 11. *Let Λ be a NS in X. For $\alpha,\beta,\gamma \in [0,1]$, the sets*

$$\begin{aligned} ULU_\Lambda(\alpha,\beta,\gamma) &= \{x \in X \mid \lambda_T \geq \alpha, \lambda_I \leq \beta, \lambda_F \geq \gamma\},\\ LUL_\Lambda(\alpha,\beta,\gamma) &= \{x \in X \mid \lambda_T \leq \alpha, \lambda_I \geq \beta, \lambda_F \leq \gamma\},\\ E_\Lambda(\alpha,\beta,\gamma) &= \{x \in X \mid \lambda_T = \alpha, \lambda_I = \beta, \lambda_F = \gamma\}\end{aligned}$$

are called a ULU-(α,β,γ)-level subset, a LUL-(α,β,γ)-level subset, and an E-(α,β,γ)-level subset of Λ, respectively. Then we see that

$$\begin{aligned} ULU_\Lambda(\alpha,\beta,\gamma) &= U(\lambda_T;\alpha) \cap L(\lambda_I;\beta) \cap U(\lambda_F;\gamma),\\ LUL_\Lambda(\alpha,\beta,\gamma) &= L(\lambda_T;\alpha) \cap U(\lambda_I;\beta) \cap L(\lambda_F;\gamma),\\ E_\Lambda(\alpha,\beta,\gamma) &= E(\lambda_T;\alpha) \cap E(\lambda_I;\beta) \cap E(\lambda_F;\gamma).\end{aligned}$$

Corollary 1. *A NS Λ in X is a neutrosophic UP-subalgebra of X if and only if for all $\alpha,\beta,\gamma \in [0,1]$, $ULU_\Lambda(\alpha,\beta,\gamma)$ is a UP-subalgebra of X where $ULU_\Lambda(\alpha,\beta,\gamma)$ is nonempty.*

Proof. It is straightforward by Theorem 19.

Corollary 2. *A NS Λ in X is a neutrosophic near UP-filter of X if and only if for all $\alpha,\beta,\gamma \in [0,1]$, $ULU_\Lambda(\alpha,\beta,\gamma)$ is a near UP-filter of X where $ULU_\Lambda(\alpha,\beta,\gamma)$ is nonempty.*

Proof. It is straightforward by Theorem 20.

Corollary 3. *A NS Λ in X is a neutrosophic UP-filter of X if and only if for all $\alpha,\beta,\gamma \in [0,1]$, $ULU_\Lambda(\alpha,\beta,\gamma)$ is a UP-filter of X where $ULU_\Lambda(\alpha,\beta,\gamma)$ is nonempty.*

Proof. It is straightforward by Theorem 21.

Corollary 4. *A NS Λ in X is a neutrosophic UP-ideal of X if and only if for all $\alpha,\beta,\gamma \in [0,1]$, $ULU_\Lambda(\alpha,\beta,\gamma)$ is a UP-ideal of X where $ULU_\Lambda(\alpha,\beta,\gamma)$ is nonempty.*

Proof. It is straightforward by Theorem 22.

Corollary 5. *A NS Λ in X is a neutrosophic strongly UP-ideal of X if and only if $E(\lambda_T,\lambda_T(0)), E(\lambda_I,\lambda_I(0))$, and $E(\lambda_F,\lambda_F(0))$ are strongly UP-ideals of X, that is, $E(\lambda_T,\lambda_T(0)) = X, E(\lambda_I,\lambda_I(0)) = X$, and $E(\lambda_F,\lambda_F(0)) = X$.*

Proof. It is straightforward by Theorem 23.

5. Conclusions

In this paper, we have introduced the notions of neutrosophic UP-subalgebras, neutrosophic near UP-filters, neutrosophic UP-filters, neutrosophic UP-ideals, and neutrosophic strongly UP-ideals of UP-algebras and investigated some of their important properties. Then, we get the diagram of generalization of NSs in UP-algebras as shown in Figure 1.

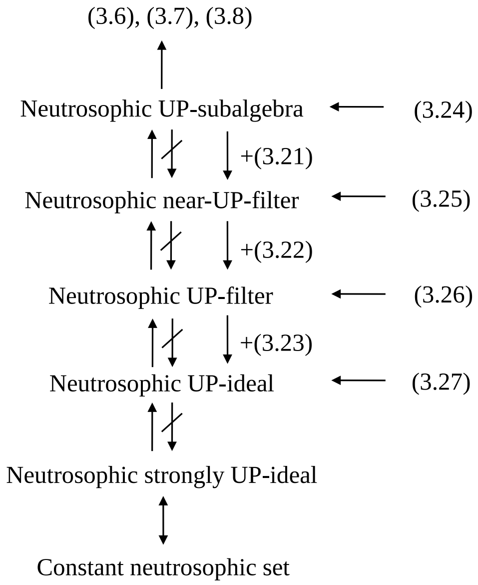

Figure 1: NSs in UP-algebras

In our future study, we will apply this notion/results to other type of NSs in UP-algebras. Also, we will study the soft set theory/cubic set theory of neutrosophic UP-subalgebras, neutrosophic near UP-filters, neutrosophic UP-filters, neutrosophic UP-ideals, and neutrosophic strongly UP-ideals.

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