EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
Vol. 13, No. 2, 2020, 314-322
ISSN 1307-5543 – www.ejpam.com
Published by New York Business Global

The problem of the optimal control with a lower coefficient for weakly nonlinear wave equation in the mixed problem

G.G Ismayilova

Sumgait State University, Sumgait, Azerbaijan

Abstract. In this paper, we consider the problem of determining the lowest coefficient of weakly nonlinear wave equation. The problem is reduced to the optimal control problem, in the new problem. In the this existence theorem of the optimal control and, the Freéchet differentiability of the functional is proved. Also the necessary condition of optimality is derived in view of variational inequality.

2020 Mathematics Subject Classifications: 49J15, 49J40

Key Words and Phrases: Coefficient, weakly nonlinear equation, optimal control,necessary condition.

1. Introduction

The problems of determining the coefficients of various partial differential equations are actual problems in connection with of applied significance. Taking into account that the coefficients of the equations of mathematical physics characterize various properties of the considered medium, finding them is undoubtedly an important problem. Such problems arise in various fields of natural science [1–4].

Recently, various methods have been used to solve such problems.One of these methods is application of the methods of optimal control theory, which is also called variational approach.

To apply this approach, a residual functional is constructed for finding the unknowns in boundary value problems using additional data.

Then we consider the problem of minimizing the constructed functional for solving a boundary value problem, moreover, the unknown in the boundary value problem is treated as a control function and the new problem is investigated as an optimal control problem by applying the methods of control theory.

DOI: https://doi.org/10.29020/nybg.ejpam.v13i2.3650

Email address: gunay_ismayilova_83@mail.ru (G.G Ismayilova)

2. Statement of the problem

Consider the problem of finding a pair of functions $\{u(x,t), v(x)\}$ from the following relations

$$\frac{\partial^2 u}{\partial t^2} - \Delta u + vu = f(x,t,u), (x,t) \in Q, \tag{1}$$

$$u = 0, (x,t) \in S, \; u|_{t=0} = u_0(x), \; \left.\frac{\partial u}{\partial t}\right|_{t=0} = u_1(x), x \in \Omega, \tag{2}$$

$$\int_0^T K(x,t)u(x,t)dt = \varphi(x), x \in \Omega, \tag{3}$$

where Δ is the Laplace operator with respect to x, $f(x,t,u), u_0(x), u_1(x), K(x,t), \varphi(x)$ are given functions. Let $Q = \Omega \times (0,T)$ be a cylinder in $R^{n+1} (n \leq 4)$, Ω be a bounded domain in R^n with a sufficiently smooth boundary $\Gamma, S = \Gamma \times (0,T)$ is the lateral surface of the cylinder $Q, T > 0$ be fixed number.

Note that problem (1) - (3) is inverse to the direct problem (1), (2) for a given function $v(x)$. We reduce this problem to the following optimal control problem: in the class of functions

$$V = \{v(x) \in L_2(\Omega)/a \leq v(x) \leq b \;\text{ a.e.on }\; \Omega\} \tag{4}$$

find the minimum of functional

$$J_0(v) = \frac{1}{2}\int_\Omega \left[\int_0^T K(x,t)u(x,t;v)dt - \varphi(x)\right]^2 dx \tag{5}$$

under constraints (1), (2), where a and b some constants, moreover $a < b$,$u(x,t;v)$ is the solution of the boundary value problem (1), (2) for $v = v(x) \in V$. We call a function $v(x)$ a control, and a set V a class of admissible controls.

Note that between problems (1) - (3) and (1), (2), (4), (5) there is exist a closely connection - if the minimum of the functional in problem (1), (2), (4), (5) is equal to zero, then the additional condition (3) is satisfied.

Let's consider the following problem to avoid the possible degeneration [[12], p.45] in the obtained necessary condition of optimality in future: find the control $v \in V$ that gives minimum to the functional.

$$J_\alpha(v) = J_0(v) + \frac{\alpha}{2}\int_\Omega |v(x)|^2 dx \tag{6}$$

under the constraints (1), (2), where $\alpha > 0$ is the given number.

This problem will be called problem (1), (2), (4), (6).

Let the following conditions be fulfilled on the data of problem (1), (2), (4), (6).

1. The function $f(x,t,u)$ satisfies the Carathéodory conditions, has a continuous partial derivative with respect to $u \in R$, for almost all $(x,t) \in Q$ and for all $u \in R$, moreover the derivative $\frac{\partial f(x,t,u)}{\partial u}$ is bounded, $f(x,t,0) \in L_2(Q)$, and the operator $\frac{\partial f(x,t,u(x,t))}{\partial u}$ acts continuously from $L_2(Q)$ to $L_2(Q)$.

2.

$$u_0 \in \overset{\circ}{W}{}_2^1(\Omega), u_1 \in L_2(\Omega), K \in L_\infty(Q), \varphi \in L_2(\Omega).$$

Since, under the imposed conditions, the function $f(x,t,u)$ satisfies the Lipschitz condition with respect to u, applying the Faedo-Galarkin method [13],[14] under conditions 1.,2. it is easy to prove that for each $v(x) \in V$ the boundary value problem (1), (2) has a unique generalized solution from

$$U = \left\{ u | u \in C\left([0,T]; \overset{\circ}{W}{}_2^1(\Omega) \right), \frac{\partial u}{\partial t} \in C\left([0,T]; L_2(\Omega)\right) \right\}$$

and following estimation is true for this solution

$$\|u\|_{\overset{\circ}{W}{}_2^1(\Omega)}(\Omega) + \left\| \frac{\partial u}{\partial t} \right\|_{L_2(\Omega)} \leq$$

$$\leq c\left[\|u_0\|_{\overset{\circ}{W}{}_2^1}(\Omega) + \|u_1\|_{L_2(\Omega)} + \|f(x,t,0)\|_{L_2(Q)} \right], \ t \in [0,T]. \tag{7}$$

Here and in the future, with c we denote various constants that are independent of the estimated quantities and of the admissible controls.

By a generalized solution of the problem (1), (2) for a given function $v(x) \in V$ we mean a function $u = u(x,t;v)$ from U such that for $t = 0$ it satisfies the condition $u(x,0) = u_0(x)$ and the integral identity

$$\int_Q \left[-\frac{\partial u}{\partial t}\frac{\partial \eta}{\partial t} + \sum_{i=1}^n \frac{\partial u}{\partial x_i}\frac{\partial \eta}{\partial x_i} + vu\eta \right] dxdt - \int_\Omega u_1(x)\eta(x,0)dx = \int_Q f(x,t,u)\eta dxdt \tag{8}$$

for all $\eta = \eta(x,t)$ from U and are equal to zero for $t = T$.

3. The existence of optimal control in problem (1), (2), (4), (6)

Theorem 1. *Let the conditions accepted in the statement of problem (1), (2), (4), (6) be satisfied. Then the set of optimal controls of problem (1), (2), (4), (6)* $V_* = \left\{ v \in V / J_\alpha(v) = J_{\alpha^*} = \inf_{v \in V} J_\alpha(v) \right\}$ *is nonempty, weakly compact in* $L_2(\Omega)$*, and any minimizing sequence* $\{v_m\}$ *weakly converges to the set* V_* *in* $L_2(\Omega)$*.*

Proof. The set V defined by relation (4) is weakly compact in $L_2(\Omega)$. We show that functional (6) is weakly lower semicontinuous on the set V. Let $v(x) \in V$ be some element and $\{v_m\} \subset V$ an arbitrary sequence such that $v_m \to v$ weakly in $L_2(\Omega)$.

Due to the unique solvability of the boundary value problem (1), (2), to each control $v_m \in V$ corresponds a unique solution $u_m = u(x,t;v_m)$ of the problem (1), (2) and, by virtue of the estimate (7), the estimate $\|u_m\|_{W^1_{2,0}(Q)} \le c$, $\forall m = 1,2,...$, holds i.e. the sequence is uniformly bounded in the norm of the space $W^1_{2,0}(Q)$. Then it follows from the embedding theorem [see [15], p. 106] that, from sequence $\{u_m\}$ can be chosen a sequence (we also denote it by $\{u_m\}$) that

$$u_m \to u \quad \text{strongly in} \;\; L_2(Q), \tag{9}$$

$$\frac{\partial u_m}{\partial t} \to \frac{\partial u}{\partial t}, \frac{\partial u_m}{\partial x_i} \to \frac{\partial u_m}{\partial x_i}, i = \overline{1,n} \;\; \text{weakly in} \;\; L_2(Q), \tag{10}$$

where $u = u(x,t) \in U$ is some element.

We show that $u(x,t) = u(x,t;v)$, i.e. the function $u(x,t)$ is a solution of problem (1), (2) corresponding to the control $v \in V$. It is clear that, the identities

$$\int_Q \left[-\frac{\partial u_m}{\partial t}\frac{\partial \eta}{\partial t} + \sum_{i=1}^n \frac{\partial u_m}{\partial x_i}\frac{\partial \eta}{\partial x_i} + v_m u_m \eta \right] dxdt -$$

$$- \int_\Omega u_1(x)\eta(x,0)dx = \int_Q f(x,t,u_m)\eta dxdt \tag{11}$$

are true for all $\eta \in U$ which are equal to zero at $t = T$.

For any $\eta \in U$ the following inequality is true:

$$\left| \int_Q v_m u_m \eta dxdt - \int_Q v u \eta dxdt \right| \le \left| \int_Q (v_m - v) u \eta dxdt \right| + \left| \int_Q v_m (u_m - u) \eta dxdt \right|.$$

Since $u, \eta \in U$ and $n \le 4$ by embedding theorem [see [13], pp. 83-84] u, $\eta \in C([0,T]; L_4(\Omega))$, therefore $u\eta \in L_2(Q)$. Take into account this inclusion, the boundedness of the sequence $\{v_m, \eta\}$ in $L_2(Q)$, and also the above established convergence sequences $\{v_m\}$ and $\{u_m\}$, weakly $L_2(\Omega)$ and strongly in $L_2(Q)$, respectively we establish from the last inequality that

$$\lim_{m\to\infty} \int_Q v_m u_m \eta dxdt = \int_Q v u \eta dxdt. \tag{12}$$

Since the function $f(x,t,u)$ has a bounded derivative with respect to u, it satisfies the Lipschitz condition with respect to the argument u. Therefore

$$|f(x,t,u)| \le L|u| + |f(x,t,0)|,$$

where $L > 0$ is the Lipschitz constant. Then it follows that the operator $Fu = f(x,t,u(x,t))$ generated by the function $f(x,t,u)$ acts continuously from $L_2(Q)$ to $L_2(Q)$ [16]. Therefore, the following relation is true:

$$\lim_{m\to\infty} \int_Q f(x,t,u_m)\eta dxdt = \int_Q f(x,t,u)\eta dxdt. \tag{13}$$

Passing to the limit in (11) as $m \to \infty$ and using (9), (10), (12), (13), we find that the function $u(x,t)$ is equal to $u_0(x)$ for $t = 0$ and satisfies the identity (8). From this and the uniqueness of the solution of problem (1), (2) corresponding to the control $v \in V$ imply that $u(x,t) = u(x,t;v)$. Thus, it follows from this that the first term in (6) is weakly continuous in $L_2(\Omega)$ on the set V . The second term in the expression of the functional is weakly lower semicontinuous in $L_2(\Omega)$. Therefore, functional (6) is weakly lower semicontinuous on the set V. Then, by virtue of Theorem 2 from [see [17], p. 49], we obtain that all the statements of Theorem 1 are true. Theorem 1 is proved.

4. The differentiability of functional (6) and the necessary condition of optimality in problem (1), (2), (4), (6)

Now we study the Frechet differentiability of functional (6) and establish the necessary condition of optimality in problem (1), (2), (4), (6).

Let $\psi = \psi(x,t;v)$ is a generalized solution from the U of adjoint problem

$$\frac{\partial^2\psi}{\partial t^2} - \Delta\psi + v\psi = \frac{\partial f(x,t,u)}{\partial u}\psi - K(x,t)\left[\int_0^T K(x,t)u(x,t)dt - \varphi(x)\right], (x,t) \in Q, \quad (14)$$

$$\psi = 0, (x,t) \in S, \left.\psi\right|_{t=T} = 0, \left.\frac{\partial\psi}{\partial t}\right|_{t=T} = 0, x \in \Omega. \quad (15)$$

By a generalized solution of the boundary value problem (14), (15) for a given $v \in V$, we mean a function $\psi = \psi(x,t;v)$ from U that is equal to zero for $t = T$ and satisfying the integral identity

$$\begin{gathered}\int_Q \left[-\frac{\partial\psi}{\partial t}\frac{\partial g}{\partial t} + \sum_{i=1}^n \frac{\partial\psi}{\partial x_i}\frac{\partial g}{\partial x_i} + v\psi g\right]dxdt = \\ = \int_Q \left\{\frac{\partial f(x,t,u)}{\partial u}\psi - K(x,t)\left[\int_0^T K(x,t)u(x,t)dt - \varphi(x)\right]\right\} gdxdt\end{gathered} \quad (16)$$

for all $g \in U$ which is equal to zero for $t = 0$.

From the results of [see [13], p. 209-215] it follows that under the above assumptions, for each given $v \in V$ problem (14), (15) has a unique generalized solution from U and the estimate is true

$$\|\psi\|_{\overset{\circ}{W}{}_2^1(\Omega)} + \left\|\frac{\partial\psi}{\partial t}\right\|_{L_2(\Omega)} \le c\left[\|u\|_{L_2(Q)} + \|\varphi\|_{L_2(\Omega)}\right], t \in [0,T].$$

Taking into account (7), we obtain

$$\begin{gathered}\|\psi\|_{\overset{\circ}{W}{}_2^1(\Omega)} + \left\|\frac{\partial\psi}{\partial t}\right\|_{L_2(\Omega)} \le \\ c\left[\|u_0\|_{\overset{\circ}{W}{}_2^1(\Omega)} + \|u_1\|_{L_2(\Omega)} + \|f(x,t,0)\|_{L_2(Q)} + \|\varphi\|_{L_2(\Omega)}\right], t \in [0,T].\end{gathered} \quad (17)$$

Theorem 2. *Suppose that the conditions of Theorem 1 are satisfied. Then functional (6) is continuously differentiable in V by Frechet and its differential at a point $v \in V$ in with increments $\delta v \in L_\infty(\Omega)$ is determined by the expression*

$$\left\langle J'_\alpha(v), \delta v \right\rangle = \int\limits_\Omega \left[\alpha v + \int\limits_0^T u\psi dt \right] \delta v(x) dx. \tag{18}$$

Proof. Let the increment $\delta v \in L_\infty(\Omega)$ of the element $v \in V$ be $v + \delta v \in V$.

Denote by $\delta u(x,t) \equiv u(x,t; v+\delta v) - u(x,t;v)$. It's clear that the function $\delta u(x,t)$ is a generalized solution from U of the boundary value problem

$$\frac{\partial^2 \delta u}{\partial t^2} - \Delta \delta u + (v+\delta v)\delta u = -u\delta v + \left[f(x,t,u+\delta u) - f(x,t,u) \right], (x,t) \in Q, \tag{19}$$

$$\delta u = 0, (x,t) \in S, \, \delta u|_{t=0} = 0, \, \left.\frac{\partial \delta u}{\partial t}\right|_{t=0} = 0, x \in \Omega. \tag{20}$$

The generalized solution U of the problem (19), (20) from is equal to zero for $t = 0$ and satisfies the identity

$$\begin{gathered} \int\limits_Q \left[\frac{\partial \delta u}{\partial t}\frac{\partial \eta}{\partial t} - \sum_{i=1}^n \frac{\partial \delta u}{\partial x_i}\frac{\partial \eta}{\partial x_i} - (v+\delta v)\delta u \eta \right] dxdt = \\ = \int\limits_Q u\eta\delta v dxdt - \int\limits_Q \left[f(x,t,u+\delta u) - f(x,t,u) \right] \eta dxdt \end{gathered} \tag{21}$$

for all $\eta = \eta(x,t)$ from U which equal to zero for $t = T$. Applying the Faedo- Galerkin method and taking into account that the function $f(x,t,u)$ satisfies the Lipschitz condition with respect to the argument, we can obtain the estimate for the solution of the problem (19), (20)

$$\|\delta u\|_{\overset{\circ}{W}{}_2^1(\Omega)} + \left\| \frac{\partial \delta u}{\partial t} \right\|_{L_2(\Omega)} \le c \|\delta v\|_{L_\infty(\Omega)}, t \in [0,T]. \tag{22}$$

We consider the increment of functional (6):

$$\begin{gathered} \Delta J_\alpha(v) = J_\alpha(v+\delta v) - J_\alpha(v) = \int\limits_\Omega \alpha v \delta v dx + \frac{\alpha}{2}\int\limits_\Omega |\delta v|^2 dx + \\ + \int\limits_\Omega \left[\int\limits_0^T K u dt - \varphi(x) \right] \int\limits_0^T K\delta u dt dx + \frac{1}{2}\int\limits_\Omega \left[\int\limits_0^T K\delta u dt \right]^2 dx. \end{gathered} \tag{23}$$

If we take $g = \delta u(x,t)$ in (16) and take $\eta = \psi(x,t;v)$ in (21) and summing the obtained relations we get:

$$\begin{gathered} \int\limits_Q K(x,t) \left[\int\limits_0^T K u dt - \varphi(x) \right] \delta u dxdt = \int\limits_Q u\psi\delta v dxdt + \int\limits_Q \psi \delta u \delta v dxdt + \\ + \int\limits_Q \frac{\partial f(x,t,u)}{\partial u} \psi \delta u dxdt - \int\limits_Q [f(x,t,u+\delta u) - f(x,t,u)]\psi dxdt. \end{gathered}$$

Taking into account this equality in (23), we obtain

$$\Delta J_\alpha(v) = \int_\Omega \left[\alpha v + \int_0^T u\psi dt\right]\delta v dx + R, \tag{24}$$

where $R = \sum_{i=1}^{4} R_i$ is the remainder term and

$$R_1 = \frac{\alpha}{2}\int_\Omega |\delta v|^2 dx,\ R_2 = \frac{1}{2}\int_\Omega \left[\int_0^T K\delta u dt\right]^2 dx,\ R_3 = \int_Q \psi\delta u\delta v dx dt,$$

$$R_4 = \int_Q \left\{\frac{\partial f(x,t,u)}{\partial u}\delta u - [f(x,t,u+\delta u) - f(x,t,u)]\right\}\psi dx dt.$$

The first term in the right-hand side of (24) is a linear bounded functional in $L_2(Q)$.

Now we estimate the remainder term of or R in (24).

First, we estimate the fourth term in the expression R.

By the Lagrange mean value theorem, we have

$$R_4 = \int_Q \left[\frac{\partial f(x,t,u)}{\partial u} - \frac{\partial f(x,t,u+\theta\delta u)}{\partial u}\right]\delta u\psi dx dt,$$

where $0 \le \theta \le 1$.

Since the operator $\frac{\partial f(x,t,u(x,t))}{\partial u}$ acts continuously from $L_2(Q)$ to $L_2(Q)$ with respect to u, from estimation (22) it follows that $\frac{\partial f(x,t,u(x,t))}{\partial u} - \frac{\partial f(x,t,u(x,t)+\theta\delta u(x,t))}{\partial u} \to 0$ strongly in $L_2(Q)$ as $\|\delta v\|_{L_\infty(\Omega)} \to 0$. Therefore, from this and estimation (22) it follows that $R_4 = 0\left(\|\delta v\|_{L_\infty(\Omega)}\right)$. Then again using estimation (22), we obtain

$$|R| \le \left|\sum_{i=1}^{4} R_i\right| \le c\left[\|\delta v\|^2_{L_\infty(\Omega)} + \|\delta u\|^2_{L_2(Q)} + \|\psi\|_{L_2(Q)}\|\delta u\|_{L_2(Q)}\|\delta v\|_{L_\infty(\Omega)}\right] +$$
$$+|R_4| \le c\|\delta v\|^2_{L_\infty(\Omega)} + |R_4|.$$

Therefore $R = 0\left(\|\delta v\|_{L_\infty(\Omega)}\right)$. Then it follows from (24) that functional (6) is differentiable by Freshet in V and formula (18) is valid.

We show that the map $v \to J'_\alpha(v)$ defined by (18) acts continuously from V to $(L_\infty(\Omega))^*$, where $(L_\infty(\Omega))^*$ is adjoint of $L_\infty(\Omega)$.

Let $\delta\psi(x,t) = \psi(x,t;v+\delta v) - \psi(x,t;v)$. It follows from (14), (15), that $\delta\psi(x,t)$ is a generalized solution from U of the boundary value problem

$$\begin{gathered}\frac{\partial^2\delta\psi}{\partial t^2} - \Delta\delta\psi + (v+\delta v)\delta\psi - \frac{\partial f(x,t,u+\delta u)}{\partial u}\delta\psi = -\psi\delta v + \\ + \left[\frac{\partial f(x,t,u+\delta u)}{\partial u} - \frac{\partial f(x,t,u)}{\partial u}\right]\psi - K(x,t)\int_0^T K\delta u dt, (x,t) \in Q,\end{gathered} \tag{25}$$

$$\delta\psi = 0, (x,t) \in S,\ \delta\psi|_{t=T} = 0,\ \left.\frac{\partial \delta\psi}{\partial t}\right|_{t=T} = 0, x \in \Omega. \tag{26}$$

Using the results of [[13], p. 209-215], it can be shown that the following estimation is true for the solution of the problem (25), (26)

$$\|\delta\psi\|_{\overset{\circ}{W}{}_2^1(\Omega)} + \left\|\frac{\partial \delta\psi}{\partial t}\right\|_{L_2(\Omega)} \leq c\,\|\delta v\|_{L_\infty(\Omega)}, t \in [0,T]. \tag{27}$$

In addition, using (18) we obtain

$$\begin{aligned}&\|J'_\alpha(v+\delta v) - J'_\alpha(v)\|_{(L_\infty(\Omega))^*} \leq c\left[\|\delta v\|_{L_\infty(\Omega)} + \|u\|_{L_2(Q)}\|\delta\psi\|_{L_2(Q)} + \right.\\ &\left. + \|\psi\|_{L_2(Q)}\|\delta u\|_{L_2(Q)} + \|\delta u\|_{L_2(Q)}\|\delta\psi\|_{L_2(Q)}\right].\end{aligned}$$

By virtue of (22), (27), the right-hand side of this inequality tends to zero as $\|\delta v\|_{L_\infty(\Omega)} \to 0$.

It follows that $v \to J^*_\alpha(v)$ is a continuous map from V to $(L_\infty(\Omega))^*$. Theorem 2 is proved.

Theorem 3. *Let the conditions of Theorem 2 be satisfied. Then, for the optimality of control $v_* = v_*(x) \in V$ in problem (1), (2), (4), (6), it is necessary that the inequality*

$$\int_\Omega \left[\alpha v_*(x) + \int_0^T u_*(x,t)\psi_*(x,t)dt\right](v(x) - v_*(x))dx \geq 0 \tag{28}$$

holds for any control $v = v(x) \in V$ here $u_*(x,t) = u(x,t;v_*)$, $\psi_*(x,t) = \psi(x,t;v_*)$ are the solutions of problems (1), (2) and (14), (15), respectively, for $v_* = v_*(x)$.

Proof. The set V defined by relation (4) is convex in $L_\infty(\Omega)$. In addition, by Theorem 2, the functional $J_\alpha(v)$ is continuously differentiable by Frechet on V and its differential at the point $v \in V$ is determined by equality (18). Then, by virtue of Theorem 5 of [[17], p. 28] inequality $\langle J'_\alpha(v_*), v - v_*\rangle \geq 0 \forall v \in V$ must be satisfied on the element $v_* \in V$. From here and (18) follows the validity of inequality (28). Theorem 3 is proved.

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