Modelling and simulating a transmission of Covid-19 disease: Niger Republic case

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Abstract. This paper focuses on the dynamics of spreads of a coronavirus disease (Covid-19). Through this paper, we study the impact of a contact rate in the transmission of the disease. We determine the basic reproductive number R_0, by using the next generation matrix method. We also determine the Disease Free Equilibrium and Endemic Equilibrium points of our model. We prove that the Disease Free Equilibrium is asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$. The asymptotical stability of Endemic Equilibrium is also establish. Numerical simulations are made to show the impact of contact rate in the spread of disease.

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1. Introduction

This paper present a model of the new disease caused by a SRAS-Cov-2 Virus. The coronavirus disease (Covid-19) start officialy on December 31th, 2019 in Wuhan (Republic of China) [8] and spread all over the world. As of February 14th, 2020 World Health Organization (WHO) has confirmed the first coronavirus disease (Covid-19) case in Egypt, after official confirmation by Egyptian Ministry of Health and population [6], it is the first case in Africa. Because of the rapid spread of the virus on a global scale, the World Health Organization (WHO) declared Covid-19 a pandemic on March 11th, 2020 [7]. As April 30th, 2020, there were 3,229,814 confirmed cases worldwide, including 1,006,987 people recovered and 228,376 deaths [11]. Republic of Niger registered its first confirmed case on Thursday March 19th, 2020 [1]. As of this date, Niger officially has 713 confirmed cases, including 435 people recovered and 32 deaths [11]. In order to overcome the spread of the disease, government and scientists proposed several strategies. Our model takes account the contact rate, a major factor in the spreading of disease. We analyse and

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propose measures to government to help them in their decision-making in the response to the disease.

The paper is organised as follow: second section is devoted to model formulation, in the third section we are interested in a Disease Free Equilibrium point and reproductive number R_0, fourth section is dedicated to analysis of the stability of Disease Free Equilibrium, in section five we are interested in the analysis of the stability of Endemic Equilibrium, in section six a Real-World implementation and numerical simulations for Niger Republic will be presented and we end this work by a conclusion.

2. Model Formulation

The total population $N(t)$ is divided in to seven compartments corresponding to theirs epidemiological status namely Susceptible individuals $S(t)$, Exposed individuals $E(t)$, Infectious individuals $I(t)$, Infectious asymptomatic individuals $I_a(t)$ (people who are infected but do not develop disease and have no symptoms), Infectious symptomatic individuals $I_s(t)$ (people infected who develop disease and have a symptoms), Infectious individuals in critical cases I_d and recovered individuals $R(t)$ (infected people who develop disease with symptoms and are in critical conditions requiring respirator assistance). Thus the total population size is given by: $N(t) = S(t) + E(t) + I(t) + I_a(t) + I_s(t) + I_d(t) + R(t)$. The parameters used are described as follow :

- μ natural mortality rate.

- μ_1 Covid-19 related mortality rate.

- β a contact rate.

- α rate moving from exposed to infectious.

- α_1 rate moving from infectious to asymptomatic infectious.

- α_2 rate moving from infectious to symptomatic infectious.

- γ rate moving from asymptomatic infectious to recovered.

- θ_1 rate moving from symptomatic infectious to recovered.

- θ_2 rate moving from symptomatic infectious to infectious in critical cases.

- Ψ rate moving from recovered to susceptible.

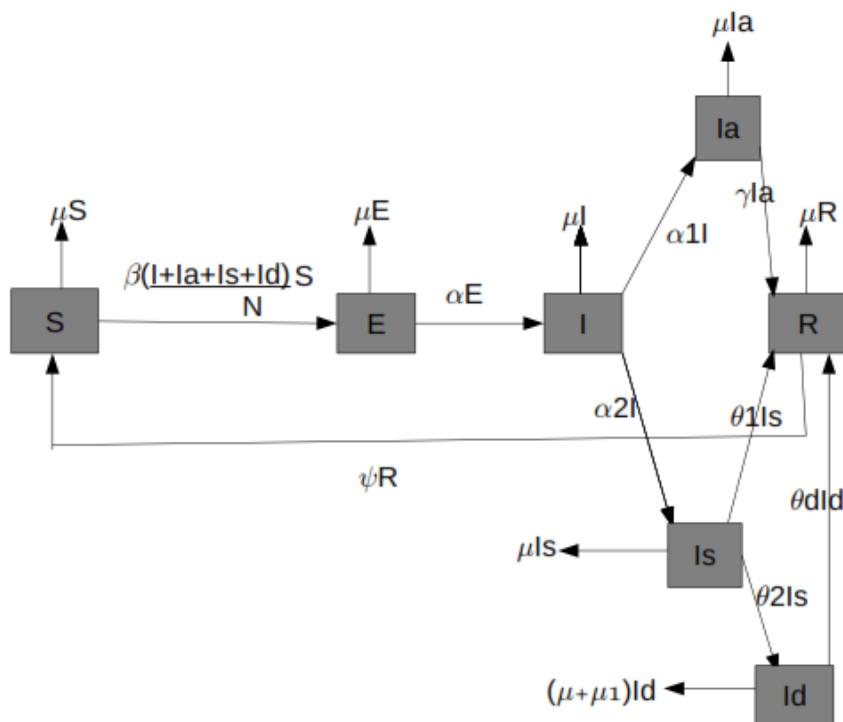The variables and model are describe in this figure:

Figure 1: Flow diagram of Covid-19 transmission dynamics

Under the above assumptions and parameters and using the compartmental analysis theory, we obtain the unique system of differential equations that described the dynamic of disease as follow:

$$
\begin{cases}
\frac{\mathrm{d}S(t)}{\mathrm{d}t} = \psi R - \beta \frac{(I(t)+I_a(t)+I_s(t)+I_d(t))}{N(t)} S(t) - \mu S(t) \\[2mm]
\frac{\mathrm{d}E(t)}{\mathrm{d}t} = \beta \frac{(I(t)+I_a(t)+I_s(t)+I_d(t))}{N(t)} S(t) - \alpha E(t) - \mu E(t) \\[2mm]
\frac{\mathrm{d}I(t)}{\mathrm{d}t} = \alpha E(t) - \alpha_1 I(t) - \alpha_2 I(t) - \mu I(t) \\[2mm]
\frac{\mathrm{d}I_a(t)}{\mathrm{d}t} = \alpha_1 I(t) - \gamma I_a(t) - \mu I_a(t) \\[2mm]
\frac{\mathrm{d}I_s(t)}{\mathrm{d}t} = \alpha_2 I(t) - \theta_1 I_s(t) - \theta_2 I_s(t) - \mu I_s(t) \\[2mm]
\frac{\mathrm{d}I_d(t)}{\mathrm{d}t} = \theta_2 I_s - \theta_d I_d(t) - \mu_1 I_d(t) - \mu I_d(t) \\[2mm]
\frac{\mathrm{d}R(t)}{\mathrm{d}t} = \gamma I_a(t) + \theta_1 I_s(t) + \theta_d I_d(t) - \psi R(t) - \mu R(t)
\end{cases}
\tag{1}
$$

with initial conditions: $S(0) \geqslant 0, \quad E(0) \geqslant 0, \quad I(0) \geqslant 0, \quad I_0(0) \geqslant 0, \quad I_s(0) \geqslant 0, \quad I_d(0) \geqslant 0, \quad R(0) \geqslant 0.$

3. Disease Free Equilibrium and reproductive number R_0

The Disease Free Equilibrium of a model (1) is determined by solving the following system:

$$\begin{cases} \frac{\mathrm{d}S(t)}{\mathrm{d}t} = 0 \\ \frac{\mathrm{d}E(t)}{\mathrm{d}t} = 0 \\ \frac{\mathrm{d}I(t)}{\mathrm{d}t} = 0 \\ \frac{\mathrm{d}I_a(t)}{\mathrm{d}t} = 0 \\ \frac{\mathrm{d}I_s(t)}{\mathrm{d}t} = 0 \\ \frac{\mathrm{d}I_d(t)}{\mathrm{d}t} = 0 \end{cases} \tag{2}$$

see [[9],[2]]. The solution of (2), in the case of absence of disease (this means that all compartments is equal to zero except susceptible compartment) and after normalization we have: $E^0 = (1, 0, 0, 0, 0, 0)$.

We determine the basic reproductive number by using the next generation matrix method [[4], [10]] at Disease Free Equilibrium. The basic reproductive number R_0 is defined as the effective number of secondary infections caused by typical infected individual during his/her entire period of infectioussness [[5],[3]]. Let X be vector of infected classes:

$$X = (E, I, I_a, I_s, I_d)^t. \tag{3}$$

$$F = \left(\beta \frac{S(I + I_a + I_s + I_d)}{N}, 0, 0, 0, 0 \right)^t \tag{4}$$

denotes the vector of terms corresponding to new infection.

$$V = \begin{pmatrix} \mu E + \alpha E \\ \mu I + \alpha_1 I + \alpha_2 I + \alpha E \\ \mu I_a + \gamma I_a - \alpha_1 I \\ \mu I_s + \theta_1 I_s + \theta_2 I_s - \alpha_2 I \\ (\mu + \mu_1) I_d + \theta_d I_d - \theta_2 I_s \end{pmatrix} \tag{5}$$

denotes the vector of terms corresponding to individuals entering a given compartment and individuals going out.

The partial derivatives of F and V with respect to E, I, I_a, I_s and I_d are given by the following matrices $\mathcal{F}$ and $\mathcal{V}$:

$$\mathcal{F} = \begin{pmatrix} 0 & \beta & \beta & \beta & \beta \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \mathcal{V} = \begin{pmatrix} \mu + \alpha & 0 & 0 & 0 & 0 \\ -\alpha & \mu + \alpha_1 + \alpha_2 & 0 & 0 & 0 \\ 0 & -\alpha_1 & \mu + \gamma & 0 & 0 \\ 0 & -\alpha_2 & 0 & \mu + \theta_1 + \theta_2 & 0 \\ 0 & 0 & 0 & -\theta_2 & \mu + \mu_1 + \theta_d \end{pmatrix} \tag{6}$$

then we have $\mathcal{V}^{-1}$ given by:

$$
\mathcal{V}^{-1} = \begin{pmatrix}
\frac{1}{\mu+\alpha} & 0 & 0 & 0 & 0 \\
\frac{\alpha}{(\mu+\alpha)(\mu+\alpha_1+\alpha_2)} & \frac{1}{\mu+\alpha_1+\alpha_2} & 0 & 0 & 0 \\
\frac{\alpha\alpha_1}{(\mu+\alpha)(\mu+\alpha_1+\alpha_2)(\mu+\gamma)} & \frac{\alpha_1}{(\mu+\alpha_1+\alpha_2)(\mu+\gamma)} & \frac{1}{\mu+\gamma} & 0 & 0 \\
\frac{\alpha\alpha_2}{(\mu+\alpha)(\mu+\alpha_1+\alpha_2)(\mu+\theta_1+\theta_2)} & \frac{\alpha_2}{(\mu+\alpha_1+\alpha_2)(\mu+\theta_1+\theta_2)} & 0 & \frac{1}{\mu+\theta_1+\theta_2} & 0 \\
\frac{\alpha\alpha_2\theta_2}{(\mu+\alpha)(\mu+\alpha_1+\alpha_2)(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)} & \frac{\alpha_2\theta_2}{(\mu+\alpha_1+\alpha_2)(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)} & 0 & \frac{\theta_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)} & \frac{1}{\mu+\mu_1+\theta_d}
\end{pmatrix}.
$$
(7)

Therefore

$$
\mathcal{F}\mathcal{V}^{-1} = \begin{pmatrix}
K & L & M & N & P \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$
(8)

where:

$$
\begin{aligned}
K &= \beta\left(\frac{\alpha}{(\mu+\alpha)(\mu+\alpha_1+\alpha_2)} + \frac{\alpha\alpha_1}{(\mu+\alpha)(\mu+\alpha_1+\alpha_2)(\mu+\gamma)} + \frac{\alpha\alpha_2}{(\mu+\alpha)(\mu+\alpha_1+\alpha_2)(\mu+\theta_1+\theta_2)}\right) + \\
&\quad \beta\left(\frac{\alpha\alpha_2\theta_2}{(\mu+\alpha)(\mu+\alpha_1+\alpha_2)(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}\right)
\end{aligned}
$$
(9)

$$
\begin{aligned}
L &= \beta\left(\frac{1}{\mu+\alpha_1+\alpha_2} + \frac{\alpha_1}{(\mu+\alpha_1+\alpha_2)(\mu+\gamma)} + \frac{\alpha_2}{(\mu+\alpha_1+\alpha_2)(\mu+\theta_1+\theta_2)}\right) + \\
&\quad \beta\left(\frac{\alpha_2\theta_2}{(\mu+\alpha_1+\alpha_2)(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}\right)
\end{aligned}
$$
(10)

$$
M = \beta\left(\frac{1}{\mu+\gamma}\right)
$$
(11)

$$
N = \beta\left(\frac{1}{\mu+\theta_1+\theta_2} + \frac{\theta_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}\right)
$$
(12)

$$
P = \beta\left(\frac{1}{\mu+\mu_1+\theta_d}\right).
$$
(13)

A spectral radius of $\mathcal{F}\mathcal{V}^{-1}$ give us the expression of R_0. It follow that the basic reproductive number is given by:

$$
R_0 = \frac{\beta\alpha}{(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}\left[1 + \frac{\alpha_1}{\mu+\gamma} + \frac{\alpha_2}{\mu+\theta_1+\theta_2} + \frac{\alpha_2\theta_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}\right]. (14)
$$

4. Stability analysis of Disease Free Equilibrium

Theorem 1. *The Disease Free Equilibrium is asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.*

Proof. We use the Jacobian matrix of system (1) at Disease Free Equilibrium (DFE) to determine the stability. The Jacobian matrix of the system is given by:

$$J_{(s,e,i,i_a,i_s,i_d)} = \begin{pmatrix} -\mu-\beta(i+i_a+i_s+i_d) & 0 & -\beta s & -\beta s & -\beta s & -\beta s \\ \beta(i+i_a+i_s+i_d) & -\mu-\alpha & \beta s & \beta s & \beta s & \beta s \\ 0 & \alpha & -\mu-\alpha_1-\alpha_2 & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & -\mu-\gamma & 0 & 0 \\ 0 & 0 & \alpha_2 & 0 & -\mu-\theta_1-\theta_2 & 0 \\ 0 & 0 & 0 & 0 & \theta_2 & -\mu-\mu_1-\theta_d \end{pmatrix}. \quad (15)$$

We obtain at DFE:

$$J_{(1,0,0,0,0,0)} = \begin{pmatrix} -\mu & 0 & -\beta & -\beta & -\beta & -\beta \\ 0 & -\mu-\alpha & \beta & \beta & \beta & \beta \\ 0 & \alpha & -\mu-\alpha_1-\alpha_2 & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & -\mu-\gamma & 0 & 0 \\ 0 & 0 & \alpha_2 & 0 & -\mu-\theta_1-\theta_2 & 0 \\ 0 & 0 & 0 & 0 & \theta_2 & -\mu-\mu_1-\theta_d \end{pmatrix}. \quad (16)$$

The characteristic polynomial of (16) is:

$$P_\lambda = (\mu+\lambda)(\lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda + a_5), \quad (17)$$

with:

$$a_1 = 5\mu + \mu_1 + \alpha + \alpha_1 + \alpha_2 + \gamma + \theta_1 + \theta_2 + \theta_d > 0, \quad (18)$$

$$a_2 = (\mu+\alpha)(\mu+\alpha_1+\alpha_2) + (2\mu+\alpha+\alpha_1+\alpha_2)(2\mu+\gamma+\theta_1+\theta_2) + (2\mu+\alpha+\alpha_1+\alpha_2)\times$$
$$(\mu+\mu_1+\theta_d) + (\mu+\gamma)(\mu+\theta_1+\theta_2) + (2\mu+\gamma+\theta_1+\theta_2)(\mu+\mu_1+\theta_d) + \alpha\beta > 0 \quad (19)$$

$$a_3 = (\mu+\alpha)(\mu+\alpha_1+\alpha_2)(2\mu+\gamma+\theta_1+\theta_2) + (\mu+\alpha)(\mu+\alpha_1+\alpha_2)(\mu+\mu_1+\theta_d) + (\mu+\gamma)\times$$
$$(\mu+\theta_1+\theta_2)(2\mu+\alpha+\alpha_1+\alpha_2) + (2\mu+\alpha+\alpha_1+\alpha_2)(2\mu+\gamma+\theta_1+\theta_2)(\mu+\mu_1+\theta_d) + (\mu+\gamma)\times$$
$$(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d) + \alpha\beta(3\mu+\mu_1+\gamma+\theta_1+\theta_2+\theta_d-\alpha_1-1) > 0 \quad (20)$$

$$a_4 = (\mu+\alpha)(\mu+\alpha_1+\alpha_2)(\mu+\gamma)(\mu+\theta_1+\theta_2) + (\mu+\alpha)(\mu+\alpha_1+\alpha_2)(\mu+\mu_1+\theta_d)\times$$
$$(2\mu+\gamma+\theta_1+\theta_2) + (\mu+\gamma)(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)(2\mu+\alpha+\alpha_1+\alpha_2) + \alpha\beta(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)$$
$$+ \alpha\beta(2\mu+\mu_1+\theta_1+\theta_2+\theta_d)(\mu+\gamma) - \alpha\beta\theta_2\alpha_2 - \alpha\beta\alpha_1(2\mu+\mu_1+\theta_1+\theta_2+\theta_d)$$
$$- \alpha\beta(2\mu+\mu_1+\gamma+\theta_d) > 0 \quad (21)$$

$a_5 = (\mu + \alpha)(\mu + \alpha_1 + \alpha_2)(\mu + \gamma)(\mu + \theta_1 + \theta_2)(\mu + \mu_1 + \theta_d) + \alpha\beta(\mu + \gamma)(\mu + \theta_1 + \theta_2) \times$
$(\mu + \mu_1 + \theta_d) - \alpha\beta\theta_2\alpha_2(\mu + \gamma) - \alpha\beta\alpha_1(\mu + \theta_1 + \theta_2)(\mu + \mu_1 + \theta_d) - \alpha\beta(\mu + \gamma)(\mu + \mu_1 + \theta_d) > 0.$

$$(22)$$

By using Routh Hurwitz criterion we get:
$\lambda_1 = -\mu < 0.$
$H_1 = a_1 > 0 \qquad H_2 = a_1 * a_2 - a_3 > 0 \qquad H_3 = a_3(a_1a_2 - a_3) - a_1(a_1a_4 - a_5)$

- If $R_0 < 1$ then $H_3 = a_3(a_1a_2 - a_3) - a_1(a_1a_4 - a_5) > 0.$ since $H_i > 0$ for $i = 1, 2, 3$ then the eigen values of the caracteristic polynomial are all negative, which mean that the Disease Free Equilibrium is asymptotically stable.

- If $R_0 > 1$ then $H_3 = a_3(a_1a_2 - a_3) - a_1(a_1a_4 - a_5) < 0.$ So that at least one of the eigen values of the caracteristic polynomial is positive. This prove that the Disease Free Equilibrium is unstable.

5. Stability analysis of Endemic Equilibrium

Theorem 2. *The system* (1) *has an unique Endemic Equilibrium* E^* *if* $R_0 > 1.$

Proof. The Endemic Equilibrium of a model (1) is determined by solving:

$$
\begin{cases}
\frac{dS(t)}{dt} = 0 \\
\frac{dE(t)}{dt} = 0 \\
\frac{dI(t)}{dt} = 0 \\
\frac{dI_a(t)}{dt} = 0 \\
\frac{dI_s(t)}{dt} = 0 \\
\frac{dI_d(t)}{dt} = 0
\end{cases}
. \qquad (23)
$$

So we obtain an Endemic Equilibrium given by $E^* = (s^*, e^*, i^*, i_a^*, i_s^*, i_d^*) \neq (0, 0, 0, 0, 0, 0)$ see [[9],[2]], where:

$$s^* = \frac{1}{R_0} \qquad (24)$$

$$e^* = \frac{\mu + \alpha_1 + \alpha_2}{\alpha} \times \frac{\frac{\mu}{R_0}}{\frac{\Psi}{\mu+\Psi}\left[\frac{\alpha_1\gamma}{\mu+\gamma} + \frac{\alpha_2\theta_1}{\mu+\theta_1+\theta_2} + \frac{\theta_2\theta_d\alpha_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}\right] - \frac{(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}{\alpha}} \qquad (25)$$

$$i^* = \frac{\frac{\mu}{R_0}}{\frac{\Psi}{\mu+\Psi}\left[\frac{\alpha_1\gamma}{\mu+\gamma} + \frac{\alpha_2\theta_1}{\mu+\theta_1+\theta_2} + \frac{\theta_2\theta_d\alpha_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}\right] - \frac{(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}{\alpha}} \qquad (26)$$

$$i_a^* = \frac{\alpha_1}{\mu+\gamma} \times \frac{\frac{\mu}{R_0}}{\frac{\Psi}{\mu+\Psi}[\frac{\alpha_1\gamma}{\mu+\gamma} + \frac{\alpha_2\theta_1}{\mu+\theta_1+\theta_2} + \frac{\theta_2\theta_d\alpha_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}] - \frac{(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}{\alpha}} \tag{27}$$

$$i_s^* = \frac{\alpha_2}{\mu+\theta_1+\theta_2} \times \frac{\frac{\mu}{R_0}}{\frac{\Psi}{\mu+\Psi}[\frac{\alpha_1\gamma}{\mu+\gamma} + \frac{\alpha_2\theta_1}{\mu+\theta_1+\theta_2} + \frac{\theta_2\theta_d\alpha_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}] - \frac{(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}{\alpha}} \tag{28}$$

$$i_d^* = \frac{\alpha_2\theta_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)} \times \frac{\frac{\mu}{R_0}}{\frac{\Psi}{\mu+\Psi}[\frac{\alpha_1\gamma}{\mu+\gamma} + \frac{\alpha_2\theta_1}{\mu+\theta_1+\theta_2} + \frac{\theta_2\theta_d\alpha_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}] - \frac{(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}{\alpha}}$$
$$\tag{29}$$

Theorem 3. *The Endemic Equilibrium E^* is asymptotically stable if $R_0 > 1$.*

Proof. We evaluate the jacobian matrix given by (15) at Endemic Equilibrium E^* and we obtain:

$$J_{(s^*,e^*,i^*,i_a^*,i_s^*,i_d^*)} = \begin{pmatrix} -\mu-i^*\frac{R_0(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}{\alpha} & 0 & -\beta s^* & -\beta s^* & -\beta s^* & -\beta s^* \\ i^*\frac{R_0(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}{\alpha} & -\mu-\alpha & \beta s^* & \beta s^* & \beta s^* & \beta s^* \\ 0 & \alpha & -\mu-\alpha_1-\alpha_2 & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & -\mu-\gamma & 0 & 0 \\ 0 & 0 & \alpha_2 & 0 & -\mu-\theta_1-\theta_2 & 0 \\ 0 & 0 & 0 & 0 & \theta_2 & -\mu-\mu_1-\theta_d \end{pmatrix} \tag{30}$$

The characteristic polynomial of matrix (30) is:

$$P_\lambda^* = \lambda^6 + a_1\lambda^5 + a_2\lambda^4 + a_3\lambda^3 + a_4\lambda^2 + a_5\lambda + a_6 \tag{31}$$

with:

$$a_1 = 6\mu+\mu_1+\gamma+\alpha+\alpha_1+\alpha_2+\theta_1+\theta_2+\theta_d+\frac{\mu}{\frac{\alpha\Psi}{(\mu+\Psi)(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}[\frac{\alpha_1\gamma}{\mu+\gamma} + \frac{\alpha_2\theta_1}{\mu+\theta_1+\theta_2} + \frac{\theta_2\theta_d\alpha_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}] - 1} > 0$$
$$\tag{32}$$

$$a_2 = (\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d) + (2\mu+\gamma+\alpha_1+\alpha_2)(2\mu+\mu_1+\theta_1+\theta_2+\theta_d) + (\mu+\alpha_1+\alpha_2) \times$$
$$(\mu+\gamma) + (2\mu+\alpha+\frac{\mu}{\frac{\alpha\Psi}{(\mu+\Psi)(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}[\frac{\alpha_1\gamma}{\mu+\gamma} + \frac{\alpha_2\theta_1}{\mu+\theta_1+\theta_2} + \frac{\theta_2\theta_d\alpha_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}] - 1})(2\mu+\mu_1+\theta_1+\theta_2+\theta_d)$$
$$+ (2\mu+\alpha+\frac{\mu}{\frac{\alpha\Psi}{(\mu+\Psi)(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}[\frac{\alpha_1\gamma}{\mu+\gamma} + \frac{\alpha_2\theta_1}{\mu+\theta_1+\theta_2} + \frac{\theta_2\theta_d\alpha_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}] - 1})(2\mu+\gamma+\alpha_1+\alpha_2)$$
$$+ (\mu+\frac{\mu}{\frac{\alpha\Psi}{(\mu+\Psi)(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}[\frac{\alpha_1\gamma}{\mu+\gamma} + \frac{\alpha_2\theta_1}{\mu+\theta_1+\theta_2} + \frac{\theta_2\theta_d\alpha_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}] - 1})(\mu+\alpha) - \frac{\alpha\beta}{R_0} > 0$$
$$\tag{33}$$

$$a_3 = (2\mu+\gamma+\alpha_1+\alpha_2)(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d) + (\mu+\alpha_1+\alpha_2)(\mu+\gamma) \times$$

$$(2\mu+\mu_1+\theta_1+\theta_2+\theta_d)+(2\mu+\alpha+\frac{\mu}{\frac{\alpha\Psi}{(\mu+\Psi)(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}[\frac{\alpha_1\gamma}{\mu+\gamma}+\frac{\alpha_2\theta_1}{\mu+\theta_1+\theta_2}+\frac{\theta_2\theta_d\alpha_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}]-1})\times$$

$$(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)+(2\mu+\alpha+\frac{\mu}{\frac{\alpha\Psi}{(\mu+\Psi)(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}[\frac{\alpha_1\gamma}{\mu+\gamma}+\frac{\alpha_2\theta_1}{\mu+\theta_1+\theta_2}+\frac{\theta_2\theta_d\alpha_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}]-1})\times$$

$$(2\mu+\gamma+\alpha_1+\alpha_2)(2\mu+\mu_1+\theta_1+\theta_2+\theta_d)+(2\mu+\alpha+\frac{\mu}{\frac{\alpha\Psi}{(\mu+\Psi)(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}[\frac{\alpha_1\gamma}{\mu+\gamma}+\frac{\alpha_2\theta_1}{\mu+\theta_1+\theta_2}+\frac{\theta_2\theta_d\alpha_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}]-1})\times$$

$$(\mu+\alpha_1+\alpha_2)(\mu+\gamma)+(\mu+\frac{\mu}{\frac{\alpha\Psi}{(\mu+\Psi)(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}[\frac{\alpha_1\gamma}{\mu+\gamma}+\frac{\alpha_2\theta_1}{\mu+\theta_1+\theta_2}+\frac{\theta_2\theta_d\alpha_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}]-1})(\mu+\alpha)\times$$

$$(2\mu+\mu_1+\theta_1+\theta_2+\theta_d)+(\mu+\frac{\mu}{\frac{\alpha\Psi}{(\mu+\Psi)(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}[\frac{\alpha_1\gamma}{\mu+\gamma}+\frac{\alpha_2\theta_1}{\mu+\theta_1+\theta_2}+\frac{\theta_2\theta_d\alpha_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}]-1})(\mu+\alpha)\times$$

$$(2\mu+\gamma+\alpha_1+\alpha_2)-\frac{\alpha\beta}{R_0}(2\mu+\mu_1+\theta_1+\theta_2+\theta_d)-\frac{\alpha\beta}{R_0}(2\mu+\gamma)-\alpha_1\frac{\alpha\beta}{R_0}-\alpha_2\frac{\alpha\beta}{R_0}>0$$

$$(34)$$

$$a_4=(\mu+\alpha_1+\alpha_2)(\mu+\gamma)(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)$$

$$+(2\mu+\alpha+\frac{\mu}{\frac{\alpha\Psi}{(\mu+\Psi)(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}[\frac{\alpha_1\gamma}{\mu+\gamma}+\frac{\alpha_2\theta_1}{\mu+\theta_1+\theta_2}+\frac{\theta_2\theta_d\alpha_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}]-1})\times$$

$$(2\mu+\gamma+\alpha_1+\alpha_2)(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)+(2\mu+\alpha+\frac{\mu}{\frac{\alpha\Psi}{(\mu+\Psi)(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}[\frac{\alpha_1\gamma}{\mu+\gamma}+\frac{\alpha_2\theta_1}{\mu+\theta_1+\theta_2}+\frac{\theta_2\theta_d\alpha_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}]-1})$$

$$(2\mu+\mu_1+\theta_1+\theta_2+\theta_d)(\mu+\alpha_1+\alpha_2)(\mu+\gamma)+(\mu+\frac{\mu}{\frac{\alpha\Psi}{(\mu+\Psi)(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}[\frac{\alpha_1\gamma}{\mu+\gamma}+\frac{\alpha_2\theta_1}{\mu+\theta_1+\theta_2}+\frac{\theta_2\theta_d\alpha_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}]-1})\times$$

$$(\mu+\alpha)(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)+(\mu+\frac{\mu}{\frac{\alpha\Psi}{(\mu+\Psi)(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}[\frac{\alpha_1\gamma}{\mu+\gamma}+\frac{\alpha_2\theta_1}{\mu+\theta_1+\theta_2}+\frac{\theta_2\theta_d\alpha_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}]-1})\times$$

$$(\mu+\alpha)(2\mu+\gamma+\alpha_1+\alpha_2)(2\mu+\mu_1+\theta_1+\theta_2+\theta_d)+(\mu+\frac{\mu}{\frac{\alpha\Psi}{(\mu+\Psi)(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}[\frac{\alpha_1\gamma}{\mu+\gamma}+\frac{\alpha_2\theta_1}{\mu+\theta_1+\theta_2}+\frac{\theta_2\theta_d\alpha_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}]-1})$$

$$(\mu+\alpha)(\mu+\alpha_1+\alpha_2)(\mu+\gamma)-\alpha_2\theta_2\frac{\alpha\beta}{R_0}-\frac{\alpha\beta}{R_0}(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)-\frac{\alpha\beta}{R_0}(2\mu+\gamma)\times$$

$$(2\mu+\mu_1+\theta_1+\theta_2+\theta_d)-\frac{\alpha\beta}{R_0}\mu(\mu+\gamma)-\frac{\alpha\beta}{R_0}\alpha_2(\mu+\mu_1+\theta_d)-\frac{\alpha\beta}{R_0}\alpha_2(2\mu+\gamma)-\frac{\alpha\beta}{R_0}\alpha_1\times$$

$$(2\mu+\mu_1+\theta_1+\theta_2+\theta_d)-\frac{\alpha\beta}{R_0}\alpha_1\mu>0 \quad (35)$$

$$a_5=(2\mu+\alpha+\frac{\mu}{\frac{\alpha\Psi}{(\mu+\Psi)(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}[\frac{\alpha_1\gamma}{\mu+\gamma}+\frac{\alpha_2\theta_1}{\mu+\theta_1+\theta_2}+\frac{\theta_2\theta_d\alpha_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}]-1})(\mu+\alpha_1+\alpha_2)(\mu+\gamma)\times$$

$$(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)+(\mu+\frac{\mu}{\frac{\alpha\Psi}{(\mu+\Psi)(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}[\frac{\alpha_1\gamma}{\mu+\gamma}+\frac{\alpha_2\theta_1}{\mu+\theta_1+\theta_2}+\frac{\theta_2\theta_d\alpha_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}]-1})\times$$

$$(\mu+\alpha)(2\mu+\gamma+\alpha_1+\alpha_2)(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)$$

$$+(\mu+\frac{\mu}{\frac{\alpha\Psi}{(\mu+\Psi)(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}[\frac{\alpha_1\gamma}{\mu+\gamma}+\frac{\alpha_2\theta_1}{\mu+\theta_1+\theta_2}+\frac{\theta_2\theta_d\alpha_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}]-1})\times$$

$$(\mu+\alpha)(\mu+\alpha_1+\alpha_2)(\mu+\gamma)(2\mu+\mu_1+\theta_1+\theta_2+\theta_d)-\alpha_2\theta_2\frac{\alpha\beta}{R_0}(2\mu+\gamma)-\frac{\alpha\beta}{R_0}(2\mu+\gamma)(\mu+\theta_1+\theta_2)\times$$

$$(\mu+\mu_1+\theta_d)-\frac{\alpha\beta}{R_0}\mu(\mu+\gamma)(2\mu+\mu_1+\theta_1+\theta_2+\theta_d)-\alpha_2\frac{\alpha\beta}{R_0}(2\mu+\gamma)(\mu+\mu_1+\theta_d)-\alpha_2\mu\frac{\alpha\beta}{R_0}(\mu+\gamma)$$

$$-\alpha_1\frac{\alpha\beta}{R_0}(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)-\alpha_1\mu\frac{\alpha\beta}{R_0}(2\mu+\mu_1+\theta_1+\theta_2+\theta_d)>0 \quad (36)$$

$$a_6=(\mu+\frac{\mu}{\frac{\alpha\Psi}{(\mu+\Psi)(\mu+\alpha)(\mu+\alpha_1+\alpha_2)}[\frac{\alpha_1\gamma}{\mu+\gamma}+\frac{\alpha_2\theta_1}{\mu+\theta_1+\theta_2}+\frac{\theta_2\theta_d\alpha_2}{(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)}]-1})(\mu+\alpha)(\mu+\alpha_1+\alpha_2)(\mu+\gamma)\times$$

$$(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)-\alpha_2\theta_2\frac{\alpha\beta}{R_0}\mu(\mu+\gamma)-\frac{\alpha\beta}{R_0}\mu(\mu+\gamma)(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)-\alpha_2\frac{\alpha\beta}{R_0}\mu\times$$

$$(\mu+\gamma)(\mu+\mu_1+\theta_d)-\alpha_1\frac{\alpha\beta}{R_0}\mu(\mu+\theta_1+\theta_2)(\mu+\mu_1+\theta_d)>0. \quad (37)$$

A simple calculus give us: $H_1=a_1>0$ $\qquad$ $H_2=a_1*a_2-a_3>0$ $\qquad$ If $R_0>1$ and $H_3=a_3(a_1a_2-a_3)-a_1(a_1a_4-a_5)>0$ and $H_4=(a_1a_2-a_3)(a_1a_6+a_3a_4-a_2a_5)-(a_1a_4-a_5)^2>0$ then the Endemic Equilibrium is asymptotically stable.
If $R_0>1$ and $H_3=a_3(a_1a_2-a_3)-a_1(a_1a_4-a_5)<0$ or $H_4=(a_1a_2-a_3)(a_1a_6+a_3a_4-a_2a_5)-(a_1a_4-a_5)^2<0$ then the Endemic Equilibrium is unstable.

6. A Real-World implementation and numerical simulations for Niger Republic

We simulate the infectious, asymptomatic infectious, symptomatic infectious and infectious in the critical cases functions when $R_0>1$ and $R_0<1$ which give us the following figures. All simulations is according to the official data from Niger Republic. Numerical simulations is generated by python environment.

(i) $R_0 = 3.65 \quad \beta = 0.2$.
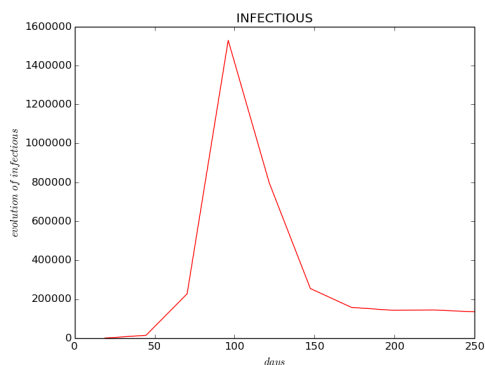
Figure 1: Infectious
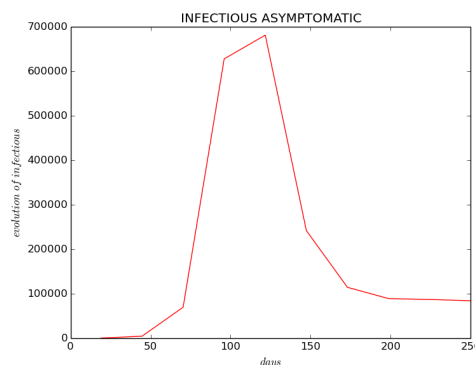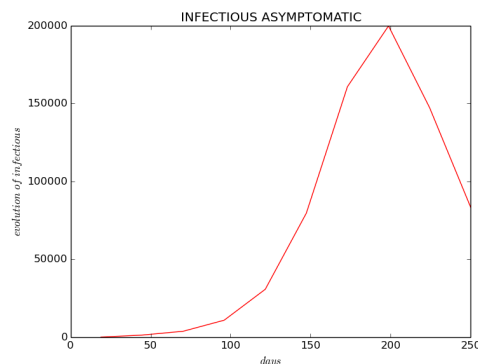

Figure 2: Infectious asymptomatic
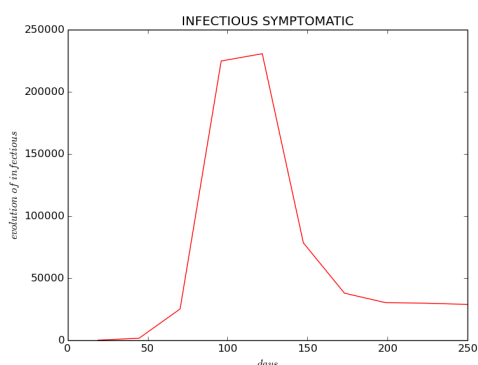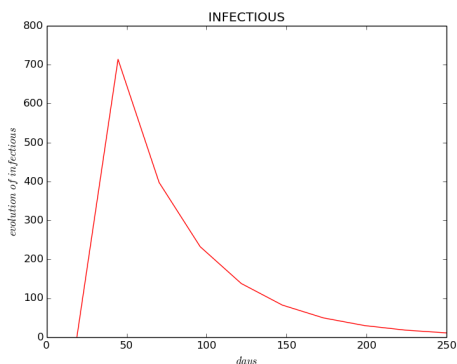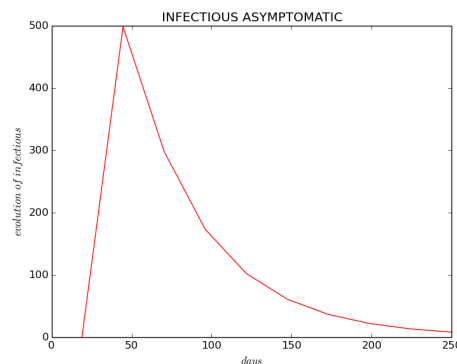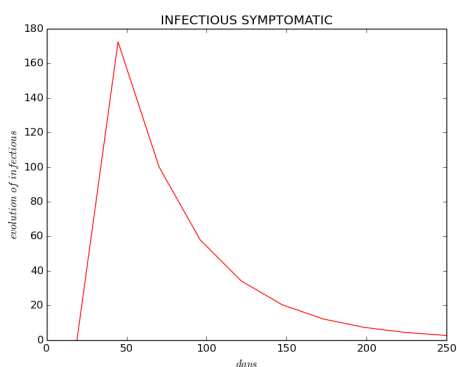

Figure 3: Infectious symptomatic
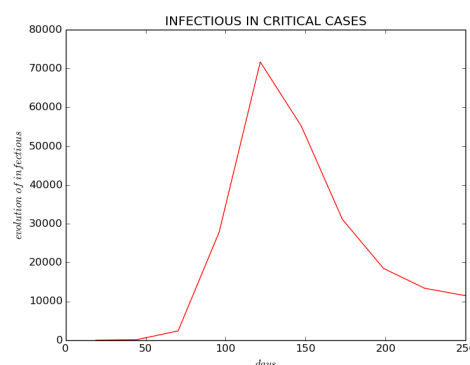
Figure 4: Infectious in critical cases

We see in Figure 1 which represents the number of infectious people that in the absence of any preventive measures, the peak of the disease would be reached on June 06th, 2020 with nearly 1,550,000 infectious cases.

Similarly in Figure 2 which represents the number of asymptomatic infectious people that in the absence of any preventive measures, the peak of the disease would be reached on June 28th, 2020 with nearly 690,000 asymptomatic infectious cases.

We see in Figure 3 which represents the number of symptomatic infectious people that in the absence of any preventive measures, the peak of the disease would be reached on June 28th, 2020 with nearly 245,000 symptomatic infectious cases. These are the cases that really need symptomatic treatment.

We see in Figure 4 which represents the number of infectious people who are in respiratory distress that in the absence of any preventive measures, the peak of the disease would be reached on June 28th, 2020 with nearly 70,000 infectious cases in distress. These are the cases that really need artificial respirators.
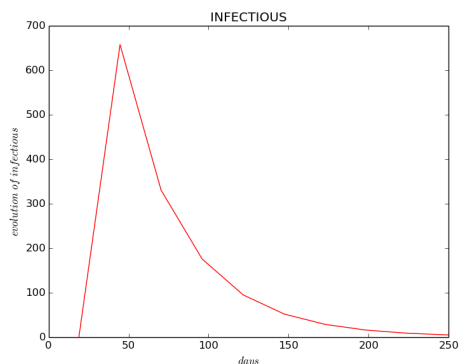
(ii) $R_0 = 1.82 \quad \beta = 0.1$.
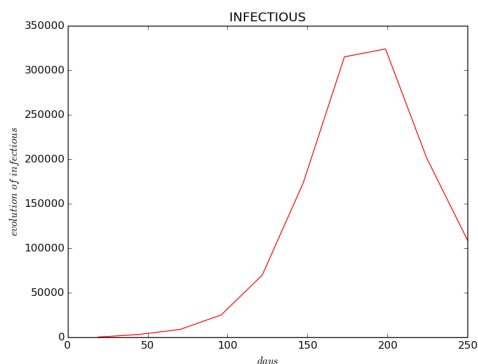
Figure 1: Infectious

Figure 2: Infectious asymptomatic
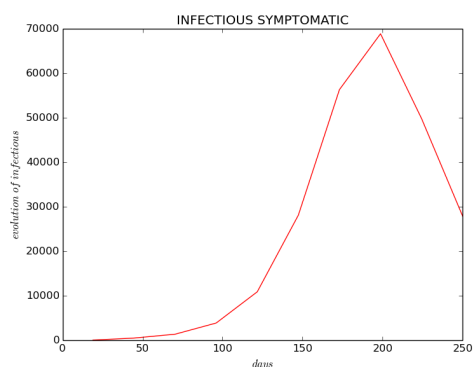
Figure 3: Infectious symptomatic
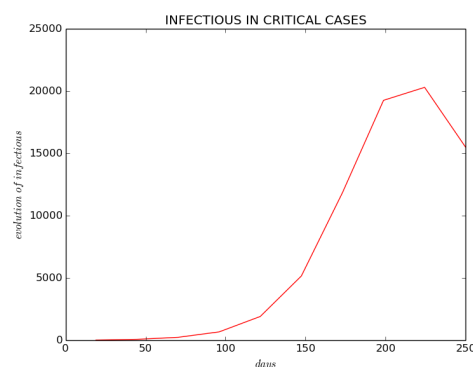
Figure 4: Infectious in critical cases

Figure 1 informs us that the peak of the number of infectious patients in the absence of any preventive measures but with a reduced contact rate compared to the previous one would be reached on September 16th, 2020 with nearly 340,000 infectious cases. Figure 2 informs us that the peak of the number of asymptomatic infectious patients in the absence of any preventive measures but with a reduced contact rate compared to the previous one would be reached on September 16th, 2020 with nearly 200,000 asymptomatic infectious cases.

Figure 3 informs us that the peak of the number of symptomatic infectious patients in the absence of any preventive measures but with a reduced contact rate compared to the previous one would be reached on September 16th, 2020 with nearly 69,000 symptomatic infectious cases. These are the cases that require symptomatic treatment.

Figure 4 informs us that the peak of the number of infectious respiratory distress patients in the absence of any preventive measures but with a reduced contact rate compared to the previous one would be reached on October 11th, 2020 with nearly 20,000 cases. These are the cases that require respiratory assistance.

(iii) $R_0 = 0.64 \quad \beta = 0.035$.
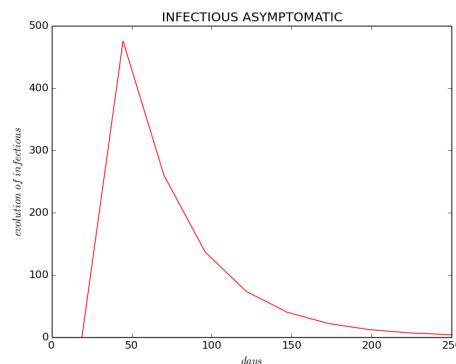
Figure 1: Infectious
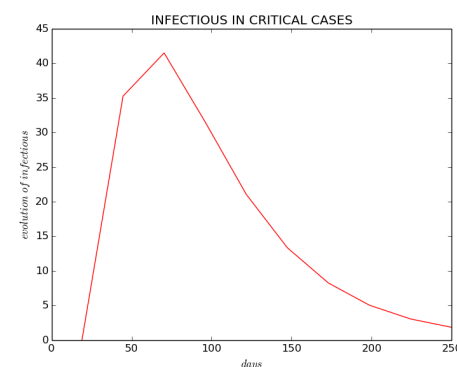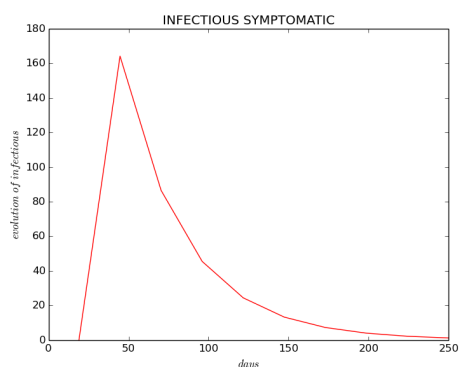

Figure 2: Infectious asymptomatic
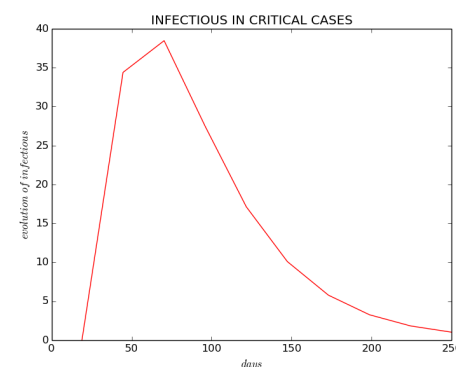

Figure 3: Infectious symptomatic

Figure 4: Infectious in critical cases

We note in Figure 1, that the peak of the number of infectious is reached on April 18th, 2020 with nearly 720 confirmed cases and the disease is expected to disappear around the beginning of November 2020. Here the prevention measures decreed by the government are partially respected.

We note in Figure 2, that the peak of the number of asymptomatic infectious is reached on April 18th, 2020 with nearly 500 confirmed cases and the disease is expected to disappear around the beginning of November 2020. Here the prevention measures decreed by the government are partially respected.

We note in Figure 3, that the peak of the number of symptomatic infectious is reached on April 18th, 2020 with nearly 175 confirmed cases and the disease is expected to disappear around the beginning of November 2020. Here the prevention measures decreed by the government are partially respected.

We note in Figure 4, that the peak of the number of infectious respiratory distress is reached on May 09th, 2020 with nearly 42 confirmed cases and the disease is expected to disappear around the beginning of November 2020. Here the prevention measures decreed by the government are partially respected.
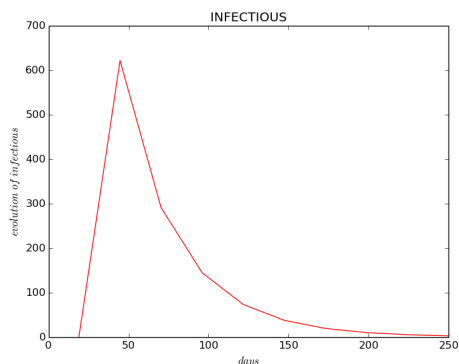
(iv) $R_0 = 0.6$ $\beta = 0.032$.
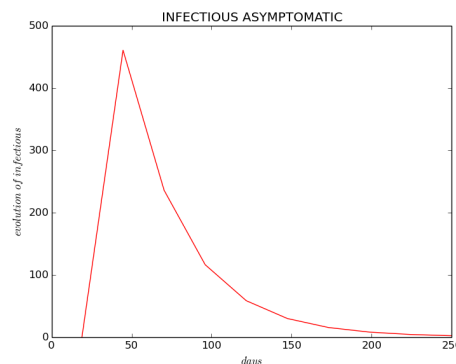
Figure 1: Infectious
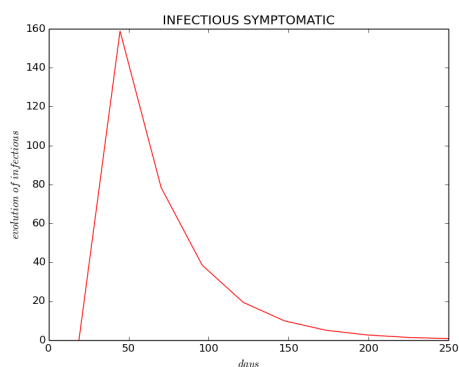

Figure 2: Infectious asymptomatic
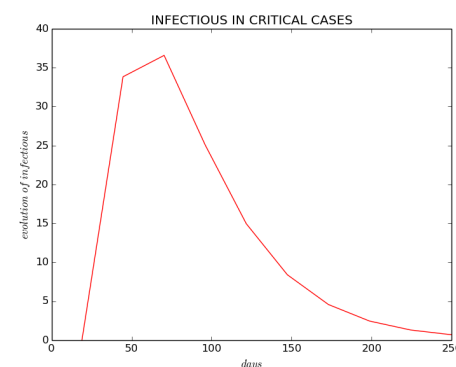

Figure 3: Infectious symptomatic

Figure 4: Infectious in critical cases

We note in Figure 1, that the peak of the number of infectious is reached on April 18th, 2020 with nearly 670 confirmed cases and the disease is expected to disappear around the beginning of November 2020. Here the prevention measures decreed by the government are moderately respected.

We note in Figure 2, that the peak of the number of asymptomatic infectious is reached on April 18th, 2020 with nearly 470 confirmed cases and the disease is expected to disappear around the beginning of November 2020. Here the prevention measures decreed by the government are moderately respected.

We note in Figure 3, that the peak of the number of symptomatic infectious is reached on April 18th, 2020 with nearly 165 confirmed cases and the disease is expected to disappear around the beginning of November 2020. Here the prevention measures decreed by the government are moderately respected.

We note in Figure 4, that the peak of the number of infectious respiratory distress is reached on May 09th, 2020 with nearly 38 confirmed cases and the disease is expected to disappear around the beginning of November 2020. Here the prevention measures decreed by the government are moderately respected.

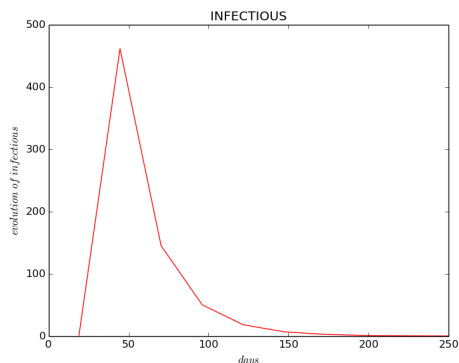(v) $R_0 = 0.54$ $\beta = 0.03$.

Figure 1: Infectious

Figure 2: Infectious asymptomatic

Figure 3: Infectious symptomatic

Figure 4: Infectious in critical cases

We note in Figure 1, that the peak of the number of infectious is reached on April 18th, 2020 with nearly 630 confirmed cases and the disease is expected to disappear around the end of October 2020. Here the prevention measures decreed by the government are mostly respected.

We note in Figure 2, that the peak of the number of asymptomatic infectious is reached on April 18th, 2020 with nearly 460 confirmed cases and the disease is expected to disappear around the end of October 2020. Here the prevention measures decreed by the government are mostly respected.

We note in Figure 3, that the peak of the number of symptomatic infectious is reached on April 18th, 2020 with nearly 160 confirmed cases and the disease is expected to disappear around the end of October 2020. Here the prevention measures decreed by the government are mostly respected.

We note in Figure 4, that the peak of the number of infectious respiratory distress is reached on May 09th, 2020 with nearly 37 confirmed cases and the disease is expected to disappear around the end of October 2020. Here the prevention measures decreed by the government are mostly respected.

(vi) $R_0 = 0.36$ $\beta = 0.02$.
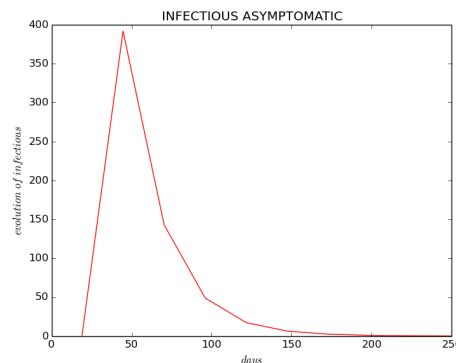

Figure 1: Infectious
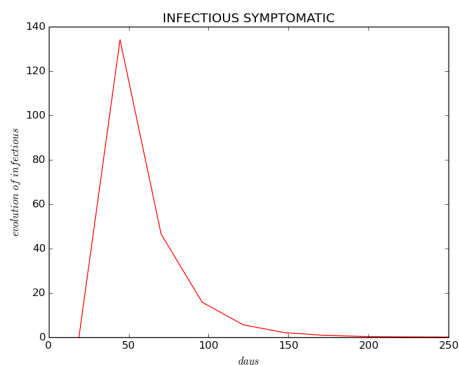

Figure 2: Infectious asymptomatic
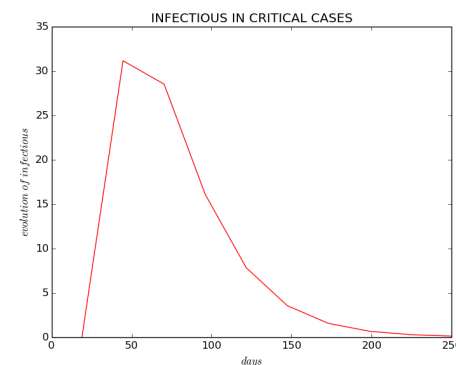

Figure 3: Infectious symptomatic

Figure 4: Infectious in critical cases

We note in Figure 1, that the peak of the number of infectious is reached on April 18th, 2020 with nearly 460 confirmed cases and the disease is expected to disappear around the end of August 2020. Here the prevention measures decreed by the government are totally respected.

We note in Figure 2, that the peak of the number of asymptomatic infectious is reached on April 18th, 2020 with nearly 390 confirmed cases and the disease is expected to disappear around the end of August 2020. Here the prevention measures decreed by the government are totally respected.

We note in Figure 3, that the peak of the number of symptomatic infectious is reached on April 18th, 2020 with nearly 135 confirmed cases and the disease is expected to disappear around the end of August 2020. Here the prevention measures decreed by the government are totally respected.

We note in Figure 4, that the peak of the number of infectious respiratory distress is reached on May 09th, 2020 with nearly 32 confirmed cases and the disease is expected to disappear around the end of October 2020. Here the prevention measures decreed by the government are totally respected.

7. Conclusion

We analyse and propose measures to government to help them in their decision-making in the response to the disease. In the cases where $R_0 > 1$ the disease will persist in the population, but the collective immunity will be reached around end of June and end of September without any prevention measures with thousands of deaths. We established that if the contact rate is large then the basic reproductive number increase, so the disease will persist in the population. The model allows us to predict the number of active infected people over time and to see the effect of the measures decreed by the government on the dynamics of the disease. At this time, the reproductive number in Niger case is around 0.6 so that $R_0 < 1$. To overcome the disease, we propose:

- Intensify communications, in particular by calling to the scientists committee, religious leaders and recovered peoples.

- Make wearing of mask compulsory in public places.

- Isolate all suspected cases with the collaboration of the population to avoid community contamination.

- More investment in scientific research for anticipating the responses to bring to the challenges of tomorrow.

Acknowledgements

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