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A view on Connectedness and Compactness in Fuzzy Soft Bitopological Spaces

Abdelhamied Farrag Sayed

Mathematics Department, Al-Lith University College, Umm Al-Qura University
P.O. Box 112, Al-Lith 21961, Makkah Al Mukarramah, Kingdom of Saudi Arabia

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Abstract. In the present paper, we introduce the notions of $(1,2)^*$-fuzzy soft b-separated sets, $(1,2)^*$-fuzzy soft b-connectedness and $(1,2)^*$-fuzzy soft b-compactness in fuzzy soft bitopological spaces. Then, some basic topological properties of these notions are investigated. Also, some illustrative examples are given to show the importance of the obtained theorems.

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Key Words and Phrases: $(1,2)^*$-fsb-separated, $(1,2)^*$-fsb-connected, $(1,2)^*$-fsb-compact

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1. Introduction

In 1965, Zadeh [36], introduced the concept of fuzzy set theory and its applications can be found in many branches of mathematical and engineering sciences including management science, control engineering, computer science and artificial intelligence (see, [5, 7]).

In 1999, Russian researcher Molodtsov [16], initiated the concept of soft sets as a new mathematical tool to deal with uncertainties while modeling problems in engineering physics, computer science, economics, social sciences and medical sciences (see, [20, 28]). In 2003, Maji, Biswas and Roy [22], studied the theory of soft sets initiated by Molodtsov. They defined equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set and absolute soft set with examples. Soft binary operations like AND, OR and also the operations of union and intersection were also defined. In 2005, D. Chen [6], presented a new definition of soft set parametrization reduction and a comparison of it with attribute reduction in rough set theory.

Recently, on soft sets, soft topological space has been studied increasingly Shabir and Naz [32] defined the theory of soft topological space over an initial universe with a fixed set of parameters. Çağman et al. [18] introduced a topology on a soft set called "soft topology" and presented the foundations of the theory of soft topological spaces. Moreover, many authors studied soft topology and its applications (e.g. [8, 9, 11]). Later Tanay and Kandemir [35] introduced fuzzy soft topological space and established the basic

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Email address: dr.afsayed@hotmail.com, afssayed@uqu.edu.sa (A. F. Sayed)

definitions of fuzzy soft topological space by incorporating the fuzzy topology and soft set. Fuzzy soft topological space was applied in various ways say, game theory, analysis, etc. Fuzzy soft set in topological space further studied by Roy [27]. The authors [13, 19, 33] are successfully applied fuzzy soft topological space in real life.

In 1963, Kelly [14], first initiated the concept of bitopological spaces and other authors have contributed to development and construction of some properties of such spaces (see, [23, 24]) as generalizations of which are in general topology.

In 2014, Ittanagi [12], introduced and studied the concept of soft bitopological spaces and other authors have contributed to development and construction of some properties of such spaces (see, [2, 3, 25, 26]).
The notion of soft bitopological space was introduced using different soft topologies on an initial universe set. On the other hand, the mixed type of soft set theory was given using different soft topologies (see, [1, 4, 10, 21, 34]).

In 2015, Mukherjee and Park [17], first introduced the notion of fuzzy soft bitopological space and they introduced the notions of $\tau_1\tau_2$-fuzzy soft open(closed) sets, $\tau_1\tau_2$-fuzzy soft interior (resp. closure) and studied some of their basic properties. Also, Sayed ([29–31]) were extension and continuation of studying in this trend by characterizing new concepts in fuzzy soft bitopological spaces. In the present paper, we introduce the notions of $(1,2)^*$-fuzzy soft b-separated sets, $(1,2)^*$-fuzzy soft b-connectedness and $(1,2)^*$-fuzzy soft b-compactness in fuzzy soft bitopological spaces. Then, some basic topological properties of these notions are investigated. Also, some illustrative examples are given to show the importance of the obtained theorems.

2. Preliminaries

In this section we are going to present the basic definitions and results of fuzzy soft set and fuzzy soft bitopological space which will be a central role in our paper.
Throughout our discussion, X refers to an initial universe, E the set of all parameters for X and $P(X)$ denotes the power set of X.

Definition 1. *[36] A fuzzy set A in a non-empty set X is characterized by a membership function $\mu_A : X \to [0,1] = I$ whose value $\mu_A(x)$ represents the "degree of membership" of x in A for every x in X. Let I^X denotes the family of all fuzzy sets on X.*

Definition 2. *[36] The empty fuzzy set on X denoted by $\tilde{0}$ is a function which maps each $x \in X$ to 0. That is, $\tilde{0}(x) = 0$ for all $x \in X$.*
A universal fuzzy set denoted by $\tilde{1}$ is a function, which maps each $x \in X$ to 1. That is, $\tilde{1}(x) = 1$ for all $x \in X$.

Definition 3. *[16] Let $A \subseteq E$. A pair (F, A) is called a soft set over X if F is a mapping given by $F : A \to P(X)$.*

Definition 4. *[15] Let $A \subseteq E$. A pair (f, A), denoted by f_A, is called a fuzzy soft set over X, where f is a mapping given by $f : A \to I^X$ defined by $f_A(e) = \mu^e_{f_A}$ where*

$$\mu^e_{f_A} = \begin{cases} \tilde{0}, & \text{if } e \notin A; \\ \text{otherwise}, & \text{if } e \in A. \end{cases}$$

$\widetilde{(X,E)}$ denotes the family of all fuzzy soft sets over (X,E).

Definition 5. *[22] A fuzzy soft set $f_A \tilde{\in} \widetilde{(X,E)}$ is said to be:*

(a) NULL fuzzy soft set, denoted by $\tilde{\phi}$, if for all $e \in A, f_A(e) = \tilde{0}$.

(b) absolute fuzzy soft set, denoted by $\tilde{E}$, if for all $e \in E, f_A(e) = \tilde{1}$.

Note that throughout our discussion in this paper, $\tilde{0}_E$ and $\tilde{1}_E$ will be denoted for $\tilde{\phi}$ and $\tilde{E}$, respectively.

Definition 6. *[27] The complement of a fuzzy soft set f_A, denoted by f^c_A where $f^c_A : E \to I^X$ is a mapping given by $\mu^e_{f^c_A} = \tilde{1} - \mu^e_{f_A}$, for all $e \in E$ and where $\tilde{1}(x) = 1$, for all $x \in X$. Clearly $(f^c_A)^c = f_A$.*

Definition 7. *[27] Let $f_A, g_B \in \widetilde{(X,E)}$. f_A is fuzzy soft subset of g_B, denoted by $f_A \tilde{\subseteq} g_B$, if $A \subseteq B$ and $\mu^e_{f_A} \leq \mu^e_{g_B}$ for all $e \in A$, that is, $\mu^e_{f_A}(x) \leq \mu^e_{g_B}(x)$ for all $x \in X$ and for all $e \in A$.*

Definition 8. *[27] Let $f_A, g_B \tilde{\in} \widetilde{(X,E)}$. The union of f_A and g_B is also a fuzzy soft set h_C, where $C = A \cup B$ and for all $e \in C, h_C(e) = \mu^e_{h_c} = \mu^e_{f_A} \vee \mu^e_{g_B}$. Here we write $h_C = f_A \tilde{\cup} g_B$.*

Definition 9. *[27] Let $f_A, g_B \tilde{\in} \widetilde{(X,E)}$. The intersection of f_A and g_B is also a fuzzy soft set d_C, where $C = A \cap B$ and for all $e \in C, d_C(e) = \mu^e_{d_c} = \mu^e_{f_A} \wedge \mu^e_{g_B}$. Here we write $d_C = f_A \tilde{\cap} g_B$.*

Definition 10. *[27] A fuzzy soft topology τ over (X,E) is a family of fuzzy soft sets over (X,E) satisfying the following properties:*

(i) $\tilde{0}_E, \tilde{1}_E \in \tau$,

(ii) if $f_A, g_B \in \tau$, then $f_A \tilde{\cap} g_B \in \tau$,

(iii) if $f_{A_\alpha} \in \tau$ for all $\alpha \in \Delta$ an index set, then $\tilde{\bigcup}_{\alpha \in \Delta} f_{A_\alpha} \in \tau$.

Definition 11. *[17] If τ is a fuzzy soft topology on (X,E), then the triple (X,E,τ) is said to be a fuzzy soft topological space. Also each member of τ is called a fuzzy soft open set in (X,E,τ).*

The complement of a fuzzy soft open set is a fuzzy soft closed set.

Definition 12. *[17] Let (X,E,τ_1) and (X,E,τ_2) be two different fuzzy soft topologies on (X,E). Then (X,E,τ_1,τ_2) is called a fuzzy soft bitopological space on which no seperation axioms are assumed unless explicitly stated.*
The members of $\tau_i (i = 1,2)$ are called $\tau_i (i = 1,2)$-fuzzy soft open sets and the complement of $\tau_i (i = 1,2)$-fuzzy soft open sets are called $\tau_i (i = 1,2)$-fuzzy soft closed sets.

Definition 13. *[17] A fuzzy soft set* $f_E \tilde{\in} \widetilde{(X, E)}$ *is called* $\tau_1\tau_2$*- fuzzy soft open set if* $f_E = g_E \tilde{\cup} h_E$ *such that* $g_E \tilde{\in} \tau_1$ *and* $h_E \tilde{\in} \tau_2$.
The complement of $\tau_1\tau_2$*- fuzzy soft open set is called* $\tau_1\tau_2$*- fuzzy soft closed set.*

The family of all $\tau_1\tau_2$- fuzzy soft open (closed) sets in (X, E, τ_1, τ_2) is denoted by $\tau_1\tau_2 FSO(X, \tau_1, \tau_2)_E$ ($\tau_1\tau_2 FSC(X, \tau_1, \tau_2)_E$), respectively.

Definition 14. *[17] Let* (X, E, τ_1, τ_2) *be a fuzzy soft bitopological space and* $f_E \tilde{\in} \widetilde{(X, E)}$. *Then the* $\tau_1\tau_2$*- fuzzy soft closure of* f_E*, denoted by* $\tau_1\tau_2 cl(f_E)$*, is the intersection of all* $\tau_1\tau_2$*- fuzzy soft closed supersets of* f_E.
Clearly, $\tau_1\tau_2 cl(f_E)$ *is the smallest* $\tau_1\tau_2$*- fuzzy soft closed set over* (X, E) *which contains* f_E.

Definition 15. *Let* (X, E, τ_1, τ_2) *be a fuzzy soft bitopological space and* $f_E \tilde{\in} \widetilde{(X, E)}$. *Then* f_E *is called* $(1,2)^*$*-fuzzy soft b-open set (briefly,* $(1,2)^*$*-*fsb*-open) if*
$f_E \tilde{\subseteq} \tau_1\tau_2 int\,(\tau_1\tau_2 cl(f_E))\,\tilde{\cup}\,\tau_1\tau_2 cl\,(\tau_1\tau_2 int(f_E))$.

Definition 16. *[30] Let* (X, E, τ_1, τ_2) *be a fuzzy soft bitopological space and* $f_E \tilde{\in} \widetilde{(X, E)}$.
(i) $(1,2)^*$*-fuzzy soft b-closure (briefly* $(1,2)^*$*-*$fsbcl(f_E)$*) of a set* f_E *in* (X, E, τ_1, τ_2) *defined by* $(1,2)^*$-$fsbcl(f_E) = \tilde{\cap}\{g_E \tilde{\supseteq} f_E : g_E$ *is a* $(1,2)^*$*-fuzzy soft b-closed set in* $(X, E, \tau_1, \tau_2)\}$.
(ii) $(1,2)^*$*-fuzzy soft b-interior (briefly* $(1,2)^*$*-*$fsbint(f_E)$*) of a set* f_E *in* (X, E, τ_1, τ_2) *defined by* $(1,2)^*$-$fsbint(f_E) = \tilde{\cup}\{g_E \tilde{\subseteq} f_E : g_E$ *is a* $(1,2)^*$*-fuzzy soft b-open set in* $(X, E, \tau_1, \tau_2)\}$.
$(1,2)^*$-$fsbcl(f_E)$ *is the smallest* $(1,2)^*$*-fuzzy soft b-closed set in* (X, E, τ_1, τ_2) *which contains* f_E *and* $(1,2)^*$-$fsbcl(f_E)$ *is the largest* $(1,2)^*$*-fuzzy soft b-closed set in* (X, E, τ_1, τ_2) *which is contained in* f_E.

Definition 17. *[31] A fuzzy soft mapping* $(\varphi, \psi) : (X, E, \tau_1, \tau_2) \to (Y, K, \sigma_1, \sigma_2)$ *is said to be* $(1,2)^*$*-fuzzy soft b-continuous (briefly* $(1,2)^*$*-*fsb*-continuous) the inverse image of every* $\sigma_1\sigma_2$*-fuzzy soft open set in* $(Y, K, \sigma_1, \sigma_2)$ *is a* $(1,2)^*$*-fuzzy soft b-open set in* (X, E, τ_1, τ_2).

Definition 18. *[31] A fuzzy soft mapping* $(\varphi, \psi) : (X, E, \tau_1, \tau_2) \to (Y, K, \sigma_1, \sigma_2)$ *is said to be* $(1,2)^*$*-fuzzy soft b-irresolute mapping (briefly,* $(1,2)^*$*-*fsb*-irresolute) if* $(\varphi, \psi)^{-1}(g_K)$ *is a* $(1,2)^*$*-fuzzy soft b-closed set in* (X, E, τ_1, τ_2) *for every* $(1,2)^*$*-fuzzy soft b-closed set* g_K *in* $(Y, K, \sigma_1, \sigma_2)$.

3. $(1,2)^*$-Fuzzy Soft b-Connectedness

In this section we introduce the concepts of $(1,2)^*$-fuzzy soft b-separated sets and $(1,2)^*$-fuzzy soft b-connectedness in fuzzy soft bitopological spaces. Also, some of the main results and properties are studied and discussed.

Definition 19. *Two non-empty fuzzy soft subsets* f_E, g_E *of* $\widetilde{(X, E)}$ *are said to be fuzzy soft disjoint if* $f_E \tilde{\cap} g_E = \tilde{0}_E$.

Definition 20. *Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space. Two non-empty fuzzy soft disjoint fuzzy soft subsets f_E, g_E of $\widetilde{(X,E)}$ are called*
(i) $(1,2)^*$-fuzzy soft separated sets over X if $\tau_1\tau_2 cl(f_E)\tilde{\cap} g_E = f_E\tilde{\cap}\tau_1\tau_2 cl(g_E) = \tilde{0}_E$.
(ii) $(1,2)^*$-fuzzy soft b-separated ($(1,2)^*$-fsb-separated)sets over X if
$((1,2)^*\text{-}fsbcl(f_E))\tilde{\cap} g_E = f_E\tilde{\cap}((1,2)^*\text{-}fsbcl(g_E)) = \tilde{0}_E$.

Remark 1. *From the fact that $(1,2)^*$-$fsbcl(f_E)\tilde{\subseteq}\tau_1\tau_2 cl(f_E)$, for every fuzzy soft subset f_E of $\widetilde{(X,E)}$, every $(1,2)^*$-fuzzy soft separated set is $(1,2)^*$-fuzzy soft b-separated. But the converse may not be true.*

Definition 21. *A $(1,2)^*$-fuzzy soft b-separation ($(1,2)^*$-fsb-separation) of a fuzzy soft bitopological space (X, E, τ_1, τ_2) is a pair of $(1,2)^*$-fuzzy soft b-separated sets f_E and g_E whose fuzzy soft union is absolute fuzzy soft set $\tilde{1}_E$(that is $f_E\tilde{\cup} g_E = \tilde{1}_E$).*

Definition 22. *Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space. Then (X, E, τ_1, τ_2) is called $(1,2)^*$-fuzzy soft b-connected space if $\tilde{1}_E$ can not be expressed as the fuzzy soft union of two $(1,2)^*$-fuzzy soft b-separated sets.*

Remark 2. *In a fuzzy soft bitopological space (X, E, τ_1, τ_2):*
(i) A fuzzy soft empty set is trivially $(1,2)^*$-fuzzy soft b-connected set.
(ii) Every fuzzy soft singleton set is $(1,2)^*$-fuzzy soft b-connected, since it can not be expressd as a fuzzy soft union of two non-empty $(1,2)^*$-fuzzy soft b-separated sets.

Theorem 1. *Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space. Then the following statements are equivalent:*
(i) (X, E, τ_1, τ_2) is a $(1,2)^*$-fuzzy soft b-connected space.
(ii) $\tilde{1}_E$ and $\tilde{0}_E$ are the only $(1,2)^*$-fuzzy soft b-clopen (that is, closed and open) sets in (X, E, τ_1, τ_2).
(iii) $\tilde{1}_E$ can not be expressed as the fuzzy soft union of two fuzzy soft disjoint non-empty $(1,2)^*$-fuzzy soft b-open sets.
(iv) $\tilde{1}_E$ can not be expressed as the fuzzy soft union of two fuzzy soft disjoint non-empty $(1,2)^*$-fuzzy soft b-closed sets.

Proof. $(i) \Rightarrow (ii)$: Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space. Let f_E be non-empty proper fuzzy soft subset of $\widetilde{(X,E)}$ that is $(1,2)^*$-fuzzy soft b-clopen. Then $\tilde{1}_E \setminus f_E$ is a non-empty $(1,2)^*$-fuzzy soft b-clopen set and $\tilde{1}_E = f_E\tilde{\cup}(\tilde{1}_E \setminus f_E)$. This is a contradiction to (X, E, τ_1, τ_2) is a $(1,2)^*$-fuzzy soft b-connected space. Therefore $\tilde{1}_E$ and $\tilde{0}_E$ are the only $(1,2)^*$-fuzzy soft b-clopen sets in (X, E, τ_1, τ_2).
$(ii) \Rightarrow (iii)$: Assume that $\tilde{1}_E$ and $\tilde{0}_E$ are the only $(1,2)^*$-fuzzy soft b-clopen sets in (X, E, τ_1, τ_2). suppose (iii) is false. Then $\tilde{1}_E = f_E\tilde{\cup} g_E$ where f_E and g_E are fuzzy soft disjoint non-empty $(1,2)^*$-fuzzy soft b-open sets. Then $g_E = \tilde{1}_E \setminus f_E$ is $(1,2)^*$-fuzzy soft b-closed and non-empty. Thus g_E is a non-empty proper $(1,2)^*$-fuzzy soft b-clopen set in (X, E, τ_1, τ_2), which contradicts (ii).
$(iii) \Rightarrow (iv)$: Assume $\tilde{1}_E$ cannot be expressed as the fuzzy soft union of two fuzzy soft disjoint non-empty $(1,2)^*$-fuzzy soft b-open sets. Suppose (iv) false. Then $(1,2)^*$-fuzzy

soft b-closed sets. Then $f_E = \tilde{1}_E \setminus g_E$ and $g_E = \tilde{1}_E \setminus f_E$ are fuzzy soft disjoint non-empty $(1,2)^*$-fuzzy soft b-open sets in (X, E, τ_1, τ_2). Thus $\tilde{1}_E$ is the fuzzy soft union of two fuzzy soft disjoint non-empty $(1,2)^*$-fuzzy soft b-open sets . This contradicts (iii).
$(iv) \Rightarrow (i)$: Suppose (X, E, τ_1, τ_2) is not $(1,2)^*$-fuzzy soft b-connected space. Then $\tilde{1}_E = f_E \tilde{\cup} g_E$ where f_E and g_E are fuzzy soft disjoint non-empty $(1,2)^*$-fuzzy soft b-open sets. Then $f_E = \tilde{1}_E \setminus g_E$ and $g_E = \tilde{1}_E \setminus f_E$ are fuzzy soft disjoint non-empty $(1,2)^*$-fuzzy soft b-closed sets in (X, E, τ_1, τ_2). This is a contradiction to (iv).

Proposition 1. *Every $(1,2)^*$-fuzzy soft b-connected space is $(1,2)^*$-fuzzy soft connected.*

Proof. Let f_E be a $(1,2)^*$-fuzzy soft b-connected set in the fuzzy soft bitopological space (X, E, τ_1, τ_2). Then there does not exist a $(1,2)^*$-fuzzy soft b-separation of f_E. Since every $\tau_1\tau_2$-fuzzy soft open set is a $(1,2)^*$-fuzzy soft b-open set, there does not exist a $(1,2)^*$-fuzzy soft separation of f_E. Hence f_E is a $(1,2)^*$-fuzzy soft connected set in the fuzzy soft bitopological space (X, E, τ_1, τ_2).

The converse is not true as shown in the following example.

Example 1. *$(1,2)^*$-fuzzy soft connectedness does not imply $(1,2)^*$-fuzzy soft b-connectedness.*
Let (X, E, τ_1, τ_2) be a fuzzy soft bitopological space, where $X = \{x, y\}, E = \{e_1, e_2\}$ and let $\tau_1 = \{\tilde{0}_E, \tilde{1}_E, f_{1_E}, f_{2_E}, f_{3_E}\}, \tau_2 = \{\tilde{0}_E, \tilde{1}_E, g_{1_E}, g_{2_E}\}$, where
$f_{1_E} = \{f_1(e_1) = \{x/0.2, y/0.0\}, f_1(e_2) = \{x/0.0, y/0.0\} = \tilde{0}\}$,
$f_{2_E} = \{f_2(e_1) = \{x/0.2, y/0.0\}, f_2(e_2) = \{x/0.7, y/0.0\}\}$,
$f_{3_E} = \{f_3(e_1) = \{x/0.2, y/0.1\}, f_3(e_2) = \{x/0.7, y/0.0\}\}$,
$g_{1_E} = \{g_1(e_1) = \{x/0.0, y/0.0\} = \tilde{0}, g_1(e_2) = \{x/0.7, y/0.0\}\}$ *and*
$g_{2_E} = \{g_2(e_1) = \{x/0.0, y/0.0\} = \tilde{0}, g_2(e_2) = \{x/0.0, y/0.4\}\}$.
Then $\tau_1\tau_2$-fuzzy soft open sets are $\{\tilde{0}_E, \tilde{1}_E, f_{1_E}, f_{2_E}, f_{3_E}, g_{1_E}, g_{2_E}\}$ and $\tau_1\tau_2$-fuzzy soft closed sets are $\{\tilde{0}_E, \tilde{1}_E, f^c_{1_E}, f^c_{2_E}, f^c_{3_E}, g^c_{1_E}, g^c_{2_E}\}$ where
$f^c_{1_E} = \{f_1(e_1) = \{x/0.0, y/0.1\}, f_1(e_2) = \{x/0.7, y/0.4\}\}$,
$f^c_{2_E} = \{f_2(e_1) = \{x/0.0, y/0.1\}, f_2(e_2) = \{x/0.0, y/0.4\}\}$,
$f^c_{3_E} = \{f_3(e_1) = \{x/0.0, y/0.0\} = \tilde{0}, f_3(e_2) = \{x/0.0, y/0.4\}\}$
$g^c_{1_E} = \{g_1(e_1) = \{x/0.2, y/0.1\}, g_1(e_2) = \{x/0.0, y/0.4\}\}$ *and*
$g^c_{2_E} = \{g_2(e_1) = \{x/0.2, y/0.1\}, g_2(e_2) = \{x/0.7, y/0.0\}\}$.
It is clear that (X, E, τ_1, τ_2) is $(1,2)^*$-fuzzy soft connected since the only $(1,2)^*$-fuzzy soft clopen sets are $\tilde{0}_E, \tilde{1}_E$. Also $(1,2)^*$-fuzzy soft b-open sets are $\{\tilde{0}_E, \tilde{1}_E, f_{1_E}, f_{2_E}, f_{3_E}, f_{4_E}, f_{5_E}, g_{1_E}, g_{2_E}, g_{3_E}, g_{4_E}\}$, where $f_{1_E}, f_{2_E}, f_{3_E}, g_{1_E}$ and $g_{2_E}\}$ are defined as above and
$f_{4_E} = \{f_4(e_1) = \{x/0.0, y/0.1\}, f_4(e_2) = \{x/0.7, y/0.4\}\}$,
$f_{5_E} = \{f_5(e_1) = \{x/0.0, y/0.1\}, f_5(e_2) = \{x/0.0, y/0.4\}\}$,
$g_{3_E} = \{g_3(e_1) = \{x/0.2, y/0.1\}, g_3(e_2) = \{x/0.0, y/0.4\}\}$ *and*
$g_{4_E} = \{g_4(e_1) = \{x/0.2, y/0.0\}, g_4(e_2) = \{x/0.0, y/0.4\}\}$.
And $(1,2)^*$-fuzzy soft b-closed sets are $\{\tilde{0}_E, \tilde{1}_E, f^c_{1_E}, f^c_{2_E}, f^c_{3_E}, f^c_{4_E}, f^c_{5_E}, g^c_{1_E}, g^c_{2_E}, g^c_{3_E}, g^c_{4_E}\}$, where $f^c_{1_E}, f^c_{2_E}, f^c_{3_E}, g^c_{1_E}$ and $g^c_{2_E}$ are obtained as above and
$f^c_{4_E} = \{f_4(e_1) = \{x/0.2, y/0.0\}, f_4(e_2) = \{x/0.0, y/0.0 = \tilde{0}\}\}$,

$f^c_{5_E} = \{f_5(e_1) = \{x/0.2, y/0.0\}, f_5(e_2) = \{x/0.7, y/0.0\}\}$,
$g^c_{3_E} = \{g_3(e_1) = \{x/0.0, y/0.0\} = \tilde{0}, g_3(e_2) = \{x/0.7, y/0.4\}\}$ *and*
$g^c_{4_E} = \{g_4(e_1) = \{x/0.0, y/0.1\}, g_4(e_2) = \{x/0.7, y/0.4\}\}$,
where $\tilde{1}_E = f_{1_E}\tilde{\cup}f_{4_E}$, *then* $(1,2)^*$-$fsbcl(f_{1_E}) = f^c_{4_E}, (1,2)^*$-$fsbcl(f_{4_E}) = g^c_{4_E}$ *and* $(1,2)^*$-$fsbcl(f_{1_E})\tilde{\cap}f_{4_E} = \tilde{0}_E, (1,2)^*$-$fsbcl(f_{4_E})\tilde{\cap}g_{4_E} = \tilde{0}_E$. *Hence* $\tilde{1}_E$ *can be expressed as a fuzzy soft union of two* $(1,2)^*$*-fuzzy soft b-separated sets* f_{1_E}, f_{4_E}. *There* (X, E, τ_1, τ_2) *is not* $(1,2)^*$*-fuzzy soft b-connected.*

Example 2. $(1,2)^*$*-fuzzy soft b-connectivity is not hereditary property.*
Consider the fuzzy soft bitopological space (X, E, τ_1, τ_2), *where* $X = \{x, y\}, E = \{e_1, e_2\}$ *and let* $\tau_1 = \{\tilde{0}_E, \tilde{1}_E, f_{1_E}, f_{2_E}\}, \tau_2 = \{\tilde{0}_E, \tilde{1}_E, g_{1_E}\}$, *where*
$f_{1_E} = \{f_1(e_1) = \{x/0.2, y/0.0\}, f_1(e_2) = \{x/0.0, y/0.0\} = \tilde{0}\}$,
$f_{2_E} = \{f_2(e_1) = \{x/0.2, y/0.0\}, f_2(e_2) = \{x/0.7, y/0.0\}\}$ *and*
$g_{1_E} = \{g_1(e_1) = \{x/0.2, y/0.1\}, g_1(e_2) = \{x/0.0, y/0.0\} = \tilde{0}\}$.
Then $\tau_1\tau_2$*-fuzzy soft open sets are* $\{\tilde{0}_E, \tilde{1}_E, f_{1_E}, f_{2_E}, g_{1_E}, h_{1_E}\}$, *where* $h_{1_E} = \{h_1(e_1) = \{x/0.2, y/0.1\}, h_1(e_2) = \{x/0.7, y/0.0\} = \tilde{0}\}$,
Also, $(1,2)^*$*-fuzzy soft b-open sets are* $\{\tilde{0}_E, \tilde{1}_E, f_{1_E}, f_{2_E}, g_{1_E}, h_{1_E}, h_{2_E}, h_{3_E}, h_{4_E}\}$, *where*
$h_{2_E} = \{h_2(e_1) = \{x/0.2, y/0.0\}, h_2(e_2) = \{x/0.0, y/0.4\}\}$,
$h_{3_E} = \{h_3(e_1) = \{x/0.2, y/0.0\}, h_3(e_2) = \{x/0.7, y/0.4\}\}$ *and*
$h_{4_E} = \{h_3(e_1) = \{x/0.0, y/0.0\}, h_3(e_2) = \{x/0.7, y/0.0\}\}$ *and* $(1,2)^*$*-fuzzy soft b-closed sets are* $\{\tilde{0}_E, \tilde{1}_E, f^c_{1_E}, f^c_{2_E}, g^c_{1_E}, h^c_{1_E}, h^c_{2_E}, h^c_{3_E}, h^c_{4_E}\}$, *where*
$f^c_{1_E} = \{f_1(e_1) = \{x/0.0, y/0.1\}, f_1(e_2) = \{x/0.7, y/0.4\}$,
$f^c_{2_E} = \{f_2(e_1) = \{x/0.0, y/0.1\}, f_2(e_2) = \{x/0.0, y/0.4\}\}$,
$g^c_{1_E} = \{g_1(e_1) = \{x/0.0, y/0.0\} = \tilde{0}, g_1(e_2) = \{x/0.7, y/0.4\}\}$,
$h^c_{1_E} = \{h_1(e_1) = \{x/0.0, y/0.0\} = \tilde{0}, h_1(e_2) = \{x/0.0, y/0.4\}\}$,
$h^c_{2_E} = \{h_2(e_1) = \{x/0.0, y/0.1\}, h_2(e_2) = \{x/0.7, y/0.0\}\}$,
$h^c_{3_E} = \{h_3(e_1) = \{x/0.0, y/0.1\}, h_3(e_2) = \{x/0.0, y/0.0\} = \tilde{0}\}$ *and*
$h^c_{4_E} = \{h_4(e_1) = \{x/0.2, y/0.1\} = \tilde{0}, h_4(e_2) = \{x/0.0, y/0.4\}\}$.
It is clear that (X, E, τ_1, τ_2) *is* $(1,2)^*$*-fuzzy soft b-connected, since the only* $(1,2)^*$*-fuzzy soft clopen sets are* $\tilde{0}_E$ *and* $\tilde{1}_E$. *Let* $Y = \{x\}\tilde{\subseteq}X$ *and* $E = \{e_1, e_2\}$. *Let* $\sigma_1 = \{\tilde{0}_E, \tilde{1}_E, f_{1_E}\}, \sigma_2 = \{\tilde{0}_E, \tilde{1}_E, h_{4_E}\}$. *Then* $\sigma_1\sigma_2$*-fuzzy soft open sets are* $\{\tilde{0}_E, \tilde{1}_E, f_{1_E}, h_{4_E}\}$. *Also* $(1,2)^*$*-fuzzy soft b-clopen sets are* $\{\tilde{0}_E, \tilde{1}_E, f_{1_E}, h_{4_E}\}$. *clearly* $(Y, E, \sigma_1, \sigma_2)$ *is not* $(1,2)^*$*-fuzzy soft b-connected; since* f_{1_E} *and* h_{4_E} *are two* $(1,2)^*$*-fuzzy soft b-clopen sets other than* $\tilde{0}_E$ *and* $\tilde{1}_E$.

Proposition 2. *Let* f_E *be a* $(1,2)^*$*-fuzzy soft b-connected set,* g_E *and* h_E *are* $(1,2)^*$*-fuzzy soft b-separated sets. If* $f_E\tilde{\subseteq}g_E\tilde{\cup}h_E$ *then either* $f_E\tilde{\subseteq}g_E$ *or* $f_E\tilde{\subseteq}h_E$.

Proof. Let f_E be a $(1,2)^*$-fuzzy soft b-connected set, g_E and h_E are $(1,2)^*$-fuzzy soft b-separated sets such that $f_E\tilde{\subseteq}g_E\tilde{\cup}h_E$. Let $f_E\tilde{\nsubseteq}g_E$ and $f_E\tilde{\nsubseteq}h_E$. Suppose $k_E = g_E\tilde{\cap}f_E \neq \tilde{0}_E$ and $l_E = h_E\tilde{\cap}f_E \neq \tilde{0}_E$ then $f_E = k_E\tilde{\cup}l_E$.
Since $k_E\tilde{\subseteq}g_E, ((1,2)^*$-$fsbcl(k_E))\tilde{\subseteq}((1,2)^*$-$fsbcl(g_E))$. Also $((1,2)^*$-$fsbcl(g_E))\tilde{\cap}h_E = \tilde{0}_E$ then $((1,2)^*$-$fsbcl(k_E))\tilde{\cap}l_E = \tilde{0}_E$. Since $l_E\tilde{\subseteq}h_E, ((1,2)^*$-$fsbcl(l_E))\tilde{\subseteq}((1,2)^*$-$fsbcl(h_E))$. Also $((1,2)^*$-$fsbcl(h_E))\tilde{\cap}g_E = \tilde{0}_E$ then $((1,2)^*$-$fsbcl(l_E))\tilde{\cap}k_E = \tilde{0}_E$. But $f_E = k_E\tilde{\cup}l_E$,

therefore f_E is not $(1,2)^*$-fuzzy soft b-connected set which is not a contradiction. Then either $f_E \tilde{\subseteq} g_E$ or $f_E \tilde{\subseteq} h_E$.

Theorem 2. *If f_E is a $(1,2)^*$-fuzzy soft b-connected set and $f_E \tilde{\subseteq} g_E \tilde{\subseteq} ((1,2)^*\text{-}fsbcl(f_E))$ then g_E is a $(1,2)^*$-fuzzy soft b-connected.*

Proof. Suppose g_E is not $(1,2)^*$-fuzzy soft b-connected then there exists two non-empty fuzzy soft sets f_{1_E} and f_{2_E} such that $((1,2)^*\text{-}fsbcl(f_{1_E}))\tilde{\cap} f_{2_E} = f_{1_E} \tilde{\cap}((1,2)^*\text{-}fsbcl(f_{2_E})) = \tilde{0}_E$ and $f_E = f_{1_E} \tilde{\cup} f_{2_E}$. Since $f_E \tilde{\subseteq} g_E$ then either $f_E \tilde{\subseteq} f_{1_E}$ or $f_E \tilde{\subseteq} f_{2_E}$.
Suppose $f_E \tilde{\subseteq} f_{1_E}$, then $((1,2)^*\text{-}fsbcl(f_E))\tilde{\subseteq}((1,2)^*\text{-}fsbcl(f_{1_E}))$, thus
$((1,2)^*\text{-}fsbcl(f_E))\tilde{\cap} f_{2_E} = f_E \tilde{\cap}((1,2)^*\text{-}fsbcl(f_{2_E})) = \tilde{0}_E$. But $f_{2_E} \tilde{\subseteq} g_E \tilde{\subseteq}((1,2)^*\text{-}fsbcl(f_E))$, thus $((1,2)^*\text{-}fsbcl(f_E))\tilde{\cap} f_{2_E} = f_{2_E}$. Therefore $f_{2_E} = \tilde{0}_E$, which is a contradiction.
If $f_E \tilde{\subseteq} f_{2_E}$, then by the same way we can prove that $f_{1_E} = \tilde{0}_E$. This is a contradiction. Thus g_E be a $(1,2)^*$-fuzzy soft b-connected.

Theorem 3. *If f_E is a $(1,2)^*$-fuzzy soft b-connected set, then $(1,2)^*\text{-}fsbcl(f_E)$ is $(1,2)^*$-fuzzy soft b-connected.*

Proof. Suppose f_E is $(1,2)^*$-fuzzy soft b-connected and $(1,2)^*\text{-}fsbcl(f_E)$ is not $(1,2)^*$-fuzzy soft b-connected. Then there exist two $(1,2)^*$-fuzzy soft b-separated sets f_{1_E} and f_{2_E} such that $(1,2)^*\text{-}fsbcl(f_E) = f_{1_E} \tilde{\cup} f_{2_E}$. But $f_E \tilde{\subseteq}(1,2)^*\text{-}fsbcl(f_E)$ then $f_E = f_{1_E} \tilde{\cup} f_{2_E}$ and since f_E is $(1,2)^*$-fuzzy soft b-connected set, then either $f_E \tilde{\subseteq} f_{1_E}$ or $f_E \tilde{\subseteq} f_{2_E}$.
If $f_E \tilde{\subseteq} f_{1_E}$ then $(1,2)^*\text{-}fsbcl(f_E)\tilde{\subseteq}(1,2)^*\text{-}fsbcl(f_{1_E})$. But $(1,2)^*\text{-}fsbcl(f_{1_E})\tilde{\cap} f_{2_E} = \tilde{0}_E$, hence $(1,2)^*\text{-}fsbcl(f_E)\tilde{\cap} f_{2_E} = \tilde{0}_E$. Since $f_{2_E} \tilde{\subseteq}(1,2)^*\text{-}fsbcl(f_E)$, then
$(1,2)^*\text{-}fsbcl(f_E)\tilde{\cap} f_{2_E} = f_{2_E}$; hence $f_{2_E} = \tilde{0}_E$ which is a contradiction.
If $f_E \tilde{\subseteq} f_{1_E}$, then by the same way we can prove that $f_{1_E} = \tilde{0}_E$, which is a contradiction. Therefore $(1,2)^*\text{-}fsbcl(f_E)$ is $(1,2)^*$-fuzzy soft b-connected.

Theorem 4. *The fuzzy soft union f_E of any family $\{f_{i_E} : i \in I\}$ of $(1,2)^*$-fuzzy soft b-connected sets having a non-empty fuzzy soft intersection is $(1,2)^*$-fuzzy soft b-connected.*

Proof. Let f_E be fuzzy soft union of any family of $(1,2)^*$-fuzzy soft b-connected sets having a non-empty fuzzy soft intersection. Suppose that $f_E = f_{1_E} \tilde{\cup} f_{2_E}$, where f_{1_E} and f_{2_E} form a $(1,2)^*$-fuzzy soft b-separation of f_E. By hypothesis, we may choose a fuzzy soft point $f_e \tilde{\in} \tilde{\bigcap}_{i\in I} f_{i_E}$. Then $f_e \tilde{\in} f_{i_E}$ for all $i \in I$. If $f_e \tilde{\in} f_E$, then either $f_e \tilde{\in} f_{1_E}$ or $f_e \tilde{\in} f_{2_E}$ but not both. Since f_{1_E} and f_{2_E} are fuzzy soft disjoint, we must have $f_{i_E} \tilde{\subseteq} f_{1_E}$, since f_{i_E} is $(1,2)^*$-fuzzy soft b-connected and it is true for all $i \in I$, and so $f_E \tilde{\subseteq} f_{i_E}$. From this we obtain that $f_{2_E} = \tilde{0}_E$; which is a contradiction. Thus, there does not exist a $(1,2)^*$-fuzzy soft b-separation of f_E. Therefore, f_E is a $(1,2)^*$-fuzzy soft b-connected set.

Theorem 5. *(i) If $\psi : (X, E, \tau_1, \tau_2) \to (Y, E, \sigma_1, \sigma_2)$ is a $(1,2)^*$-fuzzy soft b-continuous surjection and (X, E, τ_1, τ_2) is $(1,2)^*$-fuzzy soft b-connected then $(Y, E, \sigma_1, \sigma_2)$ is $(1,2)^*$-fuzzy soft connected.*
(ii) If $\psi : (X, E, \tau_1, \tau_2) \to (Y, E, \sigma_1, \sigma_2)$ is a $(1,2)^*$-fuzzy soft b-irresolute surjection and (X, E, τ_1, τ_2) is $(1,2)^*$-fuzzy soft b-connected then $(Y, E, \sigma_1, \sigma_2)$ is $(1,2)^*$-fuzzy soft b-connected.

Proof. (i) Suppose $(Y, E, \sigma_1, \sigma_2)$ is not $(1,2)^*$-fuzzy soft connected. Let $Y = f_E \tilde{\cup} g_E$, where f_E and g_E are fuzzy soft disjoint non-empty $\sigma_1\sigma_2$)-fuzzy soft open sets in $(Y, E, \sigma_1, \sigma_2)$. Since ψ is $(1,2)^*$-fuzzy soft b-continuous and onto; $\tilde{1}_E = \psi^{-1}(f_E)\tilde{\cup}\psi^{-1}(g_E)$, where $\psi^{-1}(f_E)$ and $\psi^{-1}(g_E)$ are fuzzy soft disjoint non-empty $(1,2)^*$-fuzzy soft b-open sets in (X, E, τ_1, τ_2). This contradicts the fact that (X, E, τ_1, τ_2) is $(1,2)^*$-fuzzy soft b-connected. Hence $(Y, E, \sigma_1, \sigma_2)$ is $(1,2)^*$-fuzzy soft connected.
(ii) Suppose $(Y, E, \sigma_1, \sigma_2)$ is not $(1,2)^*$-fuzzy soft b-connected. Let $Y = f_E \tilde{\cup} g_E$, where f_E and g_E are fuzzy soft disjoint non-empty $(1,2)^*$-fuzzy soft b-open sets in $(Y, E, \sigma_1, \sigma_2)$. Since ψ is $(1,2)^*$-fuzzy soft b-irresolute and onto; then $\tilde{1}_E = \psi^{-1}(f_E)\tilde{\cup}\psi^{-1}(g_E)$, where $\psi^{-1}(f_E)$ and $\psi^{-1}(g_E)$ are fuzzy soft disjoint non-empty $(1,2)^*$-fuzzy soft b-open sets in (X, E, τ_1, τ_2). This contradicts the fact that (X, E, τ_1, τ_2) is $(1,2)^*$-fuzzy soft b-connected. Hence $(Y, E, \sigma_1, \sigma_2)$ is $(1,2)^*$-fuzzy soft b-connected.

4. $(1,2)^*$-Fuzzy Soft b-Compactness

In this section $(1,2)^*$-fuzzy soft b-compactness is defined and some of the characterizations are proved.

Definition 23. *A collection $\{f_{i_E} : i \in \Lambda\}$ of $(1,2)^*$-fuzzy soft b-open sets in fuzzy soft bitopological space (X, E, τ_1, τ_2) is called a $(1,2)^*$-fuzzy soft b-cover of f_E if $f_E \tilde{\subseteq} \tilde{\bigcup}\{f_{i_E} : i \in \Lambda\}$.*

Definition 24. *A fuzzy soft bitopological space (X, E, τ_1, τ_2) is called a $(1,2)^*$-fuzzy soft b-compact if every $(1,2)^*$-fuzzy soft b-open cover of $\tilde{1}_E$ has a finite subcover.*

Definition 25. *A fuzzy soft subset f_E of fuzzy soft bitopological space (X, E, τ_1, τ_2) is said to be $(1,2)^*$-fuzzy soft b-compact relative to $\tilde{1}_E$, if for every collection $\{f_{i_E} : i \in \Lambda\}$ of $(1,2)^*$-fuzzy soft b-open subsets of (X, E, τ_1, τ_2) such that f_E if $f_E \tilde{\subseteq} \tilde{\bigcup}\{f_{i_E} : i \in \Lambda\}$ there exists a finite subset Λ_0 of Λ such that $f_E \tilde{\subseteq} \tilde{\bigcup}\{f_{i_E} : i \in \Lambda_0\}$.*

Definition 26. *A fuzzy soft subset f_E of fuzzy soft bitopological space (X, E, τ_1, τ_2) is said to be $(1,2)^*$-fuzzy soft b-compact if f_E is $(1,2)^*$-fuzzy soft b-compact as a subspace of (X, E, τ_1, τ_2).*

Theorem 6. *Every $(1,2)^*$-fuzzy soft closed subset of fuzzy $(1,2)^*$-fuzzy soft b-compact space (X, E, τ_1, τ_2) is $(1,2)^*$-fuzzy soft b-compact relative to $\tilde{1}_E$.*

Proof. Let f_E be a $(1,2)^*$-fuzzy soft closed subset of (X, E, τ_1, τ_2). Then f_E^c is a $(1,2)^*$-fuzzy soft open set in (X, E, τ_1, τ_2). Let $S = \{g_{i_E} : i \in \Lambda\}$ be a cover of f_E by $(1,2)^*$-fuzzy soft open subsets in (X, E, τ_1, τ_2). Then $S \tilde{\cup} f_E^c$ is a $(1,2)^*$-fuzzy soft b-open cover for $\tilde{1}_E$. Since (X, E, τ_1, τ_2) is a $(1,2)^*$-fuzzy soft b-compact; it has a finite subcover say $S = g_{1_E} \tilde{\cup} g_{1_E} \tilde{\cup} ... \tilde{\cup} g_{n_E} \tilde{\cup} f_E^c$, $g_{i_E} \tilde{\in} S, i = 1, 2, ..., n$. But f_E and f_E^c are fuzzy soft disjoint. Hence $f_E \tilde{\subseteq} g_{1_E} \tilde{\cup} g_{1_E} \tilde{\cup} ... \tilde{\cup} g_{n_E} \tilde{\in} S$. Thus we have shown that any $(1,2)^*$-fuzzy soft b-open cover has a finite subcover. Therefore f_E is $(1,2)^*$-fuzzy soft b-compact relative to $\tilde{1}_E$.

Theorem 7. *A* $(1,2)^*$*-fuzzy soft b-continuous image of a* $(1,2)^*$*-fuzzy soft b-compact space is* $(1,2)^*$*-fuzzy soft compact.*

Proof. Consider $\psi : (X, E, \tau_1, \tau_2) \to (Y, E, \sigma_1, \sigma_2)$ be a $(1,2)^*$-fuzzy soft b-continuous function. Let $\{f_{i_E} : i \in \Lambda\}$ be a $\sigma_1\sigma_2$-fuzzy soft open cover of $\tilde{1}_E$ in $(Y, E, \sigma_1, \sigma_2)$. Then $\{\psi^{-1}(f_{i_E}) : i \in \Lambda\}$ is a $(1,2)^*$-fuzzy soft b-open cover of $\tilde{1}_E$ in (X, E, τ_1, τ_2). Since (X, E, τ_1, τ_2) is $(1,2)^*$-fuzzy soft b-compact; it has a finite subcover say, $\{\psi^{-1}(f_{1_E}), \psi^{-1}(f_{1_E}), ..., \psi^{-1}(f_{n_E})\}$. Since ψ is onto, $\{f_{1_E}, f_{1_E}, ..., f_{n_E}\}$ is a $\sigma_1\sigma_2$-fuzzy soft open cover of $\tilde{1}_E$ in $(Y, E, \sigma_1, \sigma_2)$ and hence $(Y, E, \sigma_1, \sigma_2)$ is $(1,2)^*$-fuzzy soft compact.

Theorem 8. *If a map* $\psi : (X, E, \tau_1, \tau_2) \to (Y, E, \sigma_1, \sigma_2)$ *is a* $(1,2)^*$*-fuzzy soft b-irresolute and a fuzzy soft subset* f_E *of* (X, E, τ_1, τ_2) *is* $(1,2)^*$*-fuzzy soft compact relative to* $\tilde{1}_E$ *then the image* $\psi(f_E)$ *is* $(1,2)^*$*-fuzzy soft compact relative to* $\tilde{1}_E$ *in* $(Y, E, \sigma_1, \sigma_2)$*.*

Proof. Let $\{f_{i_E} : i \in \Lambda\}$ be a collection of $(1,2)^*$-fuzzy soft b-open sets in $(Y, E, \sigma_1, \sigma_2)$ such that $\psi(f_E) \tilde{\subseteq} \tilde{\bigcup}\{f_{i_E} : i \in \Lambda\}$. Then $f_E \tilde{\subseteq} \tilde{\bigcup}\{\psi^{-1}(f_{i_E}) : i \in \Lambda\}$, where $\psi^{-1}(f_{i_E})$ is $(1,2)^*$-fuzzy soft b-open in (X, E, τ_1, τ_2) is $(1,2)^*$-fuzzy soft compact relative to $\tilde{1}_E$ in (X, E, τ_1, τ_2), there exists a finite sub collection $\{f_{1_E}, f_{2_E}, ..., f_{n_E}\}$ such that $f_E \tilde{\subseteq} \tilde{\bigcup}\{\psi^{-1}(f_{i_E}) : i = 1, 2, 3..., n\}$ that is, $\psi(f_E) \tilde{\subseteq} \tilde{\bigcup}\{f_{i_E} : i = 1, 2, 3..., n\}$. Hence $\psi(f_E)$ is $(1,2)^*$-fuzzy soft compact relative to $\tilde{1}_E$ in $(Y, E, \sigma_1, \sigma_2)$.

5. Conclusion

In this paper, we introduced the notions of $(1,2)^*$-fuzzy soft b-separated sets, $(1,2)^*$-fuzzy soft b-connectedness and $(1,2)^*$-fuzzy soft b-compactness in fuzzy soft bitopological spaces. Then, some basic topological properties of these notions were investigated. Also, some illustrative examples were given to show the importance of the obtained theorems. We hope that this paper will be important for researchers to studying many other concepts and also the generalization for some important results in topology.

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