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A New Technique to Speed Up Group Methods for Solving Hyperbolic Telegraph Equations

Abdulkafi Mohammed Saeed

Department of Mathematics, College of Science, Qassim University, Buraydah, Kingdom of Saudi Arabia

Abstract. Numerous methods have been introduced in the literature for numerical solution of two-dimensional hyperbolic telegraph equations. Improved techniques using explicit group methods derived from the standard and skewed (rotated) finite difference operators have been developed over the last few years in solving the linear systems that arise from the discretization of the several types of partial differential equations. The preconditioning strategies play a vital role in accelerating the convergence rates of these group iterative methods. In this paper, we present a preliminary study of the formulation of new preconditioned scheme based on explicit group relaxation methods for the difference solution of the telegraph equations. The efficient and robustness of these new formulations over the existing explicit group schemes demonstrated through numerical experiments.

2020 Mathematics Subject Classifications: 35B30, 35L10, 35L20, 65M06, 65N06

Key Words and Phrases: Preconditioned technique, Finite difference method, Group iterative method, Telegraph Equations

1. Introduction

The group methods depend on rotated finite difference operator were shown to require less execution time requirements than the common point iterative methods based on the centered difference approximations for solving partial differential equations (PDEs) [1, 12–14, 20, 21]. In addition, the methods of the meshless local weak-strong forms combined with the meshless local Petrov-Galerkin are used to solve 2D linear hyperbolic equation by Dehghan and Ghesmati [6]. Besides, the operator splitting method and the spectral Galerkin method have been developed and applied for solving two dimensional hyperbolic equation [8, 10]. It is well known that preconditioners play a vital role in accelerating the convergence rates of iterative methods, several preconditioned strategies have been used for improving the convergence rate of the explicit group methods derived from the standard and skewed (rotated) finite difference operators [2–5, 15, 17–19]. Based on the existing preconditioning strategies and by combining several categories of preconditioning

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Email address: abdulkafi.ahmed@qu.edu.sa (A.M. Saeed)

techniques, we propose a new preconditioning matrix in block formulation to suit the structure of the explicit group formula for solving telegraph equations.

In the following sections of this research, we will study and discuss the formulation of two types of explicit group methods which are called Explicit group (EG) method and Explicit decoupled group(EDG) method for solving telegraph equations. These iterative methods depend on standard and rotated point iterative schemes respectively. Furthermore, the improvement of the mentioned group methods using preconditioning strategies will be introduced.

This paper is organised as follows: in section 2, we give a presentation of the formulation of explicit group iterative methods such as EG and EDG for solving telegraph equations. The proposed application of the preconditioner in block formulation to the EG and EDG is given in section 3. In section 4, The numerical examples to confirm the results obtained will be presented. Finally, we report a brief conclusion in Section 5.

2. Explicit Group(EG-EDG)Methods

Consider the telegraph equation defined in the region $\Omega = \{(x, y, t) : 0 < x, y < 1, \ t > 0\}$ of the following form [7]:

$$\frac{\partial^2 U}{\partial t^2} + 2\alpha(x,y,t)\frac{\partial U}{\partial t} + \beta^2(x,y,t) = L(x,y,t)\frac{\partial^2 U}{\partial x^2} + M(x,y,t)\frac{\partial^2 U}{\partial y^2} + F(x,y,t) \tag{1}$$

where $\alpha(x, y, t) > 0, \ \beta(x, y, t) \geq 0, \ L(x, y, t) > 0, \ M(x, y, t) > 0$. The initial and boundary conditions are given by

$$\begin{array}{l} U(x,y,0) = f_1(x,y); \ \frac{\partial U}{\partial t}(x,y,0) = f_2(x,y); \ U(0,y,t) = f_3(y,t); \\ U(1,y,t) = f_4(y,t); \ \ U(x,0,t) = f_5(y,t); \ \ U(x,1,t) = f_6(y,t)\,. \end{array}$$

Let $k > 0$ and $h > 0$ be the time step and space step respectively. We divide the interval $0 \leq x, y \leq 1$ into $(N + 1)$ subinterval and the grid points are given by $(x_i, y_j, t_m) = (ih, jh, mk)$ where $m = 1, 2, 3, ...$

Finite difference discretization of equation (1) using centred difference formula for the second partial derivatives will obtain [9]

$$\begin{array}{l} \frac{u_{i,j,m+1} - 2u_{i,j,m} + u_{i,j,m-1}}{\Delta t^2} + 2\alpha\frac{u_{i,j,m+1} - u_{i,j,m-1}}{2\,\Delta t} = \frac{1}{2}[\frac{u_{i-1,j,m+1} - 2u_{i,j,m+1} + u_{i+1,j,m+1}}{\Delta x^2} \\ + \frac{u_{i-1,j,m} - 2u_{i,j,m} + u_{i+1,j,m}}{\Delta x^2}] + \frac{1}{2}[\frac{u_{i,j-1,m+1} - 2u_{i,j,m+1} + u_{i,j+1,m+1}}{\Delta y^2} + \\ \frac{u_{i,j-1,m} - 2u_{i,j,m} + u_{i,j+1,m}}{\Delta y^2}] - \frac{\beta^2}{2}(u_{i,j,m+1} + u_{i,j,m}) + F_{i,j,m+\frac{1}{2}} \end{array} \tag{2}$$

where $x = i\,\Delta x, \ y = j\,\Delta y, \ t = m\,\Delta t; \ (i, j = 0, 1, 2, ..., n-1; \ m = 0, 1, 2, ...)$. The above equation (2) is called standard point formula and after simplification it can be written as

$$\begin{array}{l} -(\frac{r^2}{2})u_{i-1,j,m+1} + (1 + a + 2r^2 + b/2)u_{i,j,m+1} - (\frac{r^2}{2})u_{i+1,j,m+1} - (\frac{r^2}{2})u_{i,j-1,m+1} \\ -(\frac{r^2}{2})u_{i,j+1,m+1} = (\frac{r^2}{2})u_{i-1,j,m} + (2 - 2r^2 - b/2)u_{i,j,m} + (\frac{r^2}{2})u_{i+1,j,m} \\ +(\frac{r^2}{2})u_{i,j-1,m} + (\frac{r^2}{2})u_{i,j+1,m} + (a-1)u_{i,j,m-1} + \Delta t^2 F_{i,j,m+\frac{1}{2}} \end{array} \tag{3}$$

where $h = \Delta x = \Delta y = \frac{1}{N}$, $r = \frac{\Delta t}{h}$, $a = \alpha\,\Delta t$, $b = \beta^2\Delta t^2$. By rotating the x-y axis clockwise 45^0, we obtained the rotated finite difference approximation for telegraph equation (1) as follow

$$\begin{aligned}&\frac{u_{i,j,m+1}-2u_{i,j,m}+u_{i,j,m-1}}{\Delta t^2}+2\alpha\frac{u_{i,j,m+1}-u_{i,j,m-1}}{2\,\Delta t}=\frac{1}{4}[\frac{u_{i-1,j-1,m+1}-2u_{i,j,m+1}+u_{i+1,j+1,m+1}}{\Delta x^2}\\&+\frac{u_{i-1,j-1,m}-2u_{i,j,m}+u_{i+1,j+1,m}}{\Delta x^2}]+\frac{1}{4}[\frac{u_{i-1,j+1,m+1}-2u_{i,j,m+1}+u_{i+1,j-1,m+1}}{\Delta y^2}+\\&\frac{u_{i-1,j+1,m}-2u_{i,j,m}+u_{i+1,j-1,m}}{\Delta y^2}]-\frac{\beta^2}{2}(u_{i,j,m+1}+u_{i,j,m})+F_{i,j,m+\frac{1}{2}}\end{aligned} \tag{4}$$

Similarly, we can simplify equation (4) as the following

$$\begin{aligned}&-(\tfrac{r^2}{4})u_{i-1,j-1,m+1}+(1+a+r^2+b/2)u_{i,j,m+1}-(\tfrac{r^2}{4})u_{i+1,j+1,m+1}-(\tfrac{r^2}{4})u_{i-1,j-1,m+1}\\&-(\tfrac{r^2}{4})u_{i+1,j-1,m+1}=(\tfrac{r^2}{2})u_{i-1,j-1,m}+(2-r^2-b/2)u_{i,j,m}+(\tfrac{r^2}{4})u_{i+1,j+1,m}\\&+(\tfrac{r^2}{4})u_{i-1,j+1,m}+(\tfrac{r^2}{4})u_{i+,j-1,m}+(a-1)u_{i,j,m-1}+\Delta t^2F_{i,j,m+\frac{1}{2}}\end{aligned} \tag{5}$$

The formulation of EG method depend on the standard point approximation which was derived from the central finite difference discretisation as equations (2) and (3) [11, 16]. Applying equation (3) to any group of four points on a discretised solution domain will result in a 4×4 system of equations as follows:

$$\begin{pmatrix} c_1 & c_2 & 0 & c_2\\ c_2 & c_1 & c_2 & 0\\ 0 & c_2 & c_1 & c_2\\ c_2 & 0 & c_2 & c_1\end{pmatrix}\begin{pmatrix} u_{i,j,m+1}\\ u_{i+1,j,m+1}\\ u_{i+1,j+1,m+1}\\ u_{i,j+1,m+1}\end{pmatrix}=\begin{pmatrix} rhs_{i,j}\\ rhs_{i+1,j}\\ rhs_{i+1,j+1}\\ rhs_{i,j+1}\end{pmatrix} \tag{6}$$

where $c_1 = 1 + a + 2r^2 + \frac{b}{2}$, $c_2 = -\frac{r^2}{2}$,

$rhs_{i,j} = (\frac{r^2}{2})[u_{i-1,j,m+1} + u_{i,j-1,m+1}] + (\frac{r^2}{2})[u_{i-1,j,m} + u_{i,j-1,m} + u_{i+1,j,m} + +u_{i,j+1,m}]$
$+(2 - 2r^2 - \frac{b}{2})u_{i,j,m} + (a - 1)u_{i,j,m-1} + \Delta t^2\, F_{i,j,m+\frac{1}{2}}$,

$rhs_{i+1,j} = (\frac{r^2}{2})[u_{i+1,j-1,m+1} + u_{i+2,j,m+1}] + (\frac{r^2}{2})[u_{i,j,m} + u_{i+1,j-1,m} + u_{i+2,j,m}$
$+u_{i+1,j+1,m}] + (2 - 2r^2 - \frac{b}{2})u_{i+1,j,m} + (a - 1)u_{i+1,j,m-1} + \Delta t^2\, F_{i+1,j,m+\frac{1}{2}}$,

$rhs_{i+1,j+1} = (\frac{r^2}{2})[u_{i+2,j+1,m+1} + u_{i+1,j+2,m+1}] + (\frac{r^2}{2})[u_{i,j+1,m} + u_{i+1,j,m} + u_{i+2,j+1,m}$
$+u_{i+1,j+2,m}] + (2 - 2r^2 - \frac{b}{2})u_{i+1,j+1,m} + (a - 1)u_{i+1,j+1,m-1} + \Delta t^2\, F_{i+1,j+1,m+\frac{1}{2}}$,

$rhs_{i,j+1} = (\frac{r^2}{2})[u_{i-1,j+1,m+1} + u_{i,j+2,m+1}] + (\frac{r^2}{2})[u_{i-1,j+1,m} + u_{i,j,m} + u_{i+1,j+1,m}$
$+u_{i,j+2,m}] + (2 - 2r^2 - \frac{b}{2})u_{i,j+1,m} + (a - 1)u_{i,j+1,m-1} + \Delta t^2\, F_{i,j+1,m+\frac{1}{2}}$

The system of equations (6) can be inverted to the following system

$$\begin{pmatrix} u_{i,j,m+1}\\ u_{i+1,j,m+1}\\ u_{i+1,j+1,m+1}\\ u_{i,j+1,m+1}\end{pmatrix}=\begin{pmatrix} \ell_1 & \ell_2 & \ell_3 & \ell_2\\ \ell_2 & \ell_1 & \ell_2 & \ell_3\\ \ell_3 & \ell_2 & \ell_1 & \ell_2\\ \ell_2 & \ell_3 & \ell_2 & \ell_1\end{pmatrix}\begin{pmatrix} rhs_{i,j}\\ rhs_{i+1,j}\\ rhs_{i+1,j+1}\\ rhs_{i,j+1}\end{pmatrix} \tag{7}$$

where

$$\begin{aligned}\ell_1 = {}&2(4a^2 + 4ab + 16ar^2 + 8a + 16r^2 + 4 + 8r^2b + 4b + 14r^4 + b^2)/(8a^3 + 12a^2b\\&+48a^2r^2 + 24a^2 + 24ab + 88ar^4 + 96ar^2 + 48ar^2b + 24a + 8 + 6ab^2\\&+12b + 12r^2b^2 + 48r^2 + b^3 + 88r^4 + 44r^4b + 48r^6 + 48r^2b + 6b^2);\end{aligned}$$

$\ell_2 = 2r^2/(4a^2 + 4ab + 16ar^2 + 8a + 4b + 12r^4 + 16r^2 + 8r^2 b + 4 + b^2);$
$\ell_3 = 4r^4/(8a^3 + 12a^2 b + 48a^2 r^2 + 24a^2 + 24ab + 88ar^4 + 96ar^2$
$+48ar^2 b + 24a + 6ab^2 + 12r^2 b^2 + 12b + 8 + 48r^2 + b^3$
$+88r^4 + 44r^4 b + 48r^6 + 48r^2 b + 6b^2)\,.$

Similarly, the formulation of EDG method can be done by applying equation (5) to any group of four points of the solution domain will result in a 4×4 system of equations as follows:

$$\begin{pmatrix} q_1 & q_2 & 0 & 0 \\ q_2 & q_1 & 0 & 0 \\ 0 & 0 & q_1 & q_2 \\ 0 & 0 & q_2 & q_1 \end{pmatrix} \begin{pmatrix} u_{i,j,m+1} \\ u_{i+1,j+1,m+1} \\ u_{i+1,j,m+1} \\ u_{i,j+1,m+1} \end{pmatrix} = \begin{pmatrix} rhs_{i,j} \\ rhs_{i+1,j+1} \\ rhs_{i+1,j} \\ rhs_{i,j+1} \end{pmatrix} \tag{8}$$

where $q_1 = 1 + a + r^2 + \frac{b}{2}, \quad q_2 = -\frac{r^2}{4},$

$$rhs_{i,j} = (\tfrac{r^2}{4})[u_{i-1,j-1,m+1} + u_{i+1,j-1,m+1} + u_{i-1,j+1,m+1}] + (\tfrac{r^2}{4})[u_{i-1,j-1,m} + u_{i+1,j-1,m} + u_{i+1,j+1,m} + u_{i-1,j+1,m}] + (2 - r^2 - \tfrac{b}{2})u_{i,j,m} + (a-1)u_{i,j,m-1} + \Delta t^2\, F_{i,j,m+\frac{1}{2}}\,,$$

$$rhs_{i+1,j+1} = (\tfrac{r^2}{4})[u_{i+2,j,m+1} + u_{i+2,j+2,m+1} + u_{i,j+2,m+1}] + (\tfrac{r^2}{4})[u_{i,j,m} + u_{i+2,j,m} + u_{i+2,j+2,m} + u_{i,j+2,m}] + (2 - r^2 - \tfrac{b}{2})u_{i+1,j+1,m} + (a-1)u_{i+1,j+1,m-1} + \Delta t^2\, F_{i+1,j+1,m+\frac{1}{2}}\,,$$

$$rhs_{i+1,j} = (\tfrac{r^2}{4})[u_{i,j-1,m+1} + u_{i+2,j-1,m+1} + u_{i+2,j+1,m+1}] + (\tfrac{r^2}{4})[u_{i,j-1,m} + u_{i+2,j-1,m} + u_{i+2,j+1,m} + u_{i,j+1,m}] + (2 - r^2 - \tfrac{b}{2})u_{i+1,j,m} + (a-1)u_{i+1,j,m-1} + \Delta t^2\, F_{i+1,j,m+\frac{1}{2}}\,,$$

$$rhs_{i,j+1} = (\tfrac{r^2}{4})[u_{i-1,j,m+1} + u_{i+1,j+2,m+1} + u_{i-1,j+2,m+1}] + (\tfrac{r^2}{4})[u_{i-1,j,m} + u_{i+1,j,m} + u_{i+1,j+2,m} + u_{i-1,j+2,m}] + (2 - r^2 - \tfrac{b}{2})u_{i,j+1,m} + (a-1)u_{i,j+1,m-1} + \Delta t^2\, F_{i,j+1,m+\frac{1}{2}}\,.$$

The system of equations (8) can be written in an explicit decoupled system of 2×2 equations as follows

$$\begin{pmatrix} u_{i,j,m+1} \\ u_{i+1,j+1,m+1} \end{pmatrix} = \frac{1}{L} \begin{pmatrix} \xi_1 & \xi_2 \\ \xi_2 & \xi_1 \end{pmatrix} \begin{pmatrix} rhs_{i,j} \\ rhs_{i+1,j+1} \end{pmatrix}$$
$$and \tag{9}$$
$$\begin{pmatrix} u_{i+1,j,m+1} \\ u_{i,j+1,m+1} \end{pmatrix} = \frac{1}{L} \begin{pmatrix} \xi_1 & \xi_2 \\ \xi_2 & \xi_1 \end{pmatrix} \begin{pmatrix} rhs_{i+1,j} \\ rhs_{i,j+1} \end{pmatrix}$$

where $L = 16 + 32a + 32r^2 + 16b + 32ar^2 + 16a^2 + 16ab + 15r^4 + 16r^2 b + 4b^2;$
$\xi_1 = 8(2 + 2a + 2r^2 + b);\ \xi_2 = 4r^2\,.$

In the EDG method, the grid points are gathered into groups which can consists of only 2 grid points. Each value for u of every grid point is approximated by the rotated point formula. These values are calculated with a sequence from left to right and then upwards. Hence, the iteration over the solution domain is only carried out on half the mesh points. Once convergence is achieved, the solution at the other half of the points is obtained directly once using the standard point difference formula [1] .

3. The Proposed Preconditioned Technique

The convergence rates of the group iterative methods as EG and EDG depend on the spectral properties of the coefficient matrices [3, 11]. A preconditioner is a matrix that transforms the resulted system of these methods into one that is equivalent in the sense that it has the same solution, but that has more favourable spectral properties. An ongoing research in this area lies in the formulation of suitable preconditioners which can improve the convergence rates of iterative method [5, 13, 14]. Dramatic improvements are possible, but the difficulty is to construct the suitable preconditioner.
Usually the systems (7) and (9) resulted from EG and EDG methods respectively are large and sparse. By using the following preconditioner matrix

$$\Psi = \begin{pmatrix} q_1 & 0 & 0 & 0 \\ 0 & q_1 & 0 & 0 \\ 0 & 0 & 0 & c_2 \\ 0 & 0 & c_2 & 0 \end{pmatrix} \tag{10}$$

where c_2 and q_1 defined as equations (6) and (8) respectively to both EG and EDG, we will obtain new preconditioned systems. The process of obtaining the new preconditioned system depend on the structure of the coefficient matrix of the target system involves multiplying this matrix Ψ by the original system of the mentioned iterative methods to produce coefficients matrix with a spectral radius less than the spectral radius of the coefficients matrix of the original system.
By applying preconditioner matrix to any group of four points of EG scheme on a discretised solution domain will result in a 4×4 system of equations as

$$\begin{pmatrix} q_1c_1 & q_1c_2 & 0 & q_1c_2 \\ q_1c_2 & q_1c_1 & q_1c_2 & 0 \\ c_2^2 & 0 & c_2^2 & c_1c_2 \\ 0 & c_2^2 & c_1c_2 & c_2^2 \end{pmatrix} \begin{pmatrix} u_{i,j,m+1} \\ u_{i+1,j,m+1} \\ u_{i+1,j+1,m+1} \\ u_{i,j+1,m+1} \end{pmatrix} = \begin{pmatrix} rq_1hs_{i,j} \\ rq_1hs_{i+1,j} \\ rhc_2s_{i,j+1} \\ rhc_2s_{i+1,j+1} \end{pmatrix} \tag{11}$$

The resulted system is called preconditioned EG (PEG) .
By using the same preconditioner matrix and preconditioning process to any group of four points of EDG scheme on a discretised solution domain, we can write the 4×4 preconditioned EDG (PEDG) system as follows

$$\begin{pmatrix} q_1^2 & q_1q_2 & 0 & 0 \\ q_1q_2 & q_1^2 & 0 & 0 \\ 0 & 0 & c_2q_2 & c_2q_1 \\ 0 & 0 & c_2q_1 & c_2q_2 \end{pmatrix} \begin{pmatrix} u_{i,j,m+1} \\ u_{i+1,j+1,m+1} \\ u_{i+1,j,m+1} \\ u_{i,j+1,m+1} \end{pmatrix} = \begin{pmatrix} rhs_{i,j} \\ rhs_{i+1,j+1} \\ rhs_{i+1,j} \\ rhs_{i,j+1} \end{pmatrix} \tag{12}$$

In the following section, we will discuss the efficiency of the above proposed preconditioned system for solving the telegraph equations.

4. Numerical Results

In this section, we check the applicability and effectiveness of the proposed preconditioned group iterative methods in solving problems of telegraph equations. For the purpose of comparison, we use a tolerance of as the termination criteria with the convergence criteria norm. The computer processing unit is Intel(R) Core(TM) i7-7500U with memory of 8Gb and the software used to implement and generate the results was Developer C++ Version 4.9.9.2.

All the four methods (EG, EDG, PEG and PEDG) described in sections 2 and 3 were applied to the model problem (1) with $\alpha = \beta = 1$ and $L = M = 1$ and the initial and boundary conditions are given by

$$\begin{array}{ll} u(x,y,0) = \sin(x)\ \sin(y), & u_t(x,y,0) = -\sin(x)\ \sin(y), \\ u(0,y,t) = u(x,0,t) = 0, & u(1,y,t) = e^{-t}\sin(1)\ \sin(y), \\ u(x,1,t) = e^{-t}\sin(x)\ \sin(1), & \end{array}$$

with $F(x,y,t) = 2e^{-t}\ \sin(x)\ \sin(y)$. The exact solution of this model problem is $u(x,y,t) = e^{-t}\ \sin(x)\ \sin(y)$.

In addition, all the mentioned four methods were run using several mesh sizes of 20,50, 80, 98 and 118. The results are summarized in Tables 1 and 2 which showed the comparison among the unpreconditioned EG and EDG methods (original systems) and the preconditioned EG and EDG.

Table 1: Comparison of iterations(k)and elapsed time between EG and EDG methods

	Unpreconditioned EG				Unpreconditioned EDG			
h^{-1}	k	Max Error	Ave Error	Time	k	Max Error	Ave Error	Time
20	3	$7.1E-06$	$7.3E-07$	0.863	3	$6.9E-06$	$8.6E-07$	0.721
50	4	$7.7E-06$	$8.5E-07$	4.001	3	$7.4E-06$	$8.3E-07$	2.533
80	4	$7.6E-06$	$8.4E-07$	13.216	3	$7.3E-06$	$8.2E-07$	11.415
98	4	$7.6E-06$	$8.3E-07$	12.904	3	$7.3E-06$	$8.1E-07$	8.654
118	5	$7.7E-06$	$7.5E-07$	14.863	4	$7.6E-06$	$7.2E-07$	10.481

Table 2: Comparison of iterations(k)and elapsed time between PEG and PEDG methods

	Preconditioned EG				Preconditioned EDG			
h^{-1}	k	Max Error	Ave Error	Time	k	Max Error	Ave Error	Time
20	2	$6.5E-06$	$6.2E-07$	0.634	1	$6.5E-06$	$5.9E-07$	0.501
50	2	$6.8E-06$	$7.7E-07$	1.724	1	$6.6E-06$	$7.5E-07$	0.875
80	3	$6.4E-06$	$7.5E-07$	10.521	2	$6.1E-06$	$7.3E-07$	8.011
98	3	$6.6E-06$	$7.2E-07$	7.451	2	$6.1E-06$	$7.2E-07$	4.661
118	4	$6.7E-06$	$6.4E-07$	8.623	3	$6.6E-06$	$6.5E-07$	6.032

We can easily observe that the number of iterations and elapsed time significantly

reduced when using the new preconditioned methods for solving such problem.
In addition, it becomes clear that the most advanced method with regard to the number of iterations and the elapsed time among all the methods mentioned is the PEDG method. The accuracies of the proposed methods are as good as the original group iterative methods but they require lesser computing timings to achieve the results. Figure 1 shown that the proposed method (PEDG) method is the most time-reducing method due to the lower computations. This means that the new technique used has succeeded in improving the group iterative methods for solving telegraph equations.

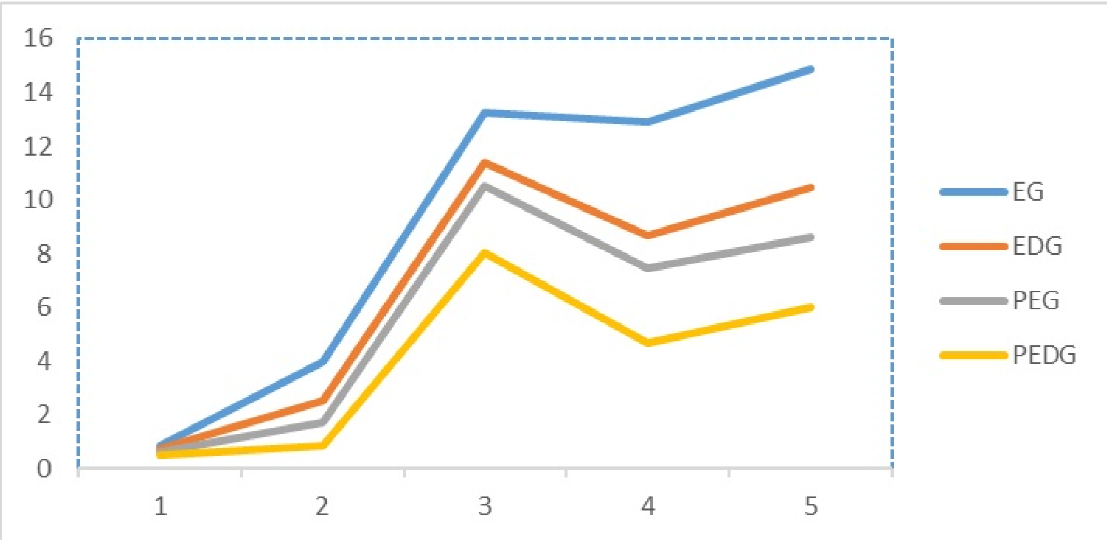

Figure 1: Comparison of elapsed time for all studied methods.

5. Conclusions

In this study, we have formulated new preconditioned iterative method based on EG and EDG methods for solving the telegraph initial boundary problems. From observation of all experimental results, it can be concluded that the proposed preconditioned EDG scheme may be a good alternative to solve this type of problems and many other numerical problems. Furthermore, the idea of this preconditioning technique can be extended to solve other types of initial boundary problems which will be reported separately in the future.

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