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Nearly Soft β - Open Sets via Soft Ditopological Spaces

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Abstract. As a result of the importance of topological space in data analysis and some applications, many researches have used various methods to expand that space, including the concept of ditopology. T. Dizman and et al. presented soft ditopolgical spaces in 2016. We define new types of nearly soft open sets in soft ditopology as soft β - open, soft β - closed, soft preopen, soft semi - open, and some related properties in this paper. Soft β - continuous and soft β - cocontinuous functions were also introduced . Finally, soft β - compact, soft β - stable and soft β - irresolute concepts were discussed, and some of the concepts were studied in this field.

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1. Introduction and Preliminaries

In the late twentieth century, Molodtsov[11] introduced the theory of soft set as a generalization of the set theory, which widely used to deal with incomplete, insufficient information for its study and analysis, which similar to the rough set theory. Soft set theory and its applications are now advancing rapidly in a variety of fields[5, 7, 8, 13–15, 19]. Maji et al.[21, 22] presented some new definitions of soft sets as well as an application of soft sets in decision making problems. Jose Carlos et al.[6] participated in the development and improvement of soft topology. The idea of a generalization of the topological space by using novel concepts as ideal, grill, filter[3, 9, 16, 24] coming as a result of the importance of topological space and used it to solve some of the measures things that were previously

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difficult to solve. The mysterious set theory and other uncertain knowledge models have led to new approaches to decision - making as [2, 4]. So, Brown et al.[12] introduced the concept of ditopological space as a generalization of topological spaces. The concept of ditopological space via the soft set theory with separation axioms of soft ditopological space introduced by Senel in 2016 [23]. Where the idea of ditopological spaces depends on two structures soft topology and soft cotopology. Also, Senel [23] introduced soft ditopological spaces as a soft generalization of ditopology concept, which depends on two structures a soft topology and a soft subspace topology. S. Dost et al. In[12] introduced the concept of β - open and β - closed in ditopological texture spaces. In this paper, we will introduce some of the nearly soft β - open sets, the study of soft β - compactness and soft β - cocompactness. Also, soft β - stable and soft β - irresolute were introduced in soft ditopological spaces and study some of their properties.

Through this section, we recall several basic notions related to soft set, soft topological space, soft cotopological space, and some of the nearly soft open sets through soft topological space, which handled in mentioned in [10–12, 17, 18, 20]. Through this paper, we notice that U refers to an universal set, E is the soft parameters and $P(U)$ is the power set of U. .

Definition 1. *[11] On universal set U, a pair (f, E) is called a soft set if and only if f is a mapping from E into the power set $P(U)$. To put it another way, the soft set is a parametrized family of subsets of the set U. Every set$f(e)$, $e \in E$ in this family can be thought of as the set of e-elements of the soft set (f, E), or as the set of e-approximate elements of the soft set..*

Definition 2. *[20] If τ is defined as the collection of soft sets over X, then τ is said to be a soft topology on X if it fulfills the following axioms: (1)$X, \Phi \in \tau$, where $\Phi(e) = \Phi$ and $X(e) = X, \forall e \in E$. (2) The union of any number of soft sets in τ belongs to τ. (3) The intersection of any two soft sets in τ belongs to τ. The triple (X, τ, E) is referred to as a soft topological space, and the members of τ are referred to as soft open sets.*

Definition 3. *Let (X, τ, E) represent a soft topological space over X and (F, A) represent a soft set over X. (1) The soft interior of (F, A) [18] is the soft set int $(F, A) = \tilde{X}\{(O, A) : (O, A)$ is the soft open and $(O, A)\tilde{\subseteq}(F, A)\}$.*
(2) The soft closure of (F, A) [20] is the soft set cl $(F, A) = \tilde{\cap}\{(C, A) : (C, A)$ is soft closed and $(F, A)\tilde{\subseteq}(C, A)\}$.

Definition 4. *[12] If κ is the collection of complement soft sets over X, then κ is said to be a soft cotopology on X if it obeys the following axioms: (1) Φ and $\tilde{X} \in \kappa$.*
(2) The intersection of any number of soft sets in $\kappa \in \kappa$.
(3) The union of any two soft sets in $\kappa \in \kappa$. The triple (X, κ, E) is referred to as a soft cotopological space, and the members of κ are referred to as soft closed sets.

Definition 5. *A soft set(F, E) of a soft topological space(X, τ, E) is said to be:*
(1) Soft β - open [17] if $(F, A) \tilde{\subseteq} cl(int(cl(F, A)))$.

(2) Soft preopen [17] if $(F,A)\tilde{\subseteq} int(cl(F,A))$.
(3) Soft α *- open [20] if* $(F,A)\tilde{\subseteq} int(cl(intl(F,A)))$.
(4) Soft semi- open [10] if $(F,A)\tilde{\subseteq} cl(int(F,A))$.
(5) Soft open [20] if its complement is soft closed.

Definition 6. *[1] A function* $f:(X,\tau)\to(Y,\sigma)$ *is said to be* β *- irresolute if the preimages of* β *- open sets are* β *- open.*

2. Soft β - open and soft β - closed sets

Definition 7. *Let* U *be a universel set and* E *be the parameters. A family* (τ,κ) *of a subsets of* $\tilde{U}_E$ *is called a soft ditopology on a soft subspace* $\tilde{U}_E$*, where* τ *is a soft topology,* κ *is a soft cotopology and the space* $(\tilde{U}_E,\tau,\kappa)$ *is called soft ditopological space. If we take* $(\tau,\kappa)=\Omega$*, then* $(\tilde{U}_E,\Omega)$ *is said to be soft ditopological space.*

Definition 8. *Let* $(\tilde{U}_E,\tau,\kappa)$ *be a soft ditopological space over* $\tilde{U}_E$ *and* f *be a soft set over* $\tilde{U}_E$ *such that* $f=\{(e,A):e\in E, A\in P(U)$ *and* $(e,A)=F:E\to P(U)\}$.

Definition 9. *Let* $\tilde{U}_E\in S$*. The power soft set of* $\tilde{U}_E$ *is defined by* $P(\tilde{U}_E)=\{f_i\tilde{\subseteq}\tilde{U}_E : i\in I\}$ *and its cardinality is defined by* $|P(\tilde{U}_E)|=2^{\sum_{e\in E}|\tilde{U}_E(e)|}$ *where* $|\tilde{U}_E(e)|$ *is the cardinality* $\tilde{U}_E(e)$

Example 1. *Let* $U=\{u_1,u_2\}$, $E=\{e_1,e_2\}$ *and* $\tilde{U}_E=\{(e_1,\{u_1,u_2\}),(e_2,\{u_1,u_2\})\}$ *then the soft sets are:* $f_1=\{(e_1,\{u_1\})\}$, $f_2=\{(e_1,\{u_2\})\}$, $f_3=\{(e_1,\{u_1,u_2\})\}$, $f_4=\{(e_2,\{u_1\})\}$, $f_5=\{(e_2,\{u_2\})\}$, $f_6=\{(e_2,\{u_1,u_2\})\}$, $f_7=\{(e_1,\{u_1\}),(e_2,\{u_1\})\}$, $f_8=\{(e_1,\{u_1\}),(e_2,\{u_2\})\}$, $f_9=\{(e_1,\{u_2\}),(e_2,\{u_1\})\}$, $f_{10}=\{(e_1,\{u_2\}),(e_2,\{u_2\})\}$, $f_{11}=\{(e_1,\{u_1\}),(e_2,\{u_1,u_2\})\}$,
$f_{12}=\{(e_1,\{u_2\}),(e_2,\{u_1,u_2\})\}$, $f_{13}=\{(e_1,\{u_1,u_2\}),(e_2,\{u_1\})\}$, $f_{14}=\{(e_1,\{u_1,u_2\}),(e_2,\{u_2\})\}$, $f_{15}=\tilde{U}_E$, $f_{16}=\Phi$. ***And*** *we get the complement the soft sets are:*
$f_1^c=\{(e_1,\{u_2\})\}$, $f_2^c=\{(e_1,\{u_1\})\}$, $f_3^c=\{(e_1,\Phi)\}$, $f_4^c=\{(e_2,\{u_2\})\}$, $f_5^c=\{(e_2,\{u_1\})\}$, $f_6^c=\{(e_2,\Phi)\}$, $f_7^c=\{(e_1,\{u_2\}),(e_2,\{u_2\})\}$, $f_8^c=\{(e_1,\{u_2\}),(e_2,\{u_1\})\}$, $f_9^c=\{(e_1,\{u_1\}),(e_2,\{u_2\})\}$, $f_{10}^c=\{(e_1,\{u_1\}),(e_2,\{u_1\})\}$, $f_{11}^c=\{(e_1,\{u_2\}),(e_2,\Phi)\}$, $f_{12}^c=\{(e_1,\{u_1\}),(e_2,\Phi)\}$, $f_{13}^c=\{(e_1,\Phi),(e_2,\{u_2\})\}$, $f_{14}^c=\{(e_1,\Phi),(e_2,\{u_2\})\}$, $f_{15}^c=\Phi$, $f_{16}^c=\tilde{U}_E$.
Also we get a soft ditopological space $(\tau,\kappa)=\{\Phi,\tilde{U}_E,\{(e_1,\{u_2\})\},\{(e_1,\{u_1\})\}\}$ *on* $\tilde{U}_E$*, such that* $\tau=\{\tilde{U}_E,\Phi,\{(e_1,\{u_2\})\}\}$ *and* $\kappa=\{\Phi,\tilde{U}_E,\{(e_1,\{u_1\})\}$.

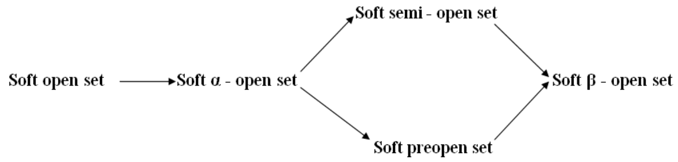
Definition 10. *A soft* β *interior of a soft set* f *is denoted by* $s\beta$ *- int* (f) *which is defined by.*
$s\beta$ *- int* $(f)=\tilde{\cup}\{h : h$ *is a soft* β *- open and* $h\tilde{\subseteq}f\}$.

A soft β closure of a soft set f is denoted by $s\beta$ - cl (f) which is defined by.
$s\beta$ - cl $(f)=\tilde{\cap}\{k : k$ is a soft β - closed and $f\tilde{\subseteq}k\}$.

Definition 11. *Let* $(\tilde{U}_E,\tau,\kappa)$ *be a soft ditopological space and* $f\in P(\tilde{U}_E)$ *then:*
(1) f *is a soft* β *- open if* $f\tilde{\subseteq}cl(int(cl(f)))$.

(2) f is a soft β - closed if $int(cl(int(f)))\tilde{\subseteq} f$.
(3) f is a soft α - open if $f\tilde{\subseteq} int(cl(int(f)))$.
(4) f is a soft preopen if $f\tilde{\subseteq} int(cl(f))$.
(5) f is a soft preclosed if $cl(int(f))\tilde{\subseteq} f$.
(6) f is a soft semi - open if $f\tilde{\subseteq} cl(int(f))$.
(7) f is a soft β - open if the complement of f is a soft β - closed.

Remark 1. *In a soft ditopological space it is easy to see the set of all soft β - open contains each of a soft semi - open, soft preopen and soft α - open, as shown in the following diagram but the converse need not be true in general as Example* 2



Diagram

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Example 2. *Let $(\tilde{U}_E, \tau, \kappa)$ be a soft ditopological space, $U = \{u_1, u_2\}$, $E = \{e_1, e_2\}$ such that*
$\tilde{U}_E = \{(e_1, \{u_1, u_2\}), (e_2, \{u_1, u_2\})\}$, $\tau = \{\tilde{U}_E, \Phi, \{(e_1, \{u_2\})\}, \{(e_2, \{u_1, u_2\})\}, \{(e_1, \{u_1, u_2\}), (e_2, \{u_1)\}, \{(e_1, \{u_2\}), (e_2, \{u_1\})\}$, $\kappa = \{\Phi, \tilde{U}_E, \{(e_1, \{u_1\}), (e_2, \Phi)\}, \{(e_1, \Phi), (e_2, \{u_2\})\}, \{(e_1, \{u_1\}), (e_2, \{u_2\})\}$*, we notice that the soft set* $\{(e_1, \Phi), (e_2, \{u_1\})\}$ *in soft ditopological space is soft preopen set and not soft α - open set. Also it is soft β - open and not soft semi - open set.*

Theorem 1. *If h is a soft closed and f is a soft β - open then $f \tilde{\cup} h$ is a soft β - open.*
Proof: *Since $f \tilde{\subseteq} cl(int(cl(f)))$, $(h \tilde{\cup} f) \tilde{\subseteq} h \tilde{\cup} cl(int(cl(f))) = cl(int(cl(h))) \tilde{\cup} cl(int(cl(f)))$ $\tilde{\subseteq} cl(int(cl((h)\tilde{\cup}(f))))$. This show that $f\tilde{\cup} h$ is soft β - open.*

The class of all soft β - open (resp. soft β - closed, soft preopen, soft semi - open, soft α - open, soft α - closed and soft preclosed)in ditopological spaces $(\tilde{U}_E, \tau, \kappa)$ denoted by SβO (resp. SβC , SPO, SSO, SαO, SαC and SPC).

Theorem 2. *Let $(\tilde{U}_E, \tau, \kappa)$ be a soft ditopological space we have:*
(1) If $f \in SPO, f \tilde{\subseteq} h \tilde{\subseteq} scl(f)$ then $h \in S\beta O$.
(2) If $f \in SPC, sint(f) \tilde{\subseteq} h \tilde{\subseteq} f$ then $h \in S\beta C$.
Proof: *(1) Since f is soft preopen $\Rightarrow f\tilde{\subseteq} int(cl(f))\tilde{\subseteq} h\tilde{\subseteq}\tilde{\cap}\{f : f$ is soft closed $\}\tilde{\subseteq} cl(int(cl(f))) \Rightarrow h \in S\beta O$.*
(2) Since f is soft preclosed $\Rightarrow cl(int(f))\tilde{\subseteq} f$, $sint(f)\tilde{\subseteq} h\tilde{\subseteq} f \Rightarrow h \in S\beta C$.

Lemma 1. *Let $(\tilde{U}_E, \tau, \kappa)$ be a soft ditopological space, then*
(1) $\tau \tilde{\subseteq} SPO \tilde{\subseteq} S\beta O$ and $\kappa \tilde{\subseteq} SPC \tilde{\subseteq} S\beta C$.
(2) SPO and $S\beta O$ are closed under arbitrary unions.
(3) SPC and $S\beta C$ are closed under arbitrary intersections.
Proof: *(1) Since the element of τ is a soft open then, $\tau \tilde{\subseteq} SPO$ and $SPO \tilde{\subseteq} S\beta O$, that is*

$\tau \tilde{\subseteq} SPO \tilde{\subseteq} S\beta O$. Similary, the element of κ is a soft closed then, $\kappa \tilde{\subseteq} SPC$ and $SPC \tilde{\subseteq} S\beta C$, that is $\kappa \tilde{\subseteq} SPC \tilde{\subseteq} S\beta C$.
(2) and (3)are obvious.

Lemma 2. *Let $(\tilde{U}_E, \tau, \kappa)$ be a soft ditopological space and f is a soft set on $\tilde{U}_E$ then :*
(1) $f \in S\beta O \Leftrightarrow f = s\beta$ - $int(f)$.
(2) $f \in S\beta C \Leftrightarrow f = s\beta$ - $cl(f)$.
Proof: *(1) Let $f = s\beta$ - $int(f)$. Since $s\beta$ - $int(f) = \tilde{\cup}\{h : h$ is a soft β - open and $h \tilde{\subseteq} f\}$ this show that $f \in \{h : h$ is a soft β - open and $h \tilde{\subseteq} f\}$ hance f is a soft β - open .*
Conversely let $f \in S\beta O$, since $f \tilde{\subseteq} f$, $f \in \{h : h$ is a soft β - open and $h \tilde{\subseteq} f\}$ further, $h \tilde{\subseteq} f$ $\forall$ f, since
$f = \tilde{\cup}\{h : h$ is a soft β - open and $h \tilde{\subseteq} f\}$.
(2) Similar (1)

Lemma 3. *Let $(\tilde{U}_E, \Omega)$ be a soft ditopological space the following hold for soft β - closure.*
(1) $s\beta$ - cl $(\Phi) = \Phi$.
(2) If $f \tilde{\subseteq} h \Rightarrow s\beta$ - $cl(f) \tilde{\subseteq} s\beta$ - $cl(h)$.

Definition 12. *A soft ditopological space $(\tilde{U}_E, \Omega)$ is called .*
(1) Soft β - compact if every cover of $\tilde{U}_E$ by soft β - open sets has a finite subcover.
(2) Soft β - cocompact if every cocover of Φ by soft β - closed sets has a finite subcocover.

Proposition 1. *Let $(\tilde{U}_E, \Omega)$ be a soft ditopological space and $(\tilde{U}_E, \Omega^c)$ is a complement of soft ditopological space. Then $h \in S\beta C \Longleftrightarrow h^c \in S\beta O$, $h \in \tilde{U}_E$.*

Proposition 2. *Let Ω^c be a complement soft ditopology on $\tilde{U}_E$. Then $(\tilde{U}_E, \Omega^c)$ is soft β - compact if and only if it is soft β - cocompact.*
Proof: *Let Ω be soft β - compact and $f = \{f_i : i \in J\} \in S\beta C$ with $\tilde{\cap} f = \Phi$. that $G = \{f_i^c : i \in J\}$*
$\in S\beta O$, Moreover $\tilde{\cup}_G = \tilde{\cup}\{f_i^c : i \in J\} = \{\tilde{\cap} f_i : i \in J\}^c = \Phi^c = \tilde{U}_E$.
Similary, if Ω is soft β - compact then it is soft β -cocompact.

Definition 13. *Let (τ, κ) be a soft ditopology on $\tilde{U}_E$.*
(1) (τ, κ) will be called $s\beta$ - stable if every $s\beta$ - closed set $h \in \Omega \setminus \{\tilde{U}_E\}$ is $s\beta$ - compact in $\tilde{U}_E$.
(2) (τ, κ) will be called $s\beta$ - costable if every $s\beta$ - open set $f \in \Omega \setminus \Phi$ is $s\beta$ - cocompact in $\tilde{U}_E$.

Example 3. *Let (τ, κ) be a soft ditopological space on $\tilde{U}_E$ such that $U = \{u_1, u_2, u_3\}$, $E = \{e_1, e_2, e_3\}$, $\tilde{U}_E = \{(e_1, \{u_1, u_2\}), (e_2, \{u_2, u_3\})\}$, $\tau = \{\Phi, \tilde{U}_E\}$ and $\kappa = \{\Phi, \{(e_1, \{u_1\}), (e_2, \{u_2\})\}\}$.*
Firstly*, we notice that, the only soft β - open are $\Phi, \tilde{U}_E$ in soft ditopolgical space $(\tilde{U}_E, \tau, \kappa)$, that is it is soft β - compact. Also, the soft $h = \{(e_1, \{u_1\}), (e_2, \{u_2\})\}$ is soft closed and soft β - closed, so it is not soft compact and not soft β - compact. If follows that (τ, κ) is not $s\beta$ - stable.*
Secondly*, we show that the space may be $s\beta$ - compact but not soft β - costable.*

Let $\tau = \{(e_1, \{u_1\}), (e_2, \{u_2\})\}$, $\kappa = \{\Phi, \tilde{U}_E\}$, the soft ditopology (τ, κ) is not $s\beta$ - compact since it is not soft compact. On the other hand (τ, κ) is $s\beta$ - stable since every $s\beta$ - closed set is closed and the only closed sets $\tilde{U}_E$ and Φ which is $s\beta$ - compact.
***Thirdly**, also we can choose τ and κ such that the soft ditopological space is $s\beta$ - costable but not $s\beta$ - compact.*

Definition 14. *A soft ditopological space is called $S\beta$ - dicompact if it is $S\beta$ - compact, $S\beta$ - cocompact, $S\beta$ - stable and $S\beta$ - costable.*

Proposition 3. *Let (τ, κ) be a soft ditopology on $\tilde{U}_E$:*
(1) Soft β - compact $\Longrightarrow$ strongly soft compact $\Longrightarrow$ soft compact.
(2) Soft β - cocompact $\Longrightarrow$ strongly soft cocompact $\Longrightarrow$ soft cocompact.
***Proof:** It is obvious, since every soft open set is soft preopen and every soft closed set is soft preclosed.*

Proposition 4. *For a soft ditopological space:*
(1) Soft β - stable $\Longrightarrow$ soft strongly stable $\Longrightarrow$ soft stable.
(2) Soft β - costable $\Longrightarrow$ strongly soft costable $\Longrightarrow$ soft costable.
Moreover, the converse is not true in general, as the following example:

Proposition 5. *Let Ω be a complemented soft ditopology on $(\tilde{U}_E)^c$. Then $(\tilde{U}_E, \Omega^c)$ is soft β - compact if and only if it is soft β - cocompact.*
***Proof:** Let $(\tilde{U}_E, \Omega)$ be a soft β - compact and let $K = \{\kappa_i \mid i \in J\}$ be a family of soft β - closed sets with $\tilde{\cap} K = \Phi$. Obvious $G = \{\kappa_i \mid i \in J\}^c$ is a family of soft β open sets. Moreover , $\tilde{\cup} G = \tilde{\cup}\{\kappa_i \mid i \in J\}^c = \tilde{U}_E$, and so we have $J^{\backslash} \subseteq J$ finite with $\tilde{\cup}\{\kappa_i \mid i \in J^{\backslash}\}^c = \tilde{U}_E$. That is $\tilde{\cap}\{\kappa_i \mid i \in J^{\backslash} = \Phi$, and so $(\tilde{U}_E, \Omega)$ is soft β - cocompact. Similarly, if $(\tilde{U}_E, \Omega)$ is soft β - compact, then it is soft β - compact.*

Definition 15. *A soft ditopological space will be called soft β - dicompact if it is soft β - compact, soft β - cocompact, soft β - stable and soft β - costable.*

Example 4. *(1) Let (τ, κ) be a soft ditopological space on $\tilde{U}_E$ such that $U = \{u_1, u_2, u_3\}$, $E = \{e_1, e_2\}, \tilde{U}_E \in S$, $\tilde{U}_E = \{(e_1, \{u_1, u_2\}), (e_2, \{u_2, u_3\})\}$, $\tau = \{\tilde{U}_E, \Phi\}, \{(e_1, \{u_1, u_2\}),$ $(e_2, \{u_3\})\}$ and $\kappa = \{\Phi, \tilde{U}_E\}$.*
Since the only soft β - open sets are $\tilde{U}_E, \Phi$ in soft ditopology $(\tilde{U}_E, \tau, \kappa)$, we have that it is soft β - compact.
(2) Let $\tau = \{\tilde{U}_E, \Phi\}$ and $\kappa = \{\Phi, \tilde{U}_E, \{(e_1, \{u_1, u_2\}), (e_2, \{u_3\})\}$, then the soft ditopology $(\tilde{U}_E, \tau, \kappa)$ is soft β - cocompact but not soft β - compact.
This example show that in general soft β - compact and soft β - cocompact are independent.

Definition 16. *Let $\Omega_1 = (\tau_1, \kappa_1)$ and $\Omega_2 = (\tau_2, \kappa_2)$ are two soft ditopological spaces on $\tilde{U}_E$.Then Ω_2 is called coarser than Ω_1 (denoted by $\Omega_2 \tilde{\subseteq} \Omega_1$ if $f \in \tau_1$ whenever $f \in \tau_2$ and $h \in \kappa_1$ whenever $f \in \kappa_2$.*

Theorem 3. *If $(\tilde{U}_E, \Omega_1)$ and $(\tilde{U}_E, \Omega_2)$ are two soft ditopological spaces. Then $(\tilde{U}_E, \Omega_1 \tilde{\cap} \Omega_2)$ is a soft ditopological space.*

Proof: *Since* $\Omega_1 = (\tau_1, \kappa_1)$ *and* $\Omega_2 = (\tau_2, \kappa_2)$ *are two a soft ditopological space on* $\tilde{U}_E$ *then* $(\tilde{U}_E, \tau_1)$ *and* $(\tilde{U}_E, \tau_2)$ *are two soft topological space* $\Rightarrow (\tilde{U}_E, (\tau_1 \tilde{\cap} \tau_2))$ *is a soft topological space* (1). *Also,* $(\tilde{U}_E, \kappa_1)$ *and* $(\tilde{U}_E, \kappa_2)$ *are two a soft cotopological space* $\Rightarrow (\tilde{U}_E, (\kappa_1 \tilde{\cap} \kappa_2))$ *is a soft ctopological space* (2). *From* (1) *and* (2), *we get* $(\tilde{U}_E, \Omega_1)$ *and* $(\tilde{U}_E, \Omega_2)$ *are two soft ditopological spaces.*

3. Soft β - continuous mappings

Definition 17. *Let* $(\tilde{U}_E, \Omega_1)$ *and* $(\tilde{V}_E, \Omega_2)$ *be two soft ditopological spaces. A soft function* $(\varphi, \psi) : (\tilde{U}_E, \Omega_1) \to (\tilde{V}_E, \Omega_2)$ *where* $\varphi : (\tilde{U}_E, \tau_1) \to (\tilde{V}_E, , \tau_2)$ *and* $\psi : (\tilde{U}_E, \kappa_1) \to (\tilde{V}_E, \kappa_2)$ *then, a mapping* (φ, ψ) *is called continuous function at a soft point* $x_p \in \tilde{U}_E$ *if* $\varphi : (\tilde{U}_E, \tau_1) \to (\tilde{V}_E, \tau_2)$ *is continuous function at* x_p, *and* $\psi : (\tilde{U}_E, \kappa_1) \to (\tilde{V}_E, \kappa_2)$ *is continuous function at* x_p.

Definition 18. *A soft function* $\Gamma = (\varphi, \psi) : (\tilde{U}_E, \Omega_1) \to (\tilde{V}_E, \Omega_2)$ *is soft continuous if and only if the inverse image of soft open in* Ω_2 *is soft open in* Ω_1.

Definition 19. *The soft function* $(\varphi, \psi) : (\tilde{U}_E, \tau_1, \kappa_1) \to (\tilde{U}_E, \tau_2, \kappa_2)$ *is called :*
(1) Soft β *- continuous if* $\varphi^{-1}(f) \in S\beta O(\tilde{U}_E)\ \forall\ f \in \tau_2$.
(2) Soft β *- cocontinuous if* $\psi^{-1}(h) \in S\beta C(\tilde{U}_E)\ \forall\ h \in \kappa_2$.
(3) Soft β *- bicontinuous if it is both soft* β *- continuous and soft* β *- cocontinuous.*
(4) Soft semi - continuous if $\varphi^{-1}(f) \in SSO(\tilde{U}_E)\ \forall\ f \in \tau_2$.
(5) Soft semi - cocontinuous if $\psi^{-1}(h) \in SSC(\tilde{U}_E)\ \forall\ h \in \kappa_2$.
(6) Soft semi - bicontinuous if it semi - continuous and semi - cocontinuous.

Example 5. *Let* $(\tilde{U}_E, \Omega_1), (\tilde{U}_E, \Omega_2)$ *be two soft ditopiogical spaces, such that* $U = \{u_1, u_2, u_3\}$, $E = \{e_1, e_2\}$, $\varphi : (\tilde{U}_E, \tau_1) \to (\tilde{U}_E, \tau_2)$ *and* $\psi : (\tilde{U}_E, \kappa_1) \to (\tilde{U}_E, \kappa_2)$, $\tau_1 = \{\Phi, \tilde{U}_E, \{(e_1, \{u_1\}), (e_2, \{u_1\})\}, \{(e_1, \{u_2\}), (e_2, \{u_2\})\}, \{(e_1, \{u_1, u_2\}), (e_2, \{u_1, u_2\})\}$, $\kappa_1 = \{\Phi, \tilde{U}_E, \{(e_1, \{u_1\}), (e_2, \{u_2\})\}, \{(e_1, \{u_1\}), (e_2, \{u_2\})\}\}$, *and* $\tau_2 = \{\Phi, \tilde{U}_E, \{(e_1, \{u_1\}), (e_2, \{u_1\})\}, \{(e_1, \{u_1, u_2\}), (e_2, \{u_1, u_2\})\}$, $\kappa_2 = \{\Phi, \tilde{U}_E, \{(e_1, \{u_2\}), (e_2, \{u_2\})\}\}$, *if we defined the mapping as* $\varphi(u_1) = u_1, \varphi(u_2) = u_3, \varphi(u_3) = u_2$ *and* $\psi(u_1) = u_1, \psi(u_2) = u_3, \psi(u_3) = u_2$, *then* φ *is a soft* β *- continuous and* ψ *is a soft* β *- cocontinuous, Consequently* Ω *is a soft* β *- bicontinuous.*

Definition 20. *A soft function* $\Gamma = (\varphi, \psi) : (\tilde{U}_E, \tau_1, \kappa_1) \to (\tilde{U}_E, \tau_2, \kappa_2)$ *is called :*
(1) Soft β *- irresolute if* $\varphi^{-1}(f)$ *is* $s\beta o(\tilde{U}_E)$ $\forall f$ *is* $s\beta o(\tilde{U}_E)$ *and* $\psi^{-1}(f)$ *is* $s\beta c(\tilde{U}_E)$ $\forall f$ *is* $s\beta c(\tilde{U}_E)$.
(2) Strongly soft β *- irresolute if* $\varphi^{-1}(f)$ *is* $sso(\tilde{U}_E)$ $\forall f$ *is* $s\beta o(\tilde{U}_E)$ *and* $\psi^{-1}(f)$ *is* $ssc(\tilde{U}_E)$ $\forall f$ *is* $s\beta c(\tilde{U}_E)$.

Proposition 6. *Let a soft function* $\Gamma_1 : (\tilde{U}_E, \tau_1, \kappa_1) \to (\tilde{V}_E, \tau_2, \kappa_2)$ *and* $\Gamma_2 : (\tilde{V}_E, \tau_2, \kappa_2) \to (\tilde{W}_E, \tau_3, \kappa_3)$ *are both soft* β *irresolute. Then* $\Gamma_1 \circ \Gamma_2 : (\tilde{U}_E, \tau_1, \kappa_1) \to (\tilde{W}_E, \tau_3, \kappa_3)$ *is soft* β *irresolute.*

Definition 21. *Let a soft function* $(\varphi, \psi) : (\tilde{U}_E, \tau_1, \kappa_1) \to (\tilde{U}_E, \tau_2, \kappa_2)$*, then:*
(1) φ *is called soft* β *- open if the image of each soft* β *open in* τ_1 *is soft* β *- open in* τ_2*.*
(2) ψ *is called soft* β *- closed if the image of each soft* β *- closed in* κ_1 *is soft* β *- closed in* κ_2*.*

4. Conclusion

In recent decades, many applications of topology have merged in different fields. Therefore we have had to expand the topological space in many ways as a result of its contribution to solving some issues. So, in this paper, we generalized some of the concepts via soft ditopology, and some properties are obtained.

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