New Spectral Idea for Conjugate Gradient Methods and Its Global Convergence Theorems

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Abstract. Recently, the unconstrained optimization conjugate gradient methods have been widely utilized, especially for problems that are known as large-scale problems. This work proposes a new spectral gradient coefficient obtained from a convex linear combination of two different gradient coefficients to solve unconstrained optimization problems. One of the most essential features of our suggested strategy is to guarantee the suitable subsidence direction of the line search precision. Furthermore, the proposed strategy is more effective than previous conjugate gradient approaches and stationary, which have been observed in the test problem. However, when it is compared to other conjugate gradient methods, such as FR methods, the proposed method confirmed the globally convergent, indicating that it can be used in scientific data computation.

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1. Introduction

The spectral gradient approaches presented by Barzilai and Browein [5] and later researched by Raydan [23] have proven to be useful and valuable in this area, which are utilized to determine the local minimizers of large-scale problems [2]. In this paper, we describe a spectral gradient technique to solve unconstrained optimization problems of the form:

$$\min_{x \in \mathbb{R}} f(x)$$  (1)

In most cases, the conjugate gradient approach creates a sequence \( \{x_k\} \) in such a way:

$$x_{k+1} = x_k + \alpha_k d_k$$  (2)
For \( k = 0 \), \( x_0 \) denotes the starting position, \( \alpha_k \) denotes the specified step size by a line search, \( d_k \) signifies the direction of search, which is specified by:

\[
d_{k+1} = -g_{k+1} + \beta_k d_k
\]

where \( d_0 = -g_0 \) for \( k = 0 \) and \( \beta_k \) is a scalar, \( g_{k+1} \) refers to the gradient \( \nabla f(x_{k+1}) \) at the new point. The most familiar \( \beta_k \) formulation is the Fletcher-Reeves (FR), Polak-Ribire, Hestenes-Stiefel (HS), Dai and Yan (DY) formulations which are supplied by: see [12][20][21][17][6]

\[
\beta_{FR}^k = \frac{g_{k+1}^T g_k}{g_k^T g_k}
\]

\[
\beta_{PRP}^k = \frac{g_{k+1}^T y_k}{||g_k||^2}
\]

\[
\beta_{HS}^k = \frac{g_{k+1}^T g_k}{d_k^T y_k}
\]

\[
\beta_{DY}^k = \frac{g_{k+1}^T g_k}{y_k^T d_k}
\]

Also, recently, Hassan [13] suggests a new formula which is denoted in equations (8), (9) respectively.

\[
\beta_k^Y = \frac{g_{k+1}^T y_k}{(f_k - f_{k+1})/\alpha_k - g_k^T d_k/2}
\]

\[
\beta_k^G = \frac{g_{k+1}^T g_k}{(f_k - f_{k+1})/\alpha_k - g_k^T d_k/2}
\]

Where the last two equations of \( \beta_k \) are used to suggest a new form of \( \beta_k \), which we call \( \beta_{NN}^k \). The other \( \beta_k \) parameters have been offered in the literature see for example [1][3][4][10][11]. To avoid the non-convergence in the nonlinear function which is used with inexact line search the following condition is used:

\[
|g_{k+1}^T g_k| > 0.2||g_{k+1}||
\]

Known as the Powell restart condition [7]. Also, to avoid the negative case of \( \beta_k \) we use Wolfe conditions to guarantee the convergence of non-linear conjugate methods for the research [24]. More information on these line search methods can be found in the literature [8][19][26][27]
Where \( d_k \) is descant direction, \( g_T d_k < 0 \), and \( 0 < \delta_1 < \delta_2 < 1 \). Dai and Yuan [6] show that the FR method satisfies the global convergence feature if the strong Wolfe conditions are satisfied. Recently, combining the well numerical results execution of PRP and HS with the globally convergent properties of the FR and DY methods [6], are the way that is adopted in this paper, which will be discussed in the second section. the systematic sequence of this paper is: The second section proposes a new spectral CG formula and the section three establishes the new method’s global convergence. The numerical experiments are shown in the fourth Section, and finally, the fifth Section presents the comparison of the results and conclusion.

2. New Spectral formula for CG Method

The new concept is to redirect the search path so that the new path guarantees the condition.

\[
y_{k+1}^T d_{k+1} \leq -c \|g_{k+1}\|^2
\]

Zhang et al. [28] suggested a conjugate gradient approach based on the modification of the Fletcher-Reeves method in such a way that the direction \( d_k \) which is given by [14]:

\[
d_{k+1} = -\theta g_{k+1} + \beta_k d_k
\]

and the scalar \( \theta \) is defined as,

\[
\theta = \frac{1 + \beta_k d_T g_{k+1}}{\|g_{k+1}\|^2}
\]

Now to improve the proposed CG method, we combine the good computational properties of \( \beta_k^Y \) and \( \beta_k^G \) method which is given in equations (8) and (9) respectively. These methods have strong convergence properties, then rewrite the new formula of \( \beta_k \) as a linear combination of (8) and (9), therefore our new formula \( \beta_k^{NN} \) becomes:

\[
\beta_k^{NN} = (1 - r) \beta_k^Y + r \beta_k^G
\]

Where \( 0 < r < 1 \), \( \beta_k^Y = \frac{g_{k+1}^T y_k}{(f_k - f_{k+1})/\alpha_k - g_k^T d_k/2} \), and \( \beta_k^G = \frac{g_{k+1}^T g_{k+1}}{(f_k - f_{k+1})/\alpha_k - g_k^T d_k/2} \)

Now by multiplying (14) by \( y_k^T \) and using the Conjugacy condition [25] \( y_k^T d_{k+1} = 0 \) in order to compute the value of \( r \) we get:

\[
y_k^T (\theta g_{k+1} + \beta_k d_k) = 0 \quad (16)
\]

\[
-\left[1 + \beta_k \frac{d_{k+1}^T g_{k+1}}{\|g_{k+1}\|^2} y_k^T g_{k+1} + \beta_k y_k^T d_k\right] = 0 \quad (17)
\]

By using (15) we get:

\[
- \left[1 + [(1 - r) \beta_k^Y + r \beta_k^G] \frac{d_{k+1}^T g_{k+1}}{\|g_{k+1}\|^2} y_k^T g_{k+1} + [(1 - r) \beta_k^Y + r \beta_k^G] y_k^T d_k\right] = 0
\]

\[
r \left[-\beta_k^G \frac{d_{k+1}^T g_{k+1}}{\|g_{k+1}\|^2} y_k^T g_{k+1} + \beta_k^Y \frac{d_{k+1}^T g_{k+1}}{\|g_{k+1}\|^2} y_k^T g_{k+1} - \beta_k^Y y_k^T d_k + \beta_k^G y_k^T d_k\right]
\]
This is the final form of \( r \) which is denoted in the equation above. The explanation of the outlines of the new algorithm might be stated as follows:

2.1. The NN-CG Algorithm

- 1\textsuperscript{st}: take \( x_1 \in \mathbb{R}^n, \varepsilon > 0, d_1 = -g_1 \) if \( ||g_1|| < \varepsilon \), then quit.
- 2\textsuperscript{nd}: if \( ||g_{k+1}|| < \varepsilon \) then come to the end; otherwise proceed to 3\textsuperscript{rd} step.
- 3\textsuperscript{rd}: By satisfying Wolfe conditions (11) and (12), find \( \alpha_k \) and take \( x_{k+1} = x_k + \alpha_k d_k \)
- 4\textsuperscript{th}: find \( \beta_k \) by equation (15), then compute \( d_{k+1} \) by (3).
- 5\textsuperscript{th}: put \( k = k + 1 \), go-to 2\textsuperscript{nd}.

3. The Global Convergence Theorem and hypothesis

The following hypothesis is required to investigate a new convergence method:

hypothesis number one:

(i) The set \( \Omega = \{ x \in \mathbb{R}^n | f(x) \leq f(x_0) \} \) is bounded when \( x_0 \in \mathbb{R}^n \), is an initial point.

(ii) There exists a constant \( W > 0 \) in an open convex set \( N \) that includes \( f \) is continuously differentiable and the gradient that satisfies Lipschitz condition is continuous. For more detail see [29]. Such as:

\[
||g(x) - g(y)|| \leq W||x - y||, \quad \forall x, y \in \Omega
\]  
(19)

Furthermore, we can deduce from the hypothesis that \( p \) and \( \ell > 0 \) are constants such that:

\[
||x|| \leq p, \quad ||g(x)|| < \ell, \quad \in \Omega
\]

Lemma 1. Consider that hypothesis number one is holding, and let any step of the type (3) where \( d_k \) is decline direction and \( \alpha_k \) fulfills conditions (11) and (12), so Zoutedijk condition hold. [30][22].
\[
\sum_{k=1}^{\infty} \frac{(g_k^T d_k)^2}{||d_k||^2} < \infty
\]  \hfill (20)

Following that, we present additional lemmas that are critical for the global convergence dissection.

**Lemma 2.** Suppose that hypothesis number one holds, and take into account any iteration of the form (2) then the direction \(d_{k+1}\) which is given in (14) satisfies the sufficient descent condition

\[
d_{k+1}^T g_{k+1} = -||g_{k+1}||^2
\]  \hfill (21)

**Proof.** From (15), for \(k = 0\) the result is hold and we have \(d_1^T g_1 = -||g_1||^2\), now when \(k > 0\) we obtain:

\[
g_{k+1}^T d_{k+1} = - \left( 1 + \beta_k \frac{g_{k+1}^T d_k}{||g_{k+1}||^2} \right) ||g_{k+1}||^2 + \beta_k \frac{g_{k+1}^T d_k}{||g_{k+1}||^2}
\]

\[
= -||g_{k+1}||^2 - \beta_k \frac{g_{k+1}^T d_k}{||g_{k+1}||^2} ||g_{k+1}||^2 + \beta_k \frac{g_{k+1}^T d_k}{||g_{k+1}||^2} = -||g_{k+1}||^2
\]  \hfill (22)

We see that (21) holds for all \(k > 0\). So, the **proof** is complete.

**Lemma 3.** Consider that hypothesis (1) and (2) hold and let \(\alpha_k\) satisfies Wolfe’s condition (11), (12) then \(\beta_k\) which is determined by (15) satisfies \(0 < \beta_k\)

**Proof.** from the line of inquiry condition (11), (12) as well as the sufficient descent condition (21), and since \(d_k^T g_k = -||g_k||^2\) it is possible to show that:

\[
\beta_k^Y = \frac{g_{k+1}^T g_k}{(f_k - f_{k+1})/\alpha_k - g_k^T d_k/2} > \frac{||g_{k+1}||^2 - g_k^T g_k}{-\delta_1 d_k^T g_k + \frac{||g_k||^2}{2}}
\]  \hfill (23)

\[
\beta_k^G = \frac{g_{k+1}^T g_k}{(f_k - f_{k+1})/\alpha_k - g_k^T d_k/2} > \frac{||g_{k+1}||^2}{-\delta_1 d_k^T g_k + \frac{||g_k||^2}{2}} > \frac{||g_{k+1}||^2}{\delta ||g_k||^2 + \frac{||g_k||^2}{2}}
\]

\[
= \frac{||g_{k+1}||^2}{(\delta + \frac{1}{2}) ||g_k||^2} > 0
\]  \hfill (24)

Then from (23), (24) and (15) we have:

\[
\beta_k^{NN} = (1 - r) \beta_k^Y + r \beta_k^G
\]

\[
= (1 - r) \frac{||g_{k+1}||^2 - g_k^T g_{k+1}}{(\delta + \frac{1}{2}) ||g_k||^2} + r \frac{||g_{k+1}||^2}{(\delta + \frac{1}{2}) ||g_k||^2} = \frac{||g_{k+1}||^2 - g_k^T g_{k+1} + r g_k^T g_{k+1}}{(\delta + \frac{1}{2}) ||g_k||^2}
\]
From Powell restart condition (10) we have

\[ \beta_{NN}^k > \frac{||g_{k+1}||^2 - 0.2||g_{k+1}||^2 + 0.2r||g_{k+1}||^2}{(\delta + \frac{1}{2})||g_k||^2} \geq \frac{0.8||g_{k+1}||^2 + 0.2r||g_{k+1}||^2}{(\delta + \frac{1}{2})||g_k||^2} \]

\[ \therefore \beta_{NN}^k = \frac{n||g_{k+1}||^2}{(\delta + \frac{1}{2})||g_k||^2} > 0 \] (25)

by assuming \( n = (0.8 + 0.2r) \), and since \( 0 < r < 1 \) then it is clearly that \( \beta_{NN}^k > 0 \)

**Theorem 1.** Assume that hypothesis number one is true, \( \{x_k\} \) is a sequence generated by the algorithm (2.1) and \( \alpha_k \) fulfill the Wolfe’s conditions (11) and (12), Then we need to prove that

\[ \lim_{k \to \infty} \inf ||g_{k+1}|| = 0 \]

**Proof.** we use the disagreement method to complete the proof which assumes that

\[ ||g_{k+1}|| > \gamma \] for \( \gamma > 0 \) from (3) it follows that:

\[ d_{k+1} + g_{k+1} = \beta_k d_k \]

by squaring the equation (3) we have:

\[ ||d_{k+1}||^2 + 2g_{k+1}^T d_{k+1} + ||g_{k+1}||^2 = \beta_k^2 ||d_k||^2 \]

Since the sufficient descent condition (21) is held, we obtain:

\[ ||d_{k+1}||^2 = (\beta_k)^2 ||d_k||^2 - ||g_{k+1}||^2 - 2g_{k+1}^T d_{k+1} \]

\[ = (\beta_k)^2 ||d_k||^2 - ||g_{k+1}||^2 + 2||g_{k+1}||^2 \]

\[ = (\beta_k)^2 ||d_k||^2 + ||g_{k+1}||^2 \] (26)

Both sides are divided by \( (g_{k+1}^T d_{k+1})^2 \) we have:

\[ \frac{||d_{k+1}||^2}{(g_{k+1}^T d_{k+1})^2} \leq \frac{||d_k||^2}{(g_{k+1}^T d_{k+1})^2} + \frac{||g_{k+1}||^2}{(g_{k+1}^T d_{k+1})^2} \]

\[ \leq (\beta_k)^2 \frac{||d_k||^2}{(g_{k+1}^T d_{k+1})^2} + \frac{1}{||g_{k+1}||^2} \] (27)

Now from (25), (27) we obtain:

\[ \frac{||d_{k+1}||^2}{(g_{k+1}^T d_{k+1})^2} \leq \left( \frac{n||g_{k+1}||^2}{(\delta + \frac{1}{2})||g_k||^2} \right)^2 \frac{||d_k||^2}{||g_{k+1}||^4} + \frac{1}{||g_{k+1}||^2} \]

\[ = \frac{n^2||d_k||^2}{(\delta + \frac{1}{2})^2 ||g_k||^4} + \frac{1}{||g_{k+1}||^2} \]

\[ = \frac{n^2||d_k||^2}{(\delta + \frac{1}{2})^2 (g_k^T d_k)^2} + \frac{1}{||g_{k+1}||^2} \] (28)
Since that:
\[ \frac{||d_i||^2}{(g_i^T d_i)^2} = \frac{1}{||g_i||^2} \]  
(29)

From this and \( ||g_{k+1}|| > \varepsilon^2 \; \forall k \) we have:
\[ \frac{||d_{k+1}||^2}{(g_{k+1}^T d_{k+1})^2} \leq \sum_{i=1}^{k+1} \frac{1}{(\delta + 1/2)^2 ||g_i||^2} \leq \frac{k}{(\delta + 1/2)^2 \varepsilon^2} \]
\[ \therefore \frac{(g_{k+1}^T d_{k+1})^2}{||d_{k+1}||^2} \geq \frac{(\delta + 1/2)^2 \varepsilon^2}{k} \]

Which indicates:
\[ \sum_{k=1}^{\infty} \frac{(g_{k+1}^T d_{k+1})^2}{||d_{k+1}||^2} \geq \sum_{k=1}^{\infty} \frac{(\delta + 1/2)^2 \varepsilon^2}{k} = \infty \]  
(30)

And this is in direct opposition to the Zoutendijk condition (20), with this contradiction we complete proof.

4. Numerical Experiments

Now we set up the numeral experiments, Compute and make a comparison between our method vs. \( \beta_k^{FR} \). The comparison is written using the Fortran 90 program, and the test function is implemented using the functions chosen from Andrei [2]. The measure of stooping the algorithm is denoted as \( ||g_{k+1}|| \leq 10^{-6} \). We use 27 test problems with different dimensions to experiment with the execution of the new method. Table 1 shows the calculation result, with NOI, NOR, and NOF which stand for the amount of iterations total, restart, and function evaluation, respectively. While Table 2 explains the execution percentage of the new method opposite FR method, also the execution was analyzed by the performance profile software which is developed by Dolan and Mor’e [9] which are seen in Figures 1, 2, 3.

![Figure 1: The performance profile of iteration.](image-url)
Figure 2: The performance profile of function evaluation.

Figure 3: The performance profile of restart.
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Table 1: Results for the NN and FR algorithm
5. Conclusion

In this paper, we searched for a new conjugate gradient method that depends on the spectral strategy. We established, under acceptable assumptions, that the global convergence for one of the offered approaches is satisfied. The arithmetical computation explained in Table 1 shows the efficiency of the proposed algorithm outperformed the regular FR method on average, according to the numerical results and Figures 1, 2, 3, which are denoted by the performance profile iteration and the number of function evaluations and CPU time respectively. Furthermore, there are exist many good progress in conjugate gradient algorithms see for example [16][15][18].

References


