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# Data envelopment analysis in the context of spherical fuzzy inputs and outputs

Kshitish Kumar Mohanta[1], Deena Sunil Sharanappa[1], Devika Dabke[2], Lakshmi Narayan Mishra[3], Vishnu Narayan Mishra[1,*]

[1] *Department of Mathematics, Indira Gandhi National Tribal University, Lalpur, Amarkantak, 484 887, Madhya Pradesh, India*
[2] *Department of Mathematics, Central University of Karnataka, Kalaburgi, 585 367, Karnataka, India*
[3] *Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore, 632 014, Tamil Nadu, India*

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**Abstract.** In this study, Data Envelopment Analysis (DEA) models are improved by employing spherical fuzzy sets (SFSs), which is an extension of generalized fuzzy sets. SFSs were recently introduced as a novel type of fuzzy set that allows decision-makers to express their level of uncertainty directly. As a result, SFSs provide a more preferred domain for decision-makers. Fundamental Charnes-Cooper-Rhodes (CCR) model is discussed on the context of spherical trapezoidal fuzzy numbers (STrFNs), which consider each data value's truth, indeterminacy, and falsehood degrees, and a unique solution technique is implemented. This method converts a spherical fuzzy DEA (SF-DEA) model into three pair of crisp DEA model, which may then be solved using one of many existing approaches. The largest optimal interval is determined for each DMU such that the efficiency score lies inside that interval. Furthermore, an example demonstrates this novel method and clearly explains the DMUs' ranking technique.

**2020 Mathematics Subject Classifications**: 90C90, 90C70, 90C08

**Key Words and Phrases**: Efficiency Analysis, Data Envelopment Analysis, Spherical Trapezoidal Fuzzy Number, CCR Model

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## 1. Introduction

The idea of fuzzy set was established by Zadeh [41] in 1965, and fuzzy set theory has been widely employed in practical applications of uncertainty modeling. Many academics have been interested in fuzzy set theory as a result of its expansion and application. The membership degree of set elements of a fuzzy set was defined by the characteristic

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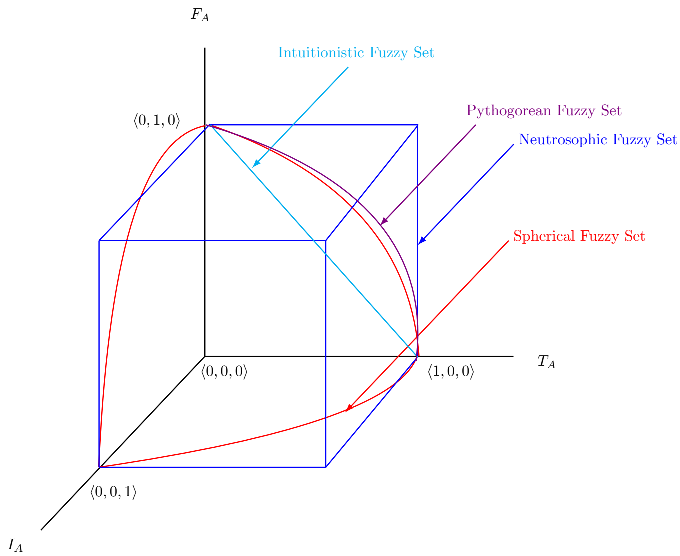
*Email addresses:* kshitishkumar.math@gmail.com (K. K. Mohanta), deena.sunil@igntu.ac.in (D. S. Sharanappa), devikash131@gmail.com (D. Dabke), vishnunarayanmishra@gmail.com (V. N. Mishra), lakshminarayanmishra04@gmail.com (L. N. Mishra)

function on the unit interval [0, 1] in the study of fuzzy set (FS) theory. The fuzzy set's non-membership degree is calculated by subtracting the membership degree from 1. Atanassov [7] in 1986 expanded Zadeh's fuzzy set notion to intuitionistic fuzzy set (IFS), and its membership and non-membership degrees are defined separately. However, the sum of IFSs' membership and non-membership degrees must fall within the range [0, 1]. Smarandache [38] in 1999 proposed neutrosophic logic and neutrosophic sets (NSs) as an extension of intuitionistic fuzzy sets. A neutrosophic set is one in which each element of the universe contains different degrees of truthiness, indeterminacy, and falsehood. They can be calculated individually, and their sum can range between 0 and 3. NSs used in solving many optimization technique and MCDM problem [26, 33]. Yager [39] in 2013 developed Pythagorean fuzzy sets which have a membership degree and a non-membership degree that satisfy the condition that the square sum of membership and non-membership degrees is at most equal to one, and are a generalisation of Intuitionistic Fuzzy Sets (IFS). Cuong and Kreinovich [10] in 2014 invented picture fuzzy sets Picture fuzzy sets-based models may be appropriate in circumstances requiring additional varieties of human opinions, such as yes, abstain, no, and rejection. Kahraman and Gundogdu [24] in 2018 proposed spherical fuzzy sets (SFS) as an extension of Pythagorean, neutrosophic, and picture fuzzy sets. SFS allows decision makers to generalise additional extensions of fuzzy sets by constructing a membership function on a spherical surface and separately assigning the parameters of that membership function to a broader domain. SFS have been applied to many multicriteria decision-making methods [4, 6, 25, 32, 36]. The difference between Intuitionistic fuzzy set, Pythagorean fuzzy set, Neutrosophic fuzzy set, and spherical fuzzy set are shown in Figure (1) where $T_A, F_A$ and $I_A$ represents the truth, falsity and indeterminate membership grade for the fuzzy set $A$.

Figure 1: Representation of the fuzzy set and it's extension in geometrically



For development, growth, and sustainability, all public or commercial organizations require an accurate performance evaluation. In today's competitive market, these businesses are under pressure to turn inputs into outputs at the lowest possible cost. This pressure motivates them to be more efficient. To be more specific, one of the major functions of government in the public sector, when the traditional disciplines of a competitive market are missing, is to offer public goods and services. As a result, identifying efficient providers can improve efficiency by acknowledging and disseminating best practices. Farrell [17] in 1957 developed the mathematical model for evaluating the efficiency of the DMUs, which was extended by Charnes et al. [9] in 1978 and developed a linear mathematical programming (LP) model to measure the comparative efficiency of the DMUs is called the Charnes-Cooper-Rhodes (CCR) model under the assumption of constant returns to scale (CRS). Banker et al. [8] in 1984 extended the pioneering work [9] and proposed a model conventionally called the Banker-Chames-Cooper (BCC) model to measure the relative efficiency under the assumption of variable returns to scale (VRS). The data envelopment analysis (DEA) is a non-parametric linear programming technique that considers the weighted sum of outputs to the weighted sum of inputs when evaluating the relative efficiency of a set of homogeneous DMUs. In the usual efficiency evaluation, it converts a single input/output ratio to a multiple input/output ratio. This approach is regarded as an effective multicriteria decision procedure and has been widely applied in various disciplines. In recent years, there has been a widespread use of DEA in a variety of industries, including banking institutions [29], the insurance business [23], financial services [30], education [35], supply chain management [20], crisis management [34], sustainability [3], energy [18] and health-care services [28].

Sengupta [37] in 1992 used fuzzy sets in DEA for the first time. The DEA techniques employing fuzzy theory may be grouped into four basic groups, according to Hatami-Marbini et al. [22]: parametric approaches, possibility approaches, ranking approaches, defuzzification approaches, and many additional approaches have been brought to fuzzy DEA advancement. Emrouznejad et al. [16] in 2014 categorized the fuzzy DEA approaches in six types: the tolerance technique, the $\alpha$-level based approach, the fuzzy ranking approach, the possible approach, the fuzzy arithmetic, and the fuzzy random/type-2 fuzzy set and reviewed the literature during the last 30 years. Zhou and Xu [42] in 2020 provides a summary of the fuzzy data envelopment analysis research and its successful implementations. Several ways to dealing with inaccurate, ambiguous, partial, and/or missing data in DEA have been proposed. To detect inaccurate input and output data, stochastic approaches [12] and interval DEA models are widely utilised. There have also been various research articles published in DEA that make use of intuitionistic fuzzy sets [5, 21]. First time, Edalatpanah [13] in 2018 extended the DEA model in the context of single value Neutrosophic number. For offering a solution to the efficiency of private institutions. Kahraman et al. [27] in 2019 presented a hybrid algorithm based on a neutrosophic analytic hierarchy process (AHP) and DEA. Abdelfattah [1] in 2019 proposed a suitable approach for solving the DEA model in which all inputs and outputs are neutrosophic number. Following that, several approaches to solving DEA models utilising neutrosophic fuzzy sets are utilised [14, 15]. Mao et al. [31] in 2020 used single- valued neutrosophic sets

(SVNSs) in DEA with undesirable output. Yang et al. [40] in 2020 used triangular single valued neutrosophic number for measuring the hospital efficiency base on data envelopment analysis. Abdelfattah [2] in 2021 developed the parametric approach in neutrosophic data envelopment analysis and measured the efficiency of the regional hospitals in Tunisia using parametric neutrosophic data envelopment analysis.

We noticed a few research gaps in this exciting field. SFSs are a generalisation and extension of Picture Fuzzy Sets that define a membership function on a spherical surface and assign the parameters of that membership function independently over a larger domain, which has not been utilised in DEA with trapezoidal inputs and outputs. Trapezoidal fuzzy numbers are the most acceptable form of a fuzzy number because it covers more ambiguity than other fuzzy numbers. The acceptability area of SFTrNs provides better information assessment flexibility as a consequence of combining the benefits of SFSs with trapezoidal fuzzy numbers. Also, for SFSs, the situation of uncertain decision-making evaluations has not been considered. In this research, a novel efficient solution strategy is provided for solving Spherical Fuzzy DEA models in which all inputs and outputs are spherical trapezoidal fuzzy numbers (STrFNs) and the reference set or peer group for inefficient DMUs is defined. We offered an example to show the method's applicability and validity.

Section (2) discusses some advanced knowledge, concepts, and arithmetic operations on SFs and STrFNs. In section (3), we create the previously proposed DEA model in spherical fuzzy environment. In section (4), offer a strategy for solving it. Section (5) presented a numerical illustration for the proposed model. Section (6) concludes with findings and future directions.

## 2. Preliminary

**Definition 1** ([19]). *Let $U$ be a universe. A spherical fuzzy set $\widehat{X}$ over $U$ is defined by*

$$\widehat{X} = \{\langle x; \phi_x, \varphi_x, \psi_x\rangle : x \in U\}, \tag{1}$$

*where $\phi_x, \varphi_x$ and $\psi_x$ are called membership function, non-membership function and hesitancy function, respectively. They are respectively defined by*

$$\phi_x, \varphi_x, \psi_x : U \to [0, 1],$$

*such that $0 \leq \phi_x^2 + \varphi_x^2 + \psi_x^2 \leq 1$.*

**Definition 2** ([11]). *A Spherical Trapezoidal Fuzzy Numbers (STrFNs) is denoted by $\widehat{X} = \langle x^L, x^{M_1}, x^{M_2}, x^U; \phi_x, \varphi_x, \psi_x\rangle$, where the three membership functions for the truth, falsity, and indeterminacy of $x$ can be defined as follows:*

$$\tau(x) = \begin{cases} \dfrac{x - x^L}{x^{M_1} - x^L}\phi_x, & \text{if} \quad x \in [x^L, x^{M_1}], \\ \phi_x, & \text{if} \quad x \in [x^{M_1}, x^{M_2}] \\ \dfrac{x^U - x}{x^U - x^{M_2}}\phi_x, & \text{if} \quad x \in [x^{M_2}, x^U], \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

$$\iota(x) = \begin{cases} \dfrac{x^{M_1} - x + (x - x^L)\varphi_x}{x^{M_1} - x^L}, & if \quad x \in [x^L, x^{M_1}] \\ \varphi_x, & if \quad x \in [x^{M_1}, x^{M_2}], \\ \dfrac{x^U - x + (x - x^{M_2})\varphi_x}{x^U - x^{M_2}}, & if \quad x \in [x^{M_2}, x^U], \\ 1, & otherwise, \end{cases} \tag{3}$$

$$\nu(x) = \begin{cases} \dfrac{x^{M_1} - x + (x - x^L)\psi_x}{x^{M_1} - x^L}, & if \quad x \in [x^L, x^{M_1}], \\ \psi_x, & if \quad x \in [x^{M_1}, x^{M_2}] \\ \dfrac{x^U - x + (x - x^{M_2})\psi_x}{x^U - x^{M_2}}, & if \quad x \in [x^{M_2}, x^U], \\ 1, & otherwise, \end{cases} \tag{4}$$

*where* $0 \leq \tau(x)^2 + \iota(x)^2 + \nu(x)^2 \leq 1, \ \forall \ x \in \widehat{X}$.

**Definition 3** ([11])**.** *Suppose* $\widehat{X}_i = \langle x_i^L, x_i^{M_1}, x_i^{M_2}, x_i^U; \phi_{x_i}, \varphi_{x_i}, \psi_{x_i}\rangle$, *for* $i = 1, 2, \cdots, n$ *are* $n$ *STrFNs. Then the arithmetic relations are defined as*

*(i)* $\widehat{X}_1 \oplus \widehat{X}_2 = \langle x_1^L + x_2^L, x_1^{M_1} + x_2^{M_1}, x_1^{M_2} + x_2^{M_2}, x_1^U + x_2^U; (\phi_{x_1}^2 + \phi_{x_2}^2 - \phi_{x_1}^2\phi_{x_2}^2)^{\frac{1}{2}}, \varphi_{x_1}\varphi_{x_2}, [(1 - \phi_{x_2}^2)\psi_{x_1}^2 + (1 - \phi_{x_1}^2)\psi_{x_2}^2 - \psi_{x_1}^2\psi_{x_2}^2]^{\frac{1}{2}}\rangle$.

*(ii)* $\widehat{X}_1 - \widehat{X}_1 = \langle x_1^L - x_2^U, x_1^{M_1} - x_2^{M_2}, x_1^{M_2} - x_2^{M_1}, x_1^U - x_2^L; (\phi_{x_1}^2 + \phi_{x_2}^2 - \phi_{x_1}^2\phi_{x_2}^2)^{\frac{1}{2}}, \varphi_{x_1}\varphi_{x_2}, [(1 - \phi_{x_2}^2)\psi_{x_1}^2 + (1 - \phi_{x_1}^2)\psi_{x_2}^2 - \psi_{x_1}^2\psi_{x_2}^2]^{\frac{1}{2}}\rangle$.

*(iii)* $\widehat{X}_1 \otimes \widehat{X}_1 = \langle x_1^L x_2^L, x_1^{M_1} x_2^{M_1}, x_1^{M_2} x_2^{M_2}, x_1^U x_2^U; \phi_{x_1}\phi_{x_2}, (\varphi_{x_1}^2 + \varphi_{x_2}^2 - \varphi_{x_1}^2\varphi_{x_2}^2)^{\frac{1}{2}}, [(1 - \varphi_{x_2}^2)\psi_{x_1}^2 + (1 - \varphi_{x_1}^2)\psi_{x_2}^2 - \psi_{x_1}^2\psi_{x_2}^2]^{\frac{1}{2}}\rangle$.

*(iv)* $\lambda\widehat{X}_1 = \begin{cases} \langle \lambda x_1^L, \lambda x_1^{M_1}, \lambda x_1^{M_2}, \lambda x_1^U; (1 - (1 - \phi_{x_1}^2)^\lambda)^{\frac{1}{2}}, \varphi_{x_1}^\lambda, [(1 - \phi_{x_1}^2)^\lambda - (1 - \phi_{x_1}^2 - \psi_{x_1}^2)^\lambda]^{\frac{1}{2}}\rangle, & \lambda > 0. \\ \langle \lambda x_1^U, \lambda x_1^{M_2}, \lambda x_1^{M_1}, \lambda x_1^L; (1 - (1 - \phi_{x_1}^2)^\lambda)^{\frac{1}{2}}, \varphi_{x_1}^\lambda, [(1 - \phi_{x_1}^2)^\lambda - (1 - \phi_{x_1}^2 - \psi_{x_1}^2)^\lambda]^{\frac{1}{2}}\rangle, & \lambda < 0. \end{cases}$

*(v)* $\sum_{i=1}^n \lambda_i \widehat{X_i} = \langle \sum_{i=1}^n \lambda_i x_i^L, \sum_{i=1}^n \lambda_i x_i^{M_1}, \sum_{i=1}^n \lambda_i x_i^{M_2}, \sum_{i=1}^n \lambda_i x_i^U; \Big(1 - \prod_{i=1}^n (1 - \phi_{x_i}^2)^{\lambda_i}\Big)^{1/2}, \prod_{i=1}^n \varphi_{x_i}^{\lambda_i}, \Big(\prod_{i=1}^n (1 - \phi_{x_i}^2)^{\lambda_i} - \prod_{i=1}^n (1 - \phi_{x_i}^2 - \psi_{x_i}^2)^{\lambda_i}\Big)^{1/2}\rangle, \quad \forall \lambda_i \geq 0.$

**Definition 4.** *The* $\alpha-$*cut,* $\beta-$*cut and* $\gamma-$*cut for a STrFN* $\widehat{X} = \langle x^L, x^{M_1}, x^{M_2}, x^U; \phi_x, \varphi_x, \psi_x\rangle$, *can be defined as*

$$\widehat{X}^{(\alpha,\beta,\gamma)} = \{x : \phi_x \geq \alpha, \varphi \leq \beta, \psi \leq \gamma\}, \tag{5}$$

*where* $0 \leq \alpha \leq \phi_x, \varphi_x \leq \beta \leq 1$ *and* $\psi_x \leq \gamma \leq 1$.

Using definition (3) and equation (5) , the lower limits $L(\alpha), L(\beta)$ and $L(\gamma)$, and upper limits $U(\alpha), U(\beta)$ and $U(\gamma)$ of $\alpha, \beta$ and $\gamma$-level cut for STrFN are defined as

$$\widehat{X}_\alpha = [L_{\widehat{X}}(\alpha), U_{\widehat{X}}(\alpha)] = \Big[x^L + \alpha(\frac{x^{M_1} - x^L}{\phi_x}), x^U - \alpha(\frac{x^U - x^{M_2}}{\phi_x})\Big],$$

$$\widehat{X}_\beta = [L_{\widehat{X}}(\beta), U_{\widehat{X}}(\beta)] = \Big[\frac{(\beta-\varphi_x)x^L + (1-\beta)x^{M_1}}{1-\varphi_x}, \frac{(\beta-\varphi_x)x^U + (1-\beta)x^{M_2}}{1-\varphi_x}\Big],$$

$$\widehat{X}_\gamma = [L_{\widehat{X}}(\gamma), U_{\widehat{X}}(\gamma)] = \Big[\frac{(\gamma-\psi_x)x^L + (1-\gamma)x^{M_1}}{1-\psi_x}, \frac{(\gamma-\psi_x)x^U + (1-\gamma)x^{M_2}}{1-\psi_x}\Big],$$

then

$$\widehat{X}^{(\alpha,\beta,\gamma)} = (\widehat{X}_\alpha, \widehat{X}_\beta, \widehat{X}_\gamma). \tag{6}$$

**Definition 5.** *Let $\widehat{X}$ and $\widehat{Y}$ are two STrFNs. The arithmatic relation for $(\alpha,\beta,\gamma)$-cut of the STrFNs can be defined as*

*(i)* $\widehat{X}_p + \widehat{Y}_p = \Big[L_{\widehat{X}}(p), U_{\widehat{X}}(p)\Big] + \Big[L_{\widehat{Y}}(p), U_{\widehat{Y}}(p)\Big] = \Big[L_{\widehat{X}}(p) + L_{\widehat{Y}}(p), U_{\widehat{X}}(p) + U_{\widehat{Y}}(p)\Big],$

*(ii)* $\widehat{X}_p - \widehat{Y}_p = \Big[L_{\widehat{X}}(p), U_{\widehat{X}}(p)\Big] - \Big[L_{\widehat{Y}}(p), U_{\widehat{Y}}(p)\Big] = \Big[L_{\widehat{X}}(p) - U_{\widehat{Y}}(p), U_{\widehat{X}}(p) - L_{\widehat{Y}}(p)\Big],$

*(iii)* $\lambda\widehat{X}_p = \begin{cases} [\lambda L_{\widehat{X}}(p), \lambda U_{\widehat{X}}(p)], & \lambda > 0, \\ 0, & \lambda = 0, \\ [\lambda U_{\widehat{X}}(p), \lambda L_{\widehat{X}}(p)], & \lambda < 0, \end{cases}$

*(iv)* $\dfrac{\widehat{X}_p}{\widehat{Y}_p} = \dfrac{[L_{\widehat{X}}(p), U_{\widehat{X}}(p)]}{[L_{\widehat{Y}}(p), U_{\widehat{Y}}(p)]} = \Big[\dfrac{L_{\widehat{X}}(p)}{U_{\widehat{Y}}(p)}, \dfrac{U_{\widehat{X}}(p)}{L_{\widehat{Y}}(p)}\Big],$

*where $p = \alpha$ or $\beta$ or $\gamma$.*

**Remark 1.** *Any real number $a \in \mathbf{R}$ may be written as a spherical triangular fuzzy number $a = \langle a, a, a, a; 1, 0, 0\rangle$.*

## 3. Spherical Fuzzy Data Envelopment Analysis (SF-DEA)

Suppose that there are $n$ decision making units (DMUs) each having $m$ inputs and $r$ outputs as represented by the vectors $\mathbf{x} \in R^m$ and $\mathbf{y} \in R^r$, respectively. We define the input matrix $X$ as $X = [x_1, \cdots, x_m] \in R^{m\times n}$, and the output matrix $Y$ as $Y = [y_l, \cdots, y_r] \in R^{r\times n}$, $x_i \in R^m$, $\forall\ i = 1, 2, \cdots, m$, $y_k \in R^r$, $\forall\ k = 1, 2, 3, \cdots, r$ and assume that $X > 0$ and $Y > 0$. Charnes et al. [9] developed this model for measuring the efficiency of $DMU_o$, $(o = 1, 2, 3, \cdots, n)$, that is,

$$\begin{aligned} \max_{u_k, v_i}\ & \theta = \frac{\sum_{k=1}^r u_k y_{ko}}{\sum_{i=1}^m v_i x_{io}}, \\ \text{subject to}\ & \frac{\sum_{k=1}^r u_k y_{kj}}{\sum_{i=1}^m v_i x_{ij}} \leq 1,\ j = 1, 2, \cdots n, \\ & u_k \geq 0, \quad k = 1, 2, \cdots, r, \\ & v_i \geq 0, \quad i = 1, 2, \cdots, m, \end{aligned} \tag{7}$$

which is equivalent to the linear programming($LP_o$) problem, i.e,

$$\begin{aligned} \max_{u_k,v_i} \ \theta &= \sum_{k=1}^{r} u_k y_{ko}, \\ \text{subject to } &\sum_{i=1}^{m} v_i x_{io} = 1, \\ &\sum_{k=1}^{r} u_k y_{kj} \le \sum_{i=1}^{m} v_i x_{ij}, \quad j = 1,2,\cdots n, \\ &u_k \ge 0, \quad k = 1,2,\cdots,r, \\ &v_i \ge 0, \quad i = 1,2,\cdots,m, \end{aligned} \tag{8}$$

which is called CCR model.
If any of the observed data for inputs and/or outputs in this model are inaccurate, unclear, or ambiguous, then the efficiency score of the $DMUo$ will be inaccurate. Let us assume that inputs and outputs are STrFNs while the variables $u_k$ and $v_i$ are real numbers; thus, $(\alpha, \beta, \gamma)-$cut approach of the CCR model will be written as follows:

$$\begin{aligned} \max_{u_k,v_i} \ \theta^{(\alpha,\beta,\gamma)} &= \sum_{k=1}^{r} u_k \widehat{y_{ko}}^{(\alpha,\beta,\gamma)}, \\ \text{subject to } &\sum_{i=1}^{m} v_i \widehat{x_{io}}^{(\alpha,\beta,\gamma)} = \widehat{1}^{(\alpha,\beta,\gamma)}, \\ &\sum_{k=1}^{r} u_k \widehat{y_{kj}}^{(\alpha,\beta,\gamma)} \le \sum_{i=1}^{m} v_i \widehat{x_{ij}}^{(\alpha,\beta,\gamma)}, \quad j = 1,2,\cdots n \\ &u_k \ge 0, \quad k = 1,2,\cdots,r, \\ &v_i \ge 0, \quad i = 1,2,\cdots,m, \end{aligned} \tag{9}$$

where $\widehat{x_{ij}} = \langle x_{ij}^L, x_{ij}^{M_1}, x_{ij}^{M_2}, x_{ij}^U, \phi_{x_{ij}}, \varphi_{x_{ij}}, \psi_{x_{ij}} \rangle$ and $\widehat{y_{kj}} = \langle y_{kj}^L, y_{kj}^{M_1}, y_{kj}^{M_2}, y_{kj}^U, \phi_{y_{kj}}, \varphi_{y_{kj}}, \psi_{y_{kj}} \rangle$ for $i = 1,2,3,\cdots,n, \quad j = 1,2,3,\cdots,m, \quad k = 1,2,3,\cdots,r,$ and $\widehat{1} = \langle 1,1,1,1;1,0,0 \rangle$ are the STrFNs and the efficiency score is lies between 0 and 1.
That implies

$$\begin{aligned} \max_{u_k,v_i} \ \theta^{(\alpha,\beta,\gamma)} &= \sum_{k=1}^{r} u_k \Bigg( \Big[ L_{\widehat{y_{ko}}}(\alpha), U_{\widehat{y_{ko}}}(\alpha) \Big], \Big[ L_{\widehat{y_{ko}}}(\beta), U_{\widehat{y_{ko}}}(\beta) \Big] \\ &\Big[ L_{\widehat{y_{ko}}}(\gamma), U_{\widehat{y_{ko}}}(\gamma) \Big] \Bigg), \\ \text{s.t. } &\sum_{i=1}^{m} v_i \Bigg( \Big[ L_{\widehat{x_{io}}}(\alpha), U_{\widehat{x_{io}}}(\alpha) \Big], \Big[ L_{\widehat{x_{io}}}(\beta), U_{\widehat{x_{io}}}(\beta) \Big], \Big[ L_{\widehat{x_{io}}}(\gamma), U_{\widehat{x_{io}}}(\gamma) \Big] \Bigg) \end{aligned} \tag{10}$$

$$
\begin{aligned}
&= \left( \left[L_{\widehat{1}}(\alpha), U_{\widehat{1}}(\alpha)\right], \left[L_{\widehat{1}}(\beta), U_{\widehat{1}}(\beta)\right], \left[L_{\widehat{1}}(\gamma), U_{\widehat{1}}(\gamma)\right] \right), \\
&\sum_{k=1}^{r} u_k \Big( \left[L_{\widehat{y_{kj}}}(\alpha), U_{\widehat{y_{kj}}}(\alpha)\right], \left[L_{\widehat{y_{kj}}}(\beta), U_{\widehat{y_{kj}}}(\beta)\right], \left[L_{\widehat{y_{kj}}}(\gamma), U_{\widehat{y_{kj}}}(\gamma)\right] \Big) \\
&\quad - \sum_{i=1}^{m} v_i \Big( \left[L_{\widehat{x_{ij}}}(\alpha), U_{\widehat{x_{ij}}}(\alpha)\right], \left[L_{\widehat{x_{ij}}}(\beta), U_{\widehat{x_{ij}}}(\beta)\right], \left[L_{\widehat{x_{ij}}}(\gamma), U_{\widehat{x_{ij}}}(\gamma)\right] \Big) \leq 0, \\
&\quad j = 1, 2, \cdots n, \\
&\text{and } u_k \geq 0, \quad k = 1, 2, \cdots, r, \\
&\quad v_i \geq 0, \quad i = 1, 2, \cdots, m.
\end{aligned}
$$

Using definition (5), we have

$$
\begin{aligned}
\max_{u_k, v_i} \theta^{(\alpha,\beta,\gamma)} &= \Bigg( \Big[ \sum_{k=1}^{r} u_k L_{\widehat{y_{ko}}}(\alpha), \sum_{k=1}^{r} u_k U_{\widehat{y_{ko}}}(\alpha) \Big] \sum_{k=1}^{r} u_k \Big[ L_{\widehat{y_{ko}}}(\beta), \\
&\qquad \sum_{k=1}^{r} u_k U_{\widehat{y_{ko}}}(\beta) \Big], \sum_{k=1}^{r} u_k \Big[ L_{\widehat{y_{ko}}}(\gamma), \sum_{k=1}^{r} u_k U_{\widehat{y_{ko}}}(\gamma) \Big] \Bigg), \\
\text{s.t. } & \Bigg( \Big[ \sum_{i=1}^{m} v_i L_{\widehat{x_{io}}}(\alpha), \sum_{i=1}^{m} v_i U_{\widehat{x_{io}}}(\alpha) \Big], \Big[ \sum_{i=1}^{m} v_i L_{\widehat{x_{io}}}(\beta), \sum_{i=1}^{m} v_i U_{\widehat{x_{io}}}(\beta) \Big] \\
&\qquad \Big[ \sum_{i=1}^{m} v_i L_{\widehat{x_{io}}}(\gamma), \sum_{i=1}^{m} v_i U_{\widehat{x_{io}}}(\gamma) \Big] \Bigg) = \Big( [1,1], [1,1], [1,1] \Big), \\
& \Big( \Big[ \sum_{k=1}^{r} u_k L_{\widehat{y_{kj}}}(\alpha), \sum_{k=1}^{r} u_k U_{\widehat{y_{kj}}}(\alpha) \Big], \Big[ \sum_{k=1}^{r} u_k L_{\widehat{y_{kj}}}(\beta), \sum_{k=1}^{r} u_k U_{\widehat{y_{kj}}}(\beta) \Big], \\
&\qquad \Big[ \sum_{k=1}^{r} u_k L_{\widehat{y_{kj}}}(\gamma), \sum_{k=1}^{r} u_k U_{\widehat{y_{kj}}}(\gamma) \Big] \Big) - \Bigg( \Big[ \sum_{i=1}^{m} v_i L_{\widehat{x_{ij}}}(\alpha), \sum_{i=1}^{m} v_i U_{\widehat{x_{ij}}}(\alpha) \Big], \\
&\qquad \Big[ \sum_{i=1}^{m} v_i L_{\widehat{x_{ij}}}(\beta), \sum_{i=1}^{m} v_i U_{\widehat{x_{ij}}}(\beta) \Big], \Big[ \sum_{i=1}^{m} v_i L_{\widehat{x_{ij}}}(\gamma), \sum_{i=1}^{m} v_i U_{\widehat{x_{ij}}}(\gamma) \Big] \Bigg) \leq 0, \\
&\qquad j = 1, 2, \cdots n, \\
\text{and} &\qquad u_k \geq 0, \quad k = 1, 2, \cdots, r, \\
&\qquad v_i \geq 0, \quad i = 1, 2, \cdots, m,
\end{aligned} \tag{11}
$$

which is the spherical fuzzy DEA model with $(\alpha, \beta, \gamma) - cut$ approach. The SF-DEA model converted into three pair of DEA models to evaluate the lower and upper bounds of the efficiency score in $(\alpha, \beta, \gamma) - cut$ approach. The mathematical model for $\alpha-$ cut approach is defined as

$$\theta_L^{\alpha *} = \inf_{\alpha \in [0,t_1]} \begin{cases} \theta_L^{\alpha} = \max_{u_k,v_i} \quad \sum_{k=1}^{r} u_k L_{\widehat{y_{ko}}}(\alpha), \\ \text{s.t} \quad \sum_{i=1}^{m} v_i U_{\widehat{x_{io}}}(\alpha) = 1, \\ \sum_{k=1}^{r} u_k L_{\widehat{y_{kj}}}(\alpha) - \sum_{i=1}^{m} v_i U_{\widehat{x_{ij}}}(\alpha) \leq 0, \\ \sum_{k=1}^{r} u_k U_{\widehat{y_{kj}}}(\alpha) - \sum_{i=1}^{m} v_i L_{\widehat{x_{ij}}}(\alpha) \leq 0, \\ \quad j = 1, 2, \cdots n, \\ \quad u_k \geq 0, \quad k = 1, 2, \cdots, r, \\ \quad v_i \geq 0, \quad i = 1, 2, \cdots, m, \end{cases} \tag{12}$$

$$\theta_U^{\alpha *} = \sup_{\alpha \in [0,t_1]} \begin{cases} \theta_U^{\alpha} = \max_{u_k,v_i} \quad \sum_{k=1}^{r} u_k U_{\widehat{y_{ko}}}(\alpha), \\ \text{s.t} \quad \sum_{i=1}^{m} v_i L_{\widehat{x_{io}}}(\alpha) = 1, \\ \sum_{k=1}^{r} u_k L_{\widehat{y_{kj}}}(\alpha) - \sum_{i=1}^{m} v_i U_{\widehat{x_{ij}}}(\alpha) \leq 0, \\ \sum_{k=1}^{r} u_k U_{\widehat{y_{kj}}}(\alpha) - \sum_{i=1}^{m} v_i L_{\widehat{x_{ij}}}(\alpha) \leq 0, \\ \quad j = 1, 2, \cdots n, \\ \quad u_k \geq 0, \quad k = 1, 2, \cdots, r, \\ \quad v_i \geq 0, \quad i = 1, 2, \cdots, m, \end{cases} \tag{13}$$

where $t_1 = \inf(\phi_{x_{ij}}, \phi_{y_{kj}}), \quad \forall\ i, j, k.$

Similarly, The mathematical model for $\beta-$cut and $\gamma$-cut approach are defined as follows.

$$\theta_L^{\beta *} = \inf_{\beta \in [t_2,1]} \begin{cases} \theta_L^{\beta} = \max_{u_k,v_i} \quad \sum_{k=1}^{r} u_k L_{\widehat{y_{ko}}}(\beta), \\ \text{s.t} \quad \sum_{i=1}^{m} v_i U_{\widehat{x_{io}}}(\beta) = 1, \\ \sum_{k=1}^{r} u_k L_{\widehat{y_{kj}}}(\beta) - \sum_{i=1}^{m} v_i U_{\widehat{x_{ij}}}(\beta) \leq 0 \\ \sum_{k=1}^{r} u_k U_{\widehat{y_{kj}}}(\beta) - \sum_{i=1}^{m} v_i L_{\widehat{x_{ij}}}(\beta) \leq 0, \\ \quad j = 1, 2, \cdots n \\ \quad u_k \geq 0, \quad k = 1, 2, \cdots, r, \\ \quad v_i \geq 0, \quad i = 1, 2, \cdots, m, \end{cases} \tag{14}$$

$$\theta_U^{\beta *} = \sup_{\beta \in [t_2,1]} \begin{cases} \theta_U^{\beta} = \max_{u_k,v_i} \quad \sum_{k=1}^{r} u_k U_{\widehat{y_{ko}}}(\beta), \\ \text{s.t} \quad \sum_{i=1}^{m} v_i L_{\widehat{x_{io}}}(\beta) = 1, \\ \sum_{k=1}^{r} u_k L_{\widehat{y_{kj}}}(\beta) - \sum_{i=1}^{m} v_i U_{\widehat{x_{ij}}}(\beta) \leq 0, \\ \sum_{k=1}^{r} u_k U_{\widehat{y_{kj}}}(\beta) - \sum_{i=1}^{m} v_i L_{\widehat{x_{ij}}}(\beta) \leq 0, \\ \quad j = 1, 2, \cdots n, \\ \quad u_k \geq 0, \quad k = 1, 2, \cdots, r, \\ \quad v_i \geq 0, \quad i = 1, 2, \cdots, m. \end{cases} \tag{15}$$

and

$$\theta_L^{\gamma *} = \inf_{\gamma \in [t_3,1]} \begin{cases} \theta_L^{\gamma} = \max_{u_k,v_i} \quad \sum_{k=1}^{r} u_k L_{\widehat{y_{ko}}}(\gamma), \\ \text{s.t} \quad \sum_{i=1}^{m} v_i U_{\widehat{x_{io}}}(\gamma) = 1, \\ \sum_{k=1}^{r} u_k L_{\widehat{y_{kj}}}(\gamma) - \sum_{i=1}^{m} v_i U_{\widehat{x_{ij}}}(\gamma) \leq 0, \\ \sum_{k=1}^{r} u_k U_{\widehat{y_{kj}}}(\gamma) - \sum_{i=1}^{m} v_i L_{\widehat{x_{ij}}}(\gamma) \leq 0, \\ \quad j = 1, 2, \cdots n, \\ \quad u_k \geq 0, \quad k = 1, 2, \cdots, r, \\ \quad v_i \geq 0, \quad i = 1, 2, \cdots, m. \end{cases} \tag{16}$$

$$\theta_U^{\gamma *} = \sup_{\gamma \in [t_3,1]} \begin{cases} \theta_U^{\gamma} = \max_{u_k,v_i} \quad \sum_{k=1}^{r} u_k U_{\widehat{y_{ko}}}(\gamma), \\ \text{s.t} \quad \sum_{i=1}^{m} v_i L_{\widehat{x_{io}}}(\gamma) = 1, \\ \sum_{k=1}^{r} u_k L_{\widehat{y_{kj}}}(\gamma) - \sum_{i=1}^{m} v_i U_{\widehat{x_{ij}}}(\gamma) \leq 0, \\ \sum_{k=1}^{r} u_k U_{\widehat{y_{kj}}}(\gamma) - \sum_{i=1}^{m} v_i L_{\widehat{x_{ij}}}(\gamma) \leq 0, \\ \quad j = 1, 2, \cdots n, \\ \quad u_k \geq 0, \quad k = 1, 2, \cdots, r, \\ \quad v_i \geq 0, \quad i = 1, 2, \cdots, m, \end{cases} \tag{17}$$

where $t_2 = \sup(\varphi_{x_{ij}}, \varphi_{y_{kj}}), \quad \forall i, j, k$ and $t_3 = \sup(\psi_{x_{ij}}, \psi_{y_{kj}}), \quad \forall\ i, j, k.$

The efficiency score in $\alpha-$cut, $\beta-$cut and $\gamma-$cut approach must be lies in the optimal interval $[\theta_L^{\alpha *}, \theta_U^{\alpha *}], [\theta_L^{\beta^*}, \theta_U^{\beta^*}]$ and $[\theta_L^{\gamma *}, \theta_U^{\gamma *}]$ respectively.

**Theorem 1.** *The lower bound of the optimal interval in* $(\alpha, \beta, \gamma)$*- cut are equal, that is*

$$\theta_L^{\alpha *} = \theta_L^{\beta^*} = \theta_L^{\gamma *}. \tag{18}$$

*Proof.* Since the $(\alpha, \beta, \gamma)$- cut for a SFTrN $\widehat{X} = \langle x^L, x^{M_1}, x^{M_2}, x^U; \phi_x, \varphi_x, \psi_x \rangle$ is defined in equation (6), we have

$$\begin{aligned} \lim_{(\alpha,\beta,\gamma) \to (0,1,1)} \widehat{X}^{(\alpha,\beta,\gamma)} &= \left( \left[ L_{\widehat{X}}(0), U_{\widehat{X}}(0) \right], \left[ L_{\widehat{X}}(1), U_{\widehat{X}}(1) \right], \left[ L_{\widehat{X}}(1), U_{\widehat{X}}(1) \right] \right), \\ &= \left( [x^L, x^U], [x^L, x^U], [x^L, x^U] \right). \end{aligned}$$

It follows that

$$\lim_{\alpha \to 0} \theta_L^{\alpha} = \lim_{\beta \to 1} \theta_L^{\beta} = \lim_{\gamma \to 1} \theta_L^{\gamma}.$$

$$\theta_L^{\alpha *} = \theta_L^{\beta^*} = \theta_L^{\gamma *}.$$

## 4. Method for Solving SF-DEA model

Let us consider the inputs and outputs of the DMUs are the STrFNs . The following steps can be used to calculate the efficiency score of the DMUs.

**Step 2:** Transform the DEA model into the SF-DEA model using the $(\alpha, \beta, \gamma)$-cut technique, as shown in equation (9) & (10).

**Step 2:** Convert three pairs of crisp DEA models as shown in the equation (12) & (13), equation (14) & (15), and equation (16) & (17).

**Step 3:** Solve this crisp DEA model and find the optimal interval $[\theta_L^{\alpha*}, \theta_U^{\alpha*}]$, $[\theta_L^{\beta^*}, \theta_U^{\beta^*}]$ and $[\theta_L^{\gamma*}, \theta_U^{\gamma*}]$ for $\alpha$-cut, $\beta-$cut and $\gamma$-cut respectively.

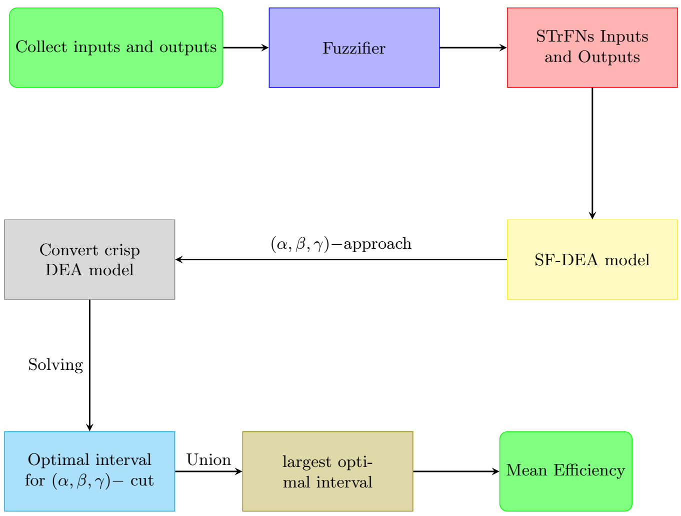
**Step 4:** The largest optimal interval $[\theta_L^*, \theta_U^*]$ for each DMU was computed by taking the union of the optimal intervals and ranking all DMUs based on the mean efficiency score of each DMUs. That is

$$[\theta_L^*, \theta_U^*] = [\theta_L^{\alpha*}, \theta_U^{\alpha*}] \cup [\theta_L^{\beta^*}, \theta_U^{\beta^*}] \cup [\theta_L^{\gamma*}, \theta_U^{\gamma*}]. \quad (19)$$

$$\text{Mean efficiency}(\theta) = \frac{\theta_L^* + \theta_U^*}{2}. \quad (20)$$

The solution method for the SF-DEA model is depicted in the flow chart shown in Figure (2).

Figure 2: Method of Solution for SF-DEA model



## 5. Numerical Example

Let us consider 12 DMUs with inputs and outputs are STrFNs, shown in Table (1) and Table (2). The efficiency score for each DMU was evaluated by proceeding with the above technique given in Section (4).

Table 1: Spherical fuzzy Inputs data

| DMU | Input 1 | Input 2 | Input 2 |
|---|---|---|---|
| D1 | $\langle 11, 15, 18, 25; 0.8, 0.3, 0.4\rangle$ | $\langle 31, 35, 40, 50; 0.5, 0.4, 0.6\rangle$ | $\langle 12, 14, 19, 21; 0.7, 0.1, 0.3\rangle$ |
| D2 | $\langle 6, 7, 11, 13; 0.7, 0.5, 0.5\rangle$ | $\langle 12, 18, 25, 30; 0.5, 0.2, 0.4\rangle$ | $\langle 10, 14, 16, 19; 0.4, 0.3, 0.6\rangle$ |
| D3 | $\langle 17, 21, 24, 28; 0.9, 0.2, 0.3\rangle$ | $\langle 48, 50, 55, 60; 0.8, 0.5, 0.3\rangle$ | $\langle 30, 35, 38, 42; 0.5, 0.2, 0.4\rangle$ |
| D4 | $\langle 11, 15, 17, 24; 0.5, 0.2, 0.5\rangle$ | $\langle 22, 25, 27, 35; 0.8, 0.2, 0.4\rangle$ | $\langle 8, 14, 18, 23; 0.7, 0.5, 0.3\rangle$ |
| D5 | $\langle 22, 25, 27, 31; 0.8, 0.4, 0.1\rangle$ | $\langle 34, 38, 41, 46; 0.6, 0.1, 0.5\rangle$ | $\langle 24, 30, 32, 38; 0.4, 0.5, 0.2\rangle$ |
| D6 | $\langle 13, 19, 24, 28; 0.6, 0.2, 0.5\rangle$ | $\langle 36, 41, 47, 51; 0.4, 0.5, 0.5\rangle$ | $\langle 12, 13, 17, 22; 0.9, 0.1, 0.2\rangle$ |
| D7 | $\langle 20, 24, 27, 32; 0.4, 0.3, 0.4\rangle$ | $\langle 41, 44, 50, 52; 0.6, 0.4, 0.3\rangle$ | $\langle 3, 7, 11, 15; 0.8, 0.2, 0.4\rangle$ |
| D8 | $\langle 11, 12, 15, 18; 0.7, 0.4, 0.6\rangle$ | $\langle 32, 34, 37, 40; 0.7, 0.4, 0.4\rangle$ | $\langle 16, 19, 21, 24; 0.7, 0.4, 0.1\rangle$ |
| D9 | $\langle 21, 24, 31, 35; 0.9, 0.3, 0.3\rangle$ | $\langle 41, 45, 47, 55; 0.5, 0.4, 0.4\rangle$ | $\langle 4, 7, 11, 17; 0.4, 0.5, 0.3\rangle$ |
| D10 | $\langle 17, 18, 21, 24; 0.9, 0.3, 0.1\rangle$ | $\langle 51, 58, 61, 65; 0.9, 0.3, 0.2\rangle$ | $\langle 21, 24, 26, 30; 0.8, 0.2, 0.4\rangle$ |
| D11 | $\langle 9, 12, 18, 22; 0.6, 0.3, 0.5\rangle$ | $\langle 13, 18, 23, 27; 0.7, 0.3, 0.3\rangle$ | $\langle 31, 34, 37, 45; 0.7, 0.3, 0.3\rangle$ |
| D12 | $\langle 18, 24, 27, 32; 0.4, 0.4, 0.2\rangle$ | $\langle 51, 54, 58, 63, 0.6, 0.2, 0.5\rangle$ | $\langle 32, 36, 39, 41; 0.5, 0.1, 0.4\rangle$ |

Table 2: Spherical fuzzy Outputs data

| DMU | Output 1 | Output 2 |
|---|---|---|
| D1 | $\langle 118, 123, 125, 135; 0.7, 0.1, 0.4\rangle$ | $\langle 134, 137, 141, 148; 0.9, 0.2, 0.3\rangle$ |
| D2 | $\langle 134, 138, 140, 144; 0.4, 0.3, 0.3\rangle$ | $\langle 182, 186, 189, 192; 0.5, 0.3, 0.2\rangle$ |
| D3 | $\langle 205, 209, 215, 220; 0.6, 0.1, 0.3\rangle$ | $\langle 141, 145, 147, 150; 0.7, 0.5, 0.4\rangle$ |
| D4 | $\langle 123, 127, 132, 134; 0.8, 0.1, 0.2\rangle$ | $\langle 128, 131, 134, 138; 0.4, 0.7, 0.3\rangle$ |
| D5 | $\langle 194, 196, 200, 215; 0.6, 0.2, 0.5\rangle$ | $\langle 184, 186, 190, 203; 0.8, 0.5, 0.2\rangle$ |
| D6 | $\langle 140, 145, 147, 152; 0.5, 0.2, 0.5\rangle$ | $\langle 94, 106, 111, 115; 0.6, 0.4, 0.1\rangle$ |
| D7 | $\langle 112, 118, 126, 131; 0.7, 0.4, 0.5\rangle$ | $\langle 170, 176, 181, 185; 0.4, 0.7, 0.2\rangle$ |
| D8 | $\langle 141, 146, 153, 155; 0.8, 0.5, 0.3\rangle$ | $\langle 129, 136, 141, 144; 0.7, 0.2, 0.4\rangle$ |
| D9 | $\langle 67, 78, 82, 88; 0.6, 0.5, 0.2\rangle$ | $\langle 211, 218, 222, 225; 0.5, 0.3, 0.6\rangle$ |
| D10 | $\langle 161, 167, 178, 181; 0.4, 0.6, 0.5\rangle$ | $\langle 141, 148, 152, 155; 0.7, 0.5, 0.1\rangle$ |
| D11 | $\langle 117, 126, 129, 137; 0.8, 0.5, 0.3\rangle$ | $\langle 125, 128, 134, 138; 0.6, 0.1, 0.3\rangle$ |
| D12 | $\langle 136, 139, 143, 147; 0.7, 0.6, 0.2\rangle$ | $\langle 185, 188, 194, 198; 0.4, 0.6, 0.6\rangle$ |

The SF-DEA model for DMU $D1$ can be written as

$$\begin{aligned}\max_{u,v}\ \ & \theta = \langle 118, 123, 125, 135; 0.7, 0.1, 0.4\rangle u_1 + \langle 134, 137, 141, 148; 0.9, 0.2, 0.3\rangle u_2,\\ \text{s.t}\ & \langle 11, 15, 18, 25; 0.8, 0.3, 0.4\rangle v_1 + \langle 31, 35, 40, 50; 0.5, 0.4, 0.6\rangle v_2\\ & \quad + \langle 12, 14, 19, 21; 0.7, 0.1, 0.3\rangle v_3 = 1,\end{aligned}$$

$$
\begin{aligned}
&\langle 118,123,125,135;0.7,0.1,0.4\rangle u_1+\langle 134,137,141,148;0.29,0.2,0.3\rangle u_2\\
&\quad\leq\langle 11,15,18,25;0.8,0.3,0.4\rangle v_1+\langle 31,35,40,50;0.5,0.4,0.6\rangle v_2\\
&\qquad+\langle 12,14,19,21;0.7,0.1,0.3\rangle v_3,\\
&\langle 134,138,140,144;0.4,0.3,0.3\rangle u_1+\langle 182,186,189,192;0.5,0.3,0.2\rangle u_2\\
&\quad\leq\langle 6,7,11,13;0.7,0.5,0.5\rangle v_1+\langle 12,18,25,30;0.5,0.2,0.4\rangle v_2\\
&\qquad+\langle 10,14,16,19;0.4,0.3,0.6\rangle v_3,\\
&\qquad\quad\vdots\\
&\langle 136,139,143,147;0.7,0.6,0.2\rangle u_1+\langle 185,188,194,198;0.4,0.6,0.6\rangle u_2\\
&\quad\leq\langle 18,24,27,32;0.4,0.4,0.2\rangle v_1+\langle 51,54,58,63;0.6,0.2,0.5\rangle v_2\\
&\qquad+\langle 32,36,39,41;0.5,0.1,0.4\rangle v_3,\\
&\text{and}\quad u_1,u_2,v_1,v_2,v_3\geq 0.
\end{aligned}
$$

**Step 1** Using equation (11), the $(\alpha,\beta,\gamma)-$cut approach of the SF-DEA model for the DMU $D_1$ can be written as follows:

$$
\begin{aligned}
\max_{u,v}\ \theta^{\alpha,\beta,\gamma}=&\Bigg(\Big[\Big(118+\frac{2\alpha}{0.7}\Big)u_1+\Big(134+\frac{3\alpha}{0.9}\Big)u_2,\Big(135-\frac{10\alpha}{0.7}\Big)u_1\\
&+\Big(148-\frac{7\alpha}{0.9}\Big)u_2\Big],\Big[\frac{(\beta-0.1)118+(1-\beta)123}{0.9}u_1\\
&+\frac{(\beta-0.2)134+(1-\beta)137}{0.8}u_2,\frac{(\beta-0.1)135+(1-\beta)125}{0.9}u_1\\
&+\frac{(\beta-0.2)148+(1-\beta)141}{0.8}u_2\Big],\Big[\frac{(\gamma-0.4)118+(1-\gamma)123}{0.6}u_1\\
&+\frac{(\gamma-0.3)134+(1-\gamma)137}{0.7}u_2,\frac{(\gamma-0.4)135+(1-\gamma)125}{0.6}u_1\\
&\quad+\frac{(\gamma-0.3)148+(1-\gamma)141}{0.7}u_2\Big]\Bigg),\\
\text{s.t}\ &\Bigg(\Big[\Big(11+\frac{4\alpha}{0.8}\Big)v_1+\Big(31+\frac{4\alpha}{0.5}\Big)v_2+\Big(12+\frac{2\alpha}{0.7}\Big)v_3,\Big(25-\frac{7\alpha}{0.8}\Big)v_1\\
&+\Big(50-\frac{10\alpha}{0.5}\Big)v_2+\Big(21-\frac{2\alpha}{0.7}\Big)v_3\Big],\Big[\frac{(\beta-0.3)11+(1-\beta)15}{0.7}v_1\\
&+\frac{(\beta-0.4)31+(1-\beta)35}{0.6}v_2+\frac{(\beta-0.1)12+(1-\beta)14}{0.9}v_3,\\
&\frac{(\beta-0.3)25+(1-\beta)18}{0.7}v_1+\frac{(\beta-0.4)50+(1-\beta)40}{0.6}v_2\\
&+\frac{(\beta-0.1)21+(1-\beta)19}{0.9}v_3\Big],\Big[\frac{(\beta-0.4)11+(1-\beta)15}{0.6}v_1\\
&+\frac{(\beta-0.6)31+(1-\beta)35}{0.4}v_2+\frac{(\beta-0.3)12+(1-\beta)14}{0.7}v_3,
\end{aligned}
$$

$$\frac{(\beta-0.4)25+(1-\beta)18}{0.6}v_1+\frac{(\beta-0.6)50+(1-\beta)40}{0.4}v_2$$
$$+\frac{(\beta-0.3)21+(1-\beta)19}{0.7}v_3\Big]\Bigg)=1,$$

$$\Bigg(\Big[\Big(118+\frac{2\alpha}{0.7}\Big)u_1+\Big(134+\frac{3\alpha}{0.9}\Big)u_2,\Big(135-\frac{10\alpha}{0.7}\Big)u_1+\Big(148-\frac{7\alpha}{0.9}\Big)u_2\Big],$$
$$\Big[\frac{(\beta-0.1)118+(1-\beta)123}{0.9}u_1+\frac{(\beta-0.2)134+(1-\beta)137}{0.8}u_2,$$
$$\frac{(\beta-0.1)135+(1-\beta)125}{0.9}u_1+\frac{(\beta-0.2)148+(1-\beta)141}{0.8}u_2\Big],$$
$$\Big[\frac{(\gamma-0.4)118+(1-\gamma)123}{0.6}u_1+\frac{(\gamma-0.3)134+(1-\gamma)137}{0.7}u_2,$$
$$\frac{(\gamma-0.4)135+(1-\gamma)125}{0.6}u_1+\frac{(\gamma-0.3)148+(1-\gamma)141}{0.7}u_2\Big]\Bigg)$$
$$\leq\Bigg(\Big[\Big(11+\frac{4\alpha}{0.8}\Big)v_1+\Big(31+\frac{4\alpha}{0.5}\Big)v_2+\Big(12+\frac{2\alpha}{0.7}\Big)v_3,\Big(25-\frac{7\alpha}{0.8}\Big)v_1$$
$$+\Big(50-\frac{10\alpha}{0.5}\Big)v_2+\Big(21-\frac{2\alpha}{0.7}\Big)v_3\Big],\Big[\frac{(\beta-0.3)11+(1-\beta)15}{0.7}v_1$$
$$+\frac{(\beta-0.4)31+(1-\beta)35}{0.6}v_2+\frac{(\beta-0.1)12+(1-\beta)14}{0.9}v_3,$$
$$\frac{(\beta-0.3)25+(1-\beta)18}{0.7}v_1+\frac{(\beta-0.4)50+(1-\beta)40}{0.6}v_2$$
$$+\frac{(\beta-0.1)21+(1-\beta)19}{0.9}v_3\Big],\Big[\frac{(\gamma-0.4)11+(1-\gamma)15}{0.6}v_1$$
$$+\frac{(\gamma-0.6)31+(1-\gamma)35}{0.4}v_2+\frac{(\gamma-0.3)12+(1-\gamma)14}{0.7}v_3,$$
$$\frac{(\gamma-0.4)25+(1-\gamma)18}{0.6}v_1+\frac{(\gamma-0.6)50+(1-\gamma)40}{0.4}v_2$$
$$+\frac{(\gamma-0.3)21+(1-\gamma)19}{0.7}v_3\Big]\Bigg),$$

$$\vdots$$

$$\Bigg(\Big[\Big(136+\frac{3\alpha}{0.7}\Big)u_1+\Big(185+\frac{3\alpha}{0.4}\Big)u_2,\Big(147-\frac{4\alpha}{0.7}\Big)u_1+\Big(198-\frac{4\alpha}{0.4}\Big)u_2\Big],$$
$$\Big[\frac{(\beta-0.6)136+(1-\beta)139}{0.4}u_1+\frac{(\beta-0.6)185+(1-\beta)188}{0.4}u_2,$$
$$\frac{(\beta-0.6)147+(1-\beta)143}{0.4}u_1+\frac{(\beta-0.6)198+(1-\beta)194}{0.4}u_2\Big],$$

$$\Big[\frac{(\gamma-0.2)136+(1-\gamma)139}{0.8}u_1+\frac{(\gamma-0.6)185+(1-\gamma)188}{0.4}u_2,$$
$$\frac{(\gamma-0.2)147+(1-\gamma)143}{0.8}u_1+\frac{(\gamma-0.6)198+(1-\gamma)194}{0.4}u_2\Big]\Bigg)$$
$$\leq\Bigg(\Big[\Big(18+\frac{6\alpha}{0.4}\Big)v_1+\Big(51+\frac{3\alpha}{0.6}\Big)v_2+\Big(32+\frac{4\alpha}{0.5}\Big)v_3,\Big(32-\frac{5\alpha}{0.4}\Big)v_1$$
$$+\Big(63-\frac{5\alpha}{0.6}\Big)v_2+\Big(41-\frac{\alpha}{0.5}\Big)v_3\Big],\Big[\frac{(\beta-0.4)18+(1-\beta)24}{0.6}v_1$$
$$+\frac{(\beta-0.2)51+(1-\beta)54}{0.8}v_2+\frac{(\beta-0.1)32+(1-\beta)36}{0.9}v_3,$$
$$\frac{(\beta-0.4)32+(1-\beta)27}{0.6}v_1+\frac{(\beta-0.2)63+(1-\beta)58}{0.8}v_2$$
$$+\frac{(\beta-0.1)41+(1-\beta)39}{0.9}v_3\Big],\Big[\frac{(\gamma-0.2)18+(1-\gamma)24}{0.8}v_1$$
$$+\frac{(\gamma-0.5)51+(1-\gamma)54}{0.5}v_2+\frac{(\gamma-0.4)32+(1-\gamma)14}{0.6}v_3,$$
$$\frac{(\gamma-0.2)32+(1-\gamma)27}{0.8}v_1+\frac{(\gamma-0.5)63+(1-\gamma)58}{0.5}v_2$$
$$+\frac{(\gamma-0.4)41+(1-\gamma)39}{0.6}v_3\Big]\Bigg),$$

$$\text{and}\quad u_1,u_2,v_1,v_2,v_3\geq 0,$$

where $\alpha\in[0,t_1],\ \beta\in[t_2,1]$ and $\gamma\in[t_3,1],\ t_1=\inf(\phi_{x_{ij}},\ \phi_{y_{kj}}),\ t_2=\sup(\varphi_{x_{ij}},\ \varphi_{y_{kj}}),\ t_3=\sup(\psi_{x_{ij}},\psi_{y_{kj}}),\ \forall\ i,j,k.$

**Step 2** The above model was transformed into three pair of crisp DEA models.

$$\theta_L^{\alpha *} = \inf_{\alpha\in[0,0.4]} \begin{cases} \theta_L^{\alpha} & = \max_{u_k, v_i} \quad \left(118+\frac{2\alpha}{0.7}\right)u_1 + \left(134+\frac{3\alpha}{0.9}\right)u_2 \\ \text{s.t} & \left(25-\frac{7\alpha}{0.8}\right)v_1 + \left(50-\frac{10\alpha}{0.5}\right)v_2 + \left(21-\frac{2\alpha}{0.7}\right)v_3 = 1, \\ & \left(118+\frac{2\alpha}{0.7}\right)u_1 + \left(134+\frac{3\alpha}{0.9}\right)u_2 - \left(25-\frac{7\alpha}{0.8}\right)v_1 - \Big(50 \\ & \quad -\frac{10\alpha}{0.5}\Big)v_2 - \left(21-\frac{2\alpha}{0.7}\right)v_3 \le 0, \\ & \quad \vdots \\ & \left(136+\frac{3\alpha}{0.7}\right)u_1 + \left(185+\frac{3\alpha}{0.4}\right)u_2 - \left(32-\frac{5\alpha}{0.4}\right)v_1 - \Big(63 \\ & \quad -\frac{5\alpha}{0.6}\Big)v_2 - \left(41-\frac{\alpha}{0.5}\right)v_3 \le 0, \\ & \left(135-\frac{10\alpha}{0.7}\right)u_1 + \left(148-\frac{7\alpha}{0.9}\right)u_2 - \left(11+\frac{4\alpha}{0.8}\right)v_1 - \Big(31 \\ & \quad +\frac{4\alpha}{0.5}\Big)v_2 - \left(12+\frac{2\alpha}{0.7}\right)v_3 \le 0, \\ & \quad \vdots \\ & \left(147-\frac{4\alpha}{0.7}\right)u_1 + \left(198-\frac{4\alpha}{0.4}\right)u_2 - \left(18+\frac{6\alpha}{0.4}\right)v_1 - \Big(51 \\ & \quad +\frac{3\alpha}{0.6}\Big)v_2 - \left(32+\frac{4\alpha}{0.5}\right)v_3 \le 0, \\ & \text{and} \quad u_1, u_2, v_1, v_2, v_3 \ge 0, \end{cases}$$

$$\theta_U^{\alpha *} = \sup_{\alpha\in[0,0.4]} \begin{cases} \theta_U^{\alpha} & = \max_{u_k, v_i} \quad \left(135-\frac{10\alpha}{0.7}\right)u_1 + \left(148-\frac{7\alpha}{0.9}\right)u_2, \\ \text{s.t} & \left(11+\frac{4\alpha}{0.8}\right)v_1 + \left(31+\frac{4\alpha}{0.5}\right)v_2 + \left(12+\frac{2\alpha}{0.7}\right)v_3 = 1, \\ & \left(118+\frac{2\alpha}{0.7}\right)u_1 + \left(134+\frac{3\alpha}{0.9}\right)u_2 - \left(25-\frac{7\alpha}{0.8}\right)v_1 - \Big(50 \\ & \quad -\frac{10\alpha}{0.5}\Big)v_2 - \left(21-\frac{2\alpha}{0.7}\right)v_3 \le 0, \\ & \quad \vdots \\ & \left(136+\frac{3\alpha}{0.7}\right)u_1 + \left(185+\frac{3\alpha}{0.4}\right)u_2 - \left(32-\frac{5\alpha}{0.4}\right)v_1 - \Big(63 \\ & \quad -\frac{5\alpha}{0.6}\Big)v_2 - \left(41-\frac{\alpha}{0.5}\right)v_3 \le 0, \\ & \left(135-\frac{10\alpha}{0.7}\right)u_1 + \left(148-\frac{7\alpha}{0.9}\right)u_2 - \left(11+\frac{4\alpha}{0.8}\right)v_1 - \Big(31 \\ & \quad +\frac{4\alpha}{0.5}\Big)v_2 - \left(12+\frac{2\alpha}{0.7}\right)v_3 \le 0, \\ & \quad \vdots \\ & \left(147-\frac{4\alpha}{0.7}\right)u_1 + \left(198-\frac{4\alpha}{0.4}\right)u_2 - \left(18+\frac{6\alpha}{0.4}\right)v_1 - \Big(51 \\ & \quad +\frac{3\alpha}{0.6}\Big)v_2 - \left(32+\frac{4\alpha}{0.5}\right)v_3 \le 0, \\ & \text{and} \quad u_1, u_2, v_1, v_2, v_3 \ge 0, \end{cases}$$

Similarly, other two pair of DEA models for $\beta$-cut and $\gamma$-cut determined using equation (14) & (15) and equation (16) & (17).

**Step 3** Solving the above three pair of DEA model using the value of $\alpha \in [0, 0.4], \beta \in [0.7, 1]$ and $\gamma \in [0.6, 1]$ and obtain the optimal interval for efficiency score of the DMU $D1$. Similarly, the optimal interval for all DMUs were calculated as shown in Table (3).

Table 3: Efficiency Score in $\alpha, \beta, \gamma-$cut

| **DMU** | $[\theta_L^{\alpha *}, \theta_U^{\alpha *}]$ | $[\theta_L^{\beta *}, \theta_U^{\beta *}]$ | $[\theta_L^{\gamma *}, \theta_U^{\gamma *}]$ |
|---|---|---|---|
| D1 | [0.327213951, 0.544156783] | [0.327213951, 0.458144777 ] | [ 0.327213951, 0.487868248] |
| D2 | [ 0.482615392, 0.802766814] | [ 0.482615392, 0.636611817] | [ 0.482615392, 0.682741675] |
| D3 | [ 0.331550137, 0.526182357] | [ 0.331550137, 0.430341206] | [ 0.331550137, 0.458495452] |
| D4 | [ 0.352868079, 0.606499314] | [ 0.352868079, 0.516346142] | [ 0.352868079, 0.568938001] |
| D5 | [0.353924801, 0.622102844] | [ 0.353924801, 0.495217527] | [ 0.353924801, 0.533268136] |
| D6 | [0.373781071, 0.59695553] | [ 0.373781071, 0.509640297] | [ 0.373781071, 0.547609621] |
| D7 | [0.410372628, 0.636753029 ] | [ 0.410372628, 0.561771929 ] | [ 0.410372628, 0.600530298] |
| D8 | [0.388465032, 0.63146065 ] | [ 0.388465032, 0.532694159 ] | [ 0.388465032, 0.571762135] |
| D9 | [ 0.450806804, 0.808444737 ] | [ 0.450806804, 0.652290118] | [ 0.450806804, 0.721735487] |
| D10 | [0.349282411, 0.566211624] | [ 0.349282411, 0.479628115 ] | [ 0.349282411, 0.510998311] |
| D11 | [0.36111111, 0.643058285] | [ 0.36111111, 0.523612246] | [ 0.36111111, 0.559335959] |
| D12 | [0.220904311, 0.344251309] | [ 0.220904311, 0.281483055] | [ 0.220904311, 0.29655302] |

**Step 4** The largest optimal interval for DMU $D1$ is obtained by using equation (19), that is, union of the optimal interval for $(\alpha, \beta, \gamma)-$cut. Similarly, the largest optimal interval for other DMUs are obtained as shown in Table (4). The DMUs were compared by taking mean of the largest optimal interval as show in Table (4) and Figure (3). The DMUs have been ranked in the following order $D2 > D9 > D7 > D8 > D11 > D5 > D6 > D4 > D10 > D1 > D3 > D12$. The DMU $D2$ is highly efficient then other DMUs and $D12$ is the least efficient.

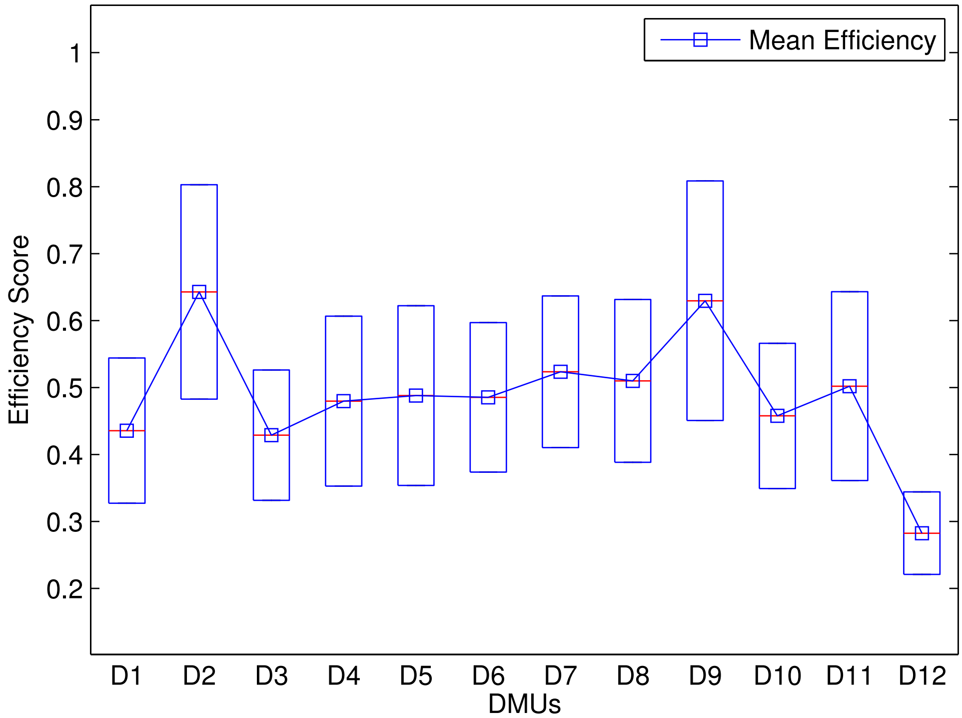
Table 4: Optimal Interval, Mean efficiency score and Ranking

| **DMU** | $[\theta_L^{t *}, \theta_U^{t *}]$ | Mean | Ranking |
|---|---|---|---|
| D1 | [0.327213951, 0.544156783] | 0.435685367020560 | 10 |
| D2 | [ 0.482615392, 0.802766814] | 0.642691102824787 | 1 |
| D3 | [ 0.331550137, 0.526182357] | 0.428866246964638 | 11 |
| D4 | [ 0.352868079, 0.606499314] | 0.479683696432378 | 8 |
| D5 | [0.353924801, 0.622102844] | 0.488013822795279 | 6 |
| D6 | [0.373781071, 0.59695553] | 0.485368300723181 | 7 |
| D7 | [0.410372628, 0.636753029 ] | 0.523562828586664 | 3 |
| D8 | [0.388465032, 0.63146065 ] | 0.509962840919284 | 4 |
| D9 | [ 0.450806804, 0.808444737 ] | 0.629625770540691 | 2 |
| D10 | [0.349282411, 0.566211624] | 0.457747017566756 | 9 |
| D11 | [0.36111111, 0.643058285] | 0.502084697510003 | 5 |
| D12 | [0.220904311, 0.344251309] | 0.282577809805037 | 12 |

## 6. Conclusion

Spherical fuzzy sets (SFSs) are a relatively new academic topic rapidly growing in popularity and being used to a wide range of decision-making concerns, particularly mathematical programming problems. This study focuses on DEA models with spherical fuzzy

Figure 3: Mean Efficiency score in SF-DEA




inputs and outputs. We introduce the Spherical fuzzy DEA (SF-DEA) models and offer a unique approach to solve them.The SF-DEA model transformed three pairs of crisp DEA models to determine the optimal interval in which the $(\alpha, \beta, \gamma)$-cut efficiency lies. The DMUs were ranked based on the mean efficiency score, with the largest optimal interval determined by combining the $(\alpha, \beta, \gamma)$-cut optimal intervals. Finally, we offer an example to demonstrate the method's applicability and validity.

Future research should use this innovative technique to solve additional DEA models, including the BCC and SBM models, and provides encouraging results. In addition, This technique also helps to solve the spherical fuzzy linear programming problem, multi-objective LP problem, and so on. The model may be considered efficient and practical based on the given findings. The use of our technique in a real-world application should be the focus of future research.

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