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Generalized residual entropy function and its applications

Mirza Abdul Khalique Baig[1,*], Javid Gani Dar[1]

[1] *P. G. Department of Statistics, University of Kashmir, Srinagar - 190006, India*

Abstract. Shannon's entropy plays an important role in the context of the information theorey. Since, this entropy is not applicable to a system which has survived for some unit of time. So, the concept of residual entropy was developed. In this paper, we study generalized information measure for residual life time distributions and characterize some life time models based on this measure. Also, a new classes of life time distributions are defined.

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Key words: Varma's entropy function, Life time distributions, Residual entropy.

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1. Introduction

Let T be a continuous random variable with probability density function $f(t)$, Varma's entropy of order α and type β is defined by

$$H_v(\alpha,\beta) = \frac{1}{\beta-\alpha}\log\int f^{\alpha+\beta-1}(t)dt \quad for \quad \beta-1<\alpha<\beta, \quad \beta\geq 1. \tag{1.1}$$

and in discrete case

$$H_v(\alpha,\beta) = \frac{1}{\beta-\alpha}\log\left(\sum_{k=1}^{n} P_k^{\alpha+\beta-1}\right) \quad for \quad \beta-1<\alpha<\beta, \quad \beta\geq 1. \tag{1.2}$$

Also

$$\lim_{\alpha\to 1,\beta=1} H_v(\alpha,\beta) = -\int f(t)\log f(t)dt \tag{1.3}$$

and in discrete case

$$\lim_{\alpha\to 1,\beta=1} H_v(\alpha,\beta) = -\sum_{k=1}^{n} P_k \log P_k \tag{1.4}$$

which is Shannon's entropy in both the cases.

Varma's entropy plays a vital role as a measure of complexity and uncertainty in different areas such as physics, electronics and engineering to describe many chaotic systems.

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*Corresponding author. *Email addresses:* baigmak@yahoo.co.in (M. A. K. Baig) jvdevi@gmail.com (J.Devi)

As argued by Ebrahimi[4], if a unit is known to have survived up to an age t, then $H(t)$ is no longer useful in measuring the uncertainty about the remaining life time of the unit. The idea is that a unit with great uncertainty is less reliable than a unit with low uncertainty. Accordingly, he introduced a measure of uncertainty known as residual entropy for the residual life time distribution. The residual entropy of continuous random variable T is defined as

$$H(T,t) = -\int_t^\infty \frac{f(x)}{R(t)} \log \frac{f(x)}{R(t)} dx \tag{1.5}$$

and in case of discrete random variable

$$H(t_j) = -\sum_{k=j}^{n} \frac{P(t_k)}{R(t_j)} \log \frac{P(t_k)}{R(t_j)} \tag{1.6}$$

where $R(t)$ is the reliability function of the random variable T.

2. Generalized Residual Entropy Function:

Let T be the non negative random variable representing component failure time with failure distribution $F(t) = P(T \leq t)$ and survival function $R(t) = 1 - F(t)$ with $R(0) = 1$. We define Varma's entropy for residual life as

$$H_v(\alpha,\beta,t) = \frac{1}{\beta-\alpha} \log\left(\frac{\int_t^\infty f^{\alpha+\beta-1}(x)}{R^{\alpha+\beta-1}(t)} dx \right), \quad \beta - 1 < \alpha < \beta, \quad \beta \geq 1. \tag{2.1}$$

or

$$(\beta-\alpha)H_v(\alpha,\beta,t) = \log\left(\int_t^\infty f^{\alpha+\beta-1}(x)dx \right) - (\alpha+\beta-1)\log R(t), \;\; \beta - 1 < \alpha < \beta, \beta \geq 1. \tag{2.2}$$

for $\beta = 1, \;\; \alpha \to 1$, (7) reduces to (5).
We now show that $H_v(\alpha,\beta,t)$ uniquely determines the $R(t)$.

THEOREM 2.1: Let T be the non negative random variable having continuous density function f and distribution function F with survival function $R(t)$. Assume $H_v(\alpha,\beta,t) < \infty, t \geq 0, \beta - 1 < \alpha < \beta, \;\; \beta \geq 1$ and increasing in t, then $H_v(\alpha,\beta,t)$ uniquely determines R(t).

Proof: Differentiating (8) with respect to t, we have

$$(\beta-\alpha)H_v^{'}(\alpha,\beta,t) = (\alpha+\beta-1)h(t) - \frac{f^{\alpha+\beta-1}(t)}{\int_t^\infty f^{\alpha+\beta-1}(x)dx} \tag{2.3}$$

where $h(t) = \frac{f(t)}{R(t)}$ is the failure rate function.
From (8) and (9), we have

$$\begin{aligned} h^{\alpha+\beta-1}(t) &= (\alpha+\beta-1)h(t)\exp\left((\beta-\alpha)H_v(\alpha,\beta,t)\right) \\ &- (\beta-\alpha)H_v^{'}(\alpha,\beta,t)\exp((\beta-\alpha)H_v(\alpha,\beta,t)). \end{aligned} \tag{2.4}$$

Hence for fixed $t > 0$, $h(t)$ is a solution of

$$\begin{aligned} g(x) &= (x)^{\alpha+\beta-1} - (\alpha+\beta-1)x \exp\left((\beta-\alpha)H_v(\alpha,\beta,t)\right) \\ &+ (\beta-\alpha)H_v^{'}(\alpha,\beta,t)\exp\left((\beta-\alpha)H_v(\alpha,\beta,t)\right) = 0. \end{aligned} \tag{2.5}$$

Differentiating both sides with respect to x, we have

$$g^{'}(x) = (\alpha+\beta-1)(x)^{\alpha+\beta-2} - (\alpha+\beta-1)\exp\left((\beta-\alpha)H_v(\alpha,\beta,t)\right). \tag{2.6}$$

For extreme value of $g(x)$, we have
$g^{'}(x) = 0$, which gives

$$x = \exp\left(\frac{\beta-\alpha}{\alpha+\beta-2}H_v(\alpha,\beta,t)\right) = x_t$$

Also

$$g^{''}(x) = (\alpha+\beta-1)(\alpha+\beta-2)x^{\alpha+\beta-3}$$

Case I: Let $\alpha+\beta > 2$, then $g^{''}(x_t) > 0$. Thus $g(x)$ attains minimum at x_t. Also, $g(0 > 0$ and $g(\infty) = \infty$. Further, $g(x)$ decreases for $0 < x < x_t$ and hence increases for $x > x_t$. So, $x = h(t)$ is the unique solution to $g(x) = 0$.
Case II: Let $\alpha+\beta < 2$, then $g^{''}(x_t) < 0$. Thus $g(x)$ attains maximum at x_t. Also, $g(0 > 0$ and $g(\infty) = -\infty$. Further, it can be easily seen that $g(x)$ decreases for $x > x_t$ and increases for $0 < x < x_t$. So, $x = h(t)$ is the unique solution to $g(x) = 0$.
Remark: For $\beta = 1, x_t = \exp(-H_v(\alpha,t))$, which is given by Baig and Dar[2].
Corollary 2.1: If $H_v(\alpha,\beta,t)$ is decreasing in t, then (11) has a unique solution if $g(x_t) = 0$. i.e, $H_v(\alpha,\beta,t) = (\frac{2-\alpha-\beta}{\beta-\alpha})\log(b-t)$ which is the Varma's residual entropy of order α and type β of the uniform distribution over (a,b). Thus the uniform distribution can be characterized by decreasing Varma's residual entropy $H_v(\alpha,\beta,t) = (\frac{2-\alpha-\beta}{\beta-\alpha})\log(b-t)$.
Proof: $H_v(\alpha,\beta,t) = (\frac{2-\alpha-\beta}{\beta-\alpha})\log(b-t)$ is the Varma's residual entropy of the uniform distribution. By putting it in (11), we have $g(x_t) = 0$. Hence $H_v(\alpha,\beta,t) = (\frac{2-\alpha-\beta}{\beta-\alpha})\log(b-t)$ is the unique solution to $g(x_t) = 0$, which proves the theorem.
Remark: For $\beta = 1,\ H_v(\alpha,t) = \log(b-t)$, which is given by Baig and Dar [2].
Corollary 2.2: Let T be the random variable having Varma's entropy of order α and type β with $\alpha+\beta > 2$, be of the form

$$H_v(\alpha,\beta,t) = \frac{1}{\beta-\alpha}\log(k) - \frac{2-\alpha-\beta}{\beta-\alpha}\log h(t) \tag{2.7}$$

where $h(t)$ is the failure rate function of T, then T has
I. Exponential distribution iff $k = \frac{1}{\alpha+\beta-1}$
II. Pareto distribution iff $k < \frac{1}{\alpha+\beta-1}$

III. Finite range distribution iff $k > \frac{1}{\alpha+\beta-1}$

Proof: (I) Let T has exponential distribution with probability distribution function

$$f(t) = \frac{1}{\theta} \exp\left(-\frac{t}{\theta}\right), t > 0, \theta > 0$$

The reliability function is given by

$$R(t) = \exp\left(-\frac{t}{\theta}\right)$$

The failure rate function is

$$h(t) = \frac{1}{\theta}$$

Therefore

$$H_v(\alpha, \beta, t) = \frac{1}{\beta - \alpha} \log\left(\frac{\int_t^\infty f^{\alpha+\beta-1}(x)}{R^{\alpha+\beta-1}(t)} dx\right), \quad \beta - 1 < \alpha < \beta, \quad \beta \geq 1$$

or

$$H_v(\alpha, \beta, t) = \frac{1}{\beta - \alpha} \log(k) - \frac{2 - \alpha - \beta}{\beta - \alpha} \log h(t)$$

where $k = \frac{1}{\alpha+\beta-1}$, $h(t) = \frac{1}{\theta}$

Thus (13) holds.

Conversely, suppose $k = \frac{1}{\alpha+\beta-1}$

$$\frac{1}{\beta - \alpha} \log(k) - \frac{2 - \alpha - \beta}{\beta - \alpha} \log h(t) = \frac{1}{\beta - \alpha} \log\left(\frac{\int_t^\infty f^{\alpha+\beta-1}(x)}{R^{\alpha+\beta-1}(t)} dx\right)$$

or

$$\int_t^\infty f^{\alpha+\beta-1}(x) dx = R^{\alpha+\beta-1}(t) \exp\left(\log(k) - (2 - \alpha - \beta) log\ h(t)\right)$$

Differentiating both sides with respect to t, we have

$$\frac{h^2(t)}{h'(t)} = \frac{k(2 - \alpha - \beta)}{1 - k(\alpha + \beta - 1)}$$

or

$$h^{-2}(t)h^{'}(t) = \frac{1-k(\alpha+\beta-1)}{k(2-\alpha-\beta)}$$

or

$$h(t) = \left(\frac{1-k(\alpha+\beta-1)}{k(\alpha+\beta-2)}t + \frac{1}{h(0)}\right)^{-1} = (at+b)^{-1} \tag{2.8}$$

where $a = \frac{1-k(\alpha+\beta-1)}{k(\alpha+\beta-2)}$ and $b = \frac{1}{h(0)}$.
Now $k = \frac{1}{\alpha+\beta-1}$, therefore $a = 0$.
Clearly (14) is the failur rate function of the exponential distribution.
(II) The density function of the Pareto distribution is given by

$$f(t) = \frac{(b)^{\frac{1}{a}}}{(at+b)^{1+\frac{1}{a}}}, \quad t \geq 0, a > 0, b > 0$$

The reliability function is given by

$$R(t) = \frac{(b)^{\frac{1}{a}}}{(at+b)^{\frac{1}{a}}}, \quad t \geq 0, a > 0, b > 0$$

The failure rate is given by

$$h(t) = (at+b)^{-1} \tag{2.9}$$

and

$$H_v(\alpha,\beta,t) = \frac{1}{\beta-\alpha}\log(k) - \frac{2-\alpha-\beta}{\beta-\alpha}\log h(t)$$

where $k = \frac{1}{(\alpha+\beta-1)+a(\alpha+\beta-2)}$ and $h(t) = (at+b)^{-1}$. Since $\alpha+\beta > 2$, therefore $k < \frac{1}{\alpha+\beta-1}$
Thus (13) holds.
Conversly, suppose $k < \frac{1}{\alpha+\beta-1}$, proceeding as in (I), (14) gives

$$h(t) = \left(\frac{1-k(\alpha+\beta-1)}{k(\alpha+\beta-2)}t + \frac{1}{h(0)}\right)^{-1} = (at+b)^{-1} \tag{2.10}$$

where $a = \left(\frac{1-k(\alpha+\beta-1)}{k(\alpha+\beta-2)}\right)$ and $b = \frac{1}{h(0)}$.
Since $k < \frac{1}{\alpha+\beta-1}$ and $\alpha+\beta > 2$, therefore $a > 0$.
Clearly (16) is the failure rate function of the Pareto distribution given in (15).
(III) The density function of the finite range distribution is given by

$$f(t) = \frac{\beta_1}{\nu}\left(1-\frac{t}{\nu}\right)^{\beta_1-1}, \beta_1 > 1, 0 \leq t \leq \nu < \infty$$

The reliability function is given by

$$f(t) = \left(1 - \frac{t}{v}\right)^{\beta_1}, \quad \beta_1 > 1, 0 \leq t \leq v < \infty$$

The failure rate function is given by

$$h(t) = \left(\frac{\beta_1}{v}\right)\left(1 - \frac{t}{v}\right)^{-1} \tag{2.11}$$

and

$$H_v(\alpha, \beta, t) = \frac{1}{\beta - \alpha}\log(k) - \frac{2 - \alpha - \beta}{\beta - \alpha}\log h(t)$$

where $k = \frac{\beta_1}{(\alpha+\beta-1)(\beta_1-1)+1}$ and $h(t) = \left(\frac{\beta_1}{v}\right)\left(1 - \frac{t}{v}\right)^{-1}$.
Since $\alpha + \beta > 2$, therefore $k > \frac{1}{\alpha+\beta-1}$.
Thus (13) holds.
Conversely, suppose $k > \frac{1}{\alpha+\beta-1}$. Proceeding as in (I), (14) gives

$$h(t) = h(0)\left(1 - \frac{k(\alpha + \beta - 1) - 1}{k(\alpha + \beta - 2)} h(0)t\right)^{-1} \tag{2.12}$$

which is the failure rate function of the distribution given in (17), iff $k > \frac{1}{\alpha+\beta-1}$.
Remark: For $\beta = 1$, (13), (14), (16), (18) reduces to

$$H_v(\alpha, t) = \frac{1}{1-\alpha}\log(k) - \log h(t), h(t) = \left(\frac{(1-k\alpha)t}{k(\alpha-1)} + \frac{1}{h(0)}\right)^{-1}$$

,

$$h(t) = \left(\frac{(1-k\alpha)t}{k(\alpha-1)} + \frac{1}{h(0)}\right)^{-1}, h(t) = h(0)\left(1 - \frac{(k\alpha-1)h(0)t}{k(\alpha-1)}\right)^{-1}$$

respectively, which is given by Baig and Dar [2].

3. New Class Of Life Time Distribution:

The survival function has increasing(decreasing) Varma's entropy for residual life of order α and type β, **IVERL**(α, β)(**DVERL**(α, β)) if $H_v(\alpha, \beta, t)$ is increasing(decreasing) in t, $t > 0$. This implies that R has **IVERL**(α, β)(**DVERL**(α, β)) if

$$\begin{aligned} H_v^{'}(\alpha, \beta, t) \quad &\geq 0 \\ &\leq 0 \end{aligned}$$

Theorem 3.1: If a distribution is **IVERL**(α,β) as well as **DVERL**(α,β) for some constant, then it must be exponential.
Proof: Since the random variable T is both **IVERL**(α,β) and **DVERL**(α,β), therfore

$$H_v(\alpha,\beta,t) = constant$$

$$\frac{1}{\beta-\alpha}\log\left(\frac{\int_t^\infty f^{\alpha+\beta-1}(x)}{R^{\alpha+\beta-1}(t)}dx\right) = k$$

or

$$\int_t^\infty f^{\alpha+\beta-1}(x)dx = R^{\alpha+\beta-1}(t)\exp(k(\beta-\alpha))$$

Differentiating both sides with respect to t, we get

$$\frac{f(t)}{h(t)} = constant$$

or

$$h(t) = constant$$

which means that the distribution is exponential.
The next theorem gives upper(lower)bounds to the failure rate function.
Theorem 3.2: If T is **IVERL**(α,β)(**DVERL**(α,β)), then
(I) $(h(t) \le (\ge)(\alpha+\beta-1)^{\frac{1}{\alpha+\beta-2}}\exp\left(-\frac{\alpha-\beta}{\alpha+\beta-2}H_v(\alpha,\beta,t)\right)$
if $\alpha+\beta > 2$.
(II) $h(t) \ge (\le)(\alpha+\beta-1)^{\frac{1}{\alpha+\beta-2}}\exp\left(-\frac{\alpha-\beta}{\alpha+\beta-2}H_v(\alpha,\beta,t)\right)$
if $\alpha+\beta < 2$.
Proof : If T is **IVERL**(α,β), then

$$H_v^{'}(\alpha,\beta,t) \ge 0$$

which gives

$$h^{\alpha+\beta-2}(t) \le (\alpha+\beta-1)\exp\left((\beta-\alpha)H_v(\alpha,\beta,t)\right).$$

Similarly, if T is **DVERL**(α,β), then

$$h^{\alpha+\beta-2}(t) \geq (\alpha+\beta-1)\exp\left((\beta-\alpha)H_v(\alpha,\beta,t)\right).$$

Case I: If $\alpha+\beta>2$ and T is **IVERL**(α,β)(**DVERL**(α,β)), then

$$h(t) \leq (\geq)(\alpha+\beta-1)^{\frac{1}{\alpha+\beta-2}}\exp\left(-\frac{\alpha-\beta}{\alpha+\beta-2}H_v(\alpha,\beta,t)\right) \tag{3.1}$$

Case II: If $\alpha+\beta<2$ and T is **IVERL**(α,β)(**DVERL**(α,β)), then

$$h(t) \geq (\leq)(\alpha+\beta-1)^{\frac{1}{\alpha+\beta-2}}\exp\left(-\frac{\alpha-\beta}{\alpha+\beta-2}H_v(\alpha,\beta,t)\right) \tag{3.2}$$

Remark: For $\beta=1$, (19) reduces to
$h(t) \leq (\geq)(\alpha)^{\frac{1}{\alpha-1}}\exp\left(-H_v(\alpha,t)\right)$, which is given by Baig and Dar[2].
Remark : For $\beta=1,\ \alpha\to 1$ (19) reduce to
$h(t) \leq (\geq)\exp(-H(T,t))$, which is given by Ebrahimi [4].

4. Applications:

Let T be a discrete random variable taking values $t_1, t_2, \cdots, t_n$ with respective probabilities $p_1, p_2, \cdots, p_n$. The discrete residual entropy is defined as

$$H(P,j) = -\sum_{k=j}^{n}\frac{p_k}{R(j)}\log\left(\frac{p_k}{R(j)}\right) \tag{4.1}$$

The Verma's residual entropy for discrete case is defined as

$$H_v(\alpha,\beta,j) = \frac{1}{\beta-\alpha}\log\left(\sum_{k=j}^{n}\frac{p_k^{\alpha+\beta-1}}{R^{\alpha+\beta-1}(j)}\right) \tag{4.2}$$

for $\beta=1, \alpha\to 1$, (22) reduces to (21).
Theorem 4.1: If T has a discrete distribution $F(t)$ with support $(t_j : t_j < t_{j+1})$ and an increasing Varma's entropy $H_v(\alpha,\beta,t)$, then $H_v(\alpha,\beta,t)$ uniquely determines $F(t)$.
Proof: We have

$$H_v(\alpha,\beta,j) = \frac{1}{\beta-\alpha}\log\left(\sum_{k=j}^{n}\frac{p_k^{\alpha+\beta-1}}{R^{\alpha+\beta-1}(j)}\right)$$

or

$$\sum_{k=j}^{n}P_k^{\alpha+\beta-1} = R^{\alpha+\beta-1}(j)\exp((\beta-\alpha)H_v(\alpha,\beta,j)) \tag{4.3}$$

For $j+1$, we have

$$\sum_{k=j+1}^{n} P_k^{\alpha+\beta-1} = R^{\alpha+\beta-1}(j+1)\exp((\beta-\alpha)H_v(\alpha,\beta,j+1)) \tag{4.4}$$

Subtracting (24) from (23), we have

$$P_j^{\alpha+\beta-1} = R^{\alpha+\beta-1}(j)\exp((\beta-\alpha)H_v(\alpha,\beta,j)) - R^{\alpha+\beta-1}(j+1)\exp((\beta-\alpha)H_v(\alpha,\beta,j+1))$$

Using $P_j = R(j) - R(j+1)$, we get

$$(R(j)-R(j+1))^{\alpha+\beta-1} = R^{\alpha+\beta-1}(j)\exp((\beta-\alpha)H_v(\alpha,\beta,j)) - R^{\alpha+\beta-1}(j+1)\exp((\beta-\alpha)H_v(\alpha,\beta,j+1))$$

or

$$\exp((\beta-\alpha)H_v(\alpha,\beta,j)) = (1-h_j)^{\alpha+\beta-1} + h_j^{\alpha+\beta-1}\exp((\beta-\alpha)H_v(\alpha,\beta,j+1))$$

where $h_j = \frac{R(j+1)}{R(j)} \in (0,1)$, which is the solution of the following equation

$$\begin{aligned} g(x) &= (1-x)^{\alpha+\beta-1} + x^{\alpha+\beta-1}\exp((\beta-\alpha)H_v(\alpha,\beta,j+1)) \\ &- \exp((\beta-\alpha)H_v(\alpha,\beta,j)) = 0 \end{aligned} \tag{4.5}$$

Differentiating both sides with respect to x, we have

$$\begin{aligned} g'(x) &= -(\alpha+\beta-1)(1-x)^{\alpha+\beta-2} \\ &+ (\alpha+\beta-1)x^{\alpha+\beta-2}\exp((\beta-\alpha)H_v(\alpha,\beta,j+1)) \end{aligned} \tag{4.6}$$

Note that $g'(x) = 0$, gives

$$x = \left[1 + \exp\left(\frac{\beta-\alpha}{\alpha+\beta-2}H_v(\alpha,\beta,j+1)\right)\right]^{-1} = x_j$$

Further, from (25) we have $g(0) \le 0$ and $g(1) \ge 0$.
Case I : Let $\alpha+\beta > 2$, then
$g'(x) > 0$ if $x < x_j$
$g'(x) = 0$ if $x = x_j$
$g'(x) < 0$ if $x > x_j$
which implies that $g(x) = 0$ has a unique solution $h_j \in (0,1)$.
Case II : Let $\alpha+\beta < 2$, then
$g'(x) > 0$ if $x > x_j$
$g'(x) = 0$ if $x = x_j$
$g'(x) < 0$ if $x < x_j$
which again shows that $g(x) = 0$ has a unique solution $h_j \in (0,1)$. Combining both the cases, we conclude that the unique solution to $g(x) = 0$ is given by $x = h_j$.
Thus $H_v(\alpha,\beta,j)$ uniquely determines $F(t)$.

Remark: For$\beta = 1, x_j = \left[1 + \exp(-H_v(\alpha, j+1)\right]^{-1}$ which is given by Baig and Dar [2]
Theorem 4.2: The discrete uniform distribution is characterized by Varam's residual entropy

$$H_v(\alpha,\beta,j) = \left(\frac{2-\alpha-\beta}{\beta-\alpha}\log(n-j+1)\right), j = 1,2,...,n$$

Proof: By putting $H_v(\alpha,\beta,j) = \left(\frac{2-\alpha-\beta}{\beta-\alpha}\log(n-j+1)\right), j = 1,2,...,n$ in (25), we have $g(x_j) = 0$. Hence $H_v(\alpha,\beta,j) = \left(\frac{2-\alpha-\beta}{\beta-\alpha}\log(n-j+1)\right), j = 1,2,...,n$ is the unique solution to $g(x_j) = 0$. Hence the theorem follows.
Remark: For $\beta = 1, H_v(\alpha, j) = \log(n-j+1), j = 1,2,\cdots,n$ which is given by Baig and Dar [2].

5. Conclusion:

We introduce and studied the concept of Varma's entropy for the life time distributions that generalizes the entropy measure given by Ebrahimi[4]. The exponential, the Pareto and the finite Range distributions which are commonly used in the reliability modeling have been characterized in terms of the Varma's entropy. The proposed residual entropy function uniquely determines the distribution function and thus the reliability function. Also, we characterize the discrete uniform distribution in terms of discrete generalized entropy.

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