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Using Kriging Technique to Interpolate and Forecasting Temperatures Spatio-Temporal Data

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Abstract. This paper deals with the forecasting temperatures' spatio-temporal data. This research aims to examine the performance of two statistical methods for interpolating and predicting Spatio-temporal. The kriging technique and a dynamic semi-parameter factor model are applied in this work. The data adopted in this work represent the temperature in Mosul city and Baghdad city in Iraq. The results of our findings show the behavior prediction is closed to the fitting model based on the cross-validation through the comparison between the kriging method and dynamic semi-parametric factor model, we are getting that kriging prediction is more efficient with the second method of the dynamic model. In conclusion, the results show that prediction is consistent with geographic basis risk, also the performance of the dynamic semi-parameter factor model appears to the extent of geographic basis risk to describe the information of the prediction model.

2020 Mathematics Subject Classifications: 62H11

Key Words and Phrases: Ordinary kriging, dynamic semi-parameter factor model, geographic basis risk, Gaussian model, spatiotemporal data

1. Introduction

Weather is an important factor in many industries, such as agriculture, the energy sector, and tourism. Unfavorable weather conditions can affect dealing with it better to avoid weather risks or to mitigate the effects of weather change. In fact, a variety of weather variables such as temperature, precipitation, snow, wind, or indicate are based on these different parameters. The person concerned is awarded in the event of a predetermined meteorological event. For example, a farmer will get insured if the total amount of rain reported at an independent weather station is less than a predetermined sample. The risks arising from natural disasters or from a geographical basis depend on two main factors: the first factor is the spatial variation of weather conditions and the second is the lack of weather data in the locations that require this information that is included in the assessment process.

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For example, temperature varies in space and time and therefore time differences must be taken into account through the application of Kriging techniques or by dynamic models of non-stationary spatiotemporal data as an alternative to standard Kriging procedures, we consider a dynamic semi-parametric factor model capable of facilitating high-dimensional data via dimensionality reduction and prediction in both time and space [12], [19]. Data analysis depends on the quality of the data when it is collected Spatio-Temporal is used to cross both space and time, where spatial refers to space, and white temporal refers to time. These data are used to study a particular phenomenon. Spatio- Temporal are used to predict unstamped spatial location or time [8]. For the analysis of different variables in temperatures data of Mosul city and Baghdad city that have been observed, the geostatistical procedures for time and space are the best towards to get a good results

There are many studies that have taken different multivariate Spatio-Temporal methods [17], [10], and Spatio- Temporal using the kriging technique to predict the depth of groundwater. And variogram functions of data sets of groundwater and Spatio-Temporal geostatistical modeling of groundwater level variations [11], [21]. Other studies dealt with the space-time covariance function [7]. Other studies dealt with dynamic Spatio-Temporal modeling [12], [19]. And many studies look at temperature by kriging techniques [3], [5], [6] and [18]. Several other papers including those that are related to numerical investigation, differential equations, and modeling can be seen in [1], [2], [13], [24].

In this work, the standard method of predicting of real data is compared with the dynamic semi-parametric factor model method [9], [4]. The purpose of this paper is to develop an empirical methodology for spatial basis risk. the dynamic physical relationship between temperature data of Mosul and Baghdad cities.

2. Methods

2.1. Kriging technique

Geostatistical technique includes kriging method to interpolate the value of observations in study filed. Kriging method developed by the French George Matheron based on the master's thesis of Denial Krige.

2.2. General equations of kriging

Let $Y(s)$ is a random field, where $Y(.) is the observations of location (s)$ and let (s_0) an unobserved location from Y_i, $\mathrm{i}=1,2,\ldots \mathrm{n}$, we suppose $Y^*\left(s_0\right)$ is the predictor of $Y\left(s_0\right)$ based on spatial data. Spatial stochastic processes defend as: $\{Y(s) : s \in D\}$ where $D \in R^P, P \geq 1$ for (s) and $(s+h)$ and the expectation $E(Y(s))=\mu$ Constant but unknown and

$$\operatorname{Var}(Y(s+h)-Y(s))=2\gamma(h) \tag{1}$$

and

$$Y\left(s_i\right), i=1,2,\ldots n \text{ then } E\left\{Y(s+h)-Y\left(s_i\right)\right\}=0 \tag{2}$$

where h is the lag (or distance) between the point. the kriging estimator $Y^*(s_0)$ is given by

$$Y^*(s_0) = \sum_{i=1}^{n} \alpha_i(s_0) Y(s_i) \tag{3}$$

Where $\alpha_i = 1, 2, \ldots n$ the weights, and s_0 is location of spatial variable $Y(s)$ and ordinary kriging error $\left(\sigma_{ok}^2\right)$

$\sigma_{ok}^2 = \operatorname{Var}\{Y^*(s_0) - Y(s_0)\}$

And the condition of biasedness is

$$E\{Y^*(s_0) - Y(s_0)\} = \sum_{i=1}^{n} \alpha_i(s_0) \mu(s_i) - \mu(s_0) = 0$$

While the variance defined as:

$$\operatorname{Var}(Y^*(s_0)) = \operatorname{Var}\left(\sum_{i=1}^{n} \alpha_i Y(s_i)\right) = \sum_{i=1}^{n} \sum_{i=j}^{n} \alpha_i \alpha_j C(s_i, s_j)$$

and kirging predictor denoted as $Y^*(s_0)$, where $C(s_i, s_j)$ is covariance function [16].

2.3. Ordinary kriging

In ordinary kriging, assume a constant but unknown mean with assumptions for applications of ordinary kriging : 1) intrinsic stationary of the field, 2) estimate the variogram function:

$$2\gamma(h) = E[Y(s) - Y(s+h)]^2$$

With the condition of $\sum_{i=1}^{n} \alpha_i = 1$ and the system of ordinary kriging are given as:

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \cdot \\ \cdot \\ \mu \end{pmatrix} = \begin{pmatrix} \gamma(s_1,s_1) \cdots \gamma(s_1,s_1) \cdots 1 \\ \cdot \quad \cdot \\ \cdot \quad \cdot \\ \gamma(s_n,s_1) \cdots \gamma(s_n,s_n) \cdots 1 \\ 1 \quad 1 \quad \ldots 0 \end{pmatrix}^{-1} \begin{pmatrix} \gamma(s_1,s^*) \\ \cdot \\ \cdot \\ \gamma(s_1,s^*) \\ 1 \end{pmatrix}$$

Where μ is Lagrange multiplication to get minimize (σ^2 krige)

And the empirical variogram function is given by:

$$2\gamma(h) = \frac{1}{N(h)} \sum_{i=1}^{N(h)} \left[Y(s_i) - Y(s_j)\right]^2 \tag{4}$$

Where $N(h)$ is number of pairs of observation $Y(s)$ denoted the value of a mature at location [15], [20].

2.4. Daily temperature model for unknown location

Kriging techniques are used to interpolate the temperature day. [22], [14].

$$Y_t = A_t + X_t$$

$$A_t = a_0 + a_1 t + a_1 \cos\left(\frac{2\pi(t-a)}{365}\right) \tag{5}$$

Where A_t is the combination of the long - term average a_0
$\sum_{i=1}^{n} \alpha_i Y(s_i)^2 - 2W\left(\sum_{i=1}^{n} \alpha_i^{-1}\right)$
Where $\alpha_1 t$ is linear trend and

$$X_t = \sum_{i=1}^{n} \beta_i \alpha_{i-1} + \sigma_t \epsilon_t \quad \text{Where} \quad \epsilon_t \sim N(0,1)$$

$$\sigma^2{}_t = c_1 + \sum_{L=1}^{t} \text{Ccos}\left(\frac{2\pi l_t}{365}\right) + \text{Csin}\left(\frac{2\pi l_t}{365}\right) \tag{6}$$

2.5. Geographic Basis Risk

Let $R(0,L)$ at location L at the time 0, the function $Q_t(L_t, L)$ at t time and location L where (L_t, L) denote a neither at t and $R(0,L)$ be modeled rely on Q_t, then $R(0,L)$ can be written as:

$$R(0,L) = \{Q_t(L_t, L) * p + F_t(L_t, s)\} * e^{-r\Delta t} - \pi 0.5 \tag{7}$$

Where P is product price and $\pi 0.5$ the produce pays, $F_t(.)$ be positive and $e^{-r\Delta t}$ is a factor and (L_t, L) and (L_t, s) is differ

Then

$$\hat{R}(0,L) = \{Q_t(L_t, L) * p + F_t(L_t, s)\} * e^{-r\Delta t} - \pi(0,L) \tag{8}$$

Where the weather derivative at location .Then the loss functions given as:

$$\text{Loss} = \left(R(0,L) - \hat{R}(0,L)\right)^2. \tag{9}$$

We can write $h_1 = [F_t(L_t, s) - F_t(L_t, L)] * e^{-r\Delta t}$

And $h_2 = \pi 0.5 - \pi(0,L)$

Then $H = h_1 - h_2$ we want to minimize $F_t(.) then \min(F_t(L_t, s) - F_t(L_t, L)$

Geographic basis risk and interpolate and describing the approach at two procedures [22], [23].

2.6. Dynamic semi parametric factor model

Many studies used the generalize a dynamic semi parametric factor model for a time such as [12], [19].

The dynamic semi parametric factor model is defined as:

$$Z_{t,i} = m_0\left(X_{t,i}\right) + \sum_{j=1}^{L} Y_{t,j} mj\left(X_{t,i}\right) + \epsilon_{t,i} \quad 1 \leq i \leq I, 1 \leq tT, X_{t,i} \in [0,1]^d \tag{10}$$

Where t is the time, i the spatial variation and L is number of factors $Z_{t,i}$ are temperature observation on t, $X_{t,i}$ denotes the coordinates of location i, j doesn't rely on t, then

$$Z_{t,i} = Y_t^T mj\left(X_{t,i}\right) + \epsilon_{t,i}$$

Where $m() = .\left(m_0, m_1, \ldots, m_j\right)^T$ unknown function (basis function) and multivariate time series with dynamic $Y_t = \left(1, Y_{t,1}, \ldots, Y_{t,j}\right)$ and the errors $\epsilon_{t,i}$

defined the basis function

$mj\left(X_{t,i}\right) = \sum_{k=1}^{K} a_{jk} Q\left(X_{t,i}\right)$ Where k is number of series a_{jk} are coefficients and $Q\left(X_{t,i}\right)$ is a B-spline basis function .The least square estimate is:

$$\left(\hat{Y}_t \hat{A}\right) = \operatorname{argmin}\left(\sum_{Y_{t,At}=1}^{T} \sum_{i=1}^{T} Z_{t,i} - Z_t AL\left(X_{t,i}\right)\right)^2$$

Where $A = a_{lk}$ matix$(l+1)xk$.

The index data can be normalize by using standard deviation calculated over the temperature data. To select the number of factors of dynamic semi parameter factor model, we can use

$$\mathrm{E(l)} = 1 - \frac{\sum_{t=1}^{T} \sum_{j=1}^{L} \left(Y_{t,j} - mj\left(x_{t,i}\right)\right) - \sum_{j=1}^{L} \left[Z_{t,i} m_0\left(x_{t,i}\right)\right]^2}{\sum_{t=1}^{T} \sum_{j=1}^{L} \left(Y_{t,j} - \bar{Y}\right)}.$$

3. Application of Temperature Data

3.1. Temperature Data

The data adopted in this paper is from two cities in Iraq (Mosul city and Baghdad city).these data represented temperatures $\left(c^0\right)$ from years (1948) to (2010) for all months, these data from Baghdad city and from years (1960) to (2010) for all months from Mosul city, Iraq.

The sample temperatures data of Baghdad city contains (10*10) data, we put a some small sample of this data:

[6.5 6.2 9.6 10.1 18.9 31.1 33.5 31.7 26.7 22.0 14.0 18.8 17.2 8.2 12.4 18.4 24.9 33.3 35.7 35.4 29.0 22.0 17.3 17.1 16.1 10.4 13.6 19.6 27.6 32.2 35.4 34.8 28.7 22.8 17.4 19.4]

And sample temperatures data of Mosul city also contains (10*10) data, we put a small sample:

[5.6 7.4 11.3 16.6 21.1 29.1 34.0 33.4 28.1 22.0 11.5 9.8 9.2 8.9 13.2 19.4 24.8 31.0 34.3 32.7 30.4 22.4 13.9 5.8 8.3 10.3 13.5 17.1 25.6 30.7 33.0 32.9 28.0 20.6 7.9 9.7 15.7 18.3 24.0 31.7 35.3 34.9];

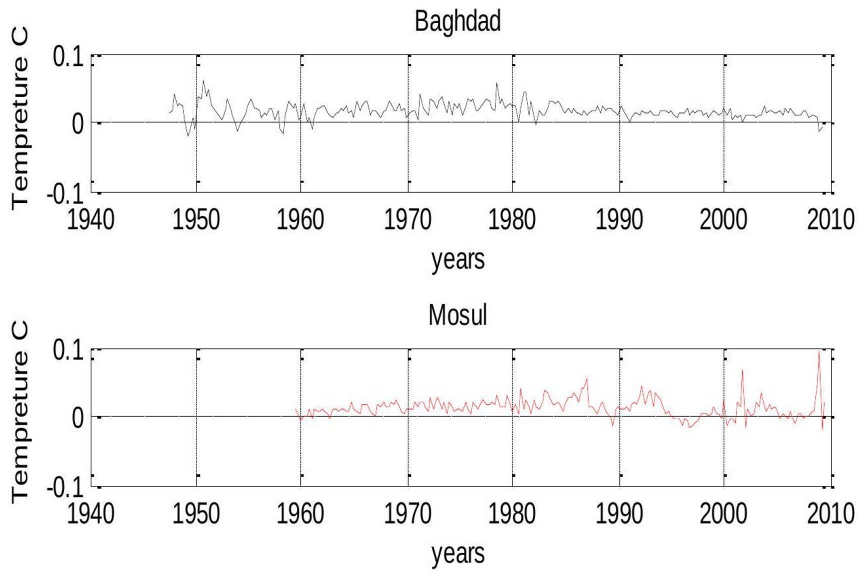

Figure 1: The temperatures of Baghdad city and Mosul city

Figure (1) shows the temperatures of two cities Baghdad and Mosul in Iraq, data represent years from years (1945-2010) of Baghdad city while y-axis is represent temperatures, and from years (1960-2010) of Mosul city with temperatures on y-axis.

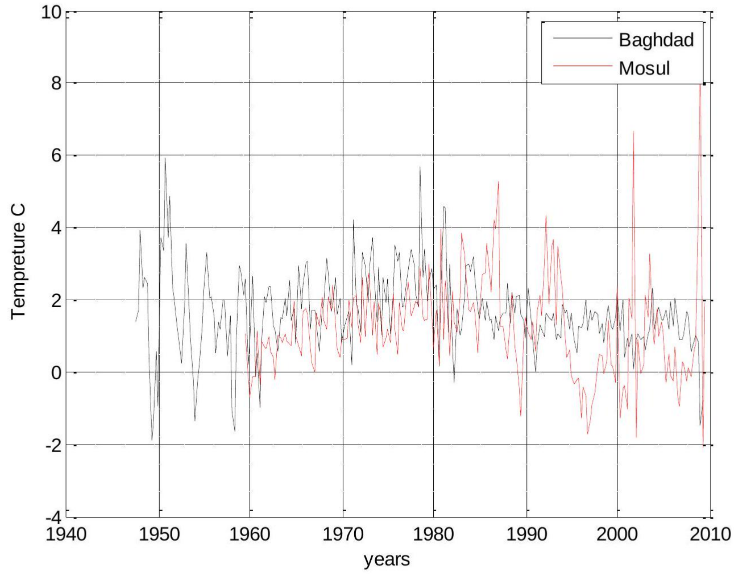

Figure 2: Data tempertures from two cities, Baghdad and Mosul

Figure (2) discribe the tempertures from years (1945-2010) of Baghdad city (black curve) and from years (1960-2010) of Mosul city (red curve).

Table 1: results of variogram function for theta $\left(0^0, 90^0, 45^0, 135^0\right)$

G11	0.0009	0.0028	0.0090	0.0183	0.0294	0.0382	0.0401	0.0183	0.0294	0.0382	0.0401
G22	0.3406	0.3290	0.5832	0.5006	0.5676	0.5225	0.3915	0.4722	0.4292	0.4385	0.3459
G33	0.0048	0.0034	0.0114	0.0209	0.0305	0.0385	0.0401	0.0348	0.0235	0.0116	0.0032
G44	0.0000	0.0032	0.0087	0.0182	0.0302	0.0382	0.0394	0.0329	0.0221	0.0112	0.0030

Table (1) shown the results of variogram function of twelve variables for Mosul city with lag (h), where G11 is results of theta $\left(0^0\right)$, G22 represent the results of variogram function of theta $\left(90^0\right)$, these two theta $(0, 90)$ have the same lag. while G33 represent variogram function for theta $\left(45^0\right)$, and G44 are the variogram function for $\left(135^0\right)$, also two theta $(45, 135)$ have the same lag.

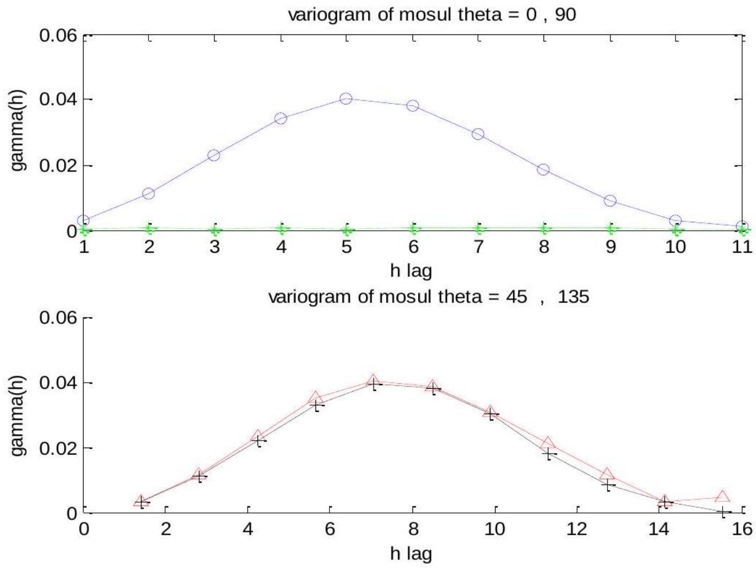

Figure 3: plots of results of variogram function for all theta

Figure (3) show the curves of variogram function on y-axis and lag (h) on x-axis, for Mosul city the first plot contains variogram function for theta 0 (blue curve) and theta 90 (green curve). In the second plot we shown two curves of theta (theta 45, red curve and theta 135 black curve) and the table (2) below describe the average of variogram function G5 represent the average of theta $(0, 90)$ while G6 represent the average of theta $(45, 135)$ (see Table (2)).

Table 2: average of variogram function

G5	0.0006	0.0015	0.0048	0.0094	0.0150	0.0193	0.0202	0.0173	0.0116	0.0059	0.0017
G6	0.0024	0.0033	0.0100	0.0195	0.0304	0.0384	0.0397	0.0339	0.0228	0.0114	0.0031

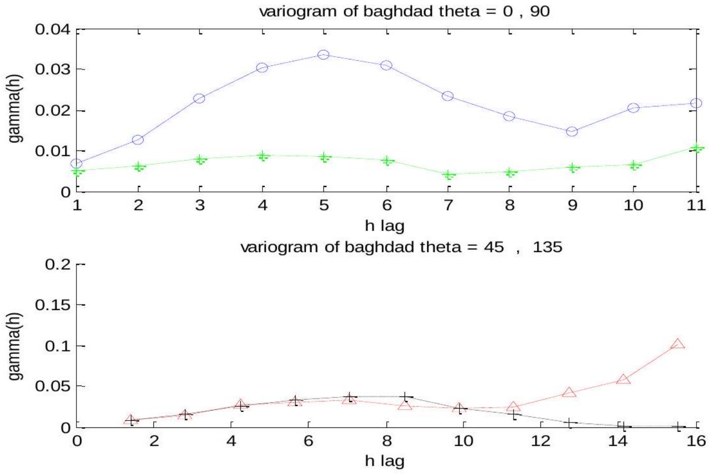

Figure 4: results of variogram function of all theta for Baghdad city

Figure (4) describes the results of variogram functions for Baghdad city in all theta. The first plot shows the variogram function for theta (0 , blue curve, and theta 90 green curve) and the second plot shown the variogram function for two thetas (45, red curve, and 90 black curve) according the results of Table (3) below.

Table 3: results of average of variorum function

$G5$	0.0006	0.0015	0.0048	0.0094	0.0150	0.0193	0.0202	0.0173	0.0116	0.0059	0.0017
$G6$	0.0024	0.0033	0.0100	0.0195	0.0304	0.0384	0.0397	0.0339	0.0228	0.0114	0.0031

And the results of the average of variogram function for Baghdad data where G5 represents the average of variogram in two directions $(0°, 90°)$ and, while G6 represents the average between two directions $(45°, 135°)$

$$\begin{aligned} \mathrm{G5} &= \begin{bmatrix} 0.0060 & 0.0094 & 0.0154 & 0.0196 & 0.0210 & 0.0193 & 0.0138 & 0.0116 & 0.0103 & 0.0135 & 0.0162 \end{bmatrix} \\ \mathrm{G6} &= \begin{bmatrix} 0.0082 & 0.0144 & 0.0255 & 0.0315 & 0.0345 & 0.0309 & 0.0225 & 0.0196 & 0.0228 & 0.0292 & 0.0510 \end{bmatrix} \end{aligned}$$

Table 4: properties of variogram functions for Baghdad city

Properties \ Theta	**0°**	**90°**	**45°**	**135°**
Min	0.0009282	0.00032	0.00031	$4.9e-005$
Max	0.0401	0.0005832	0.04008	0.03939
Mean	0.01907	0.0004473	0.02024	0.01885
Median	0.01833	0.0004385	0.02094	0.01816
Mode	0.0009282	0.000329	0.003174	$4.9e-005$
Std	0.01472	$9.013e-005$	0.01423	0.01466
Range	10	10	14.14	14.14

Table (4) shown the properties of the variogram function (min, max, mean, median, mode, standard deviation and range) for all theta $(0, 90, 45,$and 135) according to the curves of variogram function for Baghdad city.

In order to easy calculation, we transform the initial area of the rectangular coordinate area to Square unit. Further, we change the data by subtracting the mean and dividing by Standard deviation calculated for all months data and all years. After the estimate process of the model, the Mean and standard deviation are incorporated again. To obtain the number of dynamic semi parameter factor model, we use the value, "Value" which can be clear as the shown difference of the model and written as follows:

$$\mathrm{v1} = \sum_{t=1}^{T} \sum_{i=1}^{I} \left\{ Z_{(t,i)} - m_o\left(x_{t,i}\right) - \sum_{l=1}^{L} \hat{Z}_{(t,l)} \cdot \hat{m}_l\left(x_{t,i}\right) \right\}^2 \tag{11}$$

$$\mathrm{v2} = \sum_{t=1}^{T} \sum_{i=1}^{I} \left\{ Z_{(t,i)} - \bar{Z} \right\}^2 \tag{12}$$

$$\text{Value } = 1 - \frac{\mathrm{v1}}{\mathrm{v2}} \tag{13}$$

And days of increasing degree defended as:

$$D_i = \sum_{t=M}^{N} \max\left\{0, Z_{i,t} - \widehat{Z}\right\}^2$$

Where $Z_{i,t}$ denotes the daily average temperature with M first day and N last day to minimize $E\left[Y\left(x_o\right) - \hat{Y}\left(x_o\right)\right]^2$ then

$$RMSE = \sqrt{\frac{1}{K} \sum_{k=1}^{K} (\hat{l} - l)^2} \tag{14}$$

Table 5: results of cross validation of temperature

Type of Temperature	RMSE
A) Mosul.	0.16
B) Baghdad.	1.05
C) A+B	0.61

Table (5) above describes the results of cross-validation of temperature by root mean square error (RMSE) according equation (14) with Mosul, Baghdad and (Mosul, Baghdad) together. From the output of results of the cures of variogram functions for all data temperatures of Mosul city in Figure (3), there is a clear and match and it behaves the similarly to Gaussian model in all directions. These results with small errors support of the estimation. And variance of errors $\left(\sigma^2\right)$ For example $\sigma^2 = (0.0255, 0.0167, 0.0459, 0.0886, \ldots)$

for forecasting of temperatures. Also, RMSE shows the support for forecasting process. Also, data of temperature of Baghdad city, we conclude the model of Gaussian related for the kriging technique in selecting spatial prediction maps with obtaining the lowest estimate of errors and deviation from the original values of the original data. The index data can be normalized by using standard deviation calculated over the temperature data. To select the number of factors of dynamic semi parameter factor model, we can use:

$$\mathrm{E(l)} = 1 - \frac{\sum_{t=1}^{T} \sum_{j=1}^{L} \left(Y_{t,j} - mj\left(x_{t,i}\right)\right) - \sum_{j=1}^{L} \left[Z_{t,i} m_0\left(x_{t,i}\right)\right]^2}{\sum_{t=1}^{T} \sum_{j=1}^{L} \left(Y_{t,j} - \bar{Y}\right)}$$

We can obtain the results of the dynamic semi parameter factor model, where L $= (2, 3, 4)$, are $(0.986, 0.999, 0.969)$ respectively.

4. Discussion and Conclusions

In this paper, we showed how the relationship between spatio-temporal of temperatures. It can be concluded that this kriging technique, which uses dynamic secondary information, has some clear advantages over spatio-temporal. Even though the h measurement coverage over spatio-temporal the domain was high, the spatio-temporal and dynamic semi-parameter factor model shows the curves in a decrease in prediction uncertainty. Another improvement was physically more behavior of the spatio-temporal prediction. Forecasting temperatures of spatio-temporal is an important condition for design weather can be accomplished by means of a quasi-factor in the dynamic model as well as the use of kriging techniques. we compared krige's model with the dynamic semi-parameter factor model. Dynamic semi-parameter factor model uses for temperature data covering a large area of Mosul city in Iraq and evaluating its performance rather than Baghdad city.

This approach is comparison of krige's standard interpolation method, which is combined with a randomized temperature. Moreover, we compare daily and index modeling. However, the application of the dynamic semi parameter factor model should be in the context of further temperature modeling. The accuracy of forecasts can be increased by the presence of a greater number of factors or by dynamic semi parameter factor model provides a better understanding of the geographical factors that drive temperature, which can be evaluated on the basis of this data, it allows the specification quite parametric to the underlying temporal of forecasting. In the preliminary data, the temperatures in the city of Mosul are close to the Gaussian model. The data trend analysis revealed the presence of somewhat converging trends through graphs. Also, the efficiency of the graphic analysis technique in drawing variogram functions to know accurate statistical parameters. The data of temperatures followed the Gaussian model of the properties of variogram functions. The kriging technique in selecting spatial prediction maps with obtaining the lowest estimate of errors and deviation from the original values of the origin data.

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