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Mathematical Model for the effect of buoyancy forces on the stability of a fluid flow

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Abstract. Stability analysis of heat transfer (by Conduction, Convection, and Radiation) has been found for a model of flow between two horizontal plates, one of them is thermally insulated. The stability measure through the neutral curve for this model shows that an increase of the buoyancy represented by Gershoff number (Gr) leads to an increase in stability region, especially for the considerable value of Reynolds number R. The results indicated that the effects of buoyancy forces have significant contribution to the field profiles

2020 Mathematics Subject Classifications: 76E06, 80A20, 35B35, 76M10

Key Words and Phrases: Stability, Heat Transfer, Buoyancy, Gershoff Number

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1. Introduction

The field of mathematical physics known as stability theory allows for the derivation of equation-solving statements from the principle of critical values that distinguishes the many regimes of flow as well as the types of fluid motion that occur in each of these regimes. Chaotic behavior and heat transfer performance are represented very accurately [4–6].

Any system whatsoever can be disturbed, and the question is (does that disturbance die down or grows up with time-lapse) [7, 8].

The presence of buoyancy forces convection along a flat plate causes a coupling between the momentum and energy equations and, at the same time, causes the boundary layer to become non-similar [2, 3, 9, 10, 13].

Mori, Y. [11] has discussed the effects of buoyancy forces in a forced laminar convection flow over a horizontal plate, it is shown that the solution may be expanded into power series in Gr / Re whose first terms express solution for the purely forced convection flow.

A bu-Mulaweh H.I. and others [1] have discussed the measurements of velocity and temperature distributions and reported for buoyancy – opposing, laminar, mixed convection flow over a vertical back ward – facing step.

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Maulucci S.V. [12] has discussed the characteristics of spatial type waves in the boundary layer of incompressible fluid on a flat plate at high Reynolds number R.

The stability of a model of heat transfer by conduction and convection through a porous medium has been investigated by Aziz M. al [13]. The resulting analysis shows that the imposed disturbances are dying down, and the system under consideration is always stable.

The present paper is to investigate the effect of buoyancy forces one of stability of fluid flow between two plates on them is thermally insulated.

2. Formulation of the model and solution

Considering a fluid moving laminarly between two parallel horizontal wall separations of 2h distance, one of them is thermally insulated.

Choosing the coordinate framework. The x-axis is parallel to the channel and along the direction of the flow, and the y-axis is taken as the vertical coordinate measured position upward, wilts the z-axis is the direction mutually orthogonal to the other two axes.

The basic differential equation governing such a model can be summarized as follows:

2.1. Stat Equation

$$\rho = \rho_0[1 - \beta_1(T - T_1)]. \tag{1}$$

Where β_1 is the coefficient of thermal expansion, T_1 is the characteristic temperature, and ρ is the density.

2.2. Continuity Equation

A small size component by taking and calculating the force balance condition from which the continuity equation can be deduced in the general case in which the flow is compressive and unsteady flow in three -dimensions:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho_u)}{\partial x} + \frac{\partial(\rho_v)}{\partial y} + \frac{\partial(\rho_w)}{\partial z} = 0. \tag{2}$$

Where is the density and u, v, w, are the velocity components in the x, y, and z directions, respectively, which are functions of the coordinates x, y, z and time t.

2.3. Motion Equation

From the second law of motion, the motion Equation

$$\rho(\vec{v} \text{ grad } \vec{v} + \vec{g}) = \mu \nabla \vec{v} - \text{ grad } p. \tag{3}$$

Where μ viscosity, p pressure and g gravitational acceleration.

From the law of conservation of energy, the energy equation can be formulated as follow:

2.4. Energy Equation

$$\rho\left(c_v\frac{\partial T}{\partial t}+\vec{v}\ \text{grad}\ T\right)=k\nabla^2T-\ \text{div}\ \vec{q}+\varphi. \tag{4}$$

Where c_v specific heat at a constant temperature, T temperature, k coefficient of thermal conductivity and φ is viscous dissipation, and $\vec{q}=\vec{i}\,q_x+\vec{j}\,q_y+\vec{k}q_z$ is Radiative flux vector and its Component are q_x , q_y , q_z.

$$c_p\rho_0\frac{\partial T}{\partial t}=k\frac{\partial^2T}{\partial y^2}+4\alpha\sigma\left(TT_1^3-3T_1^4\right)+\mu\left(\frac{\partial u}{\partial y}\right)^2. \tag{5}$$

$$\rho_0\frac{\partial u}{\partial t}=\mu\frac{\partial^2u}{\partial y^2}+\rho g. \tag{6}$$

Where $\rho=\rho_0[1-\beta_1(T-T_1)]$, c_p specific heat at constant pressure, α is the obsorption coefficient and σ Stefan constant.

Boundary condition of the model are:

$$\begin{array}{lll} u=0 & y=h & \frac{dT}{dy}=0 \\ u=0 & y=-h & T=T_1 \end{array}. \tag{7}$$

To convert the governing partial differential equations into non-dimensional, we introduce the following non-dimensional quantities:

$$T=\theta T_1, u=u_0\cup, y=h\eta, t=t_0t^*. \tag{8}$$

$Pr=\frac{\mu c_p}{k}$ (Prandtl number),
$Ec=\frac{u_0^2}{c_pT_1}$ (Eckert number),
$Re=\frac{h\rho_0u_0}{\mu}$ (Reynolds number),
$Bo=\frac{\rho_0u_0^3}{T_1^4\sigma}$ (Boltzmann number),
$W=h\alpha$ (Bouguer number),
$Gr=\frac{g\rho_0^2B_1h^3}{\mu^2}$ (Gershoff number),
$Fr=\frac{u_0^2}{gh}$ (Frood number).
and
$A=\frac{1}{Pr.\ \ R}, C=\frac{W\ .\ \ Ec}{Bo}, D=\frac{Ec}{R}, E=\frac{1}{R}, F=\frac{Gr}{R^2}, H=\frac{1}{Fr}+F$
Equations (5) and (6) become)

$$\frac{\partial\theta}{\partial t^*}=A\frac{\partial^2\theta}{\partial\eta^2}-4C\theta+D(\frac{\partial u}{\partial\eta})^2-12C. \tag{9}$$

$$\frac{\partial u}{\partial t^*} = E\frac{\partial^2 u}{\partial \eta^2} - F\theta + H. \tag{10}$$

The boundary conditions become

$$\begin{array}{lll} \cup = 0 & \eta = -1 & \theta = 1 \\ \cup = 0 & \eta = 1 & \frac{d\theta}{d\eta} = 0^{\cdot} \end{array} \tag{11}$$

3. Steady state solution

To find the steady state solution, we shall neglect the terms which depend on the time and equations (9) and (10) become

$$A\frac{\partial^2\theta}{\partial\eta^2} - 4C\theta + D(\frac{d\cup}{d\eta})^2 - 12C = 0. \tag{12}$$

$$E\frac{d^2\cup}{d\eta^2} - F\theta + H = 0. \tag{13}$$

From equations (12) and (13) we get the following equation

$$\frac{d^4\cup}{d\eta^4} + \frac{4C}{A}(\frac{d^2\cup}{d\eta^2}) + 4\frac{CE}{AE}(\frac{H}{F} - 3) + \frac{DF}{AE}(\frac{dU}{d\eta})^2 = 0. \tag{14}$$

and the boundary conditions become

$$\left.\begin{array}{lll} \cup = 0 & \theta = 1 & \eta = -1 \\ , \cup = 0 & \frac{d\theta}{d\eta} = 0 & \eta = 1 \\ , \frac{d^2\cup}{d\eta^2} = & \frac{F-H}{E} & \eta = -1 \\ , \frac{d^3 u}{d\eta^3} = 0 & & \eta = 1. \end{array}\right\} \tag{15}$$

Now we shall use perturbation method [8] and assume the solution of (14) to be as follows.

$$\cup(\eta) = \sum_{n=0}^{\infty} \in^n \cup_n(\eta). \tag{16}$$

Since $D = \frac{EC}{R}$ and the values of EC from 0.1 to 0.01 and R is greater than 100, then the maximum value of D is 0.001, thus we assume $\in = D$ we shall neglect terms which in it D^n where $n > 2$ and thus the solution has the form

$$\cup(\eta) = \cup_0(\eta) + D\cup_1(\eta). \tag{17}$$

Differentiate equation (17) many times and compensate it in equation (14) we get.

$$\frac{d^4\cup_0}{d\eta^4} + \frac{4C}{A}\ (\frac{d^2\cup_0}{d\eta^2}) + 4\frac{CF}{AE}\ (\frac{H}{F} - 3) = 0. \tag{18}$$

$$\frac{d^4\cup_1}{d\eta^4} + \frac{4C}{A}\ (\frac{d^2\cup_1}{d\eta^2}) + \frac{F}{AE}\ (\frac{d\cup_0}{d\eta})^2 = 0. \tag{19}$$

and Boundary conditions

$$\left.\begin{array}{ll} \cup_0 = 0 & \eta = -1 \\ ,\cup_0 = 0 & \eta = 1 \\ ,\dfrac{d^2\cup_0}{d\eta^2} = \dfrac{F-H}{E} & \eta = -1 \\ ,\dfrac{d^3\cup_0}{d\eta^3} = 0 & \eta = 1. \end{array}\right\} \tag{20}$$

$$\left.\begin{array}{ll} \cup_1 = 0 & \eta = -1 \\ ,\cup_1 = 0 & \eta = 1 \\ ,\dfrac{d^2\cup_1}{d\eta^2} = 0 & \eta = -1 \\ ,\dfrac{d^3\cup_0}{d\eta^3} = 0 & \eta = 1 \quad . \end{array}\right\} \tag{21}$$

Now we found the general solution of equation (18) by found complement solution at homogenous equation of the following form.

$$\cup_{op} = A_1 + A_2\ \eta + A_3\ cos(2S_1\eta) + A_4 sin(2S_1\eta). \tag{22}$$

We found special solution of non-homogenous equation by method of variational parameters [7] which is $\cup_{oc}$

$$\cup_{oc}(\eta) = A_s\eta^2. \tag{23}$$

Where $A_5 = \frac{-F}{2E}(\frac{H}{F} - 3), S = \frac{C}{A}, S_1 = S^{\frac{1}{2}}$
and using boundary conditions we found the constant A_1, A_2, A_3, A_4, where
$A_1 = \frac{F}{2E}(\frac{H}{F} - 3) - \frac{FA}{2CE}[\frac{cos^2(2S_1)}{2cos^2\,(2S_1) - 1}]$
$A_2 = \frac{FA}{2CE}[\frac{sin^2\,(2S_1)}{2cos^2\,(2S_1) - 1}]$
$A_3 = \frac{FA}{2CE}[\frac{cos(2S_1)}{2cos^2\,(2S_1) - 1}]$
$A_4 = \frac{FA}{2CE}[\frac{sin(2S_1)}{2cos^2\,(2S_1) - 1}]$

$$\cup_{1p}(\eta) = B_1 + B_2(\eta) + B_3 cos(2S_1\eta) + B_4 sin(2S_1\eta). \tag{24}$$

and we found $\cup_{1C}(\eta)$ by method of variation which has the form

$$U_{1c}(\eta) = \frac{1}{4}S^{-1}\frac{F}{AE}\left[h_1\eta^2 + h_2\eta^3 + h_3\eta^4 + h_4\eta\cos(2S_1\eta) + h_5\eta\sin(2S_1\eta) + h_6\sin^2 (2S_1\eta) + h_7\eta^2\cos(2S_1\eta) + h_8\eta^2\sin(2S_1\eta) + h_9\eta\sin^2(2S_1\eta) + h_{10}\sin(2S_1\eta)\cos (2S_1\eta) + h_{11}\sin(2S_1\eta)\cos^2(2S_1\eta) + h_{12}\sin^2(2S_1\eta)\cos(2S_1\eta) + h_{13}sin(2S_1\eta) cos^2(2S_1\eta) + h_{14}\eta sin^2(2S_1\eta)\ cos(2S_1\eta).\right. \tag{25}$$

Where

$h_1 = \frac{1}{2}A_2^2 + 3SA_4^2 - SA_3^2 - S^{-1}A_5^2$

$h_2 = \frac{2}{3}A_2A_5$

$h_3 = \frac{1}{3}A_5^2$

$h_4 = 4A_3A_4 - 2S_1^{-1}A_2A_5 - A_2A_3 - \frac{1}{2}S_1^{-1}A_4A_5$

$h_5 = \frac{5}{2}S_1^{-1}A_3A_5 - 2S_1^{-1}A_3A_4 - A_2A_4$

$h_6 = \frac{2g}{12}A_4^2 - \frac{17}{12}A_3^2 - S_1A_4^2$

$h_7 = -2A_3A_5$

$h_8 = -A_4A_5$

$h_9 = A_2A_5 + 2S_1A_3A_4 - S_1^{-1}A_2A_5$

$h_{10} = -\frac{5}{3}A_3A_4$

$h_{11} = -(S_1^{-1}A_2A_4 - \frac{1}{4}S_1^{-1}A_4A_5)$

$h_{12} = S_1^{-1}A_2A_4 - \frac{1}{2}S_1^{-1}A_3A_4 + S_1^{-1}A_4A_5$

$h_{13} = S_1^{-1}A_3A_5 - S_1^{-1}A_4A_5$

$h_{14} = -S_1^{-1}A_4A_5$

We found $\cup_1(\eta)$ which has the form.

$\cup_1(\eta) = \cup_{1p}(\eta) + \cup_{1c}(\eta)$

By using boundary conditions, we found the constants B_1, B_2, B_3, B_4, where

$$B_1 = -B_1cos(2S_1) - \frac{1}{8S}(\frac{F}{AE})[\ A_2^2 + 6S\ A_4^2 - 2S\ A_3^2 - S\ A_5^2 + \frac{1}{3}\ A_5^2) - 4A_3A_5cos(2S_1) + (S_1^{-1}A_3A_4 - A_2A_4)sin(2S_1) + [\frac{26}{6}\ A_4^2 - \frac{17}{6}\ A_3^2 - 2S_1A_4^2]sin^2(2S_1) - \frac{1}{2S_1}A_4A_5 sin(2S_1)cos^2(2S_1) + (2S_1^{-1}A_2A_4 - S_1^{-1}A_3A_4 + 2S_1^{-1}A_4A_5)cos(2S_1)sin^2(2S_1).$$

$$B_2 = -B_4\sin(2S_1) - \frac{1}{8S}\frac{F}{AE}\left[-\frac{4}{3}A_2A_5 - \left(8A_3A_4 - 4S_1^{-1}A_2A_4 - 2A_2A_3 - S_1^{-1}A_2A_5 \quad \cos(2S_1) + A_4A_5\sin(2\right.\right.$$

$$B_3 = B_4 \tan(2S_1) + \frac{1}{16S^2} \cdot \frac{F}{AE} \cdot \frac{1}{\cos(2S_1)}$$

$$B_4 = \frac{1}{32} S_1^{-5} \frac{1}{1 - 2\sin^2(2S_1)} \frac{F}{AE} (-32S_1A_3A_4 - 32S_1A_4A_5 - 12S_1A_2A_5 - 12S_1A_2A_3 + 56 S_1A_3A_5 + 4S_1A_3A_5 - 8S_1^3A_2A_4 - 8S_1^3A_4A_5) + \Big(2S_1A_2^2 - 60S_1^3A_4^2 + \frac{56}{3}S_1^3A_3^2 + 16S_1^2A_4^2 - 4S_1^{-1}A_5^2 + 8S_1^3A_2A_5 + 16S_1^2A_3A_4 + 8S_1A_5^2 - 16S_1A_2A_5\Big) \sin(2S_1)\,(16S_1A_2A_5 + \frac{136}{3}S_1^2A_3A_4 - 12A_2A_5 + 8A_5^2\Big) \cos(2S_1) + (64S_1A_3A_4 - 48\ S_1A_4A_5 - 16S_1A_2A_5 + 24S_1 A_2A_3 + 72S_1A_3A_5 + 16S_1A_3A_5) \sin^2(2S_1) + (76S_1A_4A_5 - 126S_1A_3A_4 - 140S_1A_2A_4 + 44 A_3A_5 - 44A_4A_5 + 6S_1A_3A_5 + 64S_1^3A_3A_4 - 16S_1^3A_2A_3\Big) \sin(2S_1)\cos(2S_1) - \Big(\frac{232}{3}S_1^3A_4^2 - \frac{136}{3}S_1^3A_3^2 - 96S_1^2A_4^2 + 48S_1A_3A_5 \sin^2(2S_1)\cos^2(2S_1) + (144S_1A_2A_4 - 72S_1A_3A_4 + 144S_1A_4A_5 + 36A_4A_5 - 36A_3A_5) \sin^3(2S_1)\cos(2S_1)\Big] .$$

Thus

$$\cup(\eta) = P_1 + P_2\eta + P_3\eta^2 + P_4\eta^3 + P_5\eta^4 + P_6\cos(2S_1\eta) + P_7\sin(2S_1\eta) + P_8\eta\cos(2S_1\eta) + P_9\eta\sin(2S_1\eta) + P_{10}\sin^2(2S_1\eta) + P_{11}\eta^2\cos(2S_1\eta) + P_{12}\eta^2\sin(2S_1\eta) + P_{13}\eta\sin^2(2S_1\eta) + P_{14}\cos(2S_1\eta)\sin(2S_1\eta) + P_{15}\sin(2S_1\eta)\cos^2(2S_1\eta) + P_{16}\sin^2(2S_1\eta)\cos(2S_1\eta) + P_{17}\eta\sin(2S_1\eta)\cos^2(2S_1\eta) + P_{18}\eta\sin^2(2S_1\eta)\ \cos(2S_1\eta)] . \tag{26}$$

Where

$P_1 = A_1 + DB_1, P_2 = A_2 + DB_2, P_3 = A_5 + \frac{1}{8}S_1^{-1}kA_2^2 + \frac{3}{4}kA_4^2 - \frac{1}{4}kA_3^2 - \frac{1}{4}S_1^{-2}A_5^2$

$P_4 = \frac{1}{6}S_1^{-1}kA_2A_5, P_5 = \frac{1}{2}S_1^{-1}kA_2A_5^2, P_6 = A_3 + DB_3, P_7 = A_4 + DB_4,$

$P_8 = S_1^{-1}kA_3A_4 - \frac{1}{2}S_1^{-3}kA_2A_5 - \frac{1}{4}S_1^{-1}kA_2A_3 - \frac{1}{8}S_1^{-3}kA_3A_4,$

$P_9 = \frac{5}{8}S_1^{-1}kA_3A_5 - \frac{1}{2}S_1^{-1}kA_3A_4 - \frac{1}{4}S_1^{-1}kA_2A_4,$

$P_{10} = \frac{29}{48}S_1^{-1}kA_4^2 - \frac{17}{48}S_1^{-1}kA_3^2 - \frac{1}{4}S_1^{-1}kA_5^2, P_{11} = \frac{1}{2}S_1^{-1}kA_3A_5, P_{12} = \frac{1}{4}S_1^{-1}kA_4A_5,$

$P_{13} = \frac{1}{4}S_1^{-1}kA_2A_5 + \frac{1}{2}S_1^{-1}kA_3A_4 - \frac{1}{4}S_1^{-2}\ kA_2A_5, P_{14} = -\frac{5}{12}S_1^{-1}kA_3A_4,$

$P_{15} = -\frac{1}{4}S_1^{-3}kA_2A_3 - \frac{1}{16}S_1^{-2}kA_4A_5, P_{16} = \frac{1}{4}S_1^{-3}kA_2A_4 - \frac{1}{8}S_1^{-2}kA_3A_4 + \frac{1}{4}S_1^{-3}kA_4A_5,$

$P_{17} = \frac{1}{4}S_1^{-2}kA_3A_5 - \frac{1}{4}S_1^{-2}kA_3A_4, P_{18} = -\frac{1}{4}S_1^{-1}kA_4A_5, K = \frac{FD}{AE}$

4. Stability analysis

In the standard stability, we assume that [10]

$$\cup(\ \eta, t^*) = \cup_s(\eta) + \cup_d(\eta, t^*). \tag{27}$$

$$\theta(\ \eta, t^*) = \theta_s(\eta) + \theta_d(\eta, t^*). \tag{28}$$

Where $\cup_d$ are small perturbations or departures from equilibrium, and U_s , θ_s are the velocity and temperature in steady state distribution. Substitution of the form (27), (28) into (9), (10), we get the following equations.

$$\frac{\partial\theta_d}{\partial t^*} = A\frac{\partial^2\theta_d}{\partial\eta^2} + 4C\theta_d + 2D\frac{\partial\cup}{\partial\eta} \bullet \frac{d\cup_d}{d\eta}. \tag{29}$$

$$\frac{\partial\cup_d}{\partial t^*} = E\frac{\partial^2\cup_d}{\partial\eta^2} - F\theta_d. \tag{30}$$

$\frac{d\cup_s}{d\eta}$ which is itself $\frac{d\cup}{d\eta}$ which we get it from the derivative of equation (**??**)
The solutions of the system

$$\theta_d\ (\eta, t^*) = C_1 e^{\alpha t^* + i\beta\eta}. \tag{31}$$

$$\cup_d\ = C_2 e^{\alpha t^* + i\beta\eta}. \tag{32}$$

Where C_1, C_2 are constant which represent the amplitudes, and β is a dimensionless wave number, and $\propto$ is complex wave speed such that $\alpha = \propto_1 + i \propto_2$ is positive or negative implies growth or decay of the disturbance.

By taking the derivative of relation (31) and (32) with respect to η, t^* and then substitute the result into equation (29) and (30), we get

$$\left.\begin{array}{c}\left(\alpha + \mathrm{A}\beta^2 - 4\mathrm{C}\right) c_1 - 2\mathrm{iD}\beta\frac{d\mathrm{U}}{d\eta}c_2 = 0 \\ , Fc_1 + (\alpha + \mathrm{E}\beta)c_2 = 0.\end{array}\right\} \tag{33}$$

and thus, we transform the differential equation to an algebra equation and take the determent and we equal to zero

$$\begin{vmatrix} \alpha + \mathrm{A}\beta^2 - 4\mathrm{C} & -2\mathrm{iD}\beta\frac{d\mathrm{U}}{d\eta} \\ , F & \alpha + \beta^2 \end{vmatrix} = 0. \tag{34}$$

which gives

$$\alpha^2 + L\alpha + M + Ni = 0. \tag{35}$$

$L = A\beta^2 - 4C + E\beta^2$
$M = 4E\beta^2 – 4EC\beta^2$

$$N = \ 2D\beta \ \frac{d\cup}{d\eta}$$

Through the solution of equation (35), we get the relation between R and β for different values of Gr, 100, 300, 400 and for different locations of η, 1, 0.5, 0, -0.5, -1 as given in table (1),(2),(3) it was found that the location has a small effect on the stability of the system and then if we take one location such as η=0 the system is then become stable for $\alpha_1 < 0$ and unstable for $\alpha_1 > 0$.

The stable and unstable are shown in Figures (1), (2), (3), (4) from Table (1), (2), and (3).

Stability curves for different values at $Bo = 1,\ Pr = 0.7,\ Ec = 0.1,\ W = 0.15,\ Fr = 1000,\ Gr = 100$

Table 1: Values of Neutral Stability Curve at Bo =1, Pr=0.7, Ec=0.1, W=0.15, Fr =1000, Gr=100

R/η	-1	-0.5	0	0.5	1
100	2.05	2.06	2.06	2.06	2.06
200	3.08	3.08	3.09	3.08	3.10
300	3.64	3.64	3.64	3.67	3.67
400	4.08	4.12	4.08	4.12	4.08
500	6.10	6.11	6.09	6.08	6.07
600	5.74	5.68	5.72	5.68	5.74
700	5.41	5.43	5.42	5.41	5.42
800	6.11	6.15	6.15	6.15	6.11
900	9.18	9.18	9.18	9.18	9.13
1000	8.94	8.94	9.01	8.94	8.94

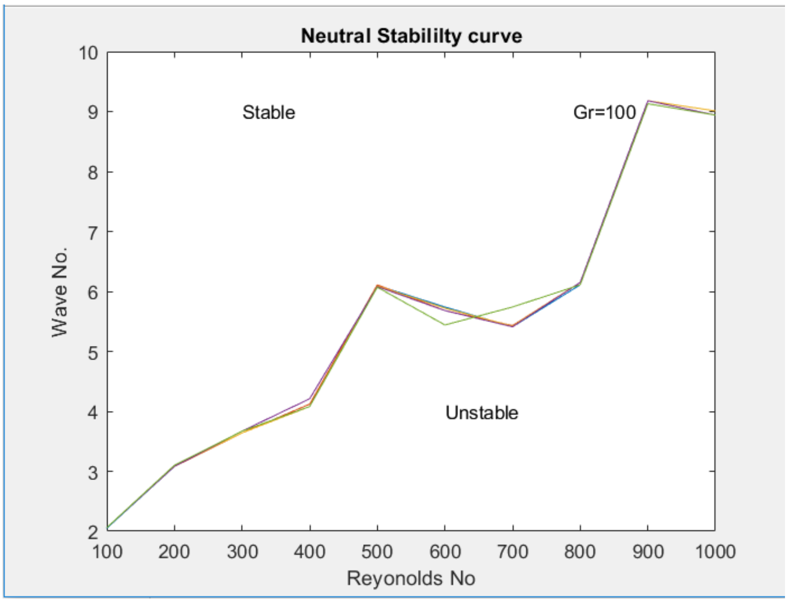

Figure 1: Neutral Stability Curve for Bo = 1, Pr = 0.7, Ec = 0.1, W = 0.15, Fr = 1000, Gr = 100

Table 2: Values of Neutral Stability Curve at Bo =1, Pr=0.7, Ec=0.1, W=0.15, Fr =1000, Gr=300

R/η	-1	-0.5	0	0.5	1
100	2.31	2.28	2.13	2.46	2.48
200	2.92	2.93	2.92	2.94	2.94
300	3.55	3.55	3.5	3.57	3.57
400	4.11	4.10	4.12	4.11	4.10
500	5.14	5.17	5.13	5.14	5.13
600	5.19	5.19	5.20	5.19	5.19
700	5.44	5.41	5.44	5.41	5.44
800	5.86	5.86	5.86	5.86	5.86
900	7.44	7.43	7.47	7.44	7.44
1000	7.48	7.48	7.45	7.48	7.48

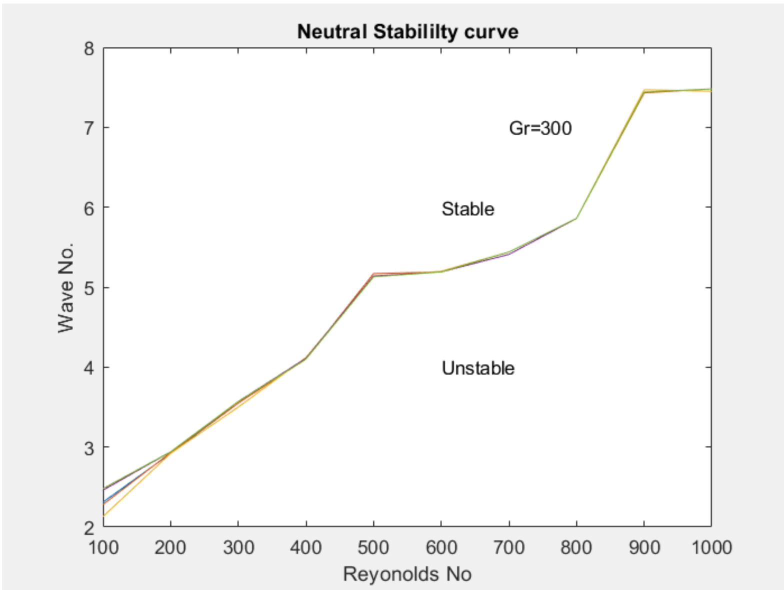

Figure 2: Neutral Stability Curve at Bo =1, Pr=0.7, Ec=0.1, W=0.15, Fr =1000, Gr=300

Table 3: Values of Neutral Stability Curve at Bo =1, Pr=0.7, Ec=0.1, W=0.15, Fr =1000, Gr=400

R/η	-1	-0.5	0	0.5	1
100	2.23	2.21	2.19	2.22	2.20
200	2.91	2.91	2.90	2.95	2.98
300	3.57	3.57	3.60	3.55	3.56
400	4.12	4.08	4.09	4.10	4.13
500	5.00	4.98	5.02	4.98	5.00
600	5.12	5.10	5.15	5.10	5.13
700	5.41	5.40	5.45	5.40	5.44
800	5.82	5.85	5.82	5.82	5.85
900	7.14	7.14	7.14	7.14	7.14
1000	7.23	7.17	7.22	7.23	7.20

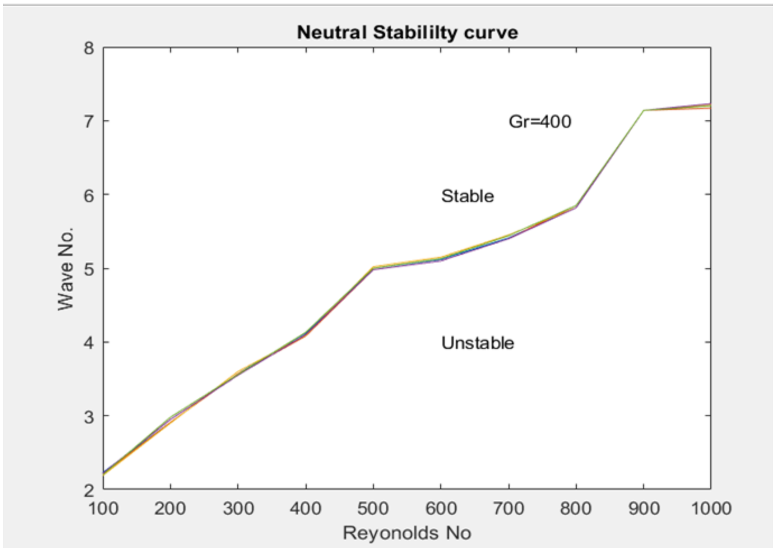

Figure 3: Neutral Stability Curve at Bo =1, Pr=0.7, Ec=0.1, W=0.15, Fr =1000, Gr=400

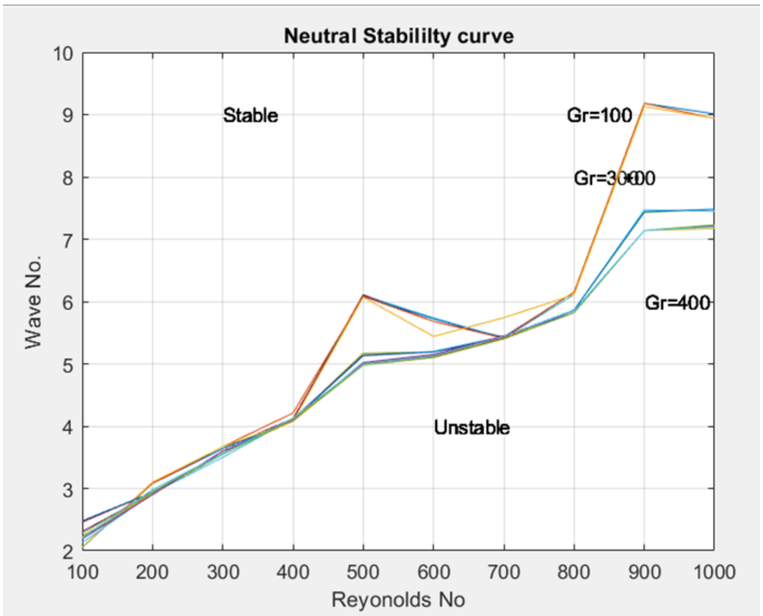

Figure 4: Stability curves for different values of Gershoff number at B0=1, Pr=0.7, Ec=0.1, W=0.15,Fr=1000

5. Conclusion

The stability of the mathematical model of disturbed fluid flow between two parallel plates has been investigated.

Analysis of the results showed the effect of buoyancy force on heat distribution and velocity, and it turns out that when the Gershoff number is increased, the speed decreases.

In the figures (1), (2), (3), and (4), the boundaries between stable and unstable states were examined and found for a perturbed system and for different values of the Gershoff numbers through the Neutral stability curve.

It has been shown through this, that the stability of the model increases with the increase in the Gershoff numbers (Gr), where the gravitational forces resist the viscous forces towards stability.

The results indicated that the effects of buoyancy forces have significant contribution on the basic variable of the model.

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