EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
Vol. 16, No. 3, 2023, 1464-1479
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

1-movable 2-Resolving Hop Domination in Graphs

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Abstract. Let G be a connected graph. A set S of vertices in G is a 1-movable 2-resolving hop dominating set of G if S is a 2-resolving hop dominating set in G and for every $v \in S$, either $S \backslash \{v\}$ is a 2-resolving hop dominating set of G or there exists a vertex $u \in \big((V(G) \backslash S) \cap N_G(v)\big)$ such that $\big(S \backslash \{v\}\big) \cup \{u\}$ is a 2-resolving hop dominating set of G. The 1-movable 2-resolving hop domination number of G, denoted by $\gamma^1_{m2Rh}(G)$ is the smallest cardinality of a 1-movable 2-resolving hop dominating set of G. In this paper, we investigate the concept and study it for graphs resulting from some binary operations. Specifically, we characterize the 1-movable 2-resolving hop dominating sets in the join, corona and lexicographic products of graphs, and determine the bounds of the 1-movable 2-resolving hop domination number of each of these graphs.

2020 Mathematics Subject Classifications: 05C69
Key Words and Phrases: 1-movable 2-resolving hop dominating set, 1-movable 2-resolving hop domination number, join, corona, edge corona, lexicographic product

1. Introduction

The concept of domination was formally studied by Claude Berge [1] in 1958 and Oystein Ore in 1962. In 2015, Natarajan and Ayyaswamy introduced and studied the concept of hop domination [14].

On the other hand, in 1975 using the term locating set, the concept of resolving sets for a connected graph was first introduced by Slater [17]. These concepts were studied much earlier in the context of the coin-weighing problem. Later that year, Harary and Melter introduced independently these concepts, but with different terminologies [8]. The term metric dimension was used by Harary and Melter instead of locating number.

Recently, 2-resolving hop dominating sets in graphs was studied in [9]. Other variations of 2-resolving hop dominating sets in graphs are found in [10, 11]. Moreover, other variations of resolving sets and hop dominating sets in graphs were also studied in [4–7, 12, 13].

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DOI: https://doi.org/10.29020/nybg.ejpam.v16i3.4770
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2. Terminology and Notation

In this study, we consider finite, simple, connected, undirected graphs. For basic graph-theoretic concepts, we then refer readers to [2] and [3]. The following concepts are found in [2], [14], and [16], respectively.

Let G be a connected graph. A vertex v in G is a *hop neighbor* of vertex u in G if $d_G(u, v) = 2$. The set $N_G(u, 2) = \{v \in V(G) : d_G(v, u) = 2\}$ is called the *open hop neighborhood* of u. The *closed hop neighborhood* of u in G is given by $N_G[u, 2] = N_G(u, 2) \cup \{u\}$. The *open hop neighborhood* of $X \subseteq V(G)$ is the set $N_G(X, 2) = \bigcup_{u \in X} N_G(u, 2)$. The *closed hop neighborhood* of X in G is the set $N_G[X, 2] = N_G(X, 2) \cup X$.

A set $S \subseteq V(G)$ is a *hop dominating set* of G if $N_G[S, 2] = V(G)$, that is, for every $v \in V(G) \backslash S$, there exists $u \in S$ such that $d_G(u, v) = 2$. The minimum cardinality of a hop dominating set of G, denoted by $\gamma_h(G)$, is called the *hop domination number* of G. Any hop dominating set with cardinality equal to $\gamma_h(G)$ is called a γ_h-set.

For an ordered set of vertices $W = \{w_1, w_2, ..., w_k\} \subseteq V(G)$ and a vertex v in G, we refer to the k-vector (ordered k-tuple)

$$r_G(v/W) = (d_G(v, w_1), d_G(v, w_2), ..., d_G(v, w_k))$$

as the *(metric) representation of* v *with respect to* W. The set W is called a *resolving set* for G if distinct vertices have distinct representations with respect to W. Hence, if W is a resolving set of cardinality k for a graph G of order n, then the set $\{r_G(v/W) : v \in V(G)\}$ consists of n distinct k-vectors. A resolving set of minimum cardinality is called a *minimum resolving set* or a *basis*, and the cardinality of a basis for G is the *dimension* $\dim(G)$ of G. An ordered set of vertices $W = \{w_1, ..., w_k\}$ is a *k-resolving set* for G if, for any distinct vertices $u, v \in V(G)$, the (metric) representations $r_G(u/W)$ and $r_G(v/W)$ of u and v, respectively, differ in at least k positions. If $k = 1$, then the k-resolving set is called a *resolving set* for G. If $k = 2$, then the k-resolving set is called a 2-resolving set for G. If G has a k-resolving set, the minimum cardinality $\dim_k(G)$ of a k-resolving set is called the *k-metric dimension* of G.

A set S of vertices in G is a 1-movable 2-resolving hop dominating set of G if S is a 2-resolving hop dominating set in G and for every $v \in S$, either $S \backslash \{v\}$ is a 2-resolving hop dominating set of G or there exists a vertex $u \in \big((V(G) \backslash S) \cap N_G(v)\big)$ such that $\big(S \backslash \{v\}\big) \cup \{u\}$ is a 2-resolving hop dominating set of G. The 1-movable 2-resolving hop domination number of G, denoted by $\gamma^1_{m2Rh}(G)$ is the smallest cardinality of a 1-movable 2-resolving hop dominating set of G. Any 1-movable 2-resolving hop dominating set of cardinality $\gamma^1_{m2Rh}(G)$ is referred to as a γ^1_{m2Rh} -set of G.

Definition 1. [6] Let G be any nontrivial connected graph and $S \subseteq V(G)$. A set $S \subseteq V(G)$ is a 2-*locating set* of G if it satisfies the following conditions:

(i) $\left|[\big(N_G(x) \backslash N_G(y)\big) \cap S] \cup [\big(N_G(y) \backslash N_G(x)\big) \cap S]\right| \geq 2$, for all $x, y \in V(G) \backslash S$ with $x \neq y$.

(ii) $\big(N_G(v) \backslash N_G(w)\big) \cap S \neq \varnothing$ or $\big(N_G(w) \backslash N_G[v]\big) \cap S \neq \varnothing$, for all $v \in S$ and for all $w \in V(G) \backslash S$.

The 2-*locating number* of G, denoted by $ln_2(G)$, is the smallest cardinality of a 2-locating set of G. A 2-locating set of G of cardinality $ln_2(G)$ is referred to as an ln_2-set of G.

Definition 2. [15] A set $D \subseteq V(G)$ is a point-wise non-dominating set of G if for each $v \in V(G)\backslash D$, there exists $u \in D$ such that $v \notin N_G(u)$. The smallest cardinality of a point-wise non-dominating set of G, denoted by $pnd(G)$, is called the point-wise non-domination number of G. Any point-wise non-dominating set D of G with $|D| = pnd(G)$, is called a pnd-set of G.

Definition 3. [9] A 2-locating set $S \subseteq V(G)$ which is point-wise non-dominating is called a 2-*locating point-wise non-dominating set* in G. The minimum cardinality of a 2-locating point-wise non-dominating set in G, denoted by $ln_2^{pnd}(G)$ is called the 2-*locating point-wise non-domination number* of G. Any 2-locating point-wise non-dominating set of cardinality $ln_2^{pnd}(G)$ is then referred to as a $ln_2^{pnd}(G)$-set in G.

Definition 4. [6] Let G be any nontrivial connected graph and $S \subseteq V(G)$. S is a $(2,2)$-*locating* ($(2,1)$-*locating*, respectively) *set* in G if S is 2-locating and $|N_G(y) \cap S| \leq |S| - 2$ ($|N_G(y) \cap S| \leq |S| - 1$, respectively), for all $y \in V(G)$. The $(2,2)$-*locating* ($(2,1)$-*locating*, respectively) *number* of G, denoted by $ln_{(2,2)}(G)$ ($ln_{(2,1)}(G)$, respectively), is the smallest cardinality of a $(2,2)$-locating ($(2,1)$-locating, respectively) set in G. A $(2,2)$-locating ($(2,1)$-locating, respectively) set in G of cardinality $ln_{(2,2)}(G)$ ($ln_{(2,1)}(G)$, respectively) is referred to as an $ln_{(2,2)}$-*set* ($ln_{(2,1)}$-*set*, respectively) in G.

Definition 5. [9] A (2,2)-locating ((2,1)-locating, respectively) set $S \subseteq V(G)$ which is a point-wise non-dominating is called a (2,2)-*locating point-wise non-dominating* ((2,1)-*locating point-wise non-dominating*, respectively) *set* in G. The minimum cardinality of a (2,2)-locating point-wise non-dominating ((2,1)-locating point-wise non-dominating, respectively) set in G, denoted by $ln_{(2,2)}^{pnd}(G)$ ($ln_{(2,1)}^{pnd}(G)$,respectively) is called the (2,2)-*locating point-wise non-domination* ((2,1)-*locating point-wise non-domination*) *number* of G. Any (2,2)-*locating point-wise non-dominating* ((2,1)-*locating point-wise non-dominating*, respectively) set of cardinality $ln_{(2,2)}^{pnd}(G)$ ($ln_{(2,1)}^{pnd}(G)$, respectively) is then referred to as a $ln_{(2,2)}^{pnd}$-set ($ln_{(2,1)}^{pnd}$-set) in G.

Definition 6. A set $S \subseteq V(G)$ is a 1-*movable* 2-*locating point-wise non-dominating set* in G if S is a 2-locating point-wise non-dominating set in G and for every $v \in S$, either $S\backslash\{v\}$ is a 2-locating point-wise non-dominating set or there exists a vertex $u \in \big((V(G)\backslash S) \cap N_G(v)\big)$ such that $\big(S\backslash\{v\}\big) \cup \{u\}$ is a 2-locating point-wise non-dominating set of G. The 1-movable 2-locating point-wise non-domination number of G, denoted by $mln_2^{pnd}(G)$ is the smallest cardinality of a 1-movable 2-locating point-wise non-dominating set of G. Any 1-movable 2-locating point-wise non-dominating set of cardinality $mln_2^{pnd}(G)$ is referred to as a mln_2^{pnd} -set of G.

Definition 7. A set $S \subseteq V(G)$ is a 1-*movable* $(2,2)$-*locating point-wise non-dominating* ($(2,1)$-*locating point-wise non-dominating, respectively*) in G if S is a $(2,2)$-locating point-wise non-dominating ($(2,1)$-*locating point-wise non-dominating, respectively*) set in G and for every $v \in S$, either $S\backslash\{v\}$ is a 2-locating point-wise non-dominating set or there exists a vertex $u \in \big((V(G)\backslash S) \cap N_G(v)\big)$ such that $\big(S\backslash\{v\}\big) \cup \{u\}$ is $(2,2)$-locating point-wise non-dominating ($(2,1)$-*locating point-wise non-dominating, respectively*) set in G. The 1-movable (2,2)-*locating point-wise non-domination* ((2,1)-*locating point-wise non-domination*) *number* of G, denoted by $mln^{pnd}_{(2,2)}(G)$ ($mln^{pnd}_{(2,1)}(G)$,respectively) is s the smallest cardinality of a 1-movable called the (2,2)-*locating point-wise non-dominating* ((2,1)-*locating point-wise non-dominating*) *number* set of G. Any 1-movable (2,2)-*locating point-wise non-dominating* ((2,1)-*locating point-wise non-dominating*, respectively) set of cardinality $mln^{pnd}_{(2,2)}(G)$ ($mln^{pnd}_{(2,1)}(G)$, respectively) is then referred to as a $mln^{pnd}_{(2,2)}$-set ($mln^{pnd}_{(2,1)}$-set) in G.

3. Preliminary Results

Remark 1. A 1-movable 2-resolving hop dominating set does not always exist in a graph G.

Example 1. A complete bipartite graph $K_{m,n}$ and a complete graph K_n do not admit 1-movable 2-resolving hop dominating set.

Remark 2. Let G be a nontrivial connected graph. If S is a 2-resolving set in G, then $\{x, y\} \subseteq S$ for every $x, y \in V(G)$ with $x \neq y$ and $d_G(x, z) = d_G(y, z)$ for each $z \in V(G)\backslash\{x, y\}$.

Proposition 1. Let G be a nontrivial connected graph. Then G admits a 1 -movable 2-resolving hop dominating set if and only if $\gamma(G) \neq 1$, $dim_2(G) \neq V(G)$ and G is a free-equidistant graph.

Proof. Suppose that G admits a 1-movable 2-resolving hop dominating set. Let S be a 1-movable 2-resolving hop dominating set of G. Suppose $\gamma(G) = 1$. Let $A = \{x \in V(G) : \{x\}$ is a dominating set of $G\}$. Then $A \neq \varnothing$ since $\gamma(G) = 1$. Since S is a hop dominating set, $A \subseteq S$. Let $x \in A$. Then $S \setminus \{x\}$ and $(S \setminus \{x\}) \cup \{y\}$ for each $y \in ((V(G) \setminus S) \cap N_G(x))$ are not hop dominating sets of G. Thus, S is not a 1-movable 2-resolving hop dominating set. Therefore, $\gamma(G) \neq 1$. If $dim_2(G) = V(G)$, then $V(G)\backslash\{z\}$ for each $z \in V(G)$ is not a 2-resolving set. Thus, S is not a 1-movable 2-resolving hop dominating set. Therefore, $dim_2(G) \neq V(G)$. If G is not a free-equidistant graph, then there exist a pair of vertices $y, w \in V(G)$ with $d_G(w, z) = d_G(y, z)$ for all $z \in V(G)\backslash\{y, w\}$. By Remark 2, $y, w \in S$. Hence, $r_G(y/(S\backslash\{y\}))$ and $r_G(w/(S\backslash\{y\}))$ differ in at most one position. Thus, S is not a 1-movable 2-resolving hop dominating set. Therefore, G is a free-equidistant graph.

Conversely, suppose that $\gamma(G) \neq 1$, $dim_2(G) \neq V(G)$ and G is a free-equidistant graph. Let $S = V(G)$. Then S is a 2-resolving hop dominating set in G. For each $x \in S$, $S \setminus \{x\}$ is a 2-resolving set in G since $dim_2(G) \neq V(G)$ and G is a free-equidistant graph. Also, since $\{x\}$ is not a dominating set, there exists $y \in (S \setminus \{x\}) \cap N_G(x, 2)$. Hence, $S \setminus \{x\}$ is a hop dominating set of G. Therefore, $S \setminus \{x\}$ is a 2-resolving hop dominating set in G for each $x \in S$. It follows that S is a 1-movable 2-resolving hop dominating set in G. Accordingly, G admits 1-movable 2-resolving hop dominating set. □

Remark 3. Every 1-movable 2-resolving hop dominating set in G is a 2-resolving hop dominating set in G. Thus, $\gamma_{2Rh}(G) \leq \gamma^1_{m2Rh}(G)$.

Proposition 2. (i) For a path P_n on n vertices $(n \geq 4)$,

$$\gamma^1_{m2Rh}(P_n) = n.$$

(ii) For a cycle C_n on n vertices,

$$\gamma^1_{m2Rh}(C_n) = \begin{cases} 3, & \text{if } n = 5; \\ \dfrac{2n+k}{3}, & \text{if } n = k(\text{mod } 3),\ 0 \leq k \leq 2 \text{ and } n > 5. \end{cases}$$

Proof. Let $P_n = [v_1, v_2, v_3, \dots v_n]$. By Proposition 1, $V(P_n)$ is a 1-movable 2-resolving hop dominating set. Suppose $S_i = V(P_n)\backslash\{v_{i_1}, v_{i_2}, \dots v_{i_k}\}$ is a 2-resolving hop dominating set in P_n for $1 \leq k \leq n-2$. Note that $0 < |N_{P_n}(v_{i_m}, 2)| \leq 2$ for each $m \in \{1, 2, \dots k\}$. If $|N_{P_n}(v_{i_m}, 2)| = 1$, then $S_i \setminus \{v_j\}$ is not a hop dominating set for $v_j \in N_{P_n}(v_{i_m}, 2)$. Suppose $|N_{P_n}(v_{i_m}, 2)| = 2$. Let $v_j, v_l \in N_{P_n}(v_{i_m}, 2)$. If $|N_{P_n}(v_j, 2)| = 1$ or $|N_{P_n}(v_l, 2)| = 1$, then $S_i\backslash\{v_j\}$ or $S_i\backslash\{v_l\}$ is not a hop dominating set in P_n. On the other hand, if $|N_{P_n}(v_j, 2)| = 2$ or $|N_{P_n}(v_l, 2)| = 2$, then $S_i \setminus \{v_p\}$ or $S_i \setminus \{v_q\}$ is not a hop dominating set where $v_p \in N_{P_n}(v_j, 2)$ and $v_q \in N_{P_n}(v_l, 2)$. Thus, S_i is not a 1-movable 2-resolving hop dominating set in P_n. Therefore, $\gamma^1_{m2Rh}(P_n) = n$.
(ii) Let $C_n = [v_1, v_2, \dots, v_n]$ and S be a γ^1_{m2Rh}- set of C_n. The case when $n = 5$ can be verified. Next, let $n > 5$ and $n \equiv k(\text{mod } 3)$ where $0 \leq k \leq 2$. Then $n = 3r + k$. Hence, $r = \dfrac{n-k}{3}$. Then the set

$$S = \{v_1, v_3, v_4, v_6, v_7, v_9, v_{10}, v_{12}, v_{13}, \dots, v_{3r+k-3}, v_{3r+k-2}, \dots, v_{3r+k}\}$$

is a γ^1_{m2Rh}- set of C_n. Therefore, $|S| = 3r + k - r = \dfrac{2n+k}{3}$. □

Now, consider the following results of 1-movable 2-locating point-wise non-dominating sets which are used in characterizing the 1-movable 2-resolving hop dominating sets in some binary operations.

Remark 4. A 1-movable 2-locating point-wise non-dominating set does not always exist in a graph G.

Example 2. A complete bipartite graph $K_{m,n}$ and a complete graph K_n do not admit 1-movable 2-locating point-wise non-dominating set.

Remark 5. Let G be a nontrivial connected graph. If S is a 2-locating set in G, then $\{x, y\} \subseteq S$ for every $x, y \in V(G)$ with $x \neq y$ and $N_G(x) = N_G(y)$.

Proposition 3. Let G be a nontrivial connected graph. Then G admits a 1-movable 2-locating set if and only if G is a point determining graph.

Proof. Let S be a 1-movable 2-locating set of G. Suppose G is not a point determining graph. Then there exist $x, y \in V(G)$ with $x \neq y$ and $N_G(x) = N_G(y)$. This implies that $x, y \in S$ by Remark 5. Thus, $S \setminus \{x\}$ and $(S \setminus \{x\}) \cup \{z\}$ are not 2-locating sets where $z \in (V(G) \setminus S) \cap N_G(x)$. Thus, G is a point determining graph.

Conversely, suppose G is a point determining graph. Let $S = V(G)$. Then S is a 2-locating set of G. Since G is a point determining graph, $S \setminus \{x\}$ is a 2-locating set for all $x \in S$. Therefore, G admits a 1-movable 2-locating set. □

Proposition 4. Let G be a nontrivial connected graph. Then G admits a 1-movable 2-locating point-wise non-dominating set if and only if G is a point determining graph where $\gamma(G) \neq 1$.

Proof. Let S be a 1-movable 2-locating point-wise non-dominating set of G. Suppose $\gamma(G) = 1$. Set $A = \{x \in V(G) : \{x\}$ is a dominating set of $G\}$. Then $A \neq \varnothing$ since $\gamma(G) = 1$. Since S is a point-wise non-dominating set, $A \subseteq S$. Let $x \in A$. Then $S \setminus \{x\}$ and $\big(S \setminus \{x\}\big) \cup \{y\}$ for each
$y \in V(G) \setminus S \cap N_G(x)$ are not point-wise non-dominating sets of G. Thus, S is not a 1-movable 2-locating point-wise non-dominating set. Therefore, $\gamma(G) \neq 1$. By Proposition 3, G is a point determining graph.

Conversely, suppose G is a point determining graph where $\gamma(G) \neq 1$. Then by Proposition 3, $S = V(G)$ is a 1-movable 2-locating set. Thus, S is a 1-movable 2-locating point-wise non-dominating set of G. Accordingly, G admits a 1-movable 2-locating point-wise non-dominating set. □

Proposition 5. Let G be a nontrivial connected graph of order $n \geq 4$. Then

(*i*)

$$mln_2^{pnd}(P_n) = \begin{cases} n, & \text{if } n = 4, 5; \\ \dfrac{2n+k}{3}, & \text{if } n = k(\text{mod } 3),\ 0 \leq k \leq 2 \text{ and } n > 5; \end{cases}$$

(*ii*)

$$\begin{aligned} mln_{(2,1)}^{pnd}(P_n) &= mln_{(2,2)}^{pnd}(P_n) \\ &= \begin{cases} n, & \text{if } n = 5; \\ \dfrac{2n+k}{3}, & \text{if } n = k(\text{mod } 3),\ 0 \leq k \leq 2 \text{ and } n > 5; \end{cases} \end{aligned}$$

(*iii*)

$$mln_2^{pnd}(C_n) = \begin{cases} 3, & \text{if } n = 5; \\ \dfrac{2n+k}{3}, & \text{if } n = k(\text{mod } 3),\ 0 \leq k \leq 2 \text{ and } n > 5; \end{cases}$$

(iv)

$$mln^{pnd}_{(2,1)}(C_n) = \begin{cases} 3, & \text{if } n = 5; \\ \dfrac{2n+k}{3}, & \text{if } n = k(\text{mod } 3),\ 0 \leq k \leq 2 \text{ and } n > 5; \end{cases}$$

(v)

$$mln^{pnd}_{(2,2)}(C_n) = \begin{cases} 5, & \text{if } n = 5; \\ \dfrac{2n+k}{3}, & \text{if } n = k(\text{mod } 3),\ 0 \leq k \leq 2 \text{ and } n > 5. \end{cases}$$

Proof. (i) Let $P_n = [v_1, v_2, \ldots, v_n]$ and S be an mln_2^{pnd}- set of P_n. The case where $n = 4, 5$ can be verified. Next, let $n > 5$ and $n \equiv k(\text{mod } 3)$ where $0 \leq k \leq 2$. Then $n = 3r + k$. Hence, $r = \dfrac{n-k}{3}$. Then the set

$$S = \{v_1, v_3, v_4, v_6, v_7, v_9, v_{10}, v_{12}, v_{13}, \ldots, v_{3r+k-3}, v_{3r+k-2}, \ldots, v_{3r+k}\}$$

is an mln_2^{pnd}- set of P_n. Therefore, $|S| = 3r + k - r = \dfrac{2n+k}{3}$.

The proofs of (ii), (iii), (iv) and (v) are similar to (i).

Remark 6. Let G be a nontrivial connected graph. Then G admits a 1-movable $(2,1)$-locating point-wise non-dominating set if and only if $\Delta(G) \leq |V(G)|| - 2$.

Remark 7. Let G be a nontrivial connected graph. Then G admits a 1-movable $(2,2)$-locating point-wise non-dominating set if and only if $\Delta(G) \leq |V(G)|| - 3$.

We now characterize the 1-movable 2-resolving hop dominating sets in some graphs under some binary operations.

4. Join of Graphs

As a consequence of Proposition 1 the next result follows.

Corollary 1. A graph G does not admit a 1-movable 2-resolving hop dominating set if and only if $G = K_1 + H$ for any nontrivial connected graph H.

Theorem 1. [9] Let G and H be nontrivial connected graphs with $\gamma(G) \neq 1$ and $\gamma(H) \neq 1$. A set $S \subseteq V(G+H)$ is a 2-resolving hop dominating set of $G + H$ if and only if $S = S_G \cup S_H$ where $S_G = V(G) \cap S$ and $S_H = V(H) \cap S$ are 2-locating point-wise non-dominating sets of G and H, respectively where S_G or S_H is a $(2,2)$-locating point-wise non-dominating set or S_G and S_H are $(2,1)$-locating point-wise non-dominating sets.

Theorem 2. Let G and H be nontrivial connected graphs with $\gamma(G) \neq 1$ and $\gamma(H) \neq 1$. A set $S \subseteq V(G+H)$ is a 1-movable 2-resolving hop dominating set of $G+H$ if and only if $S = S_G \cup S_H$ where $S_G = V(G) \cap S$ and $S_H = V(H) \cap S$ are 1-movable 2-locating point-wise non-dominating sets of G and H, respectively where S_G or S_H is a 1-movable $(2,2)$-locating point-wise non-dominating set or S_G and S_H are 1-movable $(2,1)$-locating point-wise non-dominating sets.

Proof. Suppose that $S \subseteq V(G+H)$ is a 1-movable 2-resolving hop dominating set of $G+H$. Since S is 2-resolving hop dominating set by Theorem 1, $S = S_G \cup S_H$ where S_G and S_H are 2-locating point-wise non-dominating sets of G and H, respectively where S_G or S_H is a $(2,2)$-locating (point-wise non-dominating) set or S_G and S_H are $(2,1)$-locating (point-wise non-dominating) sets. Now, let $p \in S_G$. Then $p \in S$. Thus, $S\backslash\{p\} = \big(S_G\backslash\{p\}\big) \cup S_H$ or $\big(S\backslash\{p\}\big) \cup \{z\} = \big[\big(S_G\backslash\{p\} \cup \{z\}\big)\big] \cup S_H$ for some $z \in N_G(p) \cap (V(G)\backslash S_G)$ or $(S\backslash\{p\}) \cup \{q\} = (S_G\backslash\{p\}) \cup (S_H \cup \{q\})$ for some $q \in V(H)\backslash S_H$ is a 2-resolving hop dominating set in $G+H$. Hence, by Theorem 1, $S_G\backslash\{p\}$ or $(S_G\backslash\{p\}) \cup \{z\}$ is a 2-locating point-wise non-dominating set of G. This shows that S_G is a 1-movable 2-locating point-wise non-dominating set of G. Similarly, S_H is a 1-movable 2-locating point-wise non-dominating set of H. Therefore, S_G and S_H are 1-movable 2-locating point-wise non-dominating sets of G and H, respectively where S_G or S_H is a 1-movable $(2,2)$-locating (point-wise non-dominating) set or S_G and S_H are 1-movable $(2,1)$-locating (point-wise non-dominating) sets.

Conversely, suppose that S_G and S_H satisfy the given conditions. Then by Theorem 1, $S = S_G \cup S_H$ is a 2-resolving hop dominating set in $G+H$. Let $p \in S$. If $p \in S_G$, then $S\backslash p = (S_G\backslash\{p\}) \cup S_H$ or $(S\backslash\{p\}) \cup \{w\} = \big[(S_G\backslash\{p\}) \cup \{w\}\big] \cup S_H$ for some $w \in N_G(p) \cap (V(G)\backslash S_G)$ is a 2-resolving hop dominating set in $G+H$. Similarly, suppose that $p \in S_H$. Then $S\backslash\{p\} = (S_H\backslash\{p\}) \cup S_G$ or $(S\backslash\{p\}) \cup \{w\} = \big[(S_H\backslash\{p\}) \cup \{w\}\big] \cup S_G$ for some $w \in N_H(p) \cap (V(H)\backslash S_H)$ is a 2-resolving hop dominating set in $G+H$. Therefore, S is a 1-movable 2-resolving hop dominating set in $G+H$. □

Corollary 2. Let G and H be nontrivial connected graphs with $\gamma(G) \neq 1$ and $\gamma(H) \neq 1$. Then

$$\gamma^1_{m2Rh}(G+H) = \min\{mln^{pnd}_{(2,2)}(G) + mln^{pnd}_{(2)}(H), mln^{pnd}_{(2)}(G) + mln^{pnd}_{(2,2)}(H), mln^{pnd}_{(2,1)}(G) + mln^{pnd}_{(2,1)}(H)\}.$$

5. Corona of Graphs

Theorem 3. [9] Let G and H be nontrivial connected graphs. A set $S \subseteq V(G \circ H)$ is a 2-resolving hop dominating set of $G \circ H$ if and only if

$$S = A \cup \left(\bigcup_{v \in V(G) \cap N_G(A)} S_v \right) \cup \left(\bigcup_{w \in V(G) \setminus N_G(A)} D_w \right)$$

where

(i) $A \subseteq V(G)$ such that for each $w \in V(G) \setminus A$, there exists $x \in A$ with $d_G(w, x) = 2$ or there exists $y \in V(G) \cap N_G(w)$ with $V(H^y) \cap S \neq \varnothing$;

(ii) $S_v \subseteq V(H^v)$ is a 2-locating set of H^v for all $v \in V(G) \cap N_G(A)$; and

(iii) $D_w \subseteq V(H^w)$ is a 2-locating point-wise non-dominating set of H^w for all $w \in V(G) \setminus N_G(A)$.

Theorem 4. Let G and H be nontrivial connected graphs. Then $S \subseteq V(G \circ H)$ is a 1-movable 2-resolving hop dominating set of $G \circ H$ if and only if $S \cap V(H^v) \neq \varnothing$ and

$$S = A \cup \left(\bigcup_{v \in V(G) \cap N_G(A)} S_v \right) \cup \left(\bigcup_{w \in V(G) \setminus N_G(A)} D_w \right)$$

where

(i) $A \subseteq V(G)$

(ii) $S_v \subseteq V(H^v)$ is a 1-movable 2-locating set of H^v for all $v \in V(G) \cap N_G(A)$.

(iii) $D_w \subseteq V(H^w)$ is a 1-movable 2-locating point-wise non-dominating set of H^w for all $w \in V(G) \setminus N_G(A)$.

Proof. Suppose that $S \subseteq V(G \circ H)$ is a 1-movable 2-resolving hop dominating set of $G \circ H$. Then S is a 2-resolving hop dominating set. Let $A = S \cap V(G)$ and $S_v = S \cap V(H^v)$ for all $v \in V(G) \cap N_G(A)$. By Theorem 3, S_v is a 2-locating set of H^v. Let $p \in S_v$. Since S is a 1-movable 2-resolving hop dominating set and $p \in S$, either $S \setminus \{p\}$ or $(S \setminus \{p\}) \cup \{q\}$ is a 2-resolving hop dominating set in $G \circ H$ for some $q \in (V(G \circ H) \setminus S) \cap N_{G \circ H}(p)$. Now, note that $S \setminus \{p\} = A \cup \big(S_v \setminus \{p\}\big)$ and $(S \setminus \{p\}) \cup \{q\} = A \cup \big((S_v \setminus \{p\}) \cup \{q\}\big)$ or $(S \setminus \{p\}) \cup \{q\} = \big(A \cup \{q\}\big) \cup (S_v \setminus \{p\})$. Hence, either $S_v \setminus \{p\}$ or $(S_v \setminus \{p\}) \cup \{q\}$ for some $q \in (V(H^v) \setminus S_v) \cap N_{H^v}(p)$ is a 2-locating set of H^v. Thus, S_v is a 1-movable 2-locating set of H^v. Finally, suppose $w \in V(G) \setminus N_G(A)$. Then by similar argument, D_w is a 1-movable 2-locating point-wise non-dominating set of H^w. Thus, (ii) follows.

Conversely, suppose that S is a set as described and satisfies the given conditions. Then by Theorem 3, S is a 2-resolving hop dominating set. Let $x \in S$ and let $v \in V(G)$

such that $x \in V(\langle v\rangle + H^v)$. If $x = v$, then $x \in A$. By Theorem 3, $S\backslash\{x\}$ or $\big(S\backslash\{x\}\big) \cup \{y\}$ for some $y \in \big(V(G \circ H)\backslash S\big) \cap N_{G\circ H}(x)$ is a 2-resolving hop dominating set. Next, suppose that $x \neq v$. Consider the following cases.

Case 1: $v \in V(G) \cap N_G(A)$

Then $x \in S_v$ and $S\backslash\{x\} = (S_v\backslash\{x\}) \cup \left(\bigcup_{u\in V(G)\backslash\{v\}} D_u\right) \cup A$ or $(S\backslash\{x\}) \cup \{y\}$ for some $y \in (V(G \circ H)\backslash S) \cap N_{G\circ H}(x)$ is a 2-resolving hop dominating set, by Theorem 3.

Case 2: $v \in V(G)\backslash N_G(A)$

Then $x \in D_v$ and $S\backslash\{x\} = (D_v\backslash\{x\}) \cup \left(\bigcup_{u\in V(G)\backslash\{v\}} S_u\right) \cup A$ or $(S\backslash\{x\}) \cup \{y\}$ for some $y \in (V(G \circ H)\backslash S) \cap N_{G\circ H}(x)$ is a 2-resolving hop dominating set, by Theorem 3.

Accordingly, S is a 1-movable 2-resolving hop dominating set in $G \circ H$. □

Corollary 3. Let G and H be nontrivial connected graphs where $|V(G)| = n$. Then

$$\gamma^1_{m2Rh}(G \circ H) \leq \min\{n \cdot mln_2^{pnd}(H), \gamma_t(G) + n \cdot mln_2(H)\}.$$

Proof. Let $S \subseteq V(G \circ H)$ be a 1-movable 2-resolving hop dominating set of $G \circ H$. Then $S \cap V(H^v) \neq \varnothing$ and $S \cap V(H^v)$ is a 1-movable 2-locating set for each $v \in V(G)$ and

$$S = A \cup \left(\bigcup_{v\in V(G)\cap N_G(A)} S_v\right) \cup \left(\bigcup_{w\in V(G)\backslash N_G(A)} D_w\right)$$

where $A \subseteq V(G)$ and S_v and D_w satisfy the given properties in Theorem 4. Consider the following cases for set A.

Case 1: $A = \varnothing$

Let $D_w = S \cap V(H^w)$ be an mln_2^{pnd}-set of H^w for each $w \in V(G)$. Thus, $S = \left(\bigcup_{v\in V(G)} D_w\right)$ s a 1-movable 2-resolving hop dominating set of $G \circ H$ by Theorem 4. Implying that,

$$\gamma^1_{m2Rh}(G \circ H) \leq |S| = |V(G)||D_w| \leq n \cdot (mln_2^{pnd}(H)).$$

Case 2: A is a γ_t-set of G

Let $N_G(A) = V(G)$. $S_v = S \cap V(H^v)$ be an mln_2-set of H^v for each $v \in V(G)$. Thus, $S = A \cup \left(\bigcup_{v\in V(G)} S_v\right)$ s a 1-movable 2-resolving hop dominating set of $G \circ H$ by Theorem 4. Implying that,

$$\gamma^1_{m2Rh}(G \circ H) \leq |S| = |A| + |V(G)||S_v| \leq \gamma_t(G) + n \cdot (mln_2(H)).$$

□

6. Edge Corona of Graphs

Theorem 5. Let $G \neq P_2$ and H be any nontrivial connected graphs. A set $C \subseteq V(G \diamond H)$ is a 2-resolving hop dominating set of $G \diamond H$ if and only if

$$C = A \cup \left(\bigcup_{uv \in E(G)} S_{uv} \right)$$

where

(i) $A \subseteq V(G)$;

(ii) $S_{uv} \subseteq V(H^{uv})$ is a 2-locating set of H^{uv} for all $uv \in E(G)$ or if uv is a pendant edge, then S_{uv} is a $(2,1)$-locating set of H^{uv} whenever $l(\langle\{u,v\}\rangle) \subseteq A$ and S_{uv} is a $(2,2)$-locating set of H^{uv} otherwise.

Theorem 6. Let G and H be any nontrivial connected graphs where $\gamma(G) \neq 1$ and $\Delta(H) \leq |V(H)| - 3$. A set $C \subseteq V(G \diamond H)$ is a 1-movable 2-resolving hop dominating set of $G \diamond H$ if and only if

$$C = A \cup \left(\bigcup_{uv \in E(G)} S_{uv} \right)$$

where

(i) $A \subseteq V(G)$;

(ii) $S_{uv} \subseteq V(H^{uv})$ is a 1-movable 2-locating set of H^{uv} for all $uv \in E(G)$ or if uv is a pendant edge, then S_{uv} is a 1-movable $(2,1)$-locating set of H^{uv} whenever $l(\langle\{u,v\}\rangle) \subseteq A$ and S_{uv} is a 1-movable $(2,2)$-locating set of H^{uv} otherwise.

Proof. Suppose that $C \subseteq V(G \diamond H)$ is a 1-movable 2-resolving hop dominating set of $G \diamond H$. Then C is a 2-resolving hop dominating set. Let $A = C \cap V(G)$ and $S_{uv} = C \cap V(H^{uv})$ for all $uv \in E(G)$. Then $C = A \cup \left(\bigcup_{uv \in E(G)} S_{uv} \right)$ where $A \subseteq V(G)$ and $S_{uv} \subseteq V(H^{uv})$ for all $uv \in E(G)$. By Theorem 5, S_{uv} is a 2-locating set of H^{uv} for all $uv \in E(G)$. Let $p \in S_{uv}$. Since C is a 1-movable 2-resolving hop dominating set and $p \in C$, either $C \backslash \{p\}$ or $(C \backslash \{p\}) \cup \{q\}$ is a 2-resolving hop dominating set of $G \diamond H$ for some $q \in (V(G \diamond H) \backslash C) \cap N_{G \diamond H}(p)$. Now, note that $C \backslash \{p\} = A \cup \big(S_{uv} \backslash \{p\}\big)$ and $(C \backslash \{p\}) \cup \{q\} = A \cup \big((S_{uv} \backslash \{p\}) \cup \{q\}\big)$ or $(C \backslash \{p\}) \cup \{q\} = \big(A \cup \{q\}\big) \cup (S_{uv} \backslash \{p\})$. Hence, either $S_{uv} \backslash \{p\}$ or $(S_{uv} \backslash \{p\}) \cup \{q\}$ for some $q \in (V(H^{uv}) \backslash S_{uv}) \cap N_{H^{uv}}(p)$ is a 2-locating set of H^{uv}. Thus, S_{uv} is a 1-movable 2-locating set of H^{uv}. Next, suppose that uv is a pendant edge and suppose u is an end-vertex where $u \in C$. Since $S_{uv} = C \cap V(H^{uv}) \subseteq C$ and C is a 1-movable 2-resolving set it follows by Theorem 5, S_{uv} is a 1- movable $(2,1)$-locating set of H^{uv} whenever $l(\langle\{u,v\}\rangle) \subseteq A$ and S_{uv} is a 1-movable $(2,2)$-locating set of

H^{uv} otherwise. Thus, (ii) holds.

Conversely, suppose that C is a set as described and satisfies the given conditions. By Theorem 5, C is 2-resolving hop dominating set of $G \diamond H$. Let $p \in C$. If $p \in S_{uv}$, then by assumption and Theorem 5, either $C\backslash\{p\} = A \cup \big(S_{uv}\backslash\{p\}\big)$ or $(C\backslash\{p\}) \cup \{q\} = A \cup \big((S_{uv}\backslash\{p\}) \cup \{q\}\big)$ is a 2-resolving hop dominating set of $G \diamond H$ for some $q \in (V(G \diamond H)\backslash C) \cap N_{G\diamond H}(p)$. Therefore, C is a 1-movable 2-resolving hop dominating set of $G \diamond H$. $\square$

Corollary 4. Let $\gamma(G) \neq 1$ and H a nontrivial connected graph with $|E(G)| = p$. Then the following statements hold.

(i) If G is a graph with no pendant edges, then $\gamma^1_{m2Rh}(G \diamond H) = p \cdot mln_2(H)$.

(ii) If G is a graph with $k \geq 1$ pendant edges, then

$$\begin{aligned}\gamma^1_{m2Rh}(G \diamond H) =& \min\big\{\big(p-k\big)mln_2(H) + k \cdot mln_{(2,1)}(H) + k,\\ &\big(p-k\big)mln_2(H) + k \cdot mln_{(2,2)}(H)\big\}\end{aligned}$$

and $\gamma^1_{m2Rh}(G \diamond H) = \big(p-k\big)mln_2(H) + k \cdot mln_{(2,2)}(H)$ whenever $mln_{(2,2)}(H) = mln_{(2,1)}(H)$.

7. Lexicographic Product of Graphs

Theorem 7. [9] Let G and H be nontrivial connected graphs. Then $W = \bigcup_{x\in S}[\{x\} \times T_x]$, where $S \subseteq V(G)$ and $T_x \subseteq V(H)$ for each $x \in S$, is a 2-resolving hop dominating set in $G[H]$ if and only if

(i) $S = V(G)$;

(ii) T_x is a 2-locating set in H for every $x \in V(G)$;

(iii) T_x or T_y is a $(2,1)$-locating set or one of T_x and T_y is a $(2,2)$-locating set in H whenever $x, y \in EQ_1(G)$;

(iv) T_x and T_y are $(2-locating)$ dominating sets in H or one of T_x and T_y is a 2-dominating set whenever $x, y \in EQ_2(G)$.

(v) T_x is a 2-locating point-wise non-dominating set in H for every $x \in S$ with $|N_G(x,2) \cap S| = 0$.

Theorem 8. Let G and H be nontrivial connected graphs with $\triangle(H) \leq |V(H)| - 3$. Then $W = \bigcup_{x\in S}[\{x\} \times T_x]$, where $S \subseteq V(G)$ and $T_x \subseteq V(H)$ for each $x \in S$, is a 1-movable 2-resolving hop dominating set in $G[H]$ if and only if

(i) $S = V(G)$;

(ii) T_x is a 1-movable 2-locating set of H for every $x \in V(G)$;

(*iii*) $T_x \backslash \{p\}$ or $T_x \backslash \{p\} \cup \{q\}$ is a 2-locating point-wise non-dominating set of H for every $x \in S$ with $|N_G(x, 2) \cap S| = 0$ and $p \in T_x$ and for some $q \in N_H(p)$.

(*iv*) $T_x \backslash \{p\}$ and T_y are $(2, 1)$-locating set or one of $T_x \backslash \{p\}$ and T_y is a $(2, 2)$-locating set of H whenever $x, y \in EQ_1(G)$ and for each $p \in T_x$;

(*v*) $T_x \backslash \{p\}$ or $T_x \backslash \{p\} \cup \{q\}$ or T_y is $(2-locating)$ dominating sets in H or one of $T_x \backslash \{p\}$ and T_y is a 2-dominating set whenever $x, y \in EQ_2(G)$ and for each $p \in T_x$ and for some $q \in N_H(p)$.

Proof. Suppose W is a 1-movable 2-resolving hop dominating set in $G[H]$. Then by Theorem 7, $S = V(G)$ and T_x is a 2-locating set of H for each $x \in V(G)$. Let $p \in T_x$. Then $(x, p) \in W$. Since W is a 1-movable 2-resolving hop dominating set, either

$$W \backslash \{(x, p)\} = \left(\bigcup_{v \in S \backslash \{x\}} (\{v\} \times T_v) \right) \cup [\{x\} \times (T_x \backslash \{p\})]$$

or

$$(W \backslash \{(x, p)\}) \cup \{(x, q)\} = \left(\bigcup_{z \in S \backslash \{x\}} (\{z\} \times T_z) \right) \cup [\{x\} \times (T_x \backslash \{p\} \cup \{q\})]$$

for some $q \in (V(H) \backslash T_x) \cap N_H(p)$ or

$$(W \backslash \{(x, p)\}) \cup \{(y, w)\} = \left(\bigcup_{a \in S \backslash \{(x,y)\}} (\{a\} \times T_a) \right) \cup [\{x\} \times (T_x \backslash \{p\})] \cup [\{y\} \times (T_y \backslash \{w\})]$$

for some $y \in V(G) \cap N_G(x)$ and $w \in V(H) \backslash T_y$ is a 2-resolving hop dominating set of $G[H]$.

By Theorem 7, $T_x \backslash \{p\}$ or $(T_x \backslash p) \cup \{q\}$ is a 2-locating set of H for each $p \in T_x$ and for some $q \in (V(H) \backslash T_x) \cap N_H(p)$. Hence, T_x is a 1-movable 2-locating set of H for each $x \in V(G)$ or $T_x \backslash \{p\}$ is 2-locating and (*ii*) holds.

If (*iii*) does not hold, then $W \backslash \{(x, p)\}$ and $(W \backslash \{(x, p)\} \cup \{(y, q)\})$ are not hop dominating sets of $G[H]$ for all $y \in N_G(x)$ and $q \in V(H) \backslash T_x$ or $x = y$ and $q \in N_H(p)$. This is a contradiction to W being a 1-movable 2-resolving hop dominating set of $G[H]$. Hence, (*iii*) holds.

To prove (*iv*), let x and y be adjacent vertices of G with $d_G(x, z) = d_G(y, z)$ for all $z \in V(G) \backslash \{x, y\}$. Let $p, w \in V(H), p \neq w$. Suppose (*iii*) does not hold. Then there exist $a \in V(H) \backslash (T_x \backslash \{p\})$ and $w \in V(H) \backslash T_y$ such that $N_H(a) \cap (T_x \backslash \{p\}) = T_x \backslash \{p\}$ and $N_H(w) \cap T_y = T_y$ for some adjacent vertices x and y of G and for some $p \in T_x$. Hence, both $W \backslash \{(x, p)\}$ and $(W \backslash \{(x, p)\}) \cup \{(y, w)\}$ are not 2-resolving sets, a contradiction. Thus, (*iv*) holds.

To prove (*v*), let $x, y \in V(G)$ where $d_G(x, y) = 2$ and $d_G(x, z) = d_G(y, z)$ for all $z \in V(G) \backslash \{x, y\}$. Let $p, w \in V(H), p \neq w$. Suppose one of $T_x \backslash \{p\}$ and T_y, say $T_x \backslash \{p\}$

is not a dominating set in H. Pick $p \in V(H)\backslash N_H[T_x]$ and let $w \in V(H)\backslash T_y$. Since $d_G[H]((x,a),(y,w)) = 2$, for all (y,w), it follows that $|N_H(b) \cap T_y| \geq 2$, that is, T_y is a 2-dominating set. Thus, (v) holds.

Conversely, suppose that W satisfies properties (i) to (v). By Theorem 7, W is a 2-resolving hop dominating set of $G[H]$. Let $x \in V(G)$ and $p \in T_x$. Then $(x,p) \in W$ and $W\backslash\{(x,p)\} = \left(\bigcup_{v \in S\backslash\{x\}} (\{v\} \times T_v)\right) \cup [\{x\} \times (T_x\backslash\{p\})]$ and

$$(W\backslash\{(x,p)\}) \cup \{(x,q)\} = \left(\bigcup_{z \in S\backslash\{x\}} (\{z\} \times T_z)\right) \cup [\{x\} \times (T_x\backslash\{p\} \cup \{q\})]$$

for some $q \in (V(H)\backslash T_x) \cap N_H(p)$ and

$$(W\backslash\{(x,p)\}) \cup \{(y,w)\} = \left(\bigcup_{a \in S\backslash\{(x,y)\}} (\{a\} \times T_a)\right) \cup [\{x\} \times (T_x\backslash\{p\})] \cup [\{y\} \times (T_y\backslash\{w\})]$$

for some $y \in V(G) \cap N_G(x)$ and $w \in V(H)\backslash T_y$.
By (i) to (v), for every $(x,p) \in W$ either $W\backslash\{(x,p)\}$ is a 2-resolving hop dominating set in $G[H]$ or there exists $(y,q) \in N_G[H]((x,p)) \cap (V(G[H])\backslash W)$ such that $(W\backslash\{(x,p)\}) \cup \{(y,q)\}$ is a 2-resolving hop dominating set in $G[H]$. Accordingly, W is a 1-movable 2-resolving hop dominating set in $G[H]$. □

Corollary 5. Let G and H be nontrivial connected graph with $\gamma(G) \neq 1$ and G is free-equidistant. Then $\gamma^1_{m2Rh}(G[H]) = |V(G)| \cdot mln_2(H)$.

Proof. Let $S = V(G)$ and let R_x be an mln_2-set of H for each $x \in S$. Since $\gamma(G) \neq 1$, $x \in N_G(S,2)$ for each $x \in S$. By Theorem 8, $W = \bigcup_{x \in S}[\{x\} \times R_x]$ is a 1-movable 2-resolving hop dominating set in $G[H]$. Thus,

$$\gamma^1_{m2Rh}(G[H]) \leq |W| = |V(G)||R_x| = |V(G)|mln_2(H).$$

If $W_0 = \bigcup_{x \in S}(\{x\} \times T)$ is a γ^1_{m2Rh} -set of $G[H]$, then $S_0 = V(G)$ and T_x is a 1-movable 2-locating set of H for each $x \in V(G)$ by Theorem 8. Hence,

$$\gamma^1_{m2Rh}(G[H]) = |W_0| = |V(G)||T_x| \geq |V(G)|mln_2(H).$$

Therefore, $\gamma^1_{m2Rh}(G[H]) = |V(G)| \cdot mln_2(H)$. □

8. Conclusion

1-movable 2-resolving hop domination, a variant of 2-resolving hop domination, has been introduced and studied for some graphs and graphs resulting from the join, corona and lexicographic product of two graphs. It is recommended that some bounds on the 1-movable 2-resolving hop domination be determined and that the parameter can be investigated further for graphs under other binary operations.

Acknowledgements

The authors would like to thank the Department of Science and Technology - Accelerated Science and Technology Human Resource Development Program (DOST-ASTHRDP)-Philippines, MSU-Iligan Institute of Technology.

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