EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
Vol. 16, No. 4, 2023, 2198-2207
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

Some Properties of Operations in the Collection of Intuitionistic Fuzzy Sets : A Novel Approach

Dwi Nur Yunianti[1,2,*], Noor Hidayat[1], Raden Sulaiman[2], Abdul Rouf Alghofari[1]

[1] *Department of Mathematics, Faculty of Mathematics and Natural Sciences, Brawijaya University, Malang, East Java, Indonesia*
[2] *Department of Mathematics, Faculty of Mathematics and Science, State University of Surabaya, Surabaya, East Java, Indonesia*

Abstract. In this paper, we introduce collection of intuitionistic fuzzy sets as a developing and expanding intuitionistic fuzzy theory. A collection of intuitionistic fuzzy sets is a set whose members of the universe set are intuitionistic fuzzy sets. We present intersection and union operation in the collection of of intuitionistic fuzzy sets and show that the operations hold commutative, assosiative, idempotent, and De Morgan's laws properties.

2020 Mathematics Subject Classifications: 03E72, 03G25

Key Words and Phrases: Intuitionistic fuzzy sets, Collection of intuitionistic fuzzy sets, Intersection, Union

1. Introduction

Zadeh [1] first introduced the concept of fuzzy sets which provides a solution to the weaknesses of classical set theory. The fuzzy set assigns a membership value from 0 to 1 to all elements of the considered universal set. Many researchers have done researches on fuzzy mathematics in [2], [3], [4], [5], [6], [7], [8]. Atanassov [9] proposed the idea of generalizing fuzzy sets and it called intuitionistic fuzzy sets. Intuitionistic fuzzy sets have membership and non-membership values respectively on interval [0,1] that assigned to all elements of the universal set. The developing about algebra structures on intuitionistic fuzzy sets have been explored by Ejegwa et al. [10], Macodi-Ringia and Petalcorin [11], Roh et al. [12]. Whereas, the properties of arithmetic, algebraic, model operators, and normalization of intuitionistic fuzzy sets have been researched by Ejegwa et al. [13]. Beside that, the concept of intuitionistic fuzzy sets can be applied to many fields such as medical diagnosis, pattern recognition, multi criteria decision making (see [14], [15], [16], [17], [18], [19], [20]).

*Corresponding author.
DOI: https://doi.org/10.29020/nybg.ejpam.v16i4.4939

Email addresses: dwinuryunianti@student.ub.ac.id (D. N. Yunianti), noorh@ub.ac.id (N. Hidayat), radensulaiman@unesa.ac.id (R. Sulaiman), abdul_rouf@ub.ac.id (A.R. Alghofari)

In real life, many contexts that we compare are collections of objects which already in a fixed state. Intuitionistic fuzzy sets can not be completely used to compared the contexts. So, new concept of intuitionistic fuzzy set must be defined whose the members of the universe set also have set members. In this paper, we introduce a new concept of intuitionistic fuzzy sets and we call it a collection of intuitionistic fuzzy sets. The philosophical background of the new concept is to develop and to expand the intuitionistic fuzzy theory. A collection of intuitionistic fuzzy sets is a set whose members of the universe set are intuitionistic fuzzy sets. Adopting definition of intuitionistic fuzzy set, collection of intuitionistic fuzzy sets also give membership and non-membership degree for every members of the universe set. This new concept is a theoretical basic for developing applications relate to the comparison of the set collection as the universe set. Hence, in this paper, we present definition about collection of intuitionistic fuzzy sets, and introduce basic operations in the collection of intuitionistic fuzzy sets. Those operations are intersection and union operation. Furthermore, we provide some properties that relate to those operations.

2. Preliminaries

In this section, we review definition of intuitionistic fuzzy sets and their relations and operations.

Definition 1. *[9] Let Y be a non-empty and universal set. An intuitionistic fuzzy set A of Y can be defined as:*

$$A = \{(x, \mu_A(x), v_A(x)) : x \in Y\}$$

where $\mu_A(x)$ represents the membership degree of x in A and $v_A(x)$ represents the non-membership degree of x in A, both satisfying:

$$\mu_A : Y \to [0, 1], \quad v_A : Y \to [0, 1]$$

Moreover, $\mu_A(x)$ is defined as the membership degree of x in A, and $v_A(x)$ is defined as the non-membership degree of x in A, where both of them are in the interval $[0, 1]$, and $0 \leq \mu_A(x) + v_A(x) \leq 1$.

Based on the membership and non-membership values, the hesitant degree of x in A is defined as:

$$\pi_A(x) = 1 - \mu_A(x) - v_A(x)$$

Next, we describe relations and operations between intuitionistic fuzzy sets based on Atanassov's [9] explanation.

Definition 2. *[9] Let A and B be intuitionistic fuzzy sets, respectively, of the universal set Y. The following relations and operations on intuitionistic fuzzy sets.*

1. *$A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $v_A(x) \geq v_B(x)$ for all $x \in Y$.*
2. *$A = B$ if and only if $\mu_A(x) = \mu_B(x)$ and $v_A(x) = v_B(x)$ for all $x \in Y$.*

3. $A^c = \{(x, v_A(x), \mu_A(x)) : x \in Y\}$. *So,* $\mu_{A^c}(x) = v_A(x)$ *and* $v_{A^c}(x) = \mu_A(x)$.
4. $A \cap B = \{(x, \mu_{A\cap B}(x), v_{A\cap B}(x)) : x \in Y\}$ *with* $\mu_{A\cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$ *and* $v_{A\cap B}(x) = \max\{v_A(x), v_B(x)\}$.
5. $A \cup B = \{(x, \mu_{A\cup B}(x), v_{A\cup B}(x)) : x \in Y\}$ *with* $\mu_{A\cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$ *and* $v_{A\cup B}(x) = \min\{v_A(x), v_B(x)\}$.

Example 1. *Let* Y *be the universe set where* $Y = \{a', b', c'\}$. *We have* A, B, *and* C *be three intuitionistic fuzzy sets where*

$$A = \{(a', 0.4, 0.6), (b', 0.7, 0.2), (c', 0.6, 0.1)\},$$

$$B = \{(a', 0.3, 0.7), (b', 0.2, 0.2), (c', 0.6, 0.0)\},$$

$$C = \{(a', 0.4, 0.6), (b', 0.7, 0.2), (c', 0.1, 0.1)\}.$$

We conclude that $A \cap B$ *is* $\{(a', 0.3, 0.7), (b', 0.2, 0.2), (c', 0.6, 0.1)\}$ *and*
$B \cup C$ *is* $\{(a', 0.4, 0.6), (b', 0.7, 0.2), (c', 0.6, 0.0)\}$.

Example 1 is an example of intuitionistic fuzzy sets whose elements of the universe set are not sets. In the next section, we present the definition of collection of intuitionistic fuzzy sets as a new concept of intuitionistic fuzzy sets.

3. Results

3.1. Collection Of Intuitionistic Fuzzy sets

By adopting definition of intuitionistic fuzzy sets and generalize the universe set, we give the definition of collection of intuitionistic fuzzy sets.

Definition 3. *Let* $X = \{A_i : i = 1, 2, \ldots, n\}$ *be the universe set and non empty set.* A_i *is intuitionistic fuzzy set on* Y. *A collection of intuitionistic fuzzy sets on* X *is written as*

$$A^* = \{(A_i, \mu_{A^*}(A_i), v_{A^*}(A_i)) : A_i \in X\},$$

where $\mu_{A^*} : X \to [0, 1]$ *is the membership function of* A^* *and* $v_{A^*} : X \to [0, 1]$ *is the non-membership function of* A^*. *Furthermore,* $\mu_{A^*}(A_i)$ *is the membership degree of* A_i *on* A^* *and* $v_{A^*}(A_i)$ *is the non-membership degree of* A_i *on* A^*, *where*

$$0 \leq \mu_{A^*}(A_i) + v_{A^*}(A_i) \leq 1.$$

Example 2. *A customer of three fast food restaurants* A_1, A_2, A_3 *want to give best recomendation restaurant among them based on price, taste, and product variations. If we give* $Y = \{c_1, c_2, c_3\}$ *where* c_1 *is price,* c_2 *is taste, and* c_3 *is product variations, and suppose that* A_1, A_2, A_3 *are intuitionistic fuzzy sets in* Y. *So,* A_1, A_2, A_3 *represented by*

$$A_1 = \{(c_1, 0.7, 0.2), (c_2, 0.8, 0), (c_3, 0.8, 0.1)\}$$

$$A_2 = \{(c_1, 0.2, 0.6), (c_2, 0.4, 0.5), (c_3, 0.3, 0.6)\}$$

$$A_3 = \{(c_1, 0.5, 0.5), (c_2, 0.6, 0.2), (c_3, 0.5, 0.4)\}$$

Based on the intuitionistic fuzzy sets, a customer gives best recommendation of these restaurants which represented by a collection of intuitionistic fuzzy sets. Let $X = \{A_1, A_2, A_3\}$ *be the universe set, the collection of intuitionistic fuzzy sets in* X *is*

$$A^* = \{(A_1, 0.8, 0.1), (A_2, 0.2, 0.6), (A_3, 0.4, 0.5)\}$$

Obviously, a customer shows that A_1 *is better recommendation fast food restaurant than* A_2 *and* A_3*.*

Clearly, every collection of intuitionistic fuzzy sets on X can be seen as intuitionistic fuzzy set on X where the cardinality of X is one.

Definition 4. *Let* $X = \{A_i : i = 1, 2, \ldots, n\}$ *be a non-empty universal set.* A_i *is an intuitionistic fuzzy set on* Y*. The hesitant degree of* A_i *on* A^* *is defined as*

$$\pi_{A^*}(A_i) = 1 - \mu_{A^*}(A_i) - v_{A^*}(A_i).$$

Example 3. *Based on Example 2, we have*

$$\pi_{A^*}(A_1) = 0.1, \quad \pi_{A^*}(A_2) = 0.2, \quad \pi_{A^*}(A_3) = 0.1.$$

In the next theorem, we discuss about value of hesitant degree on the interval $[0, 1]$.

Theorem 1. *The hesitant degree* $\pi_{A^*}(A_i)$ *is in the interval* $[0, 1]$ *for all* $A_i \in X$*.*

Proof. Because

$$\pi_{A^*}(A_i) = 1 - \mu_{A^*}(A_i) - v_{A^*}(A_i) = 1 - (\mu_{A^*}(A_i) + v_{A^*}(A_i))$$

and $0 \leq \mu_{A^*}(A_i) + v_{A^*}(A_i) \leq 1$. So, we have $-1 \leq -(\mu_{A^*}(A_i) + v_{A^*}(A_i)) \leq 0$. Then, $0 \leq 1 - (\mu_{A^*}(A_i) + v_{A^*}(A_i)) \leq 1$. Hence, $\pi_{A^*}(A_i) \in [0, 1]$

Definition 5. *Let* A^* *is collections of intuitionistic fuzzy sets on* X*, where*

$$A^* = \{(A_i, \mu_{A^*}(A_i), v_{A^*}(A_i)) : A_i \in X\}$$

Complement of A^* *is*

$$A^{*c} = \{(A_i, v_{A^*}(A_i), \mu_{A^*}(A_i)) : A_i \in X\}$$

More over, we can see that $\mu_{A^{*c}}(A_i) = v_{A^*}(A_i)$ and $v_{A^{*c}}(A_i) = \mu_{A^*}(A_i)$

Next, we present definition of intersection and union in the collections of intuitionistic fuzzy sets as basic operations for further development.

Definition 6. *Let A^* and B^* be collections of intuitionistic fuzzy sets on X respectively, where*

$$A^* = \{(A_i, \mu_{A^*}(A_i), v_{A^*}(A_i)) : A_i \in X\}$$

and

$$B^* = \{(A_i, \mu_{B^*}(A_i), v_{B^*}(A_i)) : A_i \in X\}.$$

The definition of intersection A^* and B^* is

$$A^* \cap B^* = \{(A_i, \mu_{A^* \cap B^*}(A_i), v_{A^* \cap B^*}(A_i)) : A_i \in X\}$$

with

$$\mu_{A^* \cap B^*}(A_i) = \min\{\mu_{A^*}(A_i), \mu_{B^*}(A_i)\}$$

and

$$v_{A^* \cap B^*}(A_i) = \max\{v_{A^*}(A_i), v_{B^*}(A_i)\}.$$

Example 4. *Let A^* and B^* be collections of intuitionistic fuzzy sets on $X = \{A_1, A_2, A_3, A_4\}$ respectively, where*

$$A^* = \{(A_1, 0.9, 0.1), (A_2, 0.1, 0.8), (A_3, 0.1, 0.5), (A_4, 0.1, 0.8)\},$$

and

$$B^* = \{(A_1, 0.5, 0.1), (A_2, 0.3, 0.6), (A_3, 0, 0.8), (A_4, 0.7, 0.1)\}.$$

The intersection of A^* and B^* is

$$A^* \cap B^* = \{(A_1, 0.5, 0.1), (A_2, 0.1, 0.8), (A_3, 0, 0.8), (A_4, 0.1, 0.8)\}.$$

Definition 7. *Let A^* and B^* be collections of intuitionistic fuzzy sets on X respectively, where*

$$A^* = \{(A_i, \mu_{A^*}(A_i), v_{A^*}(A_i)) : A_i \in X\},$$

and

$$B^* = \{(A_i, \mu_{B^*}(A_i), v_{B^*}(A_i)) : A_i \in X\}.$$

The union of A^* and B^* is

$$A^* \cup B^* = \{(A_i, \mu_{A^* \cup B^*}(A_i), v_{A^* \cup B^*}(A_i)) : A_i \in A^*\}$$

with

$$\mu_{A^* \cup B^*}(A_i) = \max\{\mu_{A^*}(A_i), \mu_{B^*}(A_i)\}$$

and

$$v_{A^* \cup B^*}(A_i) = \min\{v_{A^*}(A_i), v_{B^*}(A_i)\}.$$

Example 5. *Let A^* and B^* be collections of intuitionistic fuzzy sets on $X = \{A_1, A_2, A_3, A_4\}$ respectively, where*

$$A^* = \{(A_1, 0.9, 0.1), (A_2, 0.1, 0.8), (A_3, 0.1, 0.5), (A_4, 0.1, 0.8)\},$$

and

$$B^* = \{(A_1, 0.5, 0.1), (A_2, 0.3, 0.6), (A_3, 0, 0.8), (A_4, 0.7, 0.1)\}.$$

The union of A^* and B^* is

$$A^* \cup B^* = \{(A_1, 0.9, 0.1), (A_2, 0.3, 0.6), (A_3, 0.1, 0.5), (A_4, 0.7, 0.1)\}.$$

3.2. 3.2 Some Properties Of Intersection And Union In The Collection Of Intuitionistic Fuzzy Sets

In this section, we show that intersection and union operation hold commutative, assosiative, idempotent, and De Morgan's laws properties.

Theorem 2. *If $A^*, \varnothing$ are collections of intuitionistic fuzzy sets on X then $A^* \cap \varnothing = \varnothing$ and $A^* \cup \varnothing = A^*$.*

Proof. Let $\varnothing = \{(A_i, 0, 1) : A_i \in X\}$ so clearly that

$$A^* \cap \varnothing = \varnothing \quad \text{and} \quad A^* \cup \varnothing = A^*.$$

.

Theorem 3. *Let A^* and B^* be collections of intuitionistic fuzzy sets on X respectively. Then,*

(i) $A^* \cap B^* = B^* \cap A^*$

(ii) $A^* \cup B^* = B^* \cup A^*$

Proof.

(i) $A^* \cap B^* = \{(A_i, \mu_{A^* \cap B^*}(A_i), v_{A^* \cap B^*}(A_i)) : A_i \in X\}$
$= \{(A_i, \min\{\mu_{A^*}(A_i), \mu_{B^*}(A_i)\}, \max\{v_{A^*}(A_i), v_{B^*}(A_i)\}) : A_i \in X\}$
$= \{(A_i, \min\{\mu_{B^*}(A_i), \mu_{A^*}(A_i)\}, \max\{v_{B^*}(A_i), v_{A^*}(A_i)\}) : A_i \in X\}$
$= B^* \cap A^*$

(ii) $A^* \cup B^* = \{(A_i, \mu_{A^* \cup B^*}(A_i), v_{A^* \cup B^*}(A_i)) : A_i \in X\}$
$= \{(A_i, \max\{\mu_{A^*}(A_i), \mu_{B^*}(A_i)\}, \min\{v_{A^*}(A_i), v_{B^*}(A_i)\}) : A_i \in X\}$
$= \{(A_i, \max\{\mu_{B^*}(A_i), \mu_{A^*}(A_i)\}, \min\{v_{B^*}(A_i), v_{A^*}(A_i)\}) : A_i \in X\}$
$= B^* \cup A^*$

Based on that theorem, the intersection and union operations in the collections of intuitionistic fuzzy sets satisfy the commutative property.

Theorem 4. *If A^*, B^*, and C^* are collections of intuitionistic fuzzy sets on X respectively then:*

(i) $(A^* \cap B^*) \cap C^* = A^* \cap (B^* \cap C^*)$

(ii) $(A^* \cup B^*) \cup C^* = A^* \cup (B^* \cup C^*)$

Proof.

(i) $(A^* \cap B^*) \cap C^* = \{(A_i, \mu_{(A^* \cap B^*) \cap C^*}(A_i), v_{(A^* \cap B^*) \cap C^*}(A_i)) : A_i \in X\}$
$= \{(A_i, \min(\mu_{A^* \cap B^*}(A_i), \mu_{C^*}(A_i)), \max(v_{A^* \cap B^*}(A_i), v_{C^*}(A_i))) : A_i \in X\}$
$= \{(A_i, \min(\min(\mu_{A^*}(A_i), \mu_{B^*}(A_i)), \mu_{C^*}(A_i)), \max(\max(v_{A^*}(A_i), v_{B^*}(A_i)), v_{C^*}(A_i))) : A_i \in X\}$
$= \{(A_i, \min(\mu_{A^*}(A_i), \min(\mu_{B^*}(A_i), \mu_{C^*}(A_i))), \max(v_{A^*}(A_i), \max(v_{B^*}(A_i), v_{C^*}(A_i)))) : A_i \in X\}$
$= A^* \cap (B^* \cap C^*)$

(ii) $(A^* \cup B^*) \cup C^* = \{(A_i, \mu_{(A^* \cup B^*) \cup C^*}(A_i), v_{(A^* \cup B^*) \cup C^*}(A_i)) : A_i \in X\}$
$= \{(A_i, \max(\mu_{A^* \cup B^*}(A_i), \mu_{C^*}(A_i)), \min(v_{A^* \cup B^*}(A_i), v_{C^*}(A_i))) : A_i \in X\}$
$= \{(A_i, \max(\max(\mu_{A^*}(A_i), \mu_{B^*}(A_i)), \mu_{C^*}(A_i)), \min(\min(v_{A^*}(A_i), v_{B^*}(A_i)), v_{C^*}(A_i))) : A_i \in X\}$
$= \{(A_i, \max(\mu_{A^*}(A_i), \max(\mu_{B^*}(A_i), \mu_{C^*}(A_i))), \min(v_{A^*}(A_i), \min(v_{B^*}(A_i), v_{C^*}(A_i)))) : A_i \in X\}$
$= A^* \cup (B^* \cup C^*)$

Based on that theorem, the intersection and union operations in the collections of intuitionistic fuzzy sets satisfy the associative property.

Theorem 5. *If A^* is a collection of intuitionistic fuzzy sets on X, then:*

(i) $A^* \cap A^* = A^*$

(ii) $A^* \cup A^* = A^*$

Proof.

(i) $A^* \cap A^* = \{(A_i, \mu_{A^* \cap A^*}(A_i), v_{A^* \cap A^*}(A_i)) : A_i \in X\}$
$= \{(A_i, \min(\mu_{A^*}(A_i), \mu_{A^*}(A_i)), \max(v_{A^*}(A_i), v_{A^*}(A_i))) : A_i \in X\}$
$= \{(A_i, \mu_{A^*}(A_i), v_{A^*}(A_i)) : A_i \in X\}$
$= A^*$

(ii) $A^* \cup A^* = \{(A_i, \mu_{A^* \cup A^*}(A_i), v_{A^* \cup A^*}(A_i)) : A_i \in X\}$
$= \{(A_i, \max(\mu_{A^*}(A_i), \mu_{A^*}(A_i)), \min(v_{A^*}(A_i), v_{A^*}(A_i))) : A_i \in X\}$
$= \{(A_i, \mu_{A^*}(A_i), v_{A^*}(A_i)) : A_i \in X\}$
$= A^*$

Based on that theorem, the intersection and union operations in the collections of intuitionistic fuzzy sets satisfy the idempotent property.

Theorem 6. *If A^* and B^* are collections of intuitionistic fuzzy sets on X respectively, then:*

(i) $(A^* \cap B^*)^c = A^{*c} \cup B^{*c}$

(ii) $(A^* \cup B^*)^c = A^{*c} \cap B^{*c}$

Proof.

(i) $(A^* \cap B^*)^c = \{(A_i, v_{A^* \cap B^*}(A_i), \mu_{A^* \cap B^*}(A_i)) : A_i \in X\}$
$= \{(A_i, \max(v_{A^*}(A_i), v_{B^*}(A_i)), \min(\mu_{A^*}(A_i), \mu_{B^*}(A_i))) : A_i \in X\}$
$= \{(A_i, \max(\mu_{A^{*c}}(A_i), \mu_{B^{*c}}(A_i)), \min(v_{A^{*c}}(A_i), v_{B^{*c}}(A_i))) : A_i \in X\}$
$= A^{*c} \cup B^{*c}$

(ii) $(A^* \cup B^*)^c = \{(A_i, v_{A^* \cup B^*}(A_i), \mu_{A^* \cup B^*}(A_i)) : A_i \in X\}$
$= \{(A_i, \min(v_{A^*}(A_i), v_{B^*}(A_i)), \max(\mu_{A^*}(A_i), \mu_{B^*}(A_i))) : A_i \in X\}$
$= \{(A_i, \min(\mu_{A^{*c}}(A_i), \mu_{B^{*c}}(A_i)), \max(v_{A^{*c}}(A_i), v_{B^{*c}}(A_i))) : A_i \in X\}$
$= A^{*c} \cap B^{*c}$

Based on that theorem, the intersection and union operations in the collections of intuitionistic fuzzy sets satisfy De Morgan's law.

4. Conclusions

In this article, we give definition collection of intuitionistic fuzzy sets. Definition and some properties of intersection and union in the collection of intuitionistic fuzzy sets are given too. We can say the properties of set operations in the set theory such as commutative, assosiative, idempotent, and De Morgan's law are also hold too in the collection of intuitionistic fuzzy sets. Moreover, by using this concept, we can explore the others properties of relations and develop tools for applications related the comparison of object collection.

References

[1] L.A. Zadeh Fuzzy sets. *Information and control*, 8:338-353, 1965.

[2] J.G. Brown A note on fuzzy sets. *Information and control*, 18(1): 32-39, 1971.

[3] D. Dubois, H. Prade Fuzzy sets, probability and measurement. *European journal of operational research*, 40(2): 135-154, 1989.

[4] D. Dubois, H. Prade The three semantics of fuzzy sets. *Fuzzy sets and systems*, 90(2): 141-150, 1997.

[5] F.Y. Meng, J. Tang, H. Fujita. Consistency-based algorithms for decision-making with interval fuzzy preference relations. *IEEE transactions on fuzzy systems*, 27(10): 2052-2066, 2019.

[6] S.R. Damirchi-Darasi, M.F. Zarandi, I.B. Turksen, and M. Izadi. Type-2 fuzzy rule-based expert system for diagnosis of spinal cord disorders. *Scientia Iranica. Transaction E, industrial engineering*, 26(1):455-471, 2019.

[7] O. Uygun, S. Yalcin, A. Kiraz, and E. Furkan Erkan. A novel assessment approach to EFQM driven institutionalization using integrated fuzzy multi-criteria decision-making methods. *Scientia ironical*, 27(2): 880-892, 2020.

[8] A.I. Isah. Some algebraic structures of multi-fuzzy set. *Science world journal*, 15(1): 21-25, 2020.

[9] K.T. Atanassov Intuitionistic fuzzy sets. *In Intuitionistic fuzzy sets, Physica, Heidelberg*, 1-137, 1999.

[10] P. A.Ejegwa, J. T. Alabaa, S. Yakubu Two new algebraic properties defined over Intuitionistic fuzzy sets. *Int. J. Fuzzy Mathematical Archive*, 5: 75-78, 2014.

[11] A.P. Macodi-Ringia, G.C. Petalcorin. On Intuitionistic Fuzzy Hyper GR-ideals in Hyper GR-algebras. *European Journal of Pure and Applied Mathematics*, 13(2): 246–257, 2020.

[12] E.H. Roh, E. Yang, Y.B. Jun. Intuitionistic Fuzzy Ordered Subalgebras in Ordered BCI-algebras. *European Journal of Pure and Applied Mathematics*, 16(3): 1342–1358, 2023.

[13] P.A. Ejegwa, S.O. Akowe, P.M. Otene, J.M. Ikyule. An overview on intuitionistic fuzzy sets. *International journal of scientific and technology research*, 3(3): 142-145, 2014.

[14] E. Szmidt, J. Kacprzyk. Distances between Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 114(2000): 505-518, 2000.

[15] C.M. Hwang, M.S. Yang, W.L. Hung, M.G. Lee. A similarity measure of intuitionistic fuzzy sets based on the Sugeno integral with its application to pattern recognition. *Inf Sci*, 189:93–109, 2012.

[16] P. A. Ejegwa, A. J. Akubo, O. M. Joshua. Intuitionistic fuzzy set and its application in career determination via normalized Euclidean distance method. *European sci. Journal*, 10 (2014): 529-536, 2014.

[17] S. Singh, H. Garg. Distance measures between type-2 intuitionistic fuzzy sets and their application to multicriteria decision-making process. *Appl Intell*, 46(4):788–799, 2017.

[18] R.T. Ngan,M. Ali, L.H. Son. Equality of intuitionistic fuzzy sets:a new proximity measure and applications in medical diagnosis. *Appl Intell*, 48(2):499–525, 2018.

[19] J. Dhivya, B. Sridevi. A novel similarity measure between intuitionistic fuzzy sets based on the mid points of transformed triangular fuzzy numbers with applications to pattern recognition and medical diagnosis. *Applied Mathematics-A Journal of Chinese Universities*, 34(2):229–52, 2019.

[20] H. Garg, G. Kaur. Novel distance measures for cubic intuitionistic fuzzy sets and their applications to pattern recognitions and medical diagnosis. *Granular Computing*, 5(2):169–84, 2020.