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Almost strong $\theta(\Lambda, p)$-continuity for functions

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Abstract. Our main purpose is to introduce the concept of almost strongly $\theta(\Lambda, p)$-continuous functions. Moreover, some characterizations of almost strongly $\theta(\Lambda, p)$-continuous functions are considered.

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Key Words and Phrases: $\theta(\Lambda, p)$-open set, almost strongly $\theta(\Lambda, p)$-continuous function

1. Introduction

The notion of θ-continuous functions was introduced by Fomin [10]. Noiri [20] studied some properties of θ-continuous functions. Arya and Bhamini [1] introduced the notion of θ-semi-continuous functions. Noiri [22] investigated several characterizations of θ-semi-continuous functions. Moreover, Jafari and Noiri [15] obtained some properties of θ-semi-continuous functions. Di Maio and Noiri [18] introduced the concept of quasi-irresolute functions. It is shown in [8] that a function is quasi-irresolute if and only if it is θ-irresolute. Noiri [24] introduced and investigated the notion of θ-preirresolute functions. The notion of weakly β-irresolute functions has been defined and studied in [25]. These four classes of functions have properties similar to the class of θ-continuous functions. In 1980, Noiri [21] introduced the notion of strongly θ-continuous functions. Long et al. [17] studied some properties of strongly θ-continuous functions. In 1998, Jafari and Noiri [12] introduced and studied the concept of strongly θ-semi-continuous functions. Moreover, Jafari and Noiri [14] studied the notion of strongly sober θ-continuous functions. Noiri [23] introduced the concept of θ-precontinuous functions. In 2002, Noiri and Popa [27] introduced and investigated the notion of strongly θ-β-continuous functions. In 2005, Noiri and Popa [29] defined a new notion of strongly θ-M-continuous functions as functions from a set satisfying some minimal conditions into a set satisfying some minimal conditions. Noiri and Kang [26] introduced and studied the notion of almost strongly θ-continuous functions. Jafari and Noiri [16] investigated some properties of almost strongly θ-continuous functions. Beceren

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et al. [2] introduced and studied the concept of almost strongly θ-semi-continuous functions. Furthermore, Jafari and Noiri [13] investigated several characterizations of almost strongly θ-semi-continuous functions. Dube and Chauhan [9] introduced the notion of strongly closure semi-continuous functions which are equivalent to almost strongly θ-semi-continuous functions. These classes of functions have properties similar to the class of θ-continuous functions. Noiri and Popa [28] introduced and studied the notion of almost strongly θ-m-continuous functions as functions from a set satisfying some minimal conditions into a topological space. In [7], the present authors introduced and investigated the concept of almost (Λ, s)-continuous functions. The notions of (Λ, sp)-open sets, $s(\Lambda, sp)$-open sets, $p(\Lambda, sp)$-open sets, $\alpha(\Lambda, sp)$-open sets, $\beta(\Lambda, sp)$-open sets and $b(\Lambda, sp)$-open sets were studied in [4]. Viriyapong and Boonpok [31] investigated some characterizations of (Λ, sp)-continuous functions. Furthermore, several characterizations of pairwise almost M-continuous functions were established in [3]. In this paper, we introduce the concept of almost strongly $\theta(\Lambda, p)$-continuous functions. In particular, several characterizations of almost strongly $\theta(\Lambda, p)$-continuous functions are discussed.

2. Preliminaries

Throughout the present paper, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a topological space (X, τ), $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, represent the closure and the interior of A, respectively. A subset A of a topological space (X, τ) is said to be *preopen* [19] if $A \subseteq \mathrm{Int}(\mathrm{Cl}(A))$. The complement of a preopen set is called *preclosed*. The family of all preopen sets of a topological space (X, τ) is denoted by $PO(X, \tau)$. A subset $\Lambda_p(A)$ [11] is defined as follows: $\Lambda_p(A) = \cap\{U \mid A \subseteq U, U \in PO(X, \tau)\}$. A subset A of a topological space (X, τ) is called a Λ_p-*set* [6] (*pre-*Λ*-set* [11]) if $A = \Lambda_p(A)$. A subset A of a topological space (X, τ) is called (Λ, p)-*closed* [6] if $A = T \cap C$, where T is a Λ_p-set and C is a preclosed set. The complement of a (Λ, p)-closed set is called (Λ, p)-*open*. The family of all (Λ, p)-open (resp. (Λ, p)-closed) sets in a topological space (X, τ) is denoted by $\Lambda_pO(X, \tau)$ (resp. $\Lambda_pC(X, \tau)$). Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a (Λ, p)-*cluster point* [6] of A if $A \cap U \neq \emptyset$ for every (Λ, p)-open set U of X containing x. The set of all (Λ, p)-cluster points of A is called the (Λ, p)-*closure* [6] of A and is denoted by $A^{(\Lambda,p)}$. The union of all (Λ, p)-open sets of X contained in A is called the (Λ, p)-*interior* [6] of A and is denoted by $A_{(\Lambda,p)}$. The $\theta(\Lambda, p)$-*closure* [6] of A, $A^{\theta(\Lambda,p)}$, is defined as follows:

$$A^{\theta(\Lambda,p)} = \{x \in X \mid A \cap\ U^{(\Lambda,p)} \neq \emptyset \text{ for each } (\Lambda, p)\text{-open set } U \text{ containing } x\}.$$

A subset A of a topological space (X, τ) is called $\theta(\Lambda, p)$-*closed* [6] if $A = A^{\theta(\Lambda,p)}$. The complement of a $\theta(\Lambda, p)$-closed set is said to be $\theta(\Lambda, p)$-*open*. Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a $\theta(\Lambda, p)$-*interior point* [30] of A if $x \in U \subseteq U^{(\Lambda,p)} \subseteq A$ for some $U \in \Lambda_pO(X, \tau)$. The set of all $\theta(\Lambda, p)$-interior points of A is called the $\theta(\Lambda, p)$-*interior* [30] of A and is denoted by $A_{\theta(\Lambda,p)}$.

Lemma 1. [30] *For subsets A and B of a topological space (X, τ), the following properties hold:*

(1) $X - A^{\theta(\Lambda,p)} = [X - A]_{\theta(\Lambda,p)}$ and $X - A_{\theta(\Lambda,p)} = [X - A]^{\theta(\Lambda,p)}$.

(2) A is $\theta(\Lambda, p)$-open if and only if $A = A_{\theta(\Lambda,p)}$.

(3) $A \subseteq A^{(\Lambda,p)} \subseteq A^{\theta(\Lambda,p)}$ and $A_{\theta(\Lambda,p)} \subseteq A_{(\Lambda,p)} \subseteq A$.

(4) If $A \subseteq B$, then $A^{\theta(\Lambda,p)} \subseteq B^{\theta(\Lambda,p)}$ and $A_{\theta(\Lambda,p)} \subseteq B_{\theta(\Lambda,p)}$.

(5) If A is (Λ, p)-open, then $A^{(\Lambda,p)} = A^{\theta(\Lambda,p)}$.

A subset A of a topological space (X, τ) is said to be $s(\Lambda, p)$-*open* [6] (resp. $p(\Lambda, p)$-*open* [6], $\beta(\Lambda, p)$-*open* [6], $\alpha(\Lambda, p)$-*open* [32], $r(\Lambda, p)$-*open* [6]) if $A \subseteq [A_{(\Lambda,p)}]^{(\Lambda,p)}$ (resp. $A \subseteq [A^{(\Lambda,p)}]_{(\Lambda,p)}$, $A \subseteq [[A^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)}$, $A \subseteq [[A_{(\Lambda,p)}]^{(\Lambda,p)}]_{(\Lambda,p)}$, $A = [A^{(\Lambda,p)}]_{(\Lambda,p)}$). The family of all $s(\Lambda, p)$-open (resp. $p(\Lambda, p)$-open, $\beta(\Lambda, p)$-open, $\alpha(\Lambda, p)$-open, $r(\Lambda, p)$-open) sets in a topological space (X, τ) is denoted by $s(\Lambda, p)O(X, \tau)$ (resp. $p(\Lambda, p)O(X, \tau)$, $\beta(\Lambda, p)O(X, \tau)$, $\alpha(\Lambda, p)O(X, \tau)$, $r(\Lambda, p)O(X, \tau)$). The union of all $s(\Lambda, p)$-open (resp. $p(\Lambda, p)$-open, $\alpha(\Lambda, p)$-open) sets of X contained in A is called the $s(\Lambda, p)$-*interior* (resp. $p(\Lambda, p)$-*interior*, $\alpha(\Lambda, p)$-*interior*) of A and is denoted by $A_{s(\Lambda,p)}$ (resp. $A_{p(\Lambda,p)}$, $A_{\alpha(\Lambda,p)}$). The complement of a $s(\Lambda, p)$-open (resp. $p(\Lambda, p)$-open, $\beta(\Lambda, p)$-open, $\alpha(\Lambda, p)$-open, $r(\Lambda, p)$-open) set is called $s(\Lambda, p)$-*closed* (resp. $p(\Lambda, p)$-*closed*, $\beta(\Lambda, p)$-*closed*, $\alpha(\Lambda, p)$-*closed*, $r(\Lambda, p)$-*closed*). The family of all $s(\Lambda, p)$-closed (resp. $p(\Lambda, p)$-closed, $\beta(\Lambda, p)$-closed, $\alpha(\Lambda, p)$-closed, $r(\Lambda, p)$-closed) sets in a topological space (X, τ) is denoted by $s(\Lambda, p)C(X, \tau)$ (resp. $p(\Lambda, p)C(X, \tau)$, $\beta(\Lambda, p)C(X, \tau)$, $\alpha(\Lambda, p)C(X, \tau)$, $r(\Lambda, p)C(X, \tau)$). The intersection of all $s(\Lambda, p)$-closed (resp. $p(\Lambda, p)$-closed, $\alpha(\Lambda, p)$-closed) sets of X containing A is called the $s(\Lambda, p)$-*closure* (resp. $p(\Lambda, p)$-*closure*, $\alpha(\Lambda, p)$-*closure*) of A and is denoted by $A^{s(\Lambda,p)}$ (resp. $A^{p(\Lambda,p)}$, $A^{\alpha(\Lambda,p)}$). Let A be a subset of a topological space (X, τ). A point x of X is called a $\delta(\Lambda, p)$-*cluster point* [5] of A if $A \cap [V^{(\Lambda,p)}]_{(\Lambda,p)} \neq \emptyset$ for every (Λ, p)-open set V of X containing x. The set of all $\delta(\Lambda, p)$-cluster points of A is called the $\delta(\Lambda, p)$-*closure* [5] of A and is denoted by $A^{\delta(\Lambda,p)}$. If $A = A^{\delta(\Lambda,p)}$, then A is said to be $\delta(\Lambda, p)$-*closed* [5]. The complement of a $\delta(\Lambda, p)$-closed set is said to be $\delta(\Lambda, p)$-*open*. The union of all $\delta(\Lambda, p)$-open sets of X contained in A is called the $\delta(\Lambda, p)$-*interior* [5] of A and is denoted by $A_{\delta(\Lambda,p)}$.

3. On almost strongly $\theta(\Lambda, p)$-continuous functions

We begin this section by introducing the concept of almost strongly $\theta(\Lambda, p)$-continuous functions.

Definition 1. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be almost strongly $\theta(\Lambda, p)$-continuous functions at $x \in X$ if for each (Λ, p)-open set V of Y containing $f(x)$, there exists a (Λ, p)-open set U of X containing x such that $f(U^{(\Lambda,p)}) \subseteq V^{s(\Lambda,p)}$. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be almost strongly $\theta(\Lambda, p)$-continuous if f has the property at each point $x \in X$.*

Theorem 1. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is almost strongly $\theta(\Lambda, p)$-continuous;

(2) $f^{-1}(V)$ is $\theta(\Lambda, p)$-open in X for every $r(\Lambda, p)$-open set V of Y;

(3) $f^{-1}(K)$ is $\theta(\Lambda, p)$-closed in X for every $r(\Lambda, p)$-closed set K of Y;

(4) for each $x \in X$ and each $r(\Lambda, p)$-open set V of Y containing $f(x)$, there exists a (Λ, p)-open set U of X containing x such that $f(U^{(\Lambda,p)}) \subseteq V$;

(5) $f^{-1}(V)$ is $\theta(\Lambda, p)$-open in X for every $\delta(\Lambda, p)$-open set V of Y;

(6) $f^{-1}(K)$ is $\theta(\Lambda, p)$-closed in X for every $\delta(\Lambda, p)$-closed set K of Y;

(7) $f(A^{\theta(\Lambda,p)}) \subseteq [f(A)]^{\delta(\Lambda,p)}$ for every subset A of X;

(8) $[f^{-1}(B)]^{\theta(\Lambda,p)} \subseteq f^{-1}(B^{\delta(\Lambda,p)})$ for every subset B of Y;

(9) $f^{-1}(B_{\delta(\Lambda,p)}) \subseteq [f^{-1}(B)]_{\theta(\Lambda,p)}$ for every subset B of Y;

(10) $f^{-1}(V) \subseteq [f^{-1}(V^{s(\Lambda,p)})]_{\theta(\Lambda,p)}$ for every (Λ, p)-open set V of Y.

Proof. (1) $\Rightarrow$ (2): Let V be any $r(\Lambda, p)$-open set of Y and $x \in f^{-1}(V)$. Since f is almost strongly $\theta(\Lambda, p)$-continuous, there exists a (Λ, p)-open set U of X containing x such that $f(U^{(\Lambda,p)}) \subseteq V^{s(\Lambda,p)} = V$. Thus, $x \in U \subseteq U^{(\Lambda,p)} \subseteq f^{-1}(V)$ which implies that $x \in [f^{-1}(V)]_{\theta(\Lambda,p)}$. This shows that $f^{-1}(V) \subseteq [f^{-1}(V)]_{\theta(\Lambda,p)}$. By Lemma 1, $f^{-1}(V) = [f^{-1}(V)]_{\theta(\Lambda,p)}$ and hence $f^{-1}(V)$ is $\theta(\Lambda, p)$-open.

(2) $\Rightarrow$ (3): Let K be any $r(\Lambda, p)$-closed set of Y. By (2), we have

$$\begin{aligned} f^{-1}(K) &= X - f^{-1}(Y - K) \\ &= X - [f^{-1}(Y - K)]_{\theta(\Lambda,p)} \\ &= X - [X - f^{-1}(K)]_{\theta(\Lambda,p)} \\ &= [f^{-1}(K)]^{\theta(\Lambda,p)}. \end{aligned}$$

Thus, $f^{-1}(K)$ is $\theta(\Lambda, p)$-closed in X.

(3) $\Rightarrow$ (4): Let $x \in X$ and V be any $r(\Lambda, p)$-open set of Y containing $f(x)$. By (3), $X - f^{-1}(V) = f^{-1}(Y - V) = [f^{-1}(Y - V)]^{\theta(\Lambda,p)} = X - [f^{-1}(V)]_{\theta(\Lambda,p)}$. This implies that $f^{-1}(V) = [f^{-1}(V)]_{\theta(\Lambda,p)}$. Then, there exists a (Λ, p)-open set U of X containing x such that $U^{(\Lambda,p)} \subseteq f^{-1}(V)$; hence $f(U^{(\Lambda,p)}) \subseteq V$.

(4) $\Rightarrow$ (5): Let V be any $\delta(\Lambda, p)$-open set of Y and $x \in f^{-1}(V)$. There exists a $r(\Lambda, p)$-open set G of Y such that $f(x) \in G \subseteq V$. By (4), there exists a (Λ, p)-open set U of X containing x such that $f(U^{(\Lambda,p)}) \subseteq G$. Thus, $x \in U \subseteq U^{(\Lambda,p)} \subseteq f^{-1}(V)$ which implies that $x \in [f^{-1}(V)]_{\theta(\Lambda,p)}$. Therefore, $f^{-1}(V) \subseteq [f^{-1}(V)]_{\theta(\Lambda,p)}$ and hence $f^{-1}(V) = [f^{-1}(V)]_{\theta(\Lambda,p)}$, by Lemma 1, $f^{-1}(V)$ is $\theta(\Lambda, p)$-open.

(5) $\Rightarrow$ (6): Let K be any $\delta(\Lambda,p)$-closed set of Y. By (5), we have

$$\begin{aligned} f^{-1}(K) &= X - f^{-1}(Y-K) \\ &= X - [f^{-1}(Y-K)]_{\theta(\Lambda,p)} \\ &= [f^{-1}(K)]^{\theta(\Lambda,p)}. \end{aligned}$$

Thus, $f^{-1}(K) = [f^{-1}(K)]^{\theta(\Lambda,p)}$ and hence $f^{-1}(K)$ is $\theta(\Lambda,p)$-closed.

(6) $\Rightarrow$ (7): Let A be any subset of X. Since $[f(A)]^{\delta(\Lambda,p)}$ is $\delta(\Lambda,p)$-closed in Y, by (6) we have $f^{-1}([f(A)]^{\delta(\Lambda,p)}) = [f^{-1}([f(A)]^{\delta(\Lambda,p)})]^{\theta(\Lambda,p)}$. Let $x \notin f^{-1}([f(A)]^{\delta(\Lambda,p)})$. Then, there exists a (Λ,p)-open set U of X containing x such that $U^{(\Lambda,p)} \cap f^{-1}([f(A)]^{\delta(\Lambda,p)}) = \emptyset$. This implies that $U^{(\Lambda,p)} \cap A = \emptyset$. Thus, $x \notin A^{\theta(\Lambda,p)}$ and hence $f(A^{\theta(\Lambda,p)}) \subseteq [f(A)]^{\delta(\Lambda,p)}$.

(7) $\Rightarrow$ (8): Let B be any subset of Y. Then, by (7) we have $f([f^{-1}(B)]^{\theta(\Lambda,p)}) \subseteq B^{\delta(\Lambda,p)}$ and hence $[f^{-1}(B)]^{\theta(\Lambda,p)} \subseteq f^{-1}(B^{\delta(\Lambda,p)})$.

(8) $\Rightarrow$ (9): Let B be any subset of Y. Let $x \in f^{-1}(B_{\delta(\Lambda,p)})$. Then, $f(x) \in B_{\delta(\Lambda,p)}$ and $f(x) \notin Y - B_{\delta(\Lambda,p)} = [Y-B]^{\delta(\Lambda,p)}$. Therefore, $x \notin f^{-1}([Y-B]^{\delta(\Lambda,p)})$. By (8), $x \notin [f^{-1}(Y-B)]^{\theta(\Lambda,p)}$. There exists a (Λ,p)-open set U of X containing x such that

$$x \in U \subseteq U^{(\Lambda,p)} \subseteq f^{-1}(B).$$

Thus, $x \in [f^{-1}(B)]_{\theta(\Lambda,p)}$ and hence $f^{-1}(B_{\delta(\Lambda,p)}) \subseteq [f^{-1}(B)]_{\theta(\Lambda,p)}$.

(9) $\Rightarrow$ (10): Let V be any (Λ,p)-open set of Y. Then, we have

$$V \subseteq [V^{(\Lambda,p)}]_{(\Lambda,p)} \subseteq [V^{s(\Lambda,p)}]_{\delta(\Lambda,p)}$$

and by (9), $f^{-1}(V) \subseteq f^{-1}([V^{s(\Lambda,p)}]_{\delta(\Lambda,p)}) \subseteq [f^{-1}(V^{s(\Lambda,p)})]_{\theta(\Lambda,p)}$.

(10) $\Rightarrow$ (1): Let $x \in X$ and V be any (Λ,p)-open set of Y containing $f(x)$. Then, $x \in f^{-1}(V) \subseteq [f^{-1}(V^{s(\Lambda,p)})]_{\theta(\Lambda,p)}$. Then, there exists a (Λ,p)-open set U of X containing x such that $x \in U \subseteq U^{(\Lambda,p)} \subseteq f^{-1}(V^{s(\Lambda,p)})$ which implies that $f(U^{(\Lambda,p)}) \subseteq V^{s(\Lambda,p)}$. Thus, f is almost strongly $\theta(\Lambda,p)$-continuous.

Theorem 2. *For a function $f : (X,\tau) \to (Y,\sigma)$, the following properties are equivalent:*

(1) f is almost strongly $\theta(\Lambda,p)$-continuous;

(2) $[f^{-1}([K_{(\Lambda,p)}]^{(\Lambda,p)})]^{\theta(\Lambda,p)} \subseteq f^{-1}(K)$ for every (Λ,p)-closed set K of Y;

(3) $[f^{-1}([[B^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)})]^{\theta(\Lambda,p)} \subseteq f^{-1}(B^{(\Lambda,p)})$ for every subset B of Y;

(4) $f^{-1}(B_{(\Lambda,p)}) \subseteq [f^{-1}([[B_{(\Lambda,p)}]^{(\Lambda,p)}]_{(\Lambda,p)})]_{\theta(\Lambda,p)}$ for every subset B of Y.

Proof. (1) $\Rightarrow$ (2): Let K be any (Λ,p)-closed set of Y. Then, $Y-K$ is (Λ,p)-open in Y. Thus, by Theorem 1 and Lemma 1, we have

$$\begin{aligned} X - f^{-1}(K) &= f^{-1}(Y-K) \\ &\subseteq [f^{-1}([[Y-K]^{(\Lambda,p)}]_{(\Lambda,p)})]_{\theta(\Lambda,p)} \end{aligned}$$

$$= [X - f^{-1}([K_{(\Lambda,p)}]^{(\Lambda,p)})]_{\theta(\Lambda,p)}$$
$$= X - [f^{-1}([K_{(\Lambda,p)}]^{(\Lambda,p)})]^{\theta(\Lambda,p)}$$

and hence $[f^{-1}([K_{(\Lambda,p)}]^{(\Lambda,p)})]^{\theta(\Lambda,p)} \subseteq f^{-1}(K)$.

(2) ⇒ (3): Let B be any subset of Y. Then, $B^{(\Lambda,p)}$ is (Λ,p)-closed in Y and by (2), $[f^{-1}([[B^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)})]^{\theta(\Lambda,p)} \subseteq f^{-1}(B^{(\Lambda,p)})$.

(3) ⇒ (4): Let B be any subset of Y. Then, we have

$$\begin{aligned} f^{-1}(B_{(\Lambda,p)}) &= X - f^{-1}([Y-B]^{(\Lambda,p)}) \\ &\subseteq X - [f^{-1}([[[Y-B]^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)})]^{\theta(\Lambda,p)} \\ &= [f^{-1}([[B_{(\Lambda,p)}]^{(\Lambda,p)}]_{(\Lambda,p)})]_{\theta(\Lambda,p)} \end{aligned}$$

and hence $f^{-1}(B_{(\Lambda,p)}) \subseteq [f^{-1}([[B_{(\Lambda,p)}]^{(\Lambda,p)}]_{(\Lambda,p)})]_{\theta(\Lambda,p)}$.

(4) ⇒ (1): Let V be any $r(\Lambda,p)$-open set of Y. By (4), $f^{-1}(V) \subseteq [f^{-1}(V)]_{\theta(\Lambda,p)}$ and hence $f^{-1}(V) = [f^{-1}(V)]_{\theta(\Lambda,p)}$. Thus, $f^{-1}(V)$ is $\theta(\Lambda,p)$-open and by Theorem 1, f is almost strongly $\theta(\Lambda,p)$-continuous.

Theorem 3. *For a function $f : (X,\tau) \to (Y,\sigma)$, the following properties are equivalent:*

(1) f is almost strongly $\theta(\Lambda,p)$-continuous;

(2) $[f^{-1}(V)]^{\theta(\Lambda,p)} \subseteq f^{-1}(V^{(\Lambda,p)})$ for every $\beta(\Lambda,p)$-open set V of Y;

(3) $[f^{-1}(V)]^{\theta(\Lambda,p)} \subseteq f^{-1}(V^{(\Lambda,p)})$ for every $s(\Lambda,p)$-open set V of Y;

(4) $f^{-1}(V) \subseteq [f^{-1}([V^{(\Lambda,p)}]_{(\Lambda,p)})]_{\theta(\Lambda,p)}$ for every $p(\Lambda,p)$-open set V of Y.

Proof. (1) ⇒ (2): Let V be any $\beta(\Lambda,p)$-open set of Y. Then, $V^{(\Lambda,p)}$ is $r(\Lambda,p)$-closed. Since f is almost strongly $\theta(\Lambda,p)$-continuous, by Theorem 2 we have

$$[f^{-1}(V)]^{\theta(\Lambda,p)} \subseteq [f^{-1}([[V^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)})]^{\theta(\Lambda,p)} \subseteq f^{-1}(V^{(\Lambda,p)})$$

and hence $[f^{-1}(V)]^{\theta(\Lambda,p)} \subseteq f^{-1}(V^{(\Lambda,p)})$.

(2) ⇒ (3): This is obvious since $s(\Lambda,p)O(X,\tau) \subseteq \beta(\Lambda,p)O(X,\tau)$.

(3) ⇒ (4): Let V be any $p(\Lambda,p)$-open set of Y. Then, $Y - V$ is $p(\Lambda,p)$-closed in Y and hence $[[Y-V]_{(\Lambda,p)}]^{(\Lambda,p)} \subseteq Y - V$. Since $[[Y-V]_{(\Lambda,p)}]^{(\Lambda,p)}$ is $r(\Lambda,p)$-closed, we have $[[Y-V]_{(\Lambda,p)}]^{(\Lambda,p)}$ is $s(\Lambda,p)$-open in Y. Then by (3),

$$[f^{-1}([[Y-V]_{(\Lambda,p)}]^{(\Lambda,p)})]^{\theta(\Lambda,p)} \subseteq f^{-1}([[Y-V]_{(\Lambda,p)}]^{(\Lambda,p)}) \subseteq f^{-1}(Y-V).$$

Thus,

$$\begin{aligned} f^{-1}(V) &\subseteq X - [f^{-1}([[Y-V]_{(\Lambda,p)}]^{(\Lambda,p)})]^{\theta(\Lambda,p)} \\ &= X - [X - f^{-1}([V^{(\Lambda,p)}]_{(\Lambda,p)})]^{\theta(\Lambda,p)} \end{aligned}$$

$$= [f^{-1}([V^{(\Lambda,p)}]_{(\Lambda,p)})]_{\theta(\Lambda,p)}.$$

(4) ⇒ (1): Let V be any $r(\Lambda,p)$-open set of Y. Then, V is $p(\Lambda,p)$-open and

$$f^{-1}(V) \subseteq [f^{-1}([V^{(\Lambda,p)}]_{(\Lambda,p)})]_{\theta(\Lambda,p)} = [f^{-1}(V)]_{\theta(\Lambda,p)}.$$

Thus, $f^{-1}(V) = [f^{-1}(V)]_{\theta(\Lambda,p)}$ and by Lemma 1, $f^{-1}(V)$ is $\theta(\Lambda,p)$-open in X. It follows from Theorem 1 that f is almost strongly $\theta(\Lambda,p)$-continuous.

Lemma 2. *For a topological space (X,τ), the following properties hold:*

(1) $V^{\alpha(\Lambda,p)} = V^{(\Lambda,p)}$ for every $V \in \beta(\Lambda,p)O(X,\tau)$;

(2) $V^{p(\Lambda,p)} = V^{(\Lambda,p)}$ for every $V \in s(\Lambda,p)O(X,\tau)$;

(3) $V^{s(\Lambda,p)} = [V^{(\Lambda,p)}]_{(\Lambda,p)}$ for every $V \in p(\Lambda,p)O(X,\tau)$.

Corollary 1. *For a function $f : (X,\tau) \to (Y,\sigma)$, the following properties are equivalent:*

(1) f is almost strongly $\theta(\Lambda,p)$-continuous;

(2) $[f^{-1}(V)]^{\theta(\Lambda,p)} \subseteq f^{-1}(V^{\alpha(\Lambda,p)})$ for every $\beta(\Lambda,p)$-open set V of Y;

(3) $[f^{-1}(V)]^{\theta(\Lambda,p)} \subseteq f^{-1}(V^{p(\Lambda,p)})$ for every $s(\Lambda,p)$-open set V of Y;

(4) $f^{-1}(V) \subseteq [f^{-1}(V^{s(\Lambda,p)})]_{\theta(\Lambda,p)}$ for every $p(\Lambda,p)$-open set V of Y.

Theorem 4. *For a function $f : (X,\tau) \to (Y,\sigma)$, the following properties are equivalent:*

(1) f is almost strongly $\theta(\Lambda,p)$-continuous;

(2) $[f^{-1}([[B^{\delta(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)})]^{\theta(\Lambda,p)} \subseteq f^{-1}(B^{\delta(\Lambda,p)})$ for every subset B of Y;

(3) $[f^{-1}([[B^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)})]^{\theta(\Lambda,p)} \subseteq f^{-1}(B^{\delta(\Lambda,p)})$ for every subset B of Y;

(4) $[f^{-1}([[V^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)})]^{\theta(\Lambda,p)} \subseteq f^{-1}(V^{(\Lambda,p)})$ for every (Λ,p)-open set V of Y;

(5) $[f^{-1}([[V^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)})]^{\theta(\Lambda,p)} \subseteq f^{-1}(V^{(\Lambda,p)})$ for every $p(\Lambda,p)$-open set V of Y.

Proof. (1) ⇒ (2): Let B be any subset of Y. Then, $B^{\delta(\Lambda,p)}$ is (Λ,p)-closed in Y. By Theorem 2, $[f^{-1}([[B^{\delta(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)})]^{\theta(\Lambda,p)} \subseteq f^{-1}(B^{\delta(\Lambda,p)})$.

(2) ⇒ (3): This is obvious since $B^{(\Lambda,p)} \subseteq B^{\delta(\Lambda,p)}$ for every subset B of Y.

(3) ⇒ (4): This is obvious since $V^{(\Lambda,p)} = V^{\delta(\Lambda,p)}$ for every (Λ,p)-open set V of Y.

(4) ⇒ (5): Let V be any $p(\Lambda,p)$-open set of Y. Then, we have $V \subseteq [V^{(\Lambda,p)}]_{(\Lambda,p)}$ and $V^{(\Lambda,p)} = [[V^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)}$. Thus, by (4), $[f^{-1}([[V^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)})]^{\theta(\Lambda,p)} \subseteq f^{-1}(V^{(\Lambda,p)})$.

(5) ⇒ (1): Let K be any $r(\Lambda,p)$-closed set of Y. Then, we have $K_{(\Lambda,p)}$ is $p(\Lambda,p)$-open in Y and by (5),

$$[f^{-1}(K)]^{\theta(\Lambda,p)} = [f^{-1}([K_{(\Lambda,p)}]^{(\Lambda,p)})]^{\theta(\Lambda,p)}$$

$$= [f^{-1}([[[K_{(\Lambda,p)}]^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)})]^{\theta(\Lambda,p)}$$
$$\subseteq f^{-1}([K_{(\Lambda,p)}]^{(\Lambda,p)})$$
$$= f^{-1}(K).$$

Thus, $[f^{-1}(K)]^{\theta(\Lambda,p)} = f^{-1}(K)$ and by Lemma 1, $f^{-1}(K)$ is $\theta(\Lambda,p)$-closed in X. By Theorem 1, f is almost strongly $\theta(\Lambda,p)$-continuous.

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