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The Mohanad Transforms and Their Applications for Solving Systems of Differential Equations

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Abstract. In recent years, Mohanad transform, a mathematical approach, has drawn a lot of interest from researchers. It is useful for solving many engineering and scientific problems, such as those involving electric circuits, population growth, vibrational beams, and heat conduction. The Mohanad transform is defined and introduced in this study, along with its fundamental qualities, including linearity and convolution. It is also discussed in connection with other integral transforms and how it is used in derivatives. Additionally, we use the Mohanad transform to solve a few systems of ordinary differential equations (ODEs) and review its properties in this paper. Determining the concentration of a chemical reactant (material) in a series is a physical chemistry problem that we use in the application part. We achieve this by developing a model based on ordinary differential equations (ODEs) and then solving them using the Mohanad transform. This research proves that, with little computational effort, we can get the exact solutions of ordinary differential equations (ODEs) via the Mohanad transform. We used graphs and tables to show our answer.

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Key Words and Phrases: Mohanad transform, Laplace transform, Convolution, System of differential equations

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1. Introduction

In actuality, mathematics is a universal language and a vital resource for comprehending the world we live in [57]. Particularly, differential equations are essential to many areas of mathematics and its applications [35]. They give us the ability to explain the connections between variables and offer mathematical models that help us comprehend scientific, engineering, natural, and economic events [19]. In physics, differential equations are frequently used to describe the motion of objects and the changes in dynamic systems [16]. They have the ability to foresee and analyze the behavior of a wide range of physical systems, including celestial planets and subatomic particles. In engineering, differential equations are utilized to solve problems related to dynamics, structural analysis, and design. They provide the mathematical framework required to erect frameworks, optimize processes, and ensure system stability [25, 44].
Furthermore, industrial design, control systems, and electrical engineering all heavily rely on differential equations [46, 59]. They are necessary for industrial processes, control systems, and electrical circuit analysis and design. Engineers can maximize the efficiency and stability of these systems by creating differential equations that characterize their behavior. Differential equations are fundamental to many branches of science and technology [30, 46, 54]. It is used for expressing relationships, creating models, and resolving issues in physics, engineering, economics, the natural sciences and many others fields, they offer a potent mathematical tool for handling these issues and its explanation [26].
By converting functions from one domain to another, integral transformations are strong mathematical tools that help us in solving problems more easily and creatively in variety of domains [20]. For example, Fourier transform [21]. is a transformation that breaks down a function into its frequency components, allowing for analysis in the frequency domain. It was solved by many researchers to get the solutions of many differential equations [17, 55] The Laplace transform [18] is another significant integral transformation that transforms a function of time into a function of complex frequency, facilitating the solution of differential equations and system analysis. It has a great application in the fields of science and engineering [33, 56].
The Sumudu transform [37, 58] offered an alternative to the conventional Fourier transform and attracted attention for its exceptional capacity to accurately capture transient signals and non-stationary phenomena [14]. Numerous domains, such as pattern recognition, biomedical signal processing, image de-noising, and others wares the motivation for providing many integral transforms later, for instances: ARA integral transform [49], Aboodh transform [1], and Formable transform [52]. Moreover, double integral transforms have been effective in solving differential equations such as: double Laplace transform [22], double Laplace ARA [42, 53], double formable [51] double ARA [45] and others [7, 17, 48]. They are essential in many scientific and technological fields, improving our comprehension and facilitating effective problem-solving [10, 23, 38–40].
Developed by Mohand [37], M.T. is a revolutionary mathematical technique that has attracted a lot of attention recently. This transform provides a strong tool for signal processing and analysis. Applications including data encryption, audio compression, and image

processing benefit greatly from the transform. Signals can be transformed mathematically from the time domain to the frequency domain and back again using the Mohandas approach. By analyzing signals and comprehending their many properties, including the frequencies contained in the signal and the energy contained in each frequency, this transformation is used [2]. The Laplace transform is the foundation of the M.T. approach, however it is enhanced and modified in some ways [3]. When handling non-infinite signals or signals with restricted frequency, M.T. performs better [5, 6, 8]. A wide range of problems involving differential equations, partial differential equations, integral equations, and population development and decay are among the many uses of M.T. [24, 43]. It is also used to solve linear Volterra integral equations [4, 50] of the second sort.

The novelty in this research lies in the utilization of the Mohanad transform, a relatively recent mathematical approach, as a powerful tool for addressing a broad spectrum of scientific and engineering problems. The distinctive aspects of this study can be highlighted as follows:

The study contributes to the evolving interest in the Mohanad transform, showcasing its versatility across diverse domains such as electric circuits, population growth, vibrational beams, and heat conduction. By exploring its application in varied fields, the research expands the understanding of where and how this transform can be effectively employed.

Comprehensive Analysis: This work not only introduces the Mohanad transform but also delves into its foundational properties—highlighting linearity, convolution, and its connections with other integral transforms. The comprehensive analysis provides a robust understanding of the transform's capabilities and its relation to established mathematical tools.

Solver for ODEs: The emphasis on solving systems of ordinary differential equations (ODEs) using the Mohanad transform is a key contribution. Demonstrating its efficacy in solving these equations with minimal computational burden underscores its potential as an efficient solver for complex mathematical problems arising in various scientific and engineering disciplines.

Cross-disciplinary Application: The application of the Mohanad transform to determine chemical reactant concentrations in a series reaction bridges the gap between mathematics and physical chemistry. This cross-disciplinary application showcases the transform's ability to address real-world chemical problems, demonstrating its practical utility beyond theoretical constructs.

Efficiency and Precision: The research underscores the transformative power of the Mohanad transform by showcasing its ability to yield exact solutions to ODEs without requiring extensive computational efforts. The use of graphs and tables to present these solutions further emphasizes the precision and ease of interpretation of the results obtained.

In essence, the novelty of this research lies in the comprehensive exploration and practical application of the Mohanad transform across various scientific and engineering realms, showcasing its efficiency, precision, and applicability in solving real-world problems.

Additionally; we go over how to solve systems of ODEs with the suggested transform by a simple algorithm, we discuss a physical chemistry problem model and solve it using

M.T. the obtained results are used to examine the concentration of chemical reactants in a series of chemical reactions of reactants in a series of reactions. To obtain the numerical findings and create the figures, and create the figures, we utilize the Python software package. The novelty of this work is obvious in the new algorithm presented to solve systems of ODEs[14, 28, 47], M.T. is presented for the first time to solve these equations. The proposed method shows its simplicity and applicability to solve problems in physical chemistry[9, 15].

The utilization of the Mohanad transform (M.T) in solving a chemical application, specifically in determining chemical reactant concentrations within a series reaction, can be attributed to several reasons:

Complexity of Chemical Reactions: Chemical reactions, especially in series, can involve intricate kinetics and complex equations describing the change in concentrations over time. Oftentimes, these reactions lead to systems of ordinary differential equations (ODEs) that are challenging to solve directly. The M.T, known for its efficacy in solving ODEs, provides an alternative and efficient method to address these complexities.

Mathematical Modeling of Chemical Systems: When trying to understand chemical reactions, it's common to model them using differential equations. These equations describe how the concentrations of reactants change over time. By utilizing the M.T, researchers can translate these models into solvable equations, enabling a deeper understanding of the reaction kinetics and concentrations involved.

Accuracy and Efficiency: The M.T offers an advantage in providing exact solutions to ODEs without necessitating extensive computational efforts. Its application streamlines the process of solving these equations, making it an appealing choice when dealing with chemical systems, where precise solutions are crucial.

Visualization and Interpretation: Presenting the solutions in tables and graphs, as mentioned in the study, is beneficial for visualizing and interpreting the concentration changes of reactants over time. This graphical representation aids in comprehending the behavior of the chemical system and allows for clearer communication of results.

Broader Applicability: The versatility of the M.T extends beyond specific scientific fields. Its application in solving ODEs allows for a cross-disciplinary approach, enabling researchers from various domains, such as mathematics, physics, engineering, and chemistry, to collaborate and leverage this mathematical tool for problem-solving.

The choice of utilizing the M.T to solve a chemical application involving the determination of chemical reactant concentrations in a series reaction showcases its ability to simplify complex systems, offer accurate solutions, and facilitate a deeper understanding of chemical kinetics through mathematical modeling and analysis.

The structure of this article is as follows: The structure of this article is as follows: the primary definitions and characteristics of M.T. are presented in Section 2. In Section 3, the applications of M.T. to solve ODEs. Section 4 presents a chemical application of system of ODEs that is solved by the proposed method, finally, we get the conclusion in Section 5.

2. Basics about M.T.

In this section, we present the definition of M.T. [1, 49], and some basic properties and relations to other integral transforms. For more details about M.T..

Definition 1. *The Mohanad transform (M.T.) is defined by the integral equation:*

$$M\{\varphi(\xi)\} = \zeta^2 \int_0^\infty e^{-\zeta\xi}\varphi(\xi)d\xi = \Phi(\zeta). \tag{1}$$

Definition 2. *The inverse M.T. is given by:*

$$\varphi(\xi) = M^{-1}\{\Phi(\zeta)\} = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \frac{1}{\zeta^2} e^{\zeta\xi}\Phi(\zeta)d\zeta, \quad c \in R, \tag{2}$$

where M^{-1} is the inverse operator of M.T..

2.1. Properties of M.T.

Some properties of M.T. are presented in this section.

(i) The linearity property of M.T.

If $M\left\{\varphi_1(\xi)\right\} = \Phi_1(\zeta)$ and $M\left\{\varphi_2(\xi)\right\} = \Phi_2(\zeta)$ then $M\left\{q\varphi_1(\xi) + p\varphi_2(\xi)\right\} = q\Phi_1(\zeta) + p\Phi_2(\zeta)$, where q and p are constants.

Proof. By the definition 1 of M.T., we obtain

$$\begin{aligned} M\left\{\mathrm{q}\varphi_1(\xi) + p\varphi_2(\xi)\right\} &= \zeta^2 \int_0^\infty e^{-\zeta\xi}\left[q\varphi_2(\xi) + p\varphi_2(\xi)\right]\mathrm{d}\xi \qquad (3)\\ &= \zeta^2 \int_0^\infty e^{-\zeta\xi} q\varphi_1(\xi)\mathrm{d}\xi + \zeta^2 \int_0^\infty e^{-\zeta\xi} p\varphi_2(\xi)\mathrm{d}\xi \\ &= q\zeta^2 \int_0^\infty e^{-\zeta\xi}\varphi_1(\xi)\mathrm{d}\xi + p\zeta^2 \int_0^\infty e^{-\zeta\xi}\varphi_2(\xi)\mathrm{d}\xi \\ &= q\Phi_1(\zeta) + p\Phi_2(\zeta). \qquad (4) \end{aligned}$$

Moreover, the inverse M.T. is linear;

$$\text{If} \quad M^{-1}\left\{\Phi_1(\zeta)\right\} = \varphi_1(\xi) \quad \text{and} \quad M^{-1}\left\{\Phi_2(\zeta)\right\} = \varphi_2(\xi). \tag{5}$$

Then,

$$\begin{aligned} M^{-1}\left\{q\Phi_1(\zeta) + p\Phi_2(\zeta)\right\} &= qM^{-1}\left\{\Phi_1(\zeta)\right\} + pM^{-1}\left\{\Phi_1(\zeta)\right\} \\ &= q\varphi_1(\xi) + p\varphi_2(\xi) \qquad (6) \end{aligned}$$

(ii) Change of scale property

If M.T. of a function $\varphi(\xi)$ is $\Phi(\zeta)$, then M.T. of the function $\varphi(a\xi)$ is given by $a\Phi\left(\frac{\zeta}{a}\right)$, where a is non zero constant.

Proof. By the definition 1 of M.T., we get

$$M\{\varphi(a\xi)\} = \zeta^2 \int_0^\infty e^{-\zeta\xi}\varphi(a\xi)d\xi. \tag{7}$$

Letting $a\xi = u$, then $ad\xi = du$, then we substitute in equation (7).

$$M\{\varphi(a\xi)\} = \zeta^2 \int_0^\infty e^{-\left(\frac{\zeta}{a}\right)u}\varphi(u)\frac{du}{a} = a\left[\frac{\zeta^2}{a^2}\int_0^\infty e^{-\left(\frac{\zeta}{a}\right)u}\varphi(u)du\right] = a\Phi\left(\frac{\zeta}{a}\right). \tag{8}$$

(iii) Shifting property of M.T.

If M.T. of a function $\varphi(\xi)$ is $\Phi(\zeta)$, then M.T. of the function $e^{k\xi}\varphi(\xi)$ is given by

$$\frac{\zeta^2}{(\zeta-k)}\Phi(\zeta-k), \quad \text{where} \quad k \in R.$$

Proof. By the definition 1 of M.T., we get

$$\begin{aligned} M\left\{e^{k\xi}\varphi(\xi)\right\} &= \zeta^2\int_0^\infty e^{-\zeta\xi}e^{k\xi}\varphi(\xi)d\xi = \zeta^2\int_0^\infty e^{-(\zeta-k)\xi}\varphi(\xi)d\xi \\ &= \frac{(\zeta-K)^2}{(\zeta-K)^2}\zeta^2\int_0^\infty e^{-(\zeta-k)\xi}\varphi(\xi)d\xi = \frac{\zeta^2}{(\zeta-K)}\Phi(\zeta-k). \end{aligned} \tag{9}$$

Now, we introduce the M.T. of some basic functions in Table 1.

2.2. Relations to other integral transforms

In this section, we discuss the duality between M.T. and some other famous transforms.

- Laplace transform

If $L\{\varphi(\xi)\} = \int_0^\infty e^{-\zeta\xi}\varphi(\xi)d\xi$ is the Laplace transform of $\varphi(\xi)$, then

$$M\{\varphi(\xi)\} = \zeta^2 L\{\varphi(\xi)\}.$$

Proof.

$$M\{\varphi(\xi)\} = \zeta^2\left[\int_0^\infty e^{-\zeta\xi}\varphi(\xi)d\xi\right] = \zeta^2 L\{\varphi(\xi)\}. \tag{10}$$

Table 1: M.T. of some elementary functions.

Functions $\varphi(\xi)$	$M\{\varphi(\xi)\} = \Phi(\zeta)$
1	ζ
ξ	1
ξ^2	$\frac{2!}{\zeta}$
$\xi^\alpha, \alpha > 0$	$\frac{\Gamma(\alpha+1)}{\zeta^{\alpha-1}}$
$e^{\alpha\xi}$	$\frac{\zeta^2}{\zeta-\alpha}$
$\sin(\alpha\xi)$	$\frac{\alpha\zeta^2}{\zeta^2+\alpha^2}$
$\cos(\alpha\xi)$	$\frac{\zeta^3}{\zeta^2+\alpha^2}$
$\sinh(\alpha\xi)$	$\frac{\alpha\zeta^2}{\zeta^2-\alpha^2}$
$\cosh(\alpha\xi)$	$\frac{\zeta^3}{\zeta^2-\alpha^2}$

- Sumudu transform

If $S\{\varphi(\xi)\} = \frac{1}{\zeta}\int_0^\infty e^{-\frac{\xi}{\zeta}}\varphi(\xi)d\xi$ is the Sumudu transform of $\varphi(\xi)$, then

$$M\{\varphi(\xi)\} = \frac{1}{\zeta}S\{\varphi(\xi)\}.$$

Proof.

$$M\{\varphi(\xi)\} = \zeta^2\int_0^\infty e^{-\zeta\xi}\varphi(\xi)d\xi = \Phi(\zeta).$$

Moreover,

$$\Phi\left(\frac{1}{\zeta}\right) = \frac{1}{\zeta^2}\int_0^\infty e^{-\frac{1}{\zeta}\xi}\varphi(\xi)d\xi = \frac{1}{\zeta}\left[\frac{1}{\zeta}\int_0^\infty e^{-\xi\left(\frac{1}{\zeta}\right)}\varphi(\xi)d\xi\right] = \frac{1}{\zeta}S\{\varphi(\xi)\}. \tag{11}$$

- Aboodh transform

If $A\{\varphi(\xi)\} = \frac{1}{\zeta}\int_0^\infty e^{-\zeta\xi}\varphi(\xi)d\xi$ is the Aboodh transform of $\varphi(\xi)$, then

$$M\{\varphi(\xi)\} = \zeta^3 A\{\varphi(\xi)\}.$$

Proof.

$$M\{\varphi(\xi)\} = \zeta^2\int_0^\infty e^{-\zeta\xi}\varphi(\xi)d\xi = \Phi(\zeta).$$

Moreover,

$$M\{\varphi(\xi)\} = \zeta^3\left[\frac{1}{\zeta}\int_0^\infty e^{-\xi\zeta}\varphi(\xi)d\xi\right] = \zeta^3 A\{\varphi(\xi)\} \tag{12}$$

- Formable transform

If $B(\zeta, u) = \zeta \int_0^\infty e^{-\zeta\xi}\varphi(u\xi)d\xi$ is the Formable transform of $\varphi(u\xi)$, then

$$M\{\varphi(\xi)\} = \zeta B(\zeta, 1).$$

Proof.

$$M\{\varphi(\xi)\} = \zeta^2 \int_0^\infty e^{-\zeta\xi}\varphi(\xi)d\xi = \zeta\left[\zeta \int_0^\infty e^{-\zeta\xi}\varphi(\xi)d\xi\right] = \zeta B(\zeta, 1) \tag{13}$$

- ARA transform

If $\mathcal{G}(n, \zeta) = \zeta \int_0^\infty \xi^{n-1}e^{-\zeta\xi}\varphi(\xi)d\xi$ is the ARA transform of $\varphi(\xi)$, then

$$M\{\varphi(\xi)\} = \zeta\mathcal{G}\left\{\xi^{n-1}\varphi(\xi)\right\}$$

Proof.

$$M\{\varphi(\xi)\} = \zeta^2 \int_0^\infty e^{-\zeta t}\varphi(\xi)d\xi = \zeta\left[\zeta \int_0^\infty e^{-\zeta\xi}\xi^{n-1}\varphi(\xi)d\xi\right] = \zeta\mathcal{G}\left\{\xi^{n-1}\varphi(\xi)\right\} \tag{14}$$

2.3. M.T. for derivatives

If $M\{\varphi(\xi)\} = \Phi(\zeta)$, then

$$(i) \qquad M\left\{\varphi'(\xi)\right\} = \zeta\Phi(\zeta) - \zeta^2\varphi(0). \tag{15}$$

$$(ii) \qquad M\left\{\varphi''(\xi)\right\} = \zeta^2\Phi(\zeta) - \zeta^3\varphi(0) - \zeta^2\varphi'(0). \tag{16}$$

$$(iii) \qquad M\left\{\varphi^{(n)}(\xi)\right\} = \zeta^n\Phi(\zeta) - \sum_{k=0}^{n-1}\zeta^{n-k+1}\varphi^{(k)}(0). \tag{17}$$

Proof. To proof (i), by definition 1 of the M.T., we have

$$M\left\{\varphi'(\xi)\right\} = \zeta^2 \int_0^\infty e^{-\zeta\xi}\varphi'(\xi)d\xi. \tag{18}$$

Using integration by parts, we have

$$u = e^{-\zeta\xi} \quad \text{and} \quad dv = \varphi'(\xi), \quad du = -\zeta e^{-\zeta\xi}, \quad v = \varphi(\xi).$$

Then the equation (18) becomes,

$$M\left\{\varphi'(\xi)\right\} = \zeta\Phi(\zeta) - \zeta^2\varphi(0). \tag{19}$$

To proof (ii), we use the definition 1 of M.T., $M\{\varphi''(\xi)\} = M\left\{(\varphi'(\xi))'\right\}$.

Using part (i).

$$\begin{aligned} M\left\{\varphi''(\xi)\right\} &= \zeta M\left\{\varphi'(\xi)\right\} - \zeta^2\varphi'(0) = \zeta\left[\zeta\Phi(\zeta) - \zeta^2\varphi(0)\right] - \zeta^2\varphi'(0) \\ &= \zeta^2\Phi(\zeta) - \zeta^3\varphi(0) - \zeta^2\varphi'(0). \end{aligned} \tag{20}$$

Proof (iii). By induction, for $n = 1$ its true from part (i).
Now, assume that the equation (17) is true for $n = k$, then

$$M\left\{\varphi^{(k)}(\xi)\right\} = \zeta^k\Phi(\zeta) - \zeta^{k+1}\varphi(0) - \zeta^k\varphi'(0) - \cdots - \zeta^2\varphi^{(k-1)}(0). \tag{21}$$

Now, we prove it for $n = k + 1$, thus

$$M\left\{\varphi^{(k+1)}(\xi)\right\} = M\left\{\left(\varphi^{(k)}(\xi)\right)'\right\}.$$

Using part (i), we get

$$\begin{aligned} M\left\{\varphi^{(k+1)}(\xi)\right\} &= \zeta^k\left[\zeta\Phi(\zeta) - \zeta^2\varphi(0)\right] - \zeta^{k+1}\varphi'(0) - \zeta^k\varphi''(0) - \cdots - \zeta^2\varphi^{(k)}(0) \\ &= \zeta^n\Phi(\zeta) - \sum_{k=0}^{n-1}\zeta^{n-k+1}\varphi^{(k)}(0). \end{aligned} \tag{22}$$

3. M.T. for solving system of ODEs

In this part, we solve systems of ordinary differential equations by applying M.T..
Consider the system of first order of ODEs:

$$\left.\begin{array}{l} \frac{d\varphi_1}{d\xi} = h_{11}\varphi_1(\xi) + h_{12}\varphi_2(\xi) + h_{13}\varphi_3(\xi) + \cdots + h_{1n}\varphi_n(\xi) + \rho_1(\xi), \\ \frac{d\varphi_2}{d\xi} = h_{21}\varphi_1(\xi) + h_{22}\varphi_2(\xi) + h_{23}\varphi_3(\xi) + \cdots + h_{2n}\varphi_n(\xi) + \rho_2(\xi), \\ . \\ . \\ \frac{d\varphi_n}{d\xi} = h_{n1}\varphi_1(\xi) + h_{n2}\varphi_2(\xi) + h_{n3}\varphi_3(\xi) + \cdots + h_{nn}\varphi_n(\xi) + \rho_n(\xi). \end{array}\right\} \tag{23}$$

with the initial conditions:

$$\varphi_1(0) = b_1, \quad \varphi_2(0) = b_2, \quad \ldots, \quad \varphi_n(0) = b_n, \tag{24}$$

where $h_{11}, \quad h_{12}, \quad h_{13}, \quad \ldots, \quad h_{nn}$ are constants, and $\varphi_1(\xi), \quad \varphi_2(\xi), \quad \ldots, \quad \varphi_n(\xi)$ are unknown continuous functions, and $\rho_1(\xi), \quad \rho_2(\xi), \quad \ldots, \quad \rho_n(\xi)$ are known continuous functions.

The matrix representation of system (23) with (24) is

$$\frac{d\Phi}{d\xi} = H\varphi(\xi) + \rho(\xi), \quad \text{with} \quad \varphi(0) = B, \tag{25}$$

where

$$\frac{d\varphi}{d\xi} = \begin{bmatrix} \frac{d\varphi_1}{d\xi} \\ \frac{d\varphi_2}{d\xi} \\ \vdots \\ \frac{d\varphi_n}{d\xi} \end{bmatrix}, H = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1n} \\ h_{21} & h_{22} & \cdots & h_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n1} & h_{n2} & \cdots & h_{nn} \end{bmatrix}, \varphi(\xi) = \begin{bmatrix} \varphi_1(\xi) \\ \varphi_2(\xi) \\ \vdots \\ \varphi_n(\xi) \end{bmatrix},$$

$$\rho(\xi) = \begin{bmatrix} \rho_1(\xi) \\ \rho_2(\xi) \\ \vdots \\ \rho_n(\xi) \end{bmatrix}, \varphi(0) = \begin{bmatrix} \varphi_1(0) \\ \varphi_2(0) \\ \vdots \\ \varphi_n(0) \end{bmatrix} \text{ and } B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}.$$

By applying M.T. to system (23), we have

$$\left.\begin{array}{ll} M\{\varphi_1'(\xi)\} & = h_{11}M\{\varphi_1(\xi)\} + h_{12}M\{\varphi_2(\xi)\} + \cdots + h_{1n}M\{\varphi_n(\xi)\} + M\{\rho_1(\xi)\}, \\ M\{\varphi_2'(\xi)\} & = h_{21}M\{\varphi_1(\xi)\} + h_{22}M\{\varphi_2(\xi)\} + \cdots + h_{2n}M\{\varphi_n(\xi)\} + M\{\rho_2(\xi)\}, \\ \cdot & \\ \cdot & \\ M\{\varphi_n'(\xi)\} & = h_{n1}M\{\varphi_1(\xi)\} + h_{n2}M\{\varphi_2(\xi)\} + \cdots + h_{nn}M\{\varphi_n(\xi)\} + M\{\rho_n(\xi)\}. \end{array}\right\} \tag{26}$$

Then, we get

$$\left.\begin{array}{ll} \zeta M\{\varphi_1(\xi)\} & -\zeta^2\varphi_1(0) = h_{11}M\{\varphi_1(\xi)\} + h_{12}M\{\varphi_2(\xi)\} + \cdots + h_{1n}M\{\varphi_n(\xi)\} + M\{\rho_1(\xi)\}, \\ \zeta M\{\varphi_2(\xi)\} & -\zeta^2\varphi_2(0) = h_{21}M\{\varphi_1(\xi)\} + h_{22}M\{\varphi_2(\xi)\} + \cdots + h_{2n}M\{\varphi_n(\xi)\} + M\{\rho_2(\xi)\}, \\ & \cdot \\ & \cdot \\ \zeta M\{\varphi_n(\xi)\} & -\zeta^2\varphi_n(0) = h_{n1}M\{\varphi_1(\xi)\} + h_{n2}M\{\varphi_2(\xi)\} + \cdots + h_{nn}M\{\varphi_n(\xi)\} + M\{\rho_n(\xi)\}. \end{array}\right\} \tag{27}$$

Then, using the initial conditions (24), the system (27) becomes

$$\left.\begin{array}{l} (\zeta - h_{11})\, M\rho\{\varphi_1(\xi)\} - h_{12}M\{\varphi_2(\xi)\} - \cdots - h_{1n}M\{\varphi_n(\xi)\} = M\{\rho_1(\xi)\} + b_1\zeta^2, \\ -h_{21}M\{\varphi_1(\xi)\} + (\zeta - h_{22})\, M\{\varphi_2(\xi)\} - \cdots - h_{1n}M\{\varphi_n(\xi)\} = M\{\rho_2(\xi)\} + b_2\zeta^2 \\ \cdot \\ \cdot \\ -h_{n1}M\{\varphi_n(\xi)\} - h_{n2}M\{\varphi_2(\xi)\} - \cdots + (\zeta - h_{nn})\, M\{\varphi_n(\xi)\} = M\{\rho_n(\xi)\} + b_n\zeta^2 \end{array}\right\} \tag{28}$$

Then, the solution of system (26) can be obtained using Cramer's rule as

$$M\{\varphi_1(\xi)\} = \frac{\begin{vmatrix} M\{\rho_1(\xi)\} + b_1\zeta^2 & -h_{12} & \cdots & -h_{1n} \\ M\{\rho_2(\xi)\} + b_2\zeta^2 & (\zeta - h_{22}) & \cdots & -h_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ M\{\rho(\xi)\} + b_n\zeta^2 & -h_{nn} & \cdots & (\zeta - h_{nn}) \end{vmatrix}}{\begin{vmatrix} (\zeta - h_{11}) & -h_{12} & \cdots & -h_{1n} \\ -h_{21} & (\zeta - h_{22}) & \cdots & -h_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -h_{n1} & -h_{n2} & \cdots & (\zeta - h_{nn}) \end{vmatrix}}$$

$$M\{\varphi_2(\xi)\} = \frac{\begin{vmatrix} (\zeta - h_{11}) & M\{\rho_1(\xi)\} + b_1\zeta^2 & \cdots & -h_{1n} \\ -h_{21} & M\{\rho_2(\xi)\} + b_2\zeta^2 & \cdots & -h_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -h_{n1} & M\{\rho_n(\xi)\} + b_n\zeta^2 & \cdots & (\zeta - h_{nn}) \end{vmatrix}}{\begin{vmatrix} (\zeta - h_{11}) & -h_{12} & \cdots & -h_{1n} \\ -h_{21} & (\zeta - h_{22}) & \cdots & -h_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -h_{n1} & -h_{n2} & \cdots & (\zeta - h_{nn}) \end{vmatrix}}$$

$$\vdots$$

$$M\{\varphi_n(\xi)\} = \frac{\begin{vmatrix} (\zeta - h_{11}) & -h_{1n} & \cdots & M\{\rho_1(\xi)\} + b_1\zeta^2 \\ -h_{21} & (\zeta - h_{22}) & \cdots & M\{\rho_2(\xi)\} + b_2\zeta^2 \\ \vdots & \vdots & \ddots & \vdots \\ -h_{n1} & -h_{n2} & \cdots & M\{\rho_n(\xi)\} + b_n\zeta^2 \end{vmatrix}}{\begin{vmatrix} (\zeta - h_{11}) & -h_{12} & \cdots & -h_{1n} \\ -h_{21} & (\zeta - h_{22}) & \cdots & -h_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -h_{n1} & -h_{n2} & \cdots & (\zeta - h_{nn}). \end{vmatrix}}$$

Now, applying the inverse M.T. of $M\{\varphi_1(\xi)\}$, $M\{\varphi_2(\xi)\}$, $\ldots$, $M\{\varphi_n(\xi)\}$, then we get the values of $\varphi_1(\xi)$, $\varphi_2(\xi)$, $\ldots$, $\varphi_n(\xi)$.

Now, we introduce some examples of systems ODEs and solve them by M.T..

Example 1. *Consider the following system of ODEs*

$$\left.\begin{aligned} \tfrac{d\varphi_1}{d\xi} &= \varphi_3(\xi), \\ \tfrac{d\varphi_2}{d\xi} &= -\varphi_3(\xi), \\ \tfrac{d\varphi_3}{d\xi} &= -\varphi_1(\xi) - \varphi_2(\xi), \end{aligned}\right\} \tag{29}$$

with the initial conditions:

$$\varphi_1(0) = 0, \varphi_2(0) = 1 \text{ and } \varphi_3(0) = 0. \tag{30}$$

The matrix form of the system (29) *with the initial conditions* (30) *is given by:*

$$\frac{d\varphi}{d\xi} = H\varphi(\xi) + \rho(\xi), \text{ with } \varphi(0) = B, \tag{31}$$

where:

$$\frac{d\varphi}{d\xi} = \begin{bmatrix} \frac{d\varphi_1}{d\xi} \\ \frac{d\varphi_2}{d\xi} \\ \frac{d\varphi_3}{d\xi} \end{bmatrix}, H = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix}, \varphi(\xi) = \begin{bmatrix} \varphi_1(\xi) \\ \varphi_2(\xi) \\ \varphi_3(\xi) \end{bmatrix},$$

$$\rho(\xi) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \varphi(0) = \begin{bmatrix} \varphi_1(0) \\ \varphi_2(0) \\ \varphi_3(0) \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

By applying M.T. to the system (29), *we get*

$$\left.\begin{array}{r} M\{\varphi_1'(\xi)\} - M\{\varphi_3(\xi)\} = 0, \\ M\{\varphi_2'(\xi)\} + M\{\varphi_3(\xi)\} = 0, \\ M\{\varphi_1(\xi)\} + M\{\varphi_2(\xi)\} + M\{\varphi_3{}'(\xi)\} = 0. \end{array}\right\} \tag{32}$$

Operating M.T. on (32) *and using the initial condition* (30)

$$\left.\begin{array}{r} \zeta M\{\varphi_1(\xi)\} - \zeta^2\varphi_1(0) - M\{\varphi_3(\xi)\} = 0, \\ \zeta M\{\varphi_2(\xi)\} - \zeta^2\varphi_2(0) + M\{\varphi_3(\xi)\} = 0, \\ M\{\varphi_1(\xi)\} + M\{\varphi_2(\xi)\} + \zeta M\{\varphi_3(\xi)\} - \zeta^2\varphi_3(0) = 0. \end{array}\right\} \tag{33}$$

Simplifying the system (33), *we obtain*

$$\left.\begin{array}{r} \zeta M\{\varphi_1(\xi)\} - M\{\varphi_3(\xi)\} = 0, \\ \zeta M\{\varphi_2(\xi)\} + M\{\varphi_3(\xi)\} = \zeta^2, \\ M\{\varphi_1(\xi)\} + M\{\varphi_2(\xi)\} + \zeta M\{\varphi_3(\xi)\} = 0. \end{array}\right\} \tag{34}$$

Using Cramer's rule to solve $M\{\varphi_1(\xi)\}, M\{\varphi_2(\xi)\}$ *and* $M\{\varphi_3(\xi)\}$ *on the system* (34)

$$M\{\varphi_1(\xi)\} = \frac{\begin{vmatrix} 0 & 0 & -1 \\ \zeta^2 & \zeta & 1 \\ 0 & 1 & \zeta \end{vmatrix}}{\begin{vmatrix} \zeta & 0 & -1 \\ 0 & \zeta & 1 \\ 1 & 1 & \zeta \end{vmatrix}} = \frac{-1}{\zeta}, \tag{35}$$

$$M\left\{\varphi_2(\xi)\right\}=\frac{\begin{vmatrix}\zeta & 0 & -1\\ 0 & \zeta^2 & 1\\ 1 & 0 & \zeta\end{vmatrix}}{\begin{vmatrix}\zeta & 0 & -1\\ 0 & \zeta & 1\\ 1 & 1 & \zeta\end{vmatrix}}=\zeta+\left(\frac{1}{\zeta}\right), \tag{36}$$

$$M\left\{\varphi_3(\xi)\right\}=\frac{\begin{vmatrix}\zeta & 0 & -0\\ 0 & \zeta & \zeta^2\\ 1 & 1 & 0\end{vmatrix}}{\begin{vmatrix}\zeta & -1 & \\ 0 & \zeta & 1\\ 1 & 1 & \zeta\end{vmatrix}}=-1. \tag{37}$$

By applying the inverse M.T. on the equations (35), (36) *and* (37), *then we have*

$$\varphi_1(\xi)=M^{-1}\left\{\frac{-1}{\zeta}\right\}=\frac{-\xi^2}{2}, \tag{38}$$

$$\varphi_2(\xi)=M^{-1}\left\{\zeta+\frac{1}{\zeta}\right\}=1+\frac{\xi^2}{2}, \tag{39}$$

$$\varphi_3(\xi)=M^{-1}\{-1\}=-\xi. \tag{40}$$

Equations (38), (39) *and* (40) *give the solution of the system* (29) *with the initial condition* (30).

4. Chemical -Physical Application

This section of the study includes a chemical-physical application to estimate the concentrations c_1, c_2 and c_3 of three reactants X, Y and Z of a first-order chemical reaction in batches defined that is solved and discussed by the M.T..

$$\text{reactant } X \longrightarrow \text{ reactant } Y \longrightarrow \text{ reactant } Z.$$

$$\left.\begin{aligned}\frac{dC_1}{dt}&=-\lambda_1 C_1,\\ \frac{dC_2}{dt}&=\lambda_1 C_1-\lambda_2 C_2,\\ \frac{dC_3}{dt}&=\lambda_2 C_2.\end{aligned}\right\} \tag{41}$$

with

$$C_1(0)=\alpha, \quad C_2(0)=0 \quad \text{and} \quad C_3(0)=0. \tag{42}$$

$$\left.\begin{array}{rl} C_1 = C_1(t) = & \text{Concentration of achemical reactant} \quad X \quad \text{at time} \quad t, \\ C_2 = C_2(t) = & \text{Concentration of achemical reactant} \quad Y \quad \text{at time} \quad t, \\ C_3 = C_3(t) = & \text{Concentration of achemical reactant} \quad Z \quad \text{at time} \quad t, \\ & \text{where} \quad \lambda_1, \quad \lambda_2 = \text{rate constant} > 0. \\ C_1(0) = \alpha = & \text{The initial concentration of achemical reactant} \quad X, \\ C_2(0) = 0 = & \text{The initial concentration of achemical reactant} \quad Y, \\ C_3(0) = 0 = & \text{The initial concentration of achemical reactant} \quad Z. \end{array}\right\}$$

The matrix form of the system (41) with the initial conditions (42):

$$\frac{dC}{dt} = HC(t) + \rho(t), \quad \text{with} C(0) = B, \tag{43}$$

where:

$$\frac{dC}{dt} = \begin{bmatrix} \frac{dC_1}{dt} \\ \frac{dC_2}{dt} \\ \frac{dC_3}{dt} \end{bmatrix}, H = \begin{bmatrix} -\lambda_1 & 0 & 0 \\ \lambda_1 & -\lambda_2 & 0 \\ 0 & \lambda_2 & 0 \end{bmatrix}, C(t) = \begin{bmatrix} C_1(t) \\ C_2(t) \\ C_3(t) \end{bmatrix},$$

$$\rho(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, C(0) = \begin{bmatrix} C_1(0) \\ C_2(0) \\ C_3(0) \end{bmatrix}, \text{ and } B = \begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix}.$$

By applying M.T. on the system (41), we get:

$$\left.\begin{array}{l} M\left\{C_1'(t)\right\} + \lambda_1 M\left\{C_1(t)\right\} = 0, \\ M\left\{C_2'(t)\right\} - \lambda_1 M\left\{C_1(t)\right\} + \lambda_2 M\left\{C_2(t)\right\} = 0, \\ M\left\{C_3'(t)\right\} - \lambda_2 M\left\{C_2(t)\right\} = 0. \end{array}\right\} \tag{44}$$

Operating M.T. to (44) and using the initial conditions (42)

$$\left.\begin{array}{l} \zeta M\left\{C_1(t)\right\} - \zeta^2 C_1(0) + \lambda_1 M\left\{C_1(t)\right\} = 0, \\ \zeta M\left\{C_2(t)\right\} - \zeta^2 C_2(0) + \lambda_2 M\left\{C_2(t)\right\} - \lambda_1 M\left\{C_1(t)\right\} = 0, \\ \zeta M\left\{C_3(t)\right\} - \zeta^2 C_3(0) - \lambda_2 M\left\{C_2(t)\right\} = 0. \end{array}\right\} \tag{45}$$

Simplifying the system (45), we obtain

$$\left.\begin{array}{l} (\zeta + \lambda_1) M\left\{C_1(t)\right\} = \zeta^2 \alpha, \\ (\zeta + \lambda_2) M\left\{C_2(t)\right\} - \lambda_1 M\left\{C_1(t)\right\} = 0, \\ \zeta M\left\{C_3(t)\right\} - \lambda_2 M\left\{C_2(t)\right\} = 0. \end{array}\right\} \tag{46}$$

Using Cramer's rule to solve $M\left\{C_1(t)\right\}, M\left\{C_2(t)\right\}$ and $M\left\{C_3(t)\right\}$ on the system (46)

$$M\left\{C_1(t)\right\}=\frac{\begin{vmatrix}\zeta^2\alpha & 0 & 0\\ 0 & \zeta+\lambda_2 & 0\\ 0 & -\lambda_2 & \zeta\end{vmatrix}}{\begin{vmatrix}\zeta+\lambda_1 & 0 & 0\\ -\lambda_1 & \zeta+\lambda_2 & 0\\ 0 & -\lambda_2 & \zeta\end{vmatrix}}=\frac{\zeta^3\alpha(\zeta+\lambda_2)}{\zeta(\zeta+\lambda_2)(\zeta+\lambda_1)}=\alpha\left(\frac{\zeta^2}{(\zeta+\lambda_1)}\right), \tag{47}$$

$$M\left\{C_2(t)\right\}=\frac{\begin{vmatrix}\zeta+\lambda_1 & \zeta^2\alpha & 0\\ -\lambda_1 & 0 & 0\\ 0 & 0 & \zeta\end{vmatrix}}{\begin{vmatrix}\zeta+\lambda_1 & 0 & 0\\ -\lambda_1 & \zeta+\lambda_2 & 0\\ 0 & -\lambda_2 & \zeta\end{vmatrix}}=\frac{\zeta^3\alpha\lambda_1}{\zeta(\zeta+\lambda_2)(\zeta+\lambda_1)}=\alpha\lambda_1\left(\frac{\zeta^2}{(\zeta+\lambda_2)(\zeta+\lambda_1)}\right), \tag{48}$$

$$M\left\{C_3(t)\right\}=\frac{\begin{vmatrix}\zeta+\lambda_1 & 0 & \zeta^2\alpha\\ -\lambda_1 & \zeta+\lambda_2 & 0\\ 0 & -\lambda_2 & 0\end{vmatrix}}{\begin{vmatrix}\zeta+\lambda_1 & 0 & 0\\ -\lambda_1 & \zeta+\lambda_2 & 0\\ 0 & -\lambda_2 & \zeta\end{vmatrix}}=\alpha\lambda_1\lambda_2\left(\frac{\zeta}{(\zeta+\lambda_2)(\zeta+\lambda_1)}\right). \tag{49}$$

By applying the inverse M.T. on the equations (47), (48) and (49) then, we have

$$C_1(t)=M^{-1}\left\{\alpha\left(\frac{\zeta^2}{(\zeta+\lambda_1)}\right)\right\}=\alpha M^{-1}\left\{\frac{\zeta^2}{(\zeta+\lambda_1)}\right\}=\alpha e^{-\lambda_1 t}. \tag{50}$$

$$\begin{aligned}C_2(t)=M^{-1}\left\{\alpha\left(\frac{\zeta^2}{(\zeta+\lambda_2)(\zeta+\lambda_1)}\right)\right\} &= \alpha\lambda_1 M^{-1}\left\{\frac{\zeta^2}{(\zeta+\lambda_2)(\zeta+\lambda_1)}\right\}\\ &=\left(\frac{\alpha\lambda_1}{\lambda_2-\lambda_1}\right)\left(e^{-\lambda_1 t}-e^{-\lambda_2 t}\right).\end{aligned} \tag{51}$$

$$\begin{aligned}C_3(t) &= M^{-1}\left\{\alpha\lambda_1\lambda_2\left(\frac{\zeta}{(\zeta+\lambda_2)(\zeta+\lambda_1)}\right)\right\}=\alpha\Big(M^{-1}\{\zeta\}-\frac{\lambda_2}{\lambda_2-\lambda_1}M^{-1}\left\{\frac{\lambda_1^2}{\zeta+\lambda_1}\right\}\\ &+\frac{\lambda_1}{\lambda_2-\lambda_1}M^{-1}\left\{\frac{\lambda_2^2}{\zeta+\lambda_2}\right\}\Big)=\alpha\left(1-\left(\frac{\lambda_2}{\lambda_2-\lambda_1}\right)e^{-\lambda_1 t}+\left(\frac{\lambda_1}{\lambda_2-\lambda_1}\right)e^{-\lambda_2 t}\right)\end{aligned} \tag{52}$$

The values of concentration C_1, C_2 and C_3 conforming to diverse values of time t and for varied combinations of values of α, λ_1 and λ_2 are specified and displayed in Table 2, and Figure 1 presents graphical representations. Table 2 shows that with the increasing time t from 0 to 7 seconds, the concentration $C_1(t)$ of a chemical substance X declines for any combinations of values of α and λ_1 namely

$$\left\{\begin{array}{l} \alpha=1\left(\mathrm{~kg}/\mathrm{m}^3\right), \lambda_1=1\left(\mathrm{sec}^{-1}\right), \\ \alpha=1\left(\mathrm{~kg}/\mathrm{m}^3\right), \lambda_1=1.2\left(\mathrm{sec}^{-1}\right), \\ \alpha=1\left(\mathrm{~kg}/\mathrm{m}^3\right), \lambda_1=1.3\left(\mathrm{sec}^{-1}\right), \end{array}\right\}$$

where concentration $C_1(t)$ of a chemical substance X at time $t\left(\mathrm{~kg}/\mathrm{m}^3\right)$.

Table 2 further shows that as the value of rate constant λ_1 intensifies from 1 to $1.2\mathrm{sec}^{-1}$ and 1 to $1.3\mathrm{sec}^{-1}$, the value of the concentration $C_1(t)$ of a chemical substance X depletes for time values t stretching from 1 to 5 seconds. What's more, Table 2 determines that for high values of time, the concentration $C_1(t)$ of a chemical substance X transforms into $0\left(\mathrm{~kg}/\mathrm{m}^3\right)$. The results exhibited in Table 2 are corroborated by the diagram of Figure 1.

Table 2. Concentration $C_1(t)$ of a chemical substance X at time t of different combinations of values α and λ_1.

t(sec)	$C_1(t)\left(\mathrm{kg}/\mathrm{m}^3\right)$		
	$\alpha=1\left(\mathrm{~kg}/\mathrm{m}^3\right), \lambda_1=1\left(\mathrm{sec}^{-1}\right)$	$\alpha=1\left(\mathrm{~kg}/\mathrm{m}^3\right), \lambda_1=1.2\left(\mathrm{sec}^{-1}\right)$	$\alpha=1\left(\mathrm{~kg}/\mathrm{m}^3\right), \lambda_1=1.3\left(\mathrm{sec}^{-1}\right)$
0	1.00	1.00	1.00
1.0	0.37	0.30	0.27
2.0	0.14	0.09	0.07
3.0	0.05	0.03	0.02
4.0	0.02	0.01	0.01
5.0	0.01	0.00	0.00
6.0	0.00	0.00	0.00
7.0	0.00	0.00	0.00

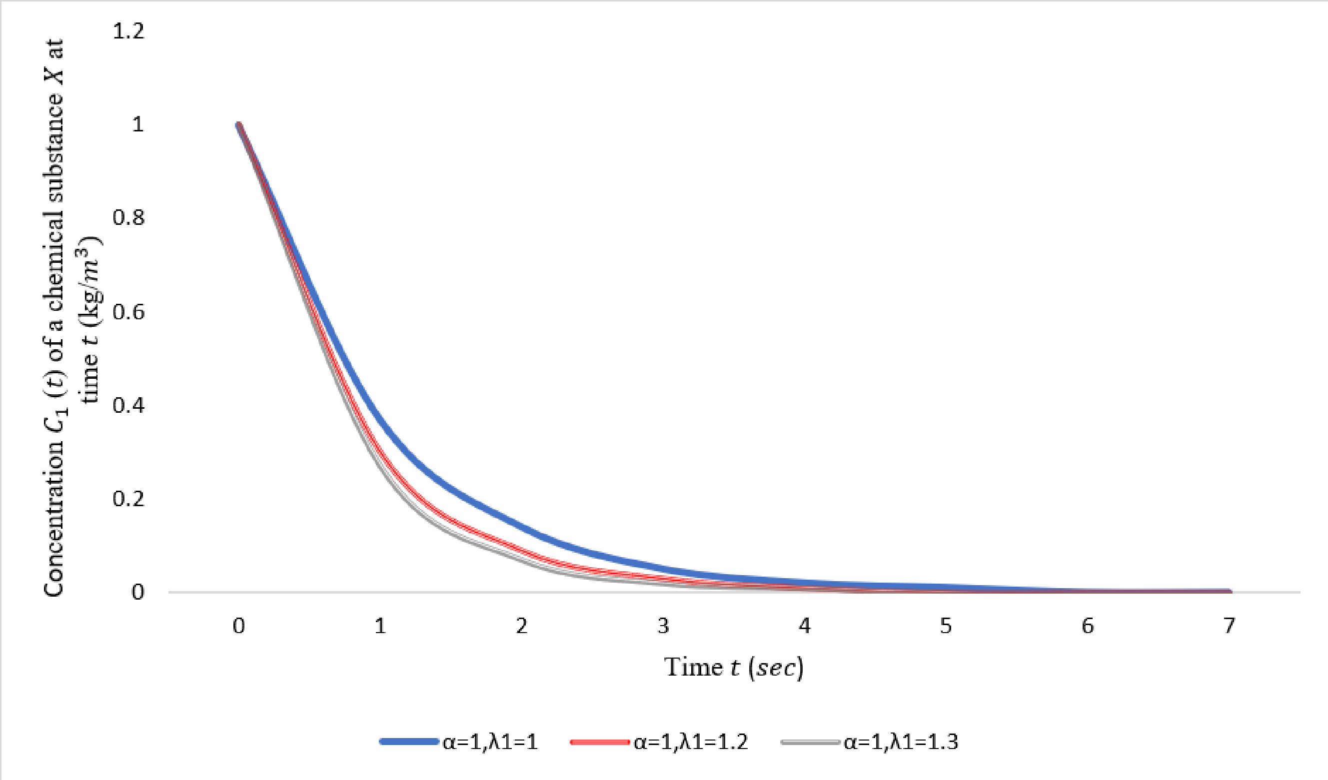

Figure 1: Concentration $C_1(t)$ of a chemical substance X at time t of different combinations of values α and λ_1.

Table 4 illustrates that the concentration $C_2(t)$ of a chemical substance Y decreases as time t increases from 0 to 6 seconds for every combination of values of α, λ_1 and λ_2.

$$\left\{\begin{array}{l}
\alpha = 1\left(\text{ kg/m}^3\right), \lambda_1 = 1.0\left(\sec^{-1}\right), \lambda_2 = 0.5\left(\sec^{-1}\right), \\
\alpha = 1\left(\text{ kg/m}^3\right), \lambda_1 = 1.0\left(\sec^{-1}\right), \lambda_2 = 1.1\left(\sec^{-1}\right), \\
\alpha = 1\left(\text{ kg/m}^3\right), \lambda_1 = 1.0\left(\sec^{-1}\right), \lambda_2 = 1.4\left(\sec^{-1}\right), \\
\alpha = 1\left(\text{ kg/m}^3\right), \lambda_1 = 1.2\left(\sec^{-1}\right), \lambda_2 = 0.5\left(\sec^{-1}\right), \\
\alpha = 1\left(\text{ kg/m}^3\right), \lambda_1 = 1.2\left(\sec^{-1}\right), \lambda_2 = 1.1\left(\sec^{-1}\right), \\
\alpha = 1\left(\text{ kg/m}^3\right), \lambda_1 = 1.2\left(\sec^{-1}\right), \lambda_2 = 1.4\left(\sec^{-1}\right), \\
\alpha = 1\left(\text{ kg/m}^3\right), \lambda_1 = 1.3\left(\sec^{-1}\right), \lambda_2 = 0.5\left(\sec^{-1}\right), \\
\alpha = 1\left(\text{ kg/m}^3\right), \lambda_1 = 1.3\left(\sec^{-1}\right), \lambda_2 = 1.1\left(\sec^{-1}\right), \\
\alpha = 1\left(\text{ kg/m}^3\right), \lambda_1 = 1.3\left(\sec^{-1}\right), \lambda_2 = 1.4\left(\sec^{-1}\right).
\end{array}\right\}$$

From the following table, it is evident that when the value of rate constant λ_1 raises from 1 to $1.3\sec^{-1}$, the value of the concentration $C_2(t)$ of a chemical substance Y raises initially, and diminishes subsequently as time t increases from 0 to 7 seconds. In addition, this table manifests that since the value of rate constant λ_2 goes from 0.5 to $1.4\sec^{-1}$, the value of a chemical substance's concentration Y drops for all time t values. Moreover, Table 3. displays that for high values of time t, the concentration $C_2(t)$ of a chemical substance Y becomes 0 kg/m^3. The diagram of Figure 2 depicts the similar observations as the Table 3.

Table 3 . Concentration $C_2(t)$ of a chemical substance Y at time t of different combinations of values α, λ_1 and λ_2.

t(sec)	$C_2(t)$ (kg/m^3)							
	$\alpha = 1$ (kg/m^3), $\lambda_1 = 1$ (sec^{-1})			$\alpha = 1$ (kg/m^3), $\lambda_1 = 1.2$ (sec^{-1})			$\alpha = 1$ (kg/m^3), $\lambda_1 = 1.3$ (sec^{-1})	
	$\lambda_2 = 0.5$ (sec^{-1})	$\lambda_2 = 1.1$ (sec^{-1})	$\lambda_2 = 1.4$ (sec^{-1})	$\lambda_2 = 0.5$ (sec^{-1})	$\lambda_2 = 1.1$ (sec^{-1})	$\lambda_2 = 1.4$ (sec^{-1})	$\lambda_2 = 0.5$ (sec^{-1})	$\lambda_2 = 1.1$ (sec^{-1})
0	0.0	0.0	0.00	0.00	0.00	0.00	0.00	0.00
1.0	0.48	0.35	0.29	0.52	0.38	0.33	0.54	0.39
2.0	0.47	0.25	0.17	0.48	0.24	0.18	0.48	0.24
3.0	0.35	0.13	0.08	0.34	0.11	0.07	0.33	0.11
4.0	0.23	0.06	0.03	0.22	0.05	0.03	0.21	0.04
5.0	0.15	0.03	0.01	0.14	0.02	0.01	0.13	0.02
6.0	0.09	0.01	0.00	0.08	0.01	0.00	0.08	0.01
7.0	0.06	0.00	0.00	0.05	0.00	0.00	0.05	0.00

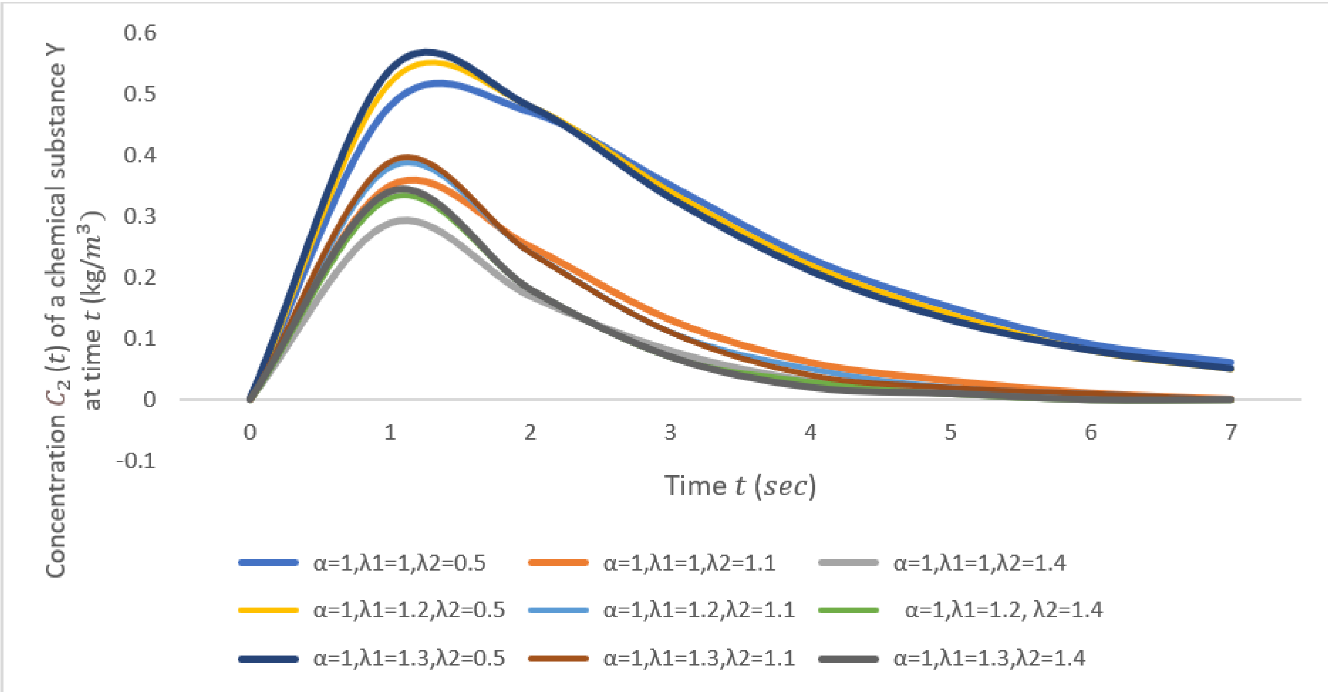

Figure 2: Concentration $C_2(t)$ of a chemical substance Y at time t of different combinations of values α, λ_1 and λ_2.

Table 4 reveals how the concentration $C_3(t)$ of a chemical substance Z elevates as time elevates from 0 to 6 seconds for all combinations of values of α, λ_1 and λ_2, namly

$$\left\{\begin{array}{l}
\alpha=1\left(\mathrm{~kg/m}^3\right), \lambda_1=1.0\left(\sec^{-1}\right), \lambda_2=0.5\left(\sec^{-1}\right), \\
\alpha=1\left(\mathrm{~kg/m}^3\right), \lambda_1=1.0\left(\sec^{-1}\right), \lambda_2=1.1\left(\sec^{-1}\right), \\
\alpha=1\left(\mathrm{~kg/m}^3\right), \lambda_1=1.0\left(\sec^{-1}\right), \lambda_2=1.4\left(\sec^{-1}\right), \\
\alpha=1\left(\mathrm{~kg/m}^3\right), \lambda_1=1.2\left(\sec^{-1}\right), \lambda_2=0.5\left(\sec^{-1}\right), \\
\alpha=1\left(\mathrm{~kg/m}^3\right), \lambda_1=1.2\left(\sec^{-1}\right), \lambda_2=1.1\left(\sec^{-1}\right), \\
\alpha=1\left(\mathrm{~kg/m}^3\right), \lambda_1=1.2\left(\sec^{-1}\right), \lambda_2=1.4\left(\sec^{-1}\right), \\
\alpha=1\left(\mathrm{~kg/m}^3\right), \lambda_1=1.3\left(\sec^{-1}\right), \lambda_2=0.5\left(\sec^{-1}\right), \\
\alpha=1\left(\mathrm{~kg/m}^3\right), \lambda_1=1.3\left(\sec^{-1}\right), \lambda_2=1.1\left(\sec^{-1}\right), \\
\alpha=1\left(\mathrm{~kg/m}^3\right), \lambda_1=1.3\left(\sec^{-1}\right), \lambda_2=1.4\left(\sec^{-1}\right).
\end{array}\right\}$$

Moreover, this table portrays that as the value of rate constant λ_1 advances from 1 to 1.3sec^{-1}, the value of the concentration $C_3(t)$ of a chemical substance Z increases for all time t values. In addition, this table unveils that while the value of rate constant λ_2 expands from 0.5 to 1.4sec^{-1}, the value of the concentration $C_3(t)$ of a chemical substance Z expands for all values of time t. In addition, Table 4 exhibits that for high time t values, the concentration $C_3(t)$ of a chemical substance Z develops into 1 kg/m^3. The graph sketch in Figure 3 renders the equivalent findings as in Table 4.

Table 4. Concentration $C_3(t)$ of a chemical substance Z at time t of different combinations of values α, λ_1 and λ_2.

t(sec)	$C_3(t)$ (kg/m^3)							
	$\alpha=1\left(\mathrm{~kg/m}^3\right), \lambda_1=1\left(\sec^{-1}\right)$			$\alpha=1\left(\mathrm{~kg/m}^3\right), \lambda_1=1.2\left(\sec^{-1}\right)$			$\alpha=1\left(\mathrm{~kg/m}^3\right), \lambda_1=1.3\left(\sec^{-1}\right)$	
	$\lambda_2=0.5$ $\left(\sec^{-1}\right)$	$\lambda_2=1.1$ $\left(\sec^{-1}\right)$	$\lambda_2=1.4$ $\left(\sec^{-1}\right)$	$\lambda_2=0.5$ $\left(\sec^{-1}\right)$	$\lambda_2=1.1$ $\left(\sec^{-1}\right)$	$\lambda_2=1.4$ $\left(\sec^{-1}\right)$	$\lambda_2=0.5$ $\left(\sec^{-1}\right)$	$\lambda_2=1.1$ $\left(\sec^{-1}\right)$
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1.0	0.15	0.28	0.33	0.18	0.32	0.37	0.18	0.34
2.0	0.40	0.62	0.68	0.43	0.67	0.73	0.45	0.69
3.0	0.60	0.82	0.86	0.64	0.86	0.90	0.65	0.87
4.0	0.75	0.92	0.95	0.77	0.94	0.96	0.78	0.95
5.0	0.84	0.97	0.98	0.86	0.98	0.99	0.87	0.98
6.0	0.90	0.99	0.99	0.92	0.99	1.00	0.92	0.99
7.0	0.94	0.99	1.00	0.95	1.00	1.00	0.95	1.00

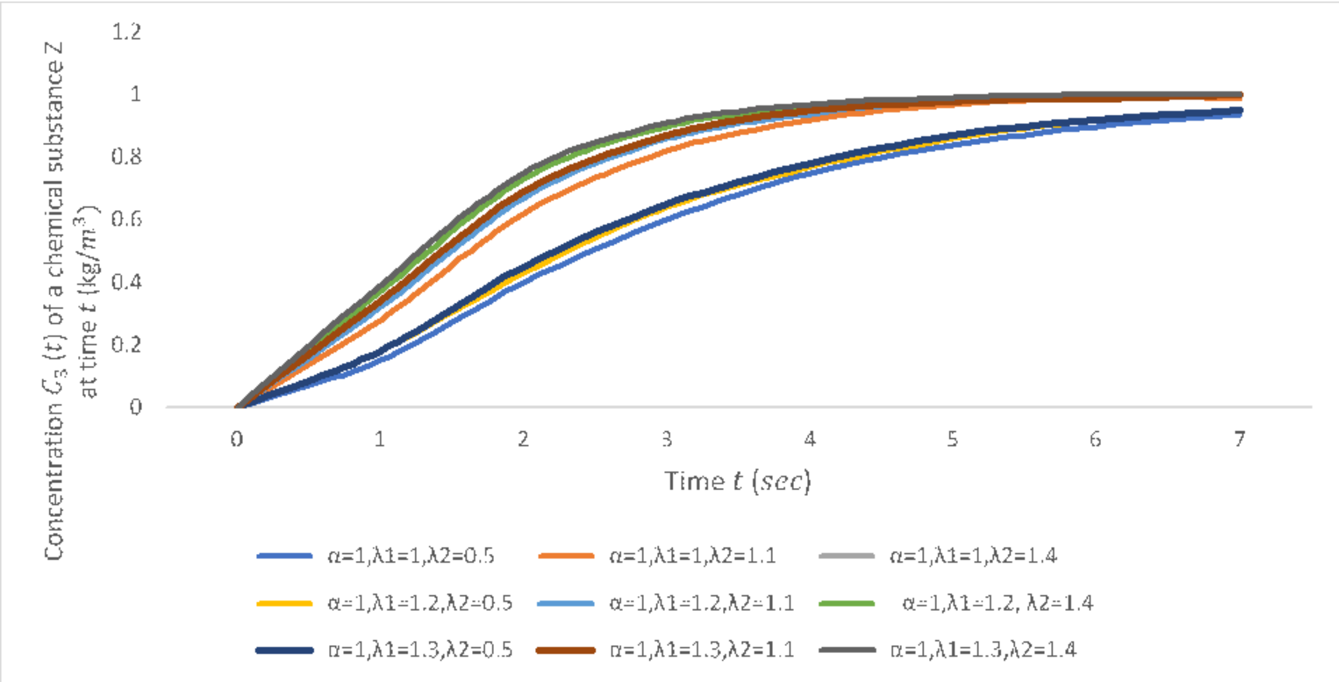

Figure 3: Concentration $C_3(t)$ of a chemical substance Z at time t of different combinations of values α, λ_1 and λ_2.

5. Conclusion

In this comprehensive research, the Mohanad transform (M.T.) emerged as a pivotal mathematical tool, its versatility and efficiency evident in solving a wide array of scientific and engineering problems. The study showcased the foundational properties of M.T. and its relation to integral transforms, affirming its robustness in tackling complex differential equations. By presenting concrete examples of solving systems of ordinary differential equations (ODEs) and applying M.T. to determine chemical reactant concentrations in a series reaction, this work not only validated its efficacy but also demonstrated its practical utility in physical chemistry. The transformative aspect of M.T. lies in its ability to yield exact solutions with minimal computational load, underscoring its precision and efficiency in problem-solving. The novelty of this research extends beyond the mere introduction of M.T., shedding light on its application across diverse fields such as electric circuits, population growth, vibrational beams, and heat conduction. By delving into its interdisciplinary utility, this study broadens the understanding of where and how M.T. can be effectively employed. Moreover, the emphasis on its cross-disciplinary application between mathematics and physical chemistry exemplifies its real-world relevance, surpassing theoretical constructs to address practical scientific challenges[11–13, 27, 29, 31, 32, 34, 36, 41]. The incorporation of graphs and tables further enhances the precision and interpretability of results, highlighting M.T.'s practical significance in both academic and industrial settings. The distinctive contribution of this research lies in its holistic exploration and practical deployment of M.T., affirming its efficiency, precision, and applicability in solving complex problems across scientific and engineering domains. This work not only solidifies the foundational understanding of M.T. but also showcases its transformative potential, paving the way for future advancements and innovative applications in diverse disciplines.

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