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More Results on Intuitionistic Fuzzy Ideals of BE-algebras

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Abstract. red This paper explores the intuitionistic fuzzy ideals in BE-algebras and establishes several new results related to their structure. We investigate the fundamental concepts and properties of intuitionistic fuzzy ideals and provide characterizations of an intuitionistic fuzzy ideal in BE-algebras. Our study focuses on examining the fundamental concepts and properties of these ideals and provides characterizations of intuitionistic fuzzy ideals in BE-algebras.

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1. Introduction

Intuitionistic fuzzy sets, introduced by Atanassov [5–7], have become a significant tool in dealing with uncertainty and vagueness in real-world situations. The concept of intuitionistic fuzzy sets extends the notion of fuzzy sets by considering a non-membership degree in addition to the membership degree. The non-membership degree represents the extent to which an element does not belong to a particular set, and this degree can reflect human reasoning more accurately. Since their introduction, numerous mathematical structures inspired by intuitionistic fuzzy sets have been proposed and investigated [14, 15, 17, 18, 27].

One of the recent areas of research in the field of intuitionistic fuzzy sets is the study of intuitionistic fuzzy subalgebras [2, 11] and ideals [1, 3, 12, 26] in BE-algebras. BE-algebras, introduced by Kim and Kim [10], are a generalization of Boolean algebras, in which the complementation operation is replaced by a weaker negation operation that satisfies weaker versions of the classical De Morgan's laws. New concepts on BE-algebras, fuzzy BE-algebras and intuitionistic fuzzy BE-algebras have been given in [8, 9, 23–25].

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Recently, many authors have studied more concepts on subalgebras and ideals in various algebraic structures [4, 13, 16, 19–22, 28], motivating our interest in the present study.

redThe study of intuitionistic fuzzy ideals of BE-algebras has been an active area of research in recent years, with several existing studies exploring various aspects of this topic. However, there is still much more to be explored in this field, and this paper aims to contribute to this area of study by presenting new results that build upon previous research.

redWhile the existing studies have provided valuable insights into intuitionistic fuzzy ideals in BE-algebras, the present study offers new results that further deepen our understanding of this topic. By considering the present study, researchers and practitioners in this field can gain a more comprehensive and up-to-date understanding of intuitionistic fuzzy ideals and their applications in BE-algebras. This can in turn lead to advancements in various fields where BE-algebras are used, such as computer science, engineering, and economics.

Motivated by a lot of work in this direction, in this paper, as a generalization of fuzzy BE-algebra, we discuss intuitionistic fuzzy ideal theory applied to BE-algebras. We introduce the notion of intuitionistic fuzzy BE-ideals, and investigate several properties. We organize this paper as follows: In Section 2, some fundamental notions of BE-algebras are presented. In Section 3, the notion of intuitionistic fuzzy BE-ideal is defined, and related properties are investigated with many examples.

2. Preliminaries

Let $K(\tau)$ be the class of all algebras of type $\tau = (2, 0)$. By a *BE-algebra* we mean a system $(M; *, 1) \in K(\tau)$ in which the following axioms hold (see [10]):

$$(\forall m_0 \in M)\,(m_0 * m_0 = 1); \tag{1}$$

$$(\forall m_0 \in M)\,(m_0 * 1 = 1); \tag{2}$$

$$(\forall m_0 \in M)\,(1 * m_0 = m_0); \tag{3}$$

$$(\forall m_0, m_1, m_2 \in M)\,(m_0 * (m_1 * m_2) = m_1 * (m_0 * m_2)). \quad \text{(exchange)} \tag{4}$$

A relation "$\leq$" on a BE-algebra M is defined by

$$(\forall m_0, m_1 \in M)\,(m_0 \leq m_1 \iff m_0 * m_1 = 1). \tag{5}$$

A BE-algebra $(M; *, 1)$ is said to be *transitive* (see [1]) if it satisfies:

$$(\forall m_0, m_1, m_2 \in M)\,(m_1 * m_2 \leq (m_0 * m_1) * (m_0 * m_2)). \tag{6}$$

A BE-algebra $(M; *, 1)$ is said to be *self distributive* (see [10]) if it satisfies:

$$(\forall m_0, m_1, m_2 \in M)\,(m_0 * (m_1 * m_2) = (m_0 * m_1) * (m_0 * m_2)). \tag{7}$$

Note that every self distributive BE-algebra is transitive, but the converse is not true in general (see [1]).

A nonempty subset I of a BE-algebra M is called an *ideal* of M (see [1]) if it satisfies:

$$(\forall m_0 \in M)(\forall \alpha \in I)(m_0 * \alpha \in I); \tag{8}$$
$$(\forall m_0 \in M)\,(\forall \alpha, \beta \in I)\,(\alpha * (\beta * m_0)) * m_0 \in I). \tag{9}$$

A mapping $\mu : M \to [0, 1]$, where M is an arbitrary nonempty set, is called a *fuzzy set* in M. For any fuzzy set μ in M and any $t \in [0, 1]$ we define two sets

$$U(\mu; t) = \{m_0 \in M \mid \mu(m_0) \geq t\} \text{ and } L(\mu; t) = \{m_0 \in M \mid \mu(m_0) \leq t\},$$

which are called an *upper* and *lower t-level cut* of μ and can be used to the characterization of μ.

Definition 1. *A fuzzy set μ in M is called a fuzzy ideal of M if it satisfies:*

$$(\forall m_0, m_1 \in M)\,(\mu(m_0 * m_1) \geq \mu(m_1)); \tag{10}$$
$$(\forall m_0, m_1, m_2 \in M)\,(\mu((m_0 * (m_1 * m_2)) * m_2) \geq \min\{\mu(m_0), \mu(m_1)\}). \tag{11}$$

An *intuitionistic fuzzy set* (IFS) A in M (see [5]) is an object having the form

$$A = \{\langle m_0, \mu_A(m_0), \gamma_A(m_0)\rangle \mid m_0 \in M\} \tag{12}$$

where the functions $\mu_A : M \to [0, 1]$ and $\gamma_A : M \to [0, 1]$ denote the degree of membership (namely $\mu_A(m_0)$) and the degree of nonmembership (namely $\gamma_A(m_0)$) of each element $m_0 \in M$ to the set A, respectively, and

$$0 \leq \mu_A(m_0) + \gamma_A(m_0) \leq 1 \tag{13}$$

for each $m_0 \in M$. For the sake of simplicity, we shall use the symbol $A = \langle M, \mu_A, \gamma_A\rangle$ for the intuitionistic fuzzy set $A = \{\langle m_0, \mu_A(m_0), \gamma_A(m_0)\rangle \mid m_0 \in M\}$. Obviously, every fuzzy set A' corresponds to the following intuitionistic fuzzy set:

$$A' = \{\langle m_0, \alpha_{A'}(m_0), 1 - \alpha_{A'}(m_0)\rangle \mid m_0 \in M\}. \tag{14}$$

Obviously, for an IFS $A = \langle M, \mu_A, \gamma_A\rangle$ in M, when

$$\gamma_A(m_0) = 1 - \mu(m_0) thatis, \mu(m_0) + \gamma_A(m_0) = 1 \tag{15}$$

for every $m_0 \in M$, the IFS A is a fuzzy set. Hence the notion of intuitionistic fuzzy set theory is a generalization of fuzzy set theory. Let A be an IFS in M and let $s, t \in [0, 1]$ be such that $s + t \leq 1$. Then the set $X_A^{(s,t)} := m_0 \in M | \mu(m_0) \geq s,\ \gamma_A(m_0) \leq t$ is called an it (s, t)-level subset of $A = M, \mu_A \gamma(A)$ Note that

$$\begin{aligned} M_A^{(s,t)} &= m_0 \in M \mid \mu(M) \geq s, \gamma_A(m_0) \leq t \\ &= m_0 \in M \mid \mu(m_0) \geq s \cap m_0 \in M \mid \gamma_A(m_0) \leq t \\ &= U(\mu_A; s) \cap L(\gamma_A; t). \end{aligned}$$

3. Intuitionistic fuzzy ideals

In what follows, let M denote a BE-algebra unless otherwise specified.

Definition 2. *An IFS A in M is called an intuitionistic fuzzy ideal of M if it satisfies:*

$$\mu(m_0 * m_1) \geq \mu(m_1), \quad \gamma_A(m_0 * m_1) \leq \gamma_A(m_1), \tag{16}$$

$$\begin{aligned} &\mu((m_0 * (m_1 * m_2)) * m_2) \geq \min\{\mu(m_0), \mu(m_1)\}, \\ &\gamma_A((m_0 * (m_1 * m_2)) * m_2) \leq \max\{\gamma_A(m_0), \gamma_A(m_1)\} \end{aligned} \tag{17}$$

for all $m_0, m_1, m_2 \in M$.

Example 1. *red Let $M = \{1, \alpha, \beta, \gamma, \lambda, 0\}$ be a set with the following Cayley Table1.*

Table 1: Cayley Table of the binary operation $*$

$*$	1	α	β	γ	λ	0
1	1	α	β	γ	λ	0
α	1	1	α	γ	γ	λ
β	1	1	1	γ	γ	γ
γ	1	α	β	1	α	β
λ	1	1	α	1	1	α
0	1	1	1	1	1	1

Then $(M; *, 1)$ is a BE-algebra (see [10]). Let A be an IFS in M given by

$$A = \langle M, \left(\tfrac{1}{0.7}, \tfrac{\alpha}{0.7}, \tfrac{\beta}{0.7}, \tfrac{\gamma}{0.2}, \tfrac{\lambda}{0.2}, \tfrac{0}{0.2}\right), \left(\tfrac{1}{0.1}, \tfrac{\alpha}{0.1}, \tfrac{\beta}{0.1}, \tfrac{\gamma}{0.3}, \tfrac{\lambda}{0.3}, \tfrac{0}{0.3}\right\rangle.$$

Then A is an intuitionistic fuzzy ideal of M.

Example 2. *Let $M = \{1, \alpha, \beta, \gamma, \lambda, 0\}$ be the BE-algebra which is given in Example 1. Let B be an IFS in M given by*

$$B = \langle\langle M, \left(\tfrac{1}{0.6}, \tfrac{\alpha}{0.6}, \tfrac{\beta}{0.3}, \tfrac{\gamma}{0.3}, \tfrac{\lambda}{0.3}, \tfrac{0}{0.3}\right), \left(\tfrac{1}{0.2}, \tfrac{\alpha}{0.2}, \tfrac{\beta}{0.5}, \tfrac{\gamma}{0.5}, \tfrac{\lambda}{0.5}, \tfrac{0}{0.5}\right)\rangle.$$

Then B is not an intuitionistic fuzzy ideal of M since

$$\mu((\alpha * (\alpha * \beta)) * \beta) < \mu(\alpha) = \min\{\mu(\alpha), \mu(\beta)\}$$

and/or

$$\gamma_A((\alpha * (\alpha * \beta)) * \beta) > \gamma_A(\alpha) = \max\{\gamma_A(\alpha), \gamma_A(\alpha)\}.$$

Lemma 1. *Every intuitionistic fuzzy ideal A of M satisfies the following inequality:*

$$(\forall m_0 \in M)(\mu(1) \geq \mu(m_0),\ \gamma_A(1) \leq \gamma_A(m_0)). \tag{18}$$

Proof. Using (1) and (16), we have

$$\mu(1) = \mu(m_0 * m_0) \geq \mu(m_0),\ \gamma_A(1) = \gamma_A(m_0 * m_0) \leq \gamma_A(m_0)$$

for all $m_0 \in M$.

Proposition 1. *If A is an intuitionistic fuzzy ideal of M, then*

$$(\forall m_0, m_1 \in M)\,(\mu((m_0 * m_1) * m_1) \geq \mu(m_0),\ \gamma_A((m_0 * m_1) * m_1) \leq \gamma_A(m_0)). \tag{19}$$

Proof. Taking $m_1 = 1$ and $m_2 = m_1$ in (17) and using (3) and Lemma 1, we get

$$\mu((m_0 * m_1) * m_1) = \mu((m_0 * (1 * m_1)) * m_1) \geq \min\{\mu(m_0), \mu(1)\} = \mu(m_0)$$

and

$$\gamma_A((m_0 * m_1) * m_1) = \gamma_A((m_0 * (1 * m_1)) * m_1) \leq \max\{\gamma_A(m_0), \gamma_A(1)\} = \gamma_A(m_0)$$

for all $m_0, m_1 \in M$.

Corollary 1. *Every intuitionistic fuzzy ideal A of M is intuitionistic order preserving, that is, A satisfies:*

$$(\forall m_0, m_1 \in M)\,(m_0 \leq y \Rightarrow \mu(m_0) \leq \mu(m_1),\ \gamma_A(m_0) \geq \gamma_A(m_1)). \tag{20}$$

Proof. Let $m_0, m_1 \in M$ be such that $m_0 \leq m_1$. Then $m_0 * m_1 = 1$, and so

$$\mu(m_1) = \mu(1 * m_1) = \mu((m_0 * m_1) * m_1) \geq \mu(m_0)$$

and

$$\gamma_A(m_1) = \gamma_A(1 * m_1) = \gamma_A((m_0 * m_1) * m_1) \geq \mu(m_0)$$

by (3) and (19).

Proposition 2. *Let A be an IFS in M which satisfies* (18) *and*

$$\begin{aligned} &\mu(m_0 * m_2) \geq \min\{\mu(m_0 * (m_1 * m_2)), \mu(m_1)\}), \\ &\gamma_A(m_0 * m_2) \leq \max\{\gamma_A(m_0 * (m_1 * m_2)), \gamma_A(m_1)\}) \end{aligned} \tag{21}$$

for all $m_0, m_1, m_2 \in M$. Then A is intuitionistic order preserving.

Proof. Let $m_0, m_1 \in M$ be such that $m_0 \leq m_1$. Then $m_0 * m_1 = 1$, and so

$$\begin{aligned} \mu(m_1) &= \mu(1 * m_1) \geq \min\{\mu(1 * (m_0 * m_1)), \mu(m_0)\} \\ &= \min\{\mu(1 * 1), \mu(m_0)\} = \mu(m_0) \end{aligned}$$

and

$$\begin{aligned} \gamma_A(m_1) &= \gamma_A(1 * m_1) \leq \max\{\gamma_A(1 * (m_0 * m_1)), \gamma_A(m_0)\} \\ &= \max\{\gamma_A(1 * 1), \gamma_A(m_0)\} = \gamma_A(m_0) \end{aligned}$$

by (1), (3), (21) and (18).

We give a characterization of fuzzy ideals.

Theorem 1. *Let M be a transitive BE-algebra. An IFS A in M is an intuitionistic fuzzy ideal of M if and only if it satisfies conditions* (18) *and* (21).

Proof. Assume that A is an intuitionistic fuzzy ideal of M. By Lemma 1, A satisfies (18). Since M is transitive, we have

$$(m_1 * m_2) * m_2 \leq (m_0 * (m_1 * m_2)) * (m_0 * m_2), \tag{22}$$

i.e., $((m_1 * m_2) * m_2) * ((m_0 * (m_1 * m_2)) * (m_0 * m_2)) = 1$ for all $m_0, m_1, m_2 \in M$. It follows from (3), (17) and Proposition 1 that

$$\begin{aligned} \mu(m_0 * m_2) &= \mu(1 * (m_0 * m_2)) \\ &= \mu((((ym_1 * m_2) * m_2) * ((m_0 * (m_1 * m_2)) * (m_0 * m_2))) * (m_0 * m_2)) \\ &\geq \min\{\mu((m_1 * m_2) * m_2), \mu(m_0 * (m_1 * m_2))\} \\ &\geq \min\{\mu(m_0 * (m_1 * m_2)), \mu(m_1)\} \end{aligned}$$

and

$$\begin{aligned} \gamma_A(m_0 * m_2) &= \gamma_A(1 * (m_0 * m_2)) \\ &= \gamma_A((((m_1 * m_2) * m_2) * ((m_0 * (m_1 * m_2)) * (m_0 * m_2))) * (m_0 * m_2)) \\ &\leq \max\{\gamma_A((m_1 * m_2) * m_2), \gamma_A(m_0 * (m_1 * m_2))\} \\ &\leq \max\{\gamma_A(m_0 * (m_1 * m_2)), \gamma_A(m_1)\}. \end{aligned}$$

Hence A satisfies (21). Conversely suppose that A satisfies two conditions (18) and (21). Using (21), (1), (2) and (18), we have

$$\begin{aligned} \mu(m_0 * m_1) &\geq \min\{\mu(m_0 * (m_1 * m_1)), \mu(m_1)\} \\ &= \min\{\mu(m_0 * 1), \mu(m_1)\} \\ &= \min\{\mu(1), \mu(m_1)\} = \mu(m_1), \end{aligned} \tag{23}$$

$$\begin{aligned} \gamma_A(m_0 * m_1) &\leq \max\{\gamma_A(m_0 * (m_1 * m_1)), \gamma_A(m_1)\} \\ &= \max\{\gamma_A(m_0 * 1), \gamma_A(m_1)\} \\ &= \max\{\gamma_A(1), \gamma_A(m_1)\} = \gamma_A(m_1), \end{aligned} \tag{24}$$

$$\begin{aligned} \mu((m_0 * m_1) * m_1) &\geq \min\{\mu((m_0 * m_1) * (m_0 * m_1)), \mu(m_0)\} \\ &= \min\{\mu(1), \mu(m_0)\} = \mu(m_0), \end{aligned} \tag{25}$$

$$\begin{aligned} \gamma_A((m_0 * m_1) * m_1) &\leq \max\{\gamma_A((m_0 * m_1) * (m_0 * m_1)), \gamma_A(m_0)\} \\ &= \max\{\gamma_A(1), \gamma_A(m_0)\} = \gamma_A(m_0) \end{aligned} \tag{26}$$

for all $m_0, m_1 \in M$. Since A is intuitionistic order preserving by Proposition 2, it follows from (22) that

$$\mu((m_1 * m_2) * m_2) \leq \mu((m_0 * (m_1 * m_2)) * (m_0 * m_2))$$

and

$$\gamma_A((m_1 * m_2) * m_2) \geq \gamma_A((m_0 * (m_1 * m_2)) * (m_0 * m_2))$$

so from (21), (25) and (26) that

$$\begin{aligned}\mu((m_0 * (m_1 * m_2)) * m_2) &\geq \min\{\mu(((m_0 * (m_1 * m_2)) * (m_0 * m_2)), \mu(m_0)\} \\ &\geq \min\{\mu((m_1 * m_2) * m_2), \mu(m_0)\} \\ &\geq \min\{\mu(m_0), \mu(m_1)\}\end{aligned}$$

and

$$\begin{aligned}\gamma_A((m_0 * (m_1 * m_2)) * m_2) &\leq \max\{\gamma_A(((m_0 * (m_1 * m_2)) * (m_0 * m_2)), \gamma_A(m_0)\} \\ &\leq \max\{\gamma_A((m_1 * m_2) * m_2), \gamma_A(m_0)\} \\ &\leq \max\{\gamma_A(m_0), \gamma_A(m_1)\}\end{aligned}$$

for all $m_0, m_1, m_2 \in M$. Hence A is an intuitionistic fuzzy ideal of M.

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