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Formulas and Properties of 2-Hop Domination in Some Graphs

Javier A. Hassan[1,*], Anabel E. Gomorez[2], Ladznar S. Laja[1], Eman C. Ahmad[2]

[1] *Mathematics and Statistics Department, College of Arts and Sciences, MSU Tawi-Tawi College of Technology and Oceanography, Bongao, Tawi-Tawi, Philippines*
[2] *Department of Mathematics and Statistics, College of Science and Mathematics, Western Mindanao State University, Zamboanga City, Philippines*

Abstract. In this paper, 2-hop domination parameter is introduced and investigated on some special graphs and on the join of two graphs. Characterizations of 2-hop dominating sets in some special graphs are formulated to derive bounds or formulas of the parameter of these graphs. Moreover, new variant of pointwise non-domination is introduced to characterize 2-hop dominating sets in the join of two graphs. This characterization is used to calculate the exact value of 2-hop domination number of the join of two graphs.

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1. Introduction

Hop domination is the variant of the standard domination and was introduced by Natarajan et al. in [12]. Hop domination has applications in various fields such as network design, communication networks, and facility location problems. Researchers continue to explore various aspects of hop domination, including its computational complexity, structural properties, and applications in real-world networks. As graph theory and its applications continue to evolve, hop domination remains an active area of research, contributing to our understanding of network dynamics and optimization. Researchers in the field had further investigated this concept, and its variants. They have obtained some significant results that contributed a lot to the hop domination theory (see [1–11]).

In this paper, new variant of hop domination called 2-hop domination is introduced and studied on some special graphs and on the join of two graphs. The researchers believe that this parameter and its results would contribute positively to the field of graph theory and would help other researchers in the field for more research directions in the future.

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Email address: `javierhassan@msutawi-tawi.edu.ph` (J. Hassan),
`anabel.gamorez@wmsu.edu.ph` (A. Gamorez), `ladznarlaja@msutawi-tawi.edu.ph` (L. Laja),
`ahmad.eman@wmsu.edu.ph` (E. Ahmad)

2. Terminology and Notation

Let $G = (V(G), E(G))$ be a simple and undirected graph. The *distance* $d_G(a, b)$ in G of two vertices a, b is the length of a shortest *a-b path* in G.

Two vertices x, y of G are *adjacent* or *neighbors*, if $d_G(x, y) = 1$. The *open neighborhood* of x in G is the set $N_G(x) = \{y \in V(G) : d_G(x, y) = 1\}$. The *closed neighborhood* of x in G is the set $N_G[x] = N_G(x) \cup \{x\}$. If $X \subseteq V(G)$, the *open neighborhood* of X in G is the set $N_G(X) = \bigcup_{x \in X} N_G(x)$. The *closed neighborhood* of X in G is the set $N_G[X] = N_G(X) \cup X$.

A vertex a in G is a *hop neighbor* of a vertex b in G if $d_G(a, b) = 2$. The set

$$N_G^2(a) = \{b \in V(G) : d_G(a, b) = 2\}$$

is called the *open hop neighborhood* of a. The *closed hop neighborhood* of a in G is given by $N_G^2[a] = N_G^2(a) \cup \{a\}$. The *open hop neighborhood* of $S \subseteq V(G)$ is the set $N_G^2(S) = \bigcup_{a \in S} N_G^2(a)$. The *closed hop neighborhood* of S in G is the set $N_G^2[S] = N_G^2(S) \cup S$.

A subset S of $V(G)$ is a *hop dominating* of G if for every $a \in V(G)\backslash S$, there exists $b \in S$ such that $d_G(a, b) = 2$. The minimum cardinality among all hop dominating sets of G, denoted by $\gamma_h(G)$, is called the *hop domination number* of G. Any hop dominating set with cardinality equal to $\gamma_h(G)$ is called a γ_h*-set* of G.

A subset C of $V(G)$ is a *pointwise non-dominating set* if for every $v \in V(G)\backslash C$, there exists $u \in C$ such that $v \notin N_G(u)$. The minimum cardinality of a pointwise non-dominating set in G, denoted by $pnd(G)$, is called a *pointwise non-domination number* of G.

Let G and H be any two graphs. The *join* of G and H, denoted by $G + H$ is the graph with vertex set $V(G + H) = V(G) \cup V(H)$ and edge set

$$E(G + H) = E(G) \cup E(H) \cup \{uv : u \in V(G), v \in V(H)\}.$$

3. Results

We begin this section by introducing the concept of 2-hop domination in a graph.

Definition 1. Let G be a simple and undirected graph. A subset P of a vertex-set $V(G)$ of G is called a 2-hop dominating if for every $x \in V(G)\backslash P$, x has at least two hop neighbors in P. The 2-hop domination number of G, denoted by $\gamma_{2h}(G)$, is the minimum cardinality of a 2-hop dominating set of G.

To further understand the aforementioned concept, consider the following example:

Example 1. Consider the graph G below.

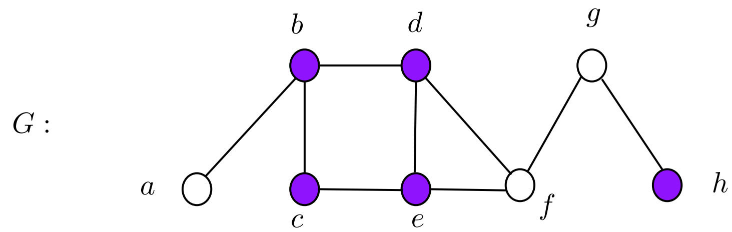

Figure 1: Graph G with $\gamma_{2h}(G) = 5$

Let $P = \{b, c, d, e, h\}$. Then $V(G)\backslash P = \{a, f, g\}$. Observe that vertex a has two hop neighbors c and d in P, f has three hop neighbors b, c and h in P, and g has two neighbors d and e in P. Thus, P is a 2-hop dominating set of G. Moreover, it can be verified that $\gamma_{2h}(G) = 5$.

Remark 1. *Let G be a graph. Then*

(i) a 2-hop dominating set P of G is always a hop dominating; and

(ii) G admits a 2-hop domination.

Theorem 1. *Let G be a graph. Then*

(i) $\gamma_h(G) \leq \gamma_{2h}(G)$, and this bound is tight;

(ii) $1 \leq \gamma_{2h}(G) \leq |V(G)|$; and

(iii) if G has $|V(G)| \geq 2$, then $\gamma_{2h}(G) \geq 2$.

Proof. (i) Let G be a graph and let P be a minimum 2-hop dominating set of G. Then $\gamma_{2h}(G) = |P|$. Since every 2-hop dominating is a hop dominating, it follows that $\gamma_h(G) \leq |P| = \gamma_{2h}(G)$. For tightness, consider K_n. Then $\gamma_{2h}(K_n) = n = \gamma_h(G)$.

(ii) Since $\gamma_h(G) \geq 1$ for any graph G, by(i), $\gamma_{2h}(G) \geq 1$. Now, Since every 2-hop dominating set P is always a subset of $V(G)$, it follows that $\gamma_{2h}(G) \leq |V(G)|$. Therefore, $1 \leq \gamma_{2h}(G) \leq |V(G)|$.

(iii) Suppose that $\gamma_{2h}(G) = 1$. Then $P = \{a\} \subseteq V(G)$ is minimum 2-hop dominating set of G for some $a \in V(G)$. Since every 2-hop dominating is a hop dominating, $N_G^2[a] = V(G)$. Assume that there is $b \in V(G)\backslash P$ such that $b \in N_G^2[a]$. Let $x \in N_G(a) \cap N_G(b)$. Then $x \notin N_G^2[a]$, a contradiction. Therefore, $N_G^2[a] = \{a\}$, and so G is trivial. Consequently, $|V(G)| = 1$, a contradiction. $\square$

Theorem 2. *Let G be a graph. Then $\gamma_{2h}(G) = 2$ if and only if $G = K_2$ or $G = \overline{K}_2$.*

Proof. Suppose that $\gamma_{2h}(G) = 2$, say $P = \{a, b\}$ is a minimum 2-hop dominating set of G. Then $N_G^2[P] = V(G)$ since every 2-hop dominating set P is a hop dominating. Assume that G is connected. If $d_G(a, b) = 2$, then there exists $y \in V(G)\backslash P$

such that $y \in N_G(a) \cap N_G(b)$. However, $y \notin N_G^2[P]$, a contradiction. Similarly, when $d_G(a,b) = 3, 4, \ldots, n-1$, where n is the order of G. Thus, $d_G(a,b) = 1$, and so $G = K_2$.

Now, assume that G is disconnected. Let $G_1, \ldots, G_k$, $k \geq 2$, be conponents of G. Since $\gamma_{2h}(G) = 2$, it follows that $k = 2$. That is, there are only 2 components of G. If G_1 is non-trivial, then $\gamma_{2h}(G_1) \geq 2$ by Theorem 1 (*iii*). Since G has two components, it follows that $\gamma_{2h}(G) \geq 3$, a contradiction. Similarly, when G_2 is non-trivial. Therefore, both G_1 and G_2 are trivial, and so $G = \overline{K}_2$.

Conversely, suppose that $G = K_2$. Then $\gamma_h(G) = 2$. Since $\gamma_{2h}(G) \geq \gamma_h(G)$, it follows that $\gamma_{2h}(G) \geq 2$. Since $|V(G)| = 2$, $\gamma_{2h}(G) = 2$ by Theorem 1(ii). Similarly, if $G = \overline{K}_2$, then $\gamma_{2h}(G) = 2$. □

Theorem 3. *Let G be a graph. Then*

(i) $\gamma_{2h}(G) = |V(G)|$ if and only if $|N_G^2[x]| \leq 2$ for every $x \in V(G)$; and

(ii) If $\gamma_h(G) = |V(G)|$, then $\gamma_{2h}(G) = |V(G)|$. However, the converse is not true.

Proof. (i) Suppose that $\gamma_{2h}(G) = |V(G)|$. Then $V(G)$ is the minimum 2-hop dominating set of G, that is, $N_G^2[V(G)] = V(G)$. Assume that $|N_G^2[x]| \geq 3$ for some $x \in V(G)$. Then there exist $u, v \in V(G)$ such that $u, v \in N_G^2(x)$. Let $P = V(G) \backslash \{x\}$. Then P is a 2-hop dominating set of G. Thus, $\gamma_{2h}(G) \leq |V(G)| - 1$, a contradiction. Therefore, $|N_G^2[x]| \leq 2$ for all $x \in V(G)$.

Conversely, suppose that $|N_G^2[x]| \leq 2$ for all $x \in V(G)$. If $N_G^2[x] = \{x\}$ for all $x \in V(G)$, then we are done. Assume that $|N_G^2[x]| = 2$ for all $x \in V(G)$. Then there exists a unique $y \in V(G)$ such that $y \in N_G^2(x)$. Let P be a 2-hop dominating set of G. Since P is a hop dominating, either x or y is in P. Assume that $x \in P$. Assume further that $y \notin P$. Since P is a 2-hop dominating, there exists $w \in P$ such that $d_G(w,y) = 2$. It follows that $w, x \in N_G^2(y)$. Thus $|N_G^2[y]| \geq 3$, a contradiction. Therefore, $y \in P$. Similarly, when $y \in P$, then $x \in P$. Since x is arbitrary, it follows that $V(G)$ is the minimum 2-hop dominating set of G. Consequently, $\gamma_{2h}(G) = |V(G)|$.

(ii) Suppose that $\gamma_h(G) = |V(G)|$. Then by Theorem 1, $\gamma_{2h}(G) = |V(G)|$. To see that the converse is not true, conisder P_4. Then $\gamma_{2h}(P_4) = 4$ by (i). However, $\gamma_h(P_4) = 2$. □

The following definition will be used to characterize 2-hop dominating sets in the join of two graphs.

Definition 2. Let G be a graph. A subset N of a vertex-set $V(G)$ of G is called a *2-pointwise non-dominating* if for every $x \in V(G) \backslash N$, there exist at least two distinct vertices $a, b \in N$ such that $x \notin N_G(a)$ and $x \notin N_G(b)$. The minimum cardinality of a 2-pointwise non-dominating set of G is the *2-pointwise non-domination number* of G, and is denoted by $pnd_2(G)$.

Remark 2. *Let G be a graph. Then* *(i) every 2-pointwise non-dominating set is a pointwise non-dominating;*

(ii) $pnd(G) \leq pnd_2(G)$*;*

(iii) $pnd_2(K_n) = n$ *for all positive integer* $n \geq 1$*;*

(iv) $pnd_2(\overline{K}_n) = \begin{cases} 1 \ , \ n = 1 \\ 2 \ , \ n \geq 2; and \end{cases}$

(v) $1 \leq pnd_2(G) \leq |V(G)|$.

Proposition 1. *Let n be a positive integer. Then*

(i) $pnd_2(P_n) = \begin{cases} n \ , \ 1 \leq n \leq 3 \\ 3 \ , otherwise \end{cases}$

(ii) $pnd_2(C_n) = \begin{cases} n \ , \ n = 3, 4 \\ 3 \ , \ otherwise \end{cases}$

Proof. (i) Since $pnd(P_n) = n$ for $n = 1, 2$, it follows that $pnd_2(P_n) = n$ for $n = 1, 2$. For $n = 3$, let $V(P_3) = \{v_1, v_2, v_3\}$. Since $pnd(P_3) = 2$, $pnd_2(P_3) \geq 2$. If $pnd_2(P_3) = 2$, then there exists $v_i \in V(P_3)$ such that $v_i \notin P$, where P is a minimum 2-pointwise non-dominating set of P_3 for some $i \in \{1, 2, 3\}$. Suppose that $v_i = v_1$. Then v_3 is the only vertex in P such that $v_1 \notin N_G(v_3)$, which is a contradiction. Similarly, if $v_i = v_3$. Now, assume that $v_i = v_2$. Observe that vertices v_1 and v_3 are both adjacent to v_2, that is, $v_1, v_3 \in N_{P_3}(v_2)$, a contradiction. Therefore, $pnd_2(P_3) = 3$.

Suppose that $n \geq 4$. Let $V(P_n) = \{v_1, v_2, v_3, \ldots, v_n\}$. Since $pnd(P_n) = 2$ for all $n \geq 4$, it follows that $pnd_2(P_n) \geq 2$ for all $n \geq 4$. Assume that $pnd_2(P_n) = 2$ for all $n \geq 4$. Let $Q = \{v_i, v_j\}$ be a minimum 2-pointwise non-dominating set of P_n, where $j < i, i, j \in \{1, 2, \ldots, n\}$. Assume that $\langle Q \rangle$ is connected. If $v_i = v_n$, then $v_j = v_{n-1}$. Thus, $v_i = v_n$ is the only vertex in Q such that $v_{n-2} \notin N_{P_n}(v_i)$, a contradiction. If $v_i = v_2$, then $v_j = v_1$. Thus, $v_j = v_1$ is the only vertex in Q such that $v_3 \notin N_{P_n}(v_j)$, a contradiction. Similarly, when $v_i = v_r$, where $r \in \{3, 4, \ldots, n-1\}$.

Next, assume that $\langle Q \rangle$ is disconnected. If $d_{P_n}(v_i, v_j) = 2$, then there exists $v_s \in V(P_n) \backslash Q$ such that $v_s \in N_{P_n}(v_i) \cap N_{P_n}(v_j)$ for some $s \neq i, j$, which is a contradiction. Thus, $d_{P_n}(v_i, v_j) \geq 3$. Let $v_k \in N_{P_n}(v_i)$. Then v_j is the only vertex in Q such that $v_k \notin N_{P_n}(v_j)$, a contradiction. Therefore, $pnd_2(P_n) \geq 3$ for all $n \geq 4$.

Now, let $S = \{v_1, v_2, v_3\} \subseteq V(P_n)$. Let $v_t \in V(P_n) \backslash S$, where $t \in \{4, 5, \ldots, n\}$. Then $v_t \notin N_{P_n}(v_1)$ and $v_t \notin N_{P_n}(v_2)$. It follows that S is a 2-pointwise non-dominating set of P_n. Consequently, $pnd_2(P_n) = 3$ for all $n \geq 4$.

(ii) Clearly, $pnd_2(C_3) = 3$. Let $n = 4$ and let $V(C_n) = \{a_1, a_2, a_3, a_4\}$. Since $pnd(C_4) = 2$, $pnd_2(C_4) \geq 2$. Suppose that $pnd_2(C_4) = 2$. Let S be a minimum 2-pointwise non-dominating set of C_4. Since 2-pointwise non-dominating is a pointwise non-dominating, $\langle S \rangle$ is connected. Thus, $\{a_1, a_2\}$, $\{a_2, a_3\}$, $\{a_3, a_4\}$ or $\{a_1, a_4\}$ is the possible set S. If $S = \{a_1, a_2\}$, then a_1 is the only vertex in S such that $a_3 \notin N_{C_n}(a_1)$, a contradiction.

Next, suppose that $S = \{a_2, a_3\}$, then a_2 is the only vertex in S such that $a_4 \notin N_{C_4}(a_2)$, a contradiction. Similarly, when $S = \{a_3, a_4\}$, or $S = \{a_1, a_4\}$. Therefore, $pnd_2(C_4) \geq 3$. Suppose that $pnd_2(C_4) = 3$. Then there exists $a_i \in V(C_4)$ which is not in 2-pointwise non-dominating set R of C_4 for some $i \in \{1, 2, 3, 4\}$. Assume that $a_i = a_1$. However, a_3 is the only vertex in R such that $a_i \notin N_{C_4}(a_3)$, a contradiction. Similarly, when $a_i = a_2, a_3$ or a_4. Consequently, $pnd_2(C_4) = 4$.

Now, assume that $n \geq 5$. Let $V(C_n) = \{a_1, a_2, \ldots, a_n\}$ and consider $B = \{a_1, a_2, a_3\}$. Then, B is a minimum 2-pointwise non-dominating set of C_n. Thus, $pnd_2(C_n) = 3$ for all $n \geq 5$. □

Theorem 4. *Let G and H be two graphs. Then a subset P of a vertex-set of $G + H$ is a 2-hop dominating if and only if $P = P_G \cup P_H$, where $P_G \subseteq V(G)$ and $P_H \subseteq V(H)$ are 2-pointwise non-dominating sets of G and H, respectively.*

Proof. Suppose that P is a 2-hop dominating set of $G + H$. Assume that $P_G = \varnothing$. Then $P = P_H \subseteq V(H)$. Since $N_{G+H}[P_H] \subseteq V(H)$, $N_{G+H}[P_H] \neq V(G + H)$, which is a contradiction. Therefore, $P_G \neq \varnothing$. Similarly, $P_H \neq \varnothing$. Now, let $x \in V(G) \backslash P_G$. Since P is a 2-hop dominating set in $G + H$, there exist $y, w \in P_G \subseteq P$ such that $d_{G+H}(x, y) = 2$ and $d_{G+H}(w, x) = 2$. Thus, $x \notin N_G(y)$ and $x \notin N_G(w)$. Hence, P_G is 2-pointwise non-dominating set of G. Similarly, P_H is a 2-pointwise non-dominating set of H.

Conversely, suppose that $P = P_G \cup P_H$, where P_G and P_H are 2-pointwise non-dominating sets of G and H, respectively. Let $a \in V(G+H) \backslash P$. Then either $a \in V(G) \backslash P_G$ or $a \in V(H) \backslash P_H$. Suppose that $a \in V(G) \backslash P_G$. Since P_G is a 2-pointwise non-dominating set of G, there exist $u, v \in P_G \subseteq P$ such that $d_G(a, u) \geq 2$ and $d_G(a, v) \geq 2$. It follows that $a \in N^2_{G+H}[u]$ and $a \in N^2_{G+H}[v]$. Hence, P is a 2-hop dominating set of $G + H$. Similarly, when $a \in V(H) \backslash P_H$, then P is a 2-hop dominating set of $G + H$. □

Corollary 1. *Let G and H be two graphs. Then*

$$\gamma_{2h}(G + H) = pnd_2(G) + pnd_2(H).$$

In particular, each of the following holds:

$$(i)\ \ \gamma_{2h}(P_n + P_m) = \begin{cases} n + m & , \textit{if } 1 \leq n, m \leq 3 \\ n + 3 & , \textit{if } 1 \leq n \leq 3 \textit{ and } m \geq 4 \\ m + 3 & , \textit{if } 1 \leq m \leq 3 \textit{ and } n \geq 4 \\ 6 & , \textit{if } n, m \geq 4. \end{cases}$$

$$(ii)\ \ \gamma_{2h}(C_n+C_m)=\begin{cases} n+m\ , \textit{if}\ n,m=3,4\\ n+3\ , \textit{if}\ n=3,4\ \textit{and}\ m\geq 5\\ m+3\ , \textit{if}\ m=3,4\ \textit{and}\ n\geq 5\\ 6\qquad , \textit{if}\ n,m\geq 5.\end{cases}$$

(iii) $\gamma_{2h}(K_n+K_m)=n+m$ *for all positive integer* $n,m\geq 1$.

$$(iv)\ \ \gamma_{2h}(F_n)=\gamma_{2h}(K_1+P_n)=\begin{cases} n+1\ , \textit{if}\ \ 1\leq n\leq 3\\ 4\qquad , \textit{if}\ \ n\geq 4\end{cases}$$

$$(v)\ \ \gamma_{2h}(W_n)=\gamma_{2h}(K_1+C_n)=\begin{cases} n+1\ , \textit{if}\ \ n=3,4\\ 4\qquad , \textit{if}\ \ n\geq 5\end{cases}$$

Proof. Let P be a minimum 2-hop dominating set of $G+H$. Then by Theorem 4, $P=P_G\cup P_H$, where P_G and P_H are 2-pointwise non-dominating sets of G and H, respectively. Thus, $pnd_2(G)\leq |P_G|$ and $pnd_2(H)\leq |P_H|$. Hence,

$$\gamma_{2h}(G+H)=|P|=|P_G|+|P_H|\geq pnd_2(P_G)+pnd_2(P_H).$$

On the other hand, let $P=P_G\cup P_H$, where P_G and P_H are minimum 2-pointwise non-dominating sets of G and H, respectively. Then by Theorem 4, P is a 2-hop dominating set of $G+H$. Thus, $\gamma_{2h}(G+H)\leq |P|=|P_G|+|P_H|=pnd_2(G)+pnd_2(H)$. Consequently,

$$\gamma_{2h}(G+H)=pnd_2(G)+pnd_2(H).$$

Moreover, (i),(ii), (iii),(iv) and (v) follow from Remark 2 and Proposition 1. □

4. Conclusion

The 2-hop domination parameter has been introduced and initially investigated in this paper. This new parameter is always defined on any simple and undirected graph. Its properties and its connections with hop domination have been presented. Moreover, this parameter has been investigated on the join of two graphs. The 2-hop dominating sets in the join of two graphs have been characterized and used to derive some formulas of the parameter. Interested researchers may further investigate this concept on other graphs that were not considered in this study. They may also consider the possibility of applying this newly defined parameter to another field.

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