A Novel Fractional Edge Detector Based on Generalized Fractional Operator

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\begin{abstract}
This work pioneers a novel approach in image edge detection through the utilization of the generalized fractional operator. By harnessing the global attributes inherent in fractional derivatives, it aims to enhance the extraction of intricate edge details. This is accomplished by creating the mask by using fractional derivative and adapt the mask by another parameter, yielding compelling and informative edge representations, as validated by experimental results. This advancement not only augments computer vision and image analysis techniques but also holds promise for refining image processing methodologies. Future endeavors may explore its adaptability across diverse imaging domains like medical and satellite imagery, while integration into deep learning frameworks could elevate its potential for advanced feature extraction and deeper image understanding. Additionally, optimizing its computational efficiency would broaden its scope for real-time deployment in fields such as robotics and autonomous systems.

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\textbf{Key Words and Phrases:} Fractional-order, Edge detection, finite difference
\end{abstract}

1. Introduction

Edge detection is a foundational process in the realm of image processing, playing a pivotal role in various applications such as image segmentation, image compression, and image reconstruction [8, 13, 27, 32, 33, 44, 47]. Its primary objective is to pinpoint and delineate abrupt transitions in intensity or color within an image, often signifying object...
boundaries or other salient image features. Broadly categorized into two main methodologies, edge detection methods can be classified as either gradient-based or Laplacian-based [3, 7, 28, 29, 31, 35, 37, 39, 42, 43].

Gradient-based techniques identify edges by scrutinizing the gradient, or the first derivative, of the image. These methods aim to locate points in the image where intensity changes occur rapidly, typically marked by maximum or minimum values in the first derivative. However, one limitation of gradient-based approaches is their tendency to produce thicker edge representations, potentially leading to the loss of finer image details[6, 11, 34].

In contrast, generalized fractional Laplace operator concentrate on identifying zero-crossings in the fractional derivative. These zero-crossings frequently coincide with the positions of edges. Generalized fractional Laplace operator-based methods exhibit greater sensitivity to finer image details but may be more susceptible to the influence of noise in the image.

To tackle the challenge of striking a balance between edge thickness and noise sensitivity, fractional-order derivatives have been introduced[10, 18, 22]. Fractional derivatives expand upon the concept of integer derivatives, including first and second derivatives, by encompassing an infinite number of terms. This results in fractional derivatives serving as global operators, allowing them to consider information from a broader set of neighboring pixels. Consequently, fractional derivatives have the potential to retain more image edge details while ameliorating the effects of noise to some extent[5, 24, 45]. Significantly, researchers such as Pu et al. have proposed fractional differential masks designed to enhance image textures [17, 38, 41], and Bai and Feng have introduced fractional-order anisotropic diffusion models tailored for noise removal [26]. In this context, this paper introduces an innovative fractional-order Gr unwald-Letnikov operator with the specific aim of enhancing the structural features within an image, addressing the trade-off between edge thickness and sensitivity to noise. Experimental results showcase the promise of this operator in preserving image edge details, comparing favorably to traditional first-order and second-order Laplacian operators [30].

In comparison to other methods[23, 25, 36, 40, 46], the distinctiveness of this approach lies in its utilization of fractional derivatives and the development of a thresholding technique based on the mean fractional-order gradient, which collectively contribute to improved edge extraction and representation. The experimental validation further strengthens the credibility of the proposed method, suggesting its viability for practical applications in image processing and computer vision.

In summation, edge detection is a fundamental aspect of image processing, with the choice of method contingent on the specific application and the desired equilibrium between edge thickness and sensitivity to noise. Fractional-order derivatives present an intriguing avenue for achieving this equilibrium, with the proposed generalized fractional operator demonstrating substantial potential in safeguarding image edge details.
2. Basic Definitions and Tools

**Definition 1** [30]. Grünwald-Letnikov fractional derivative and integral definitions: Let \( \alpha \) be any arbitrary positive real number, define \( \alpha \) order derivative and \( \alpha \) order integral of a continuous function \( f(x) \):

\[
 a^G D_x^\alpha f(x) = \lim_{h \to 0} \frac{1}{n^\alpha} \sum_{k=0}^{\infty} (-1)^k \binom{\alpha}{k} f(x - kh)
\]
and

\[
a^G D_x^{-\alpha} f(x) = \lim_{h \to 0} \frac{1}{n^\alpha} \sum_{k=0}^{\infty} \binom{\alpha}{k} f(x - kh).
\]

(1)

Riemann-Liouville fractional derivative and integral definitions [16]: Let \( m \) be positive integer, \( n < \alpha < n + 1 \), define Riemann-Liouville \( \alpha \) order derivative and \( \alpha \) order integral of a continuous function \( f(x) \):

\[
a^R D_x^\alpha f(x) = \left( \frac{d}{dt} \right)^{n+1} \int_a^x (x - \tau)^{n-\alpha} f(\tau) d\tau.
\]
and

\[
a^R D_x^{-\alpha} f(x) = \frac{1}{\Gamma(n - \alpha)} \int_a^x (x - \tau)^{n-\alpha} f(\tau) d\tau.
\]

(2)

Define Liouville-Caputo fractional derivative and integral definitions[16]: Suppose that \( f(x) \) has \( m \) order continuous derivative, define Caputo’s \( \alpha \) order derivative and \( \alpha \) order integral:

\[
a^C D_x^\alpha f(x) = \frac{1}{\Gamma(n - \alpha)} \int_a^x \frac{f^n(\tau)}{(x - \tau)^{\alpha+1-n}} d\tau, (n - 1 \leq \alpha \leq n),
\]
and

\[
a^C D_x^{-\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x (x - \tau)^{\alpha-1} f(\tau) d\tau.
\]

(3)

**Definition 2** [20]. If \( f \) is a continuous function, then the generalized fractional integral denoted by \( I_{a+}^{\alpha,\rho} f(x) \), \( \alpha > 0 \), and \( \rho > 0 \), is given by the following:

\[
 I_{a+}^{\alpha,\rho} f(x) = \frac{\rho^{1-\alpha}}{\Gamma(\alpha)} \int_a^x (x^\rho - s^\rho)^{\alpha-1} f(s) ds, \alpha > 0, x > a.
\]

For \( n - 1 < \alpha \leq n \) where \( n \in \mathbb{N} \).
3. Edge Detection Operator

An essential area of image processing is edge detection. It contains methods for locating pixels in a digital image where the brightness intensity differs significantly from neighboring pixels. Numerous effective algorithms are suggested in the literature to identify edges. The majority of these algorithms are built using second-order differential operators, like the Laplace operator, and first-order operators, like the Sobel, Prewitt, and Roberts operators.

This feature might cause the matching masks to perform poorly in real-world applications. The performance of these algorithms to extract edges from images can also be hindered by noise [15, 19].

The Prewitt operator is a widely used filter for identifying an image’s edges. The foundation of this strategy is the central difference is used to approximate the first-order derivative in this method. The image is convoluted using the following two kernels to obtain the method’s results:

\[
\begin{align*}
    h_x &= \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \\
    h_y &= \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}
\end{align*}
\]

The Sobel operator, which is based on central finite differences, is another significant filter. In contrast to the Prewitt operator, the method’s primary goal is to increase the number of partnerships with pixels that are closer to the mask’s center. The following convolution kernels are employed in this method:

\[
\begin{align*}
    f_x &= \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \\
    f_y &= \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}
\end{align*}
\]

Although the Sobel operator will identify many false edges with a coarse edge width, it can also provide more accurate edge direction information. The Sobel operator is more sensitive to the diagonal edges than the horizontal and vertical edges, whereas the Prewitt operator is more sensitive to the horizontal and vertical edges. The integral differential operators of integer-order operators provide the foundation of each of the aforementioned kernels. Considering the notions of fractional difference has resulted in some significant advancements in this field. Fractional differential operators have produced amazing results in recent years when used to increase image quality, image texture enhancement, image noise reduction, and image edge analysis [4, 12]. One of the key equations for fractional differential operator expansion in images processing is to use the following general form:

\[
D^n f(s) \approx c_0 f(s) + c_1 f(s - 1) + c_2 f(s - 2) + c_3 f(s - 3) + \ldots, \quad (4)
\]
where the successive coefficients in the expansion of Eq.(4) are denoted by \( c_1 \), \( c_2 \), and \( c_3 \).

In our method, we generalized the fractional operator in Eq.(4) to the new following operator

\[
D^{\alpha,\rho} f(s) \approx c_0 f(s) + c_1 f(s-1) + c_2 f(s-2) + c_3 f(s-3)....
\]

(5)

In light of this definition, the expansion can be extended in the following ways to the two-dimensional spaces of the images in the \( x \) and \( y \) directions:

\[
D^{\alpha,\rho}_x f(x, y) \approx c_0 f(x, y) + c_1 f(x-1, y) + c_2 f(x-2, y) + c_3 f(x-3, y)....
\]

\[
D^{\alpha,\rho}_y f(x, y) \approx c_0 f(x, y) + c_1 f(x, y-1) + c_2 f(x, y-2) + c_3 f(x, y-3)....
\]

(6)

In the remainder of the paper, the two structures in Eq.(5) will be used to make new fractional-order mask.

In this article, we build a \( 3 \times 3 \) fractional integral mask by the following way:

\[
f_x = \begin{bmatrix}
-c_0 & 0 & c_0 \\
-c_1 & 0 & c_1 \\
-c_2 & 0 & c_2
\end{bmatrix}
\]

\[
f_y = \begin{bmatrix}
c_0 & c_1 & c_2 \\
0 & 0 & 0 \\
-c_0 & -c_1 & -c_2
\end{bmatrix}
\]

This kernel’s design applies the vertical and horizontal pixels surrounding the central pixel. This kernel is a great tool for extracting texture and edges from images because of this important feature. Following the creation of these kernels, the gradient moduli are typically approximated as follows using the absolute values:

\[
|E| = |E_x| + |E_y|
\]

where:

\[
E_x = I(x, y) * f_x, \quad E_y = I(x, y) * f_y
\]

and \( I(x, y) \) is the pixel value of the gray scale image.

4. **New edge detector based on Generalized Fractional-order.**

The application of fractional-order derivative has been observed in various scientific domains see [41, 43], such as image processing.

To construct our method we approximate the fractional integral in definition (2) by the following way:

\[
I^{\alpha,\rho}_{a+}f(x) = \frac{\rho^{1-\alpha}}{\Gamma(\alpha)} \int_a^x (x^\rho - s^\rho)^{\alpha-1} f(s) ds, \alpha > 0, x > a.
\]

(7)

let \( f : [0, b] \rightarrow R \) and \( I^{\alpha,\rho}_{a+}f \) is to be determined on \([0, b]\), choose \( n + 1 \) grid points:

\( 0 = x_0 < x_1 < x_2... < x_n = b \).
We can define the function \( f_n = [0, b] \rightarrow R \) using piecewise constant approach:

\[
    f_n(x) = f(x_j), \quad x \in [x_j, x_{j+1}].
\]

(8)

So we can uses \( I_{a+}^{\alpha, \rho} f_n \) as an approximation for \( I_{a+}^{\alpha, \rho} f \).

After doing some necessarily mathematical (see [12, 21])calculations finally we get the flowing formula:

\[
    I_{a+}^{\alpha, \rho} f(x_j) = \frac{h^\alpha \rho^{\alpha-2}}{\Gamma(\alpha + 1)} \sum_{k=0}^{j-1} f(x_k) [ (j^\rho - k^\rho)^\alpha - (j^\rho - (k + 1)^\rho)^\alpha ].
\]

(9)

where \( h = \frac{b}{n} \) is equal step size in the grid \([x_j], x_j = jh\).

Now we can using Eq.(9) to find the coefficients \( c_0, c_1, c_2 \) in Eq.(5):

\[
    \begin{align*}
    I_{a+}^{\alpha, \rho} f(x_j) &= \frac{h^\alpha \rho^{\alpha-2}}{\Gamma(\alpha + 1)} [(j^\rho - (j - 1)^\rho)^\alpha] f(x_{j-1}) \\
    I_{a+}^{\alpha, \rho} f(x_{j-1}) &= \frac{h^\alpha \rho^{\alpha-2}}{\Gamma(\alpha + 1)} [( (j - 1)^\rho - (j - 2)^\rho)^\alpha] f(x_{j-2}) \\
    I_{a+}^{\alpha, \rho} f(x_{j-2}) &= \frac{h^\alpha \rho^{\alpha-2}}{\Gamma(\alpha + 1)} [( (j - 2)^\rho - (j - 3)^\rho)^\alpha] f(x_{j-3}).
    \end{align*}
\]

(10)

Now, by taking \( h = 1 \) in Eq.(10) we obtain the following expansions:

\[
    \begin{align*}
    I_x^{\alpha, \rho} f(x, y) &= \frac{h^\alpha \rho^{\alpha-2}}{\Gamma(\alpha + 1)} [(j^\rho - (j - 1)^\rho)^\alpha] f(x, y) \\
    &\quad + \frac{h^\alpha \rho^{\alpha-2}}{\Gamma(\alpha + 1)} [( (j - 1)^\rho - (j - 2)^\rho)^\alpha] f(x - 1, y) \\
    &\quad + \frac{h^\alpha \rho^{\alpha-2}}{\Gamma(\alpha + 1)} [( (j - 2)^\rho - (j - 3)^\rho)^\alpha] f(x - 2, y),
\end{align*}
\]

(11)

\[
    \begin{align*}
    I_y^{\alpha, \rho} f(x, y) &= \frac{h^\alpha \rho^{\alpha-2}}{\Gamma(\alpha + 1)} [(j^\rho - (j - 1)^\rho)^\alpha] f(x, y) \\
    &\quad + \frac{h^\alpha \rho^{\alpha-2}}{\Gamma(\alpha + 1)} [( (j - 1)^\rho - (j - 2)^\rho)^\alpha] f(x, y - 1) \\
    &\quad + \frac{h^\alpha \rho^{\alpha-2}}{\Gamma(\alpha + 1)} [( (j - 2)^\rho - (j - 3)^\rho)^\alpha] f(x, y - 2).
\end{align*}
\]

(12)
Therefore, the coefficients for the Generalized Fractional-order are determined as follows:

\[ c_0 = \frac{h^\alpha \rho^{\alpha-2}}{\Gamma(\alpha+1)} [(j^\rho - (j-1)^\rho)^\alpha] \]

\[ c_1 = \frac{h^\alpha \rho^{\alpha-2}}{\Gamma(\alpha+1)} [((j-1)^\rho - (j-2)^\rho)^\alpha] \]

\[ c_2 = \frac{h^\alpha \rho^{\alpha-2}}{\Gamma(\alpha+1)} [((j-2)^\rho - (j-3)^\rho)^\alpha]. \]

(13)

By using the obtained result in Eq.(13), the new mask taking the following form:

\[ f_x = \begin{array}{ccc}
-\frac{\rho^{\alpha-2}}{\Gamma(\alpha+1)} [(j^\rho - (j-1)^\rho)^\alpha] & 0 & -\frac{\rho^{\alpha-2}}{\Gamma(\alpha+1)} [(j^\rho - (j-1)^\rho)^\alpha] \\
-\frac{\rho^{\alpha-2}}{\Gamma(\alpha+1)} [((j-1)^\rho - (j-2)^\rho)^\alpha] & 0 & -\frac{\rho^{\alpha-2}}{\Gamma(\alpha+1)} [((j-1)^\rho - (j-2)^\rho)^\alpha] \\
-\frac{\rho^{\alpha-2}}{\Gamma(\alpha+1)} [((j-2)^\rho - (j-3)^\rho)^\alpha] & 0 & -\frac{\rho^{\alpha-2}}{\Gamma(\alpha+1)} [((j-2)^\rho - (j-3)^\rho)^\alpha] \\
\end{array} \]

\[ f_y = \begin{array}{ccc}
\frac{\rho^{\alpha-2}}{\Gamma(\alpha+1)} [(j^\rho - (j-1)^\rho)^\alpha] & \frac{\rho^{\alpha-2}}{\Gamma(\alpha+1)} [((j-1)^\rho - (j-2)^\rho)^\alpha] & \frac{\rho^{\alpha-2}}{\Gamma(\alpha+1)} [((j-2)^\rho - (j-3)^\rho)^\alpha] \\
0 & 0 & 0 \\
-\frac{\rho^{\alpha-2}}{\Gamma(\alpha+1)} [(j^\rho - (j-1)^\rho)^\alpha] & -\frac{\rho^{\alpha-2}}{\Gamma(\alpha+1)} [((j-1)^\rho - (j-2)^\rho)^\alpha] & -\frac{\rho^{\alpha-2}}{\Gamma(\alpha+1)} [((j-2)^\rho - (j-3)^\rho)^\alpha] \\
\end{array} \]

The following table illustrate the algorithm for the new fractional edge detection.

<table>
<thead>
<tr>
<th>Algorithm: The edge detection using new generalized fractional operator</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. convert the input image to gray scale.</td>
</tr>
<tr>
<td>2. choose suitable ( \alpha \in [0,1] ), and ( \rho \geq 0 ) to get the new fractional edge detector.</td>
</tr>
<tr>
<td>3. use Eq.(13) to obtain the new mask.</td>
</tr>
</tbody>
</table>
5. Experimental Work

The outcomes of edge identification in three photos collected using the Sobel, Prewitt, and generalized fractional edge detector techniques are covered in this section. The contrast, PSNR, and MSE values of each edge-detected image are then compared.

5.1. Edge detection

Edge detection is the process of identifying variations in intensity within an image field that are clearly distinct.

Edge detection uses sharp changes in intensity values at the object’s two boundary areas to identify the object’s edge. A group of connected pixels that define the boundary between two areas is called an edge. An edge contains important information that can be expressed by the object’s dimensions or form. The gradient-based operator (first derivative), also known as the Sobel and Prewitt operators, is the first edge in some detector implementations.

5.2. Input image

Next, the MATLAB 2021 software is used to input the images that have been gathered for research. Table 1 labels the images that were used in RGB color mode. The degree of difficulty associated with each image’s edge detection determines which images are used as a dataset. In addition, the RGB color mode of the images is switched to grayscale.

<table>
<thead>
<tr>
<th>sample-1</th>
<th>sample-2</th>
<th>sample-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Image 1]</td>
<td>![Image 2]</td>
<td>![Image 3]</td>
</tr>
</tbody>
</table>

Table 1: Image samples

5.3. Image conversion and edge detection procedure

Technique for edge detection and image conversion. The image that was imported into the MATLAB application has now been converted to grayscale, as shown in Table 2.

This is necessary since the edge detection procedure can only be applied to images in grayscale color mode. After being converted to grayscale, the image moves on to the next stage, edge detection utilizing Sobel, Prewitt, and generalized fractional detector.
5.4. Sobel operator

The first edge detection method, the sobel operator, requires first converting the image to grayscale in order to do edge detection.

5.5. Prewitt operator

In order to perform edge detection using the second edge detection technique, the Prewitt operator, the image must first be converted to grayscale. The code that uses the Prewitt operator to achieve edge detection after the convolution formula has been applied displays the edge detection image.

5.6. Generalized fractional operator

The third edge detection technique is the generalized fractional operator. After converting the original image to gray scale, edges are detected using the generalized fractional operator.
Fig. 1a: Sobel mask.
Fig. 1b: Prewitt mask.
Fig. 1c: GF-at $\alpha = 0.35, \rho = 1$

Fig. 1d: GF-at $\alpha = 0.35, \rho = 1.5$
Fig. 1e: GF-at $\alpha = 0.35, \rho = 2$
Fig. 1f: GF-at $\alpha = 0.40, \rho = 1$

Fig. 1g: GF-at $\alpha = 0.40$ and $\rho = 1.5$
Fig. 1h: GF-at $\alpha = 0.40$ and $\rho = 2$
Fig. 2a: Sobel mask.  
Fig. 2b: Prewitt mask.  
Fig. 2c: GF at $\alpha = 0.35$, $\rho = 1$.

Fig. 2d: GF at $\alpha = 0.35$, $\rho = 1.5$.  
Fig. 2e: GF at $\alpha = 0.35$, $\rho = 2$.  
Fig. 2f: GF at $\alpha = 0.40$, $\rho = 1$.  

Fig. 2g: GF at $\alpha = 0.40$, $\rho = 1.5$.  
Fig. 2h: GF at $\alpha = 0.40$, $\rho = 2$.  

Fig.3a: Sobel mask.  

Fig.3b: Prewitt mask.  

Fig.3c: GF-at $\alpha = 0.35, \rho = 1$

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Fig.3d: GF-at $\alpha = 0.35, \rho = 1.5$  

Fig.3e: GF-at $\alpha = 0.35, \rho = 2$  

Fig.3f: GF-at $\alpha = 0.40, \rho = 1$  

---

Fig.3g: GF-at $\alpha = 0.40, \rho = 1.5$  

Fig.3h: GF-at $\alpha = 0.40, \rho = 2$
After successfully identifying the image’s edges, a full complete discussion will be created for the previously results following the successful identification of the image’s edges.

The MSE, PSNR, and contrast values are measured for all results are shown in Figs.1–3.

5.7. Calculation of MSE

Using the 24 binary images obtained from the edge detection process, the MSE was computed. The results from employing the four detection operators—Sobel, Prewitt and Generalized Fractional—to determine the MSE value of each edge-detected image are presented in Table3.

<table>
<thead>
<tr>
<th>$\alpha = \rho$ values</th>
<th>Sobel</th>
<th>Prewitt</th>
<th>G-F- method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 0.35$, $\rho = 1$</td>
<td>46409.129685</td>
<td>41860.148297</td>
<td>49443.110742</td>
</tr>
<tr>
<td>$\alpha = 0.35$, $\rho = 1.5$</td>
<td>46409.129685</td>
<td>41860.148297</td>
<td>40738.306791</td>
</tr>
<tr>
<td>$\alpha = 0.35$, $\rho = 2$</td>
<td>46409.129685</td>
<td>41860.148297</td>
<td>38375.214925</td>
</tr>
</tbody>
</table>

MSE values of sample-1-case-2

| $\alpha = 0.40$, $\rho = 1$ | 46409.129685 | 41860.148297 | 49560.930656 |
| $\alpha = 0.40$, $\rho = 1.5$ | 46409.129685 | 41860.148297 | 41194.688260 |
| $\alpha = 0.40$, $\rho = 2$ | 46409.129685 | 41860.148297 | 38784.421049 |

MSE values of sample-2-case-1

| $\alpha = 0.35$, $\rho = 1$ | -44986.437879 | 38359.543095 | 49091.969362 |
| $\alpha = 0.35$, $\rho = 1.5$ | 44986.437879 | 38359.543095 | 36823.880878 |
| $\alpha = 0.35$, $\rho = 2$ | 44986.437879 | 38359.543095 | 33490.523683 |

MSE values of sample-2-case-2

| $\alpha = 0.40$, $\rho = 1$ | 44986.437879 | 38359.543095 | 49258.289239 |
| $\alpha = 0.40$, $\rho = 1.5$ | 44986.437879 | 38359.543095 | 37474.303856 |
| $\alpha = 0.40$, $\rho = 2$ | 44986.437879 | 38359.543095 | 34077.450522 |

MSE values of sample-3-case-1

| $\alpha = 0.35$, $\rho = 1$ | 46796.856189 | 40360.121740 | 51137.669257 |
| $\alpha = 0.35$, $\rho = 1.5$ | 46796.856189 | 40360.121740 | 38921.258446 |
| $\alpha = 0.35$, $\rho = 2$ | 46796.856189 | 40360.121740 | 35602.534842 |

MSE values of sample-3-case-2

| $\alpha = 0.35$, $\rho = 1$ | 46796.856189 | 40360.121740 | 51303.137432 |
| $\alpha = 0.35$, $\rho = 1.5$ | 46796.856189 | 40360.121740 | 39565.565217 |
| $\alpha = 0.35$, $\rho = 2$ | 46796.856189 | 40360.121740 | 36182.208594 |

Table 3: The MSE Results
5.8. Calculation of PSNR

After implementing edge detection to the four research material images, the next step is finding the PSNR value of the 24 binary images from the edge detection that completed in the first stage. Table 4 shows the outcomes of each edge-detected image’s application of the three detection operators: Sobel, Prewitt and Generalized Fractional operator.

<table>
<thead>
<tr>
<th>PSNR values of sample-1-case-1</th>
<th>α = ρ values</th>
<th>Sobel</th>
<th>Prewitt</th>
<th>G-F- method</th>
</tr>
</thead>
<tbody>
<tr>
<td>α = 0.35, ρ = 1</td>
<td>-46.6660</td>
<td>-46.2180</td>
<td>-46.9411</td>
<td></td>
</tr>
<tr>
<td>α = 0.35, ρ = 1.5</td>
<td>-46.6660</td>
<td>-46.2180</td>
<td>-46.1000</td>
<td></td>
</tr>
<tr>
<td>α = 0.35, ρ = 2</td>
<td>-46.6660</td>
<td>-46.2180</td>
<td>-45.8405</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PSNR values of sample-1-case-2</th>
<th>α = ρ values</th>
<th>Sobel</th>
<th>Prewitt</th>
<th>G-F- method</th>
</tr>
</thead>
<tbody>
<tr>
<td>α = 0.40, ρ = 1</td>
<td>-46.6660</td>
<td>-46.2180</td>
<td>-46.9514</td>
<td></td>
</tr>
<tr>
<td>α = 0.40, ρ = 1.5</td>
<td>-46.6660</td>
<td>-46.2180</td>
<td>-46.1484</td>
<td></td>
</tr>
<tr>
<td>α = 0.40, ρ = 2</td>
<td>-46.6660</td>
<td>-46.2180</td>
<td>-45.8666</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PSNR values of sample-2-case-1</th>
<th>α = ρ values</th>
<th>Sobel</th>
<th>Prewitt</th>
<th>G-F- method</th>
</tr>
</thead>
<tbody>
<tr>
<td>α = 0.35, ρ = 1</td>
<td>-46.5308</td>
<td>-45.8387</td>
<td>-46.9101</td>
<td></td>
</tr>
<tr>
<td>α = 0.35, ρ = 1.5</td>
<td>-46.5308</td>
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<th>Sobel</th>
<th>Prewitt</th>
<th>G-F- method</th>
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<th>Sobel</th>
<th>Prewitt</th>
<th>G-F- method</th>
</tr>
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<td>α = 0.35, ρ = 1</td>
<td>-46.7022</td>
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<td>-47.0874</td>
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<th>G-F- method</th>
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Table 4: The PSNR Results

5.9. Calculation of contrast

The contrast value of the image is evaluated via MATLAB’s texture analysis feature. The outcomes of computing each image’s contrast value after its edges were found using
the Generalized Fractional, Prewitt, and Sobel detection operators are displayed in Table 5.

<table>
<thead>
<tr>
<th>α = ρ values</th>
<th>Sobel</th>
<th>Prewitt</th>
<th>G-F- method</th>
</tr>
</thead>
<tbody>
<tr>
<td>α = 0.35, ρ = 1</td>
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<td>2.2535</td>
<td>4.0256</td>
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<td>3.6973</td>
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</tbody>
</table>

Contrast values of sample-1-case-2

<table>
<thead>
<tr>
<th>α = ρ values</th>
<th>Sobel</th>
<th>Prewitt</th>
<th>G-F- method</th>
</tr>
</thead>
<tbody>
<tr>
<td>α = 0.40, ρ = 1</td>
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<td>2.2535</td>
<td>4.0256</td>
</tr>
<tr>
<td>α = 0.40, ρ = 1.5</td>
<td>2.3183</td>
<td>2.2535</td>
<td>3.7879</td>
</tr>
<tr>
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Contrast values of sample-2-case-1

<table>
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<th>Sobel</th>
<th>Prewitt</th>
<th>G-F- method</th>
</tr>
</thead>
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<td>α = 0.35, ρ = 1</td>
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<td>6.1596</td>
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<tr>
<td>α = 0.35, ρ = 1.5</td>
<td>4.3387</td>
<td>4.1220</td>
<td>5.9591</td>
</tr>
<tr>
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<td>4.3387</td>
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<td>5.9107</td>
</tr>
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</table>

Contrast values of sample-2-case-2

<table>
<thead>
<tr>
<th>α = ρ values</th>
<th>Sobel</th>
<th>Prewitt</th>
<th>G-F- method</th>
</tr>
</thead>
<tbody>
<tr>
<td>α = 0.40, ρ = 1</td>
<td>4.3387</td>
<td>4.1220</td>
<td>6.1596</td>
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<td>4.3387</td>
<td>4.1220</td>
<td>5.9795</td>
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<tr>
<td>α = 0.40, ρ = 2</td>
<td>4.3387</td>
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<td>5.9488</td>
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Contrast values of sample-3-case-1

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<th>α = ρ values</th>
<th>Sobel</th>
<th>Prewitt</th>
<th>G-F- method</th>
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<tbody>
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<td>α = 0.35, ρ = 1</td>
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Contrast values of sample-3-case-2

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<th>α = ρ values</th>
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<th>Prewitt</th>
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<td>α = 0.40, ρ = 2</td>
<td>4.3387</td>
<td>4.1220</td>
<td>5.9488</td>
</tr>
</tbody>
</table>

Table 5: The Contrast Results
6. Performance Comparison

Various techniques such as Sobel, Prewitt, and Generalized Fractional operators can be used to identify edges in an image.

The edge detection quality analysis’s results show that the Generalized Fractional operator produces the best edge detection, with an average PSNR value of -45.8405, according to the MSE, PSNR, and Contrast values on the Sobel, Prewitt, and Generalized Fractional operators. The average value of the Sobel and Prewitt operators is -46.6660 dB, -46.2180 dB, and -45.5148 dB, while the Sobel and Prewitt operators are -46.7022 dB, -46.0595 dB.

The Generalized Fractional operator has the lowest MSE value, 38375.214925, but the Sobel and Prewitt operators have average MSE values of 46409.129685 and 41860.148297, respectively. The operator with the highest average score, 5.9488, is the Generalized Fractional operator, according on the comparison of Contrast values.

However, the Contrast setting just serves as a supporting parameter to draw attention to the differences in the output of each edge detection operator.

7. Conclusion

This work pioneers a novel approach in image edge detection through the utilization of the Generalized Fractional. By harnessing the global attributes inherent in fractional derivatives, it aims to enhance the extraction of intricate edge details. This is accomplished by creating the mask by using fractional derivative and adapt the mask by another parameter, yielding compelling and informative edge representations. Given that the Generalized Fractional edge detection approach has the lowest MSE value and the highest PSNR and Contrast value among the other two ways, it can be argued that it is the best method. This advancement not only augments computer vision and image analysis techniques but also holds promise for refining image processing methodologies. Future endeavors may explore its adaptability across diverse imaging domains like medical and satellite imagery, while integration into deep learning frameworks could elevate its potential for advanced feature extraction and deeper image understanding. Additionally, optimizing its computational efficiency would broaden its scope for real-time deployment in fields such as robotics and autonomous systems. Future research could extend its use to various imaging domains, integrate it into deep learning for advanced analysis, and optimize its efficiency for real-time applications in robotics and autonomous systems. Further exploration might involve applying this technique to new fractional models and comparing its effectiveness with existing numerical methods [1, 2, 9, 14].

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REFERENCES


