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Some Properties of Contra-b-Continuous and Almost Contra-b-Continuous Functions

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Abstract. The notion of contra-continuous functions was introduced and investigated by Dontchev [5]. In this paper we apply the notion of b-open sets in topological space to present and study a new class of function called almost contra-b-continuous functions as a new generalization of contra-continuity.

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1. Introduction and Preliminaries

In 1996, Dontchev [5] introduced contra-continuous functions. Jafari and Noiri [8] introduced and studied contra-precontinuous functions. Ekici [10] introduced and studied almost contra-precontinuous functions. Recently [13] introduced and studied contra b-continuous functions. In this paper, we introduce a new class of functions called almost contra-b-continuous function. Moreover, we obtain basic properties and preservation theorem of almost contra-b-continuous function, contra-b-continuous function and relationships between almost contra-b-continuous function and b-regular graphs.

Throughout the paper, the space X and Y (or (X,τ) and (Y,σ)) stand for topological spaces with no separation axioms assumed unless otherwise stated. Let A be a subset of X. The closure of A and the interior of A will be denoted by $Cl(A)$ and $Int(A)$, respectively.

Definition 1.1. A subset A of a space X is said to be

1. regular open [27] if $A = Int(Cl(A))$;
2. α-open [20] if $A \subseteq Int(Cl(Int(A)))$;
3. semi-open [24] if $A \subseteq Cl(Int(A))$;
4. pre-open [25] if $A \subseteq Int(Cl(A))$;
5. β-open [2] if $A \subseteq Cl(Int(Cl(A)))$;
6. b-open [3] if $A \subseteq Cl(Int(A)) \cup Int(Cl(A))$.

The complement of a b-open set is said to be b-closed [3]. The intersection of all b-closed sets of X containing A is called the b-closure of A and is denoted by $bCl(A)$. The union of all b-open sets of X contained A is called b-interior of A and is denoted by $bInt(A)$. The family of all b-open (resp. α-open, semi-open, preopen, β-open, regular open, b-closed, preclosed, β-closed, regular closed, closed) subsets of a space X is denoted by $BO(X)$ (resp. $\alpha O(X)$, $SO(X)$, $PO(X)$, $\beta O(X)$, $RO(X)$, $BC(X)$, $PC(X)$, $\beta C(X)$, $RC(X)$, $C(X)$ respectively) and the collection of all b-open subsets of X containing a fixed point x is denoted by $BO(X,x)$. The sets $\alpha O(X,x)$, $SO(X,x)$, $PO(X,x)$, $\beta O(X,x)$, $RO(X,x)$ and $C(X,x)$ are defined analogously.

Definition 1.2. A function $f : X \to Y$ is called b-continuous [11] if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in BO(X,x)$ such that $f(U) \subseteq V$.

Definition 1.3. A function $f : X \to Y$ is called contra-continuous [5] (resp. contra-precontinuous [8], contra-β-continuous [4], contra-b-continuous [13]) if $f^{-1}(V)$ closed (resp. preclosed, β-closed, b-closed) in X for each open set V of Y.

2. Contra-b-Continuous Functions

In this section, we obtain some properties of contra-b-continuous functions, (for more properties the reader should refer to [1, 13, 17, 20]).

Lemma 2.1. *[3] Let (X,τ) be a topological space.*

1. *The intersection of an open set and a b-open set is a b-open set.*
2. *The union of any family of b-open sets is a b-open set.*

Recall that for a function $f : X \to Y$, the subset $\{(x, f(x)) : x \in X\} \subseteq X \times Y$ is called the graph of f and is denoted by $G(f)$.

Definition 2.2. [13] The graph $G(f)$ of a function $f : X \to Y$ is said to be contra-b-closed graph if for each $(x, y) \in (X,Y) - G(f)$, there exist $U \in BO(X,x)$ and a closed set V of Y containing y such that $(U \times V) \cap G(f) = \phi$.

Definition 2.3. [5] A space X is said to be strongly S-closed if every closed cover of X has a finite subcover

Theorem 2.4. *If (X, τ_b) is a topological space and $f : X \to Y$ has a contra b-closed graph, then the inverse image of a strongly S-closed set A of Y is b-closed in X.*

Proof. Assume that A is a strongly S-closed set of Y and $x \notin f^{-1}(A)$. For each $a \in A$, $(x, a) \notin G(f)$. By Lemma 3.3 in [13] there exist $U_a \in BO(X, x)$ and $V_a \in C(Y, a)$ such that $f(U_a) \cap V_a = \phi$. Since $\{A \cap V_a : a \in A\}$ is a closed cover of the subspace A, since A is S-closed, then there exists a finite subset $A_0 \subseteq A$ such that $A \subseteq \cup\{V_a : a \in A_0\}$. Set $U = \cap\{U_a : a \in A_0\}$, but (X, τ_b) is a topological space, then $U \in BO(X, x)$ and $f(U) \cap A \subseteq f(U_a) \cap [\cup(V_a : a \in A_0)] = \phi$. Therefore $U \cap f^{-1}(A) = \phi$ and hence and hence $x \notin bCl(f^{-1}(A))$. This show that $f^{-1}(A)$ is b-closed.

Theorem 2.5. *Let Y be a strongly S-closed space. If (X, τ_b) is a topological space and a function $f : X \to Y$ has a contra b-closed graph, then f is contra b-continuous.*

Proof. Suppose that Y is strongly S-closed and $G(f)$ is contra b-closed. First we show that an open set of Y is strongly S-closed. Let U be an open set of Y and $\{V_i : i \in I\}$ be a cover of U by closed sets V_i of U. For each $i \in I$, there exists a closed set K_i of X such that $V_i = K_i \cap U$. Then the family $\{K_i : i \in I\} \cup (Y - U)$ is a closed cover of Y. Since Y is strongly S-closed, there exists a finite subset $I_0 \subseteq I$ such that $Y = \cup\{K_i : i \in I_0\} \cup (Y - U)$. Therefore we obtain $U = \cup\{V_i : i \in I_0\}$. This shows that U is strongly S-closed. By Theorem 2.4 $f^{-1}(U)$ is b-closed in X for every open U in Y. Therefore, f is contra b-continuous.

Theorem 2.6. *Let $f : X \to Y$ be a function and $g : X \to X \times Y$ the graph function of f, defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is contra b-continuous, then f is contra b-continuous.*

Proof. Let U be an open set in Y, then $X \times U$ is an open set in $X \times Y$. Since g is contra b-continuous. It follows that $f^{-1}(U) = g^{-1}(X \times U)$ is an b-closed in X. Thus, f is contra b-continuous.

Theorem 2.7. *If $f : X \rightarrow Y$ is contra b-continuous, $g : X \rightarrow Y$ is contra continuous, and Y is Urysohn, then $E = \{x \in X : f(x) = g(x)\}$ is b-closed in X.*

Proof. Let $x \in X - E$. Then $f(x) \neq g(x)$. Since Y is Urysohn, there exist open sets V and W such that $f(x) \in V$, $g(x) \in W$ and $Cl(V) \cap Cl(W) = \phi$. Since f is contra b-continuous, then $f^{-1}(Cl(V))$ is b-open in X and g is contra continuous, then $g^{-1}(Cl(W))$ is open set in X. Let $U = f^{-1}(Cl(V))$ and $G = g^{-1}(Cl(W))$. Then U and G contain x. Set $A = U \cap G$. A is b-open in X. And $f(A) \cap g(A) \subseteq f(U) \cap g(G) \subseteq Cl(V) \cap Cl(W) = \phi$. Hence $f(A) \cap g(A) = \phi$ and $A \cap E = \phi$ where A is b-open therefore $x \notin bCl(E)$. Thus E is b-closed in X.

A subset A of a topological space X is said to be b-dense in X if $bCl(A) = X$.

Theorem 2.8. *Let $f : X \rightarrow Y$ is contra b-continuous and $g : X \rightarrow Y$ is contra continuous. If Y is Urysohn, and $f = g$ on b-dense set $A \subseteq X$, then $f = g$ on X.*

Proof. Since f is contra b-continuous, g contra continuous functions and Y is Urysohn by the previous Theorem $E = \{x \in X : f(x) = g(x)\}$ is b-closed in X. We have $f = g$ on b-dense set $A \subseteq X$. Since $A \subseteq E$ and A is b-dense set in X, then $X = bCl(A) \subseteq bCl(E) = E$. Hence $f = g$ on X.

Definition 2.9. [11] A space X is called b-connected provided that X is not the union of two disjoint nonempty b-open sets.

Theorem 2.10. *If $f : X \rightarrow Y$ is a contra b-continuous function from a b-connected space X onto any space Y, then Y is not a discrete space.*

Proof. Suppose that Y is discrete. Let A be a proper nonempty open and closed subset of Y. Then $f^{-1}(A)$ is a proper nonempty b-clopen subset of X, which is a contradiction to the fact that X is b-connected.

3. Almost Contra-b-Continuous Functions

In this section, we introduce a new type of continuity called almost contra-b-continuity which is weaker than almost contra-precontinuity [8] and stronger than almost contra-β-continuity [4].

Definition 3.1. A function $f : X \to Y$ is said to be almost contra-b-continuous (resp. almost contra-precontinuous [8], almost contra-β-continuous [4]) $f^{-1}(V) \in BC(X)$ (resp. $f^{-1}(V) \in PC(X)$, $f^{-1}(V) \in \beta C(X)$) for every $V \in RO(X)$.

Lemma 3.2. *[3] Let A be a subset of a space X. Then*

1. $bCl(A) = sCl(A) \cap pCl(A) = A \cup [Int(Cl(A)) \cap Cl(Int(A))]$;
2. $bInt(A) = sInt(A) \cup pInt(A) = A \cap [Int(Cl(A)) \cup Cl(Int(A))$.

Lemma 3.3. *For a subset V of a topological space Y, the following hold:*
$\alpha Cl(V) = Cl(V)$ for every $V \in BO(Y)$.

Proof. Since $V \in BO(Y)$, then

$$
\begin{aligned}
V &= bInt(V) = sInt(V) \cup pInt(V) \\
&= bInt(V) = V \cap [Int(Cl(V)) \cup Cl(Int(V)) \\
&= Cl(V \cap [Int(Cl(V)) \cup Cl(Int(V))) \\
&\subseteq Cl(V) \cap Cl(Int(Cl(V)) \cup Cl(Int(V))) \\
&\subseteq Cl(V) \cap Cl(Int(Cl(V))) \cup Cl(Int(V)) \\
&\subseteq (Cl(V) \cap Cl(Int(Cl(V)))) \cup (Cl(V) \cap (Cl(Int(V))) \\
&\subseteq Cl(Int(Cl(V))) \cup Cl(Int(V)) \\
&\subseteq Cl(Int(Cl(V))) \\
&\subseteq Cl(Int(Cl(V))) \cup V = \alpha Cl(V)
\end{aligned}
$$

and also $\alpha Cl(V) \subseteq Cl(V)$ for every subset $V \subseteq X$. Hence $\alpha Cl(V) = Cl(V)$ for every $V \in BO(Y)$.

Definition 3.4. A subfamily m_X of the power set $P(X)$ of a nonempty set X is called a minimal structure (briefly m-structure) on X [18] if; $\phi \in m_X$ and $X \in m_X$.

By $(X; m_X)$, we denote a nonempty set X with a minimal structure m_X on X and call it an m-space. Each member of m_X is said to be m_X-open (or briefly m-open) and the complement of an m_X-open set is said to be m_X-closed (or briefly m-closed).

Definition 3.5. A function $f : (X; m_X) \to (Y; \sigma)$ is said to be almost contra m-continuous [18] if $f^{-1}(V) = m_X$-$Cl(f^{-1}(V))$ for every regular open set V of $(Y; \sigma)$.

Theorem 3.6. *[19] For a function $f : (X, m_X) \to (Y, \sigma)$, the following properties are equivalent:*

1. *f is almost contra m-continuous;*

2. *$f^{-1}(Cl(V)) = m_X - Int(f^{-1}(Cl(V)))$ for every $V \in \beta(Y)$;*

3. $f^{-1}(Cl(V)) = m_X - Int(f^{-1}(Cl(V)))$ *for every* $V \in SO(Y)$*;*

4. $f^{-1}(Int(Cl(V))) = m_X - Int(f^{-1}(Int(Cl(V))))$ *for every* $V \in PO(Y)$*.*

For more properties of almost contra m-continuous the reader should refer to [19].

Corollary 3.7. *For a function* $f : X \to Y$*, the following properties are equivalent:*

1. f *is almost contra* b*-continuous;*

2. $f^{-1}(Cl(V))$ *is* b*-open in* X *for every* $V \in BO(Y)$*;*

3. $f^{-1}(\alpha Cl(V))$ *is* b*-open in* X *for every* $V \in BO(Y)$*.*

Definition 3.8. [26] A function $f : X \to Y$ is said to be R-map if $f^{-1}(V)$ is regular open in X for each regular open set V of Y.

Recall that a function $f : X \to Y$ is almost-continuous if $f^{-1}(V)$ is open in X for each regular open set V of Y.

Theorem 3.9. *If a function* $f : X \to Y$ *is almost contra-*b*-continuous and almost continuous, then* f *is R-map.*

Proof. Let V be any regular open set in Y. Since f is almost contra-b-continuous and contra continuous $f^{-1}(V)$ is b-closed and open, Thus
$bCl(f^{-1}(V)) = f^{-1}(V) = Int((f^{-1}(V)),$
by Lemma 3.2 we have

$$\begin{aligned} bCl(f^{-1}(V)) &= f^{-1}(V) \cup [Cl(Int(f^{-1}(V))) \cap Int(Cl(f^{-1}(V)))] \\ &= f^{-1}(V) \cup [Cl(f^{-1}(V)) \cap Int(Cl(f^{-1}(V)))] \\ &= f^{-1}(V) \cup Int(Cl(f^{-1}(V))) \\ &= Int(Cl(f^{-1}(V))) = f^{-1}(V). \end{aligned}$$

We obtain that f is R-map.

Definition 3.10. [11] A space X is said to be b-compact (resp. b-Lindelöf, countably b-compact) if every b-open (resp. b-open, countable b-open) cover of X has a finite (resp. a countable, a finite) subcover.

Definition 3.11. A space X is said to be S-Lindelöf [10] (resp. S-closed [21], countably S-closed [6]) if every regular closed (resp. regular closed, countable regular closed) cover of X has a countable (resp. a finite, a finite) subcover.

Theorem 3.12. *Let $f : X \to Y$ be an almost contra-b-continuous surjection. The following statements hold:*

1. *if X is b-compact, then Y is S-closed;*
2. *if X is b-Lindelöf, then Y is S-Lindelöf;*
3. *if X is countably b-compact, then Y is countably S-closed.*

Proof. We prove only (1). Let $\{V_\alpha : \alpha \in I\}$ be any regular closed cover of Y. Since f is almost contra-b -continuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a b-open cover of X and hence there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$ therefore we have $Y = \cup\{V_\alpha : \alpha \in I_0\}$ and Y is S-closed.

Definition 3.13. [22] A space X is said to be nearly compact (resp. nearly countably compact, nearly Lindelöf) if every regular open (resp. countable regular open, regular open) cover of X has a finite (resp. a finite, a countably) subcover.

Theorem 3.14. *Let $f : X \to Y$ be an almost contra-b-continuous and almost continuous surjection and X is S-closed(resp. nearly compact, nearly Lindelöf, nearly countably compact, countably S-closed, S-Lindelöf) then Y is S-closed (resp. nearly countably compact, nearly Lindelöf, nearly countably compact, countably S-closed, S-Lindelöf).*

Proof. Let V be any regular closed set on Y. Then since f is almost contra-b-continuous and almost continuous, then by Theorem 3.9 f is R-map. Hence $f^{-1}(V)$ is regular closed in X. Let $\{V_\alpha : \alpha \in I\}$ be any regular closed cover of Y. Then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a regular closed cover of X and since X is S-closed, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. since f is surjection, we obtain $Y = \cup\{V_\alpha : \alpha \in I_0\}$. This shows that Y is S-closed. The other proofs are similar.

Theorem 3.15. *If $f : X \to Y$ is contra-b-continuous and A is b-compact relative to X, then $f(A)$ is strongly S-closed in Y.*

Proof. Let $\{V_i : i \in I\}$ be any cover of $f(A)$ by closed sets of the subspace $f(A)$. for $i \in I$, there exists a closed set A_i of Y such that $V_i = A_i \cap f(A)$. For each $x \in A$, there exists $i(x) \in I$ such that $f(x) \in A_{i(x)}$ and by Theorem 3.1 in [13], there exists $U_x \in BO(X,x)$ such that $f(U_x) \subseteq A_{i(x)}$. Since the family $\{U_x : x \in A\}$ is a cover of A by b-open sets of X, there exists a finite subset A_0 of A such that $A \subseteq \cup\{U_x : x \in A_0\}$. Therefore, we obtain $f(A) \subseteq \cup\{f(U_x) : x \in A_0\}$. which is a subset of $\cup\{A_{i(x)} : x \in A_0\}$. Thus $f(A) = \cup\{V_{i(x)} : x \in A_0\}$ and hence $f(A)$ is strongly S-closed.

Corollary 3.16. *If $f : X \to Y$ is contra-b-continuous surjection and X is b-compact then Y is strongly S-closed.*

Definition 3.17. A function $f : X \to Y$ is almost weakly continuous [12] (resp. almost weakly b-continuous) if for each $x \in X$ and each open set V containing $f(x)$ there exists $U \in PO(X,x)$ (resp. $U \in BO(X,x)$) such that $f(U) \subseteq Cl(V)$.

Definition 3.18. [9] A function $f : X \to Y$ is (θ, s)-continuous if the preimage of every regular open subset of Y is closed in X.

The following examples will show that the concepts of almost, contra-b-continuity, almost contra-b-continuity, almost weak b-continuity, almost contra-precontinuity, almost weak-continuity are independent from each other.

Example 3.19. *Let* $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ *and* $\sigma = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$. *Then* $RC(X, \tau) = \{X, \phi, \{b, c\}, \{a, c\}\}$ *and* $BO(X, \sigma) = \{X, \phi, \{b\}, \{c\}, \{b, c\}, \{a, c\}, \{a, b\}\}$, $PO(X, \sigma) = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$. *Let* $f : (X, \sigma) \to (X, \tau)$ *be the identity function. Then* f *is almost contra-b-continuous function which is not almost contra-precontinuous, since* $\{a, c\}$ *is a regular closed set of* (X, τ) *and* $f^{-1}(\{a, c\}) = \{a, c\} \notin PO(X, \sigma)$.

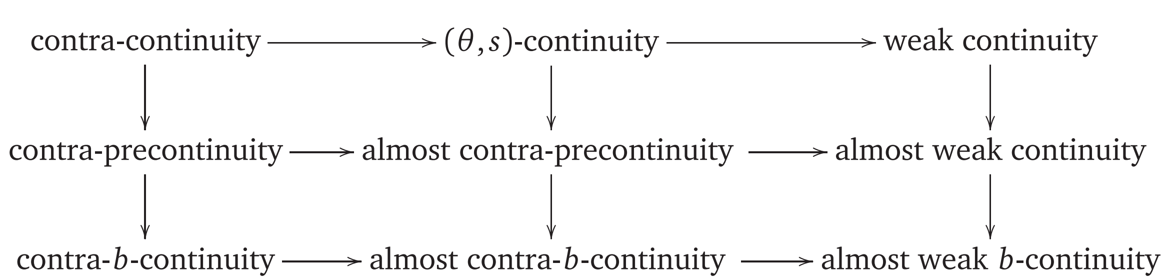
Example 3.20. *Let* $X = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. *Then* $RC(X, \tau) = \{X, \phi, \{b, c\}, \{a, c\}\}$ *and* $BO(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{b, c\}, \{a, c\}, \{a, b\}\}$, *Let* $f : (X, \tau) \to (X, \tau)$ *be the identity function. Then* f *is almost contra-b-continuous function which is not contra-b-continuous, since* $\{c\}$ *is a closed set of* (X, τ) *and* $f^{-1}(\{c\}) = \{c\} \notin BO(X, \sigma)$.

Example 3.21. *Let* $X = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. *Then* $RC(X, \tau) = \{X, \phi, \{b, c\}, \{a, c\}\}$ *and* $Y = \{1, 2\}$ $(Y, \sigma) = \{Y, \phi, \{1\}\}$, $bO(Y, \sigma) = \{Y, \phi, \{1\}\}$ *Let* $f : (Y, \sigma) \to (X, \tau)$ *be defined by* $f(1) = a$ *and* $f(2) = c$. *Then function is almost weak-b-continuous, and* f *is not almost contra-b-continuous since* $\{b, c\}$ *is a regular closed set of* (X, τ) *and* $f^{-1}(\{b, c\}) = \{2\} \notin BO(Y, \sigma)$.

Example 3.22. *In Example 3.19* f *is almost weak-b-continuous, and it is not almost weak-continuous, since if* $a \in X$ *and* $f(a) \in \{a\}$ *it clear that does not exist* $U \in PO(X, x)$ *such that* $f(\{U\}) \subseteq Cl(\{a\}) = \{a, c\}$.

We have the following relation for the functions defined above:

DIAGRAM I



4. b-Regular Graphs

We introduce the following relatively new definition:

Definition 4.1. The graph $G(f)$ of a function $f : X \to Y$ is said to be b-regular (resp. strongly contra-b-closed) graph if for each $(x, y) \in (X, Y) - G(f)$, there exist $U \in BO(X, x)$ and a regular open (resp, regular closed) set V of Y containing y such that $(U \times V) \cap G(f) = \phi$.

Lemma 4.2. *[13] The graph $G(f)$ of $f : X \to Y$ is contra-b- closed in $X \times Y$ if and only if for each $(x, y) \in (X \times Y) - G(f)$, there exists $U \in BO(X, x)$ and $V \in C(Y, y)$ such that $f(U) \cap V = \phi$.*

Theorem 4.3. *[13] If $f : X \to Y$ is contra-b-continuous and Y is Urysohn, then $G(f)$ is contra-b-closed in $X \times Y$.*

The following results can be easily verified.

Lemma 4.4. *Let $G(f)$ be the graph of f, for any subset $A \subseteq X$ and $B \subseteq Y$, we have $f(A) \cap B = \phi$ if and only if $(A \times B) \cap G(f) = \phi$*

Lemma 4.5. *The graph $G(f)$ of $f : X \to Y$ is b-regular in $X \times Y$ if and only if for each $(x, y) \in (X \times Y) - G(f)$, there exists $U \in BO(X, x)$ and $V \in RO(Y, y)$ such that $f(U) \cap V = \phi$.*

Lemma 4.6. *If A and B are open sets with $A \cap B = \phi$, then $Cl(A) \cap Int(Cl(B)) = \phi$*

Theorem 4.7. *If $f : X \to Y$ is almost contra-b-continuous and Y is T_2, then $G(f)$ is b-regular in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y) - G(f)$. It follows that $f(x) \neq y$. Since Y is T_2, there exists open sets V and W containing $f(x)$ and y respectively, such that $V \cap W = \phi$. Then $Int(Cl(V)) \cap Int(Cl(W)) = \phi$. Since f is almost contra-b-continuous, we have

$f^{-1}(Int(Cl(V)))$ is b-closed in X containing x. Take $U = f^{-1}(Int(Cl(V)))$. Then $f(U) \subseteq Int(Cl(V))$. Therefore $f(U) \cap Int(Cl(W)) = \phi$, and $Int(Cl(W)) \in RO(Y)$. Hence $G(f)$ is b-regular in $X \times Y$.

Definition 4.8. [11] A space X is said to be b-T_1 if each pair of distinct points x and y of X, there exists b-open sets U and V containing x and y respectively such that $y \notin U$ and $x \notin V$.

Theorem 4.9. *Let $f : X \to Y$ have a b-regular graph. If f is injection then X is b-T_1.*

Proof. Let x and y be any two distinct points of X. Since f is injection, then we have $(x, f(y)) \in (X \times Y) - G(f)$. By definition of b-regular graph, there exist a b-closed set U of X and $V \in RO(Y)$ such that $(x, f(y)) \in U \times V$ and $f(U) \cap V = \phi$. therefore we have $U \cap f^{-1}(V) = \phi$ and $y \notin U$. Thus $y \in X - U$ and $x \notin X - U$ and $X - U \in BO(X)$. This implies that X is b-T_1.

Definition 4.10. [23] A space X is said to be weakly Hausdorff if each element of X is an intersection of regular closed sets.

Theorem 4.11. *Let $f : X \to Y$ have a b-regular graph. If f is surjection then Y is weakly Hausdorff.*

Proof. Let y_1 and y_2 be any two distinct points of Y. Since f is surjective $f(x) = y_1$ for some $x \in X$ and $(x, y_2) \in (X \times Y) - G(f)$. By definition of b-regular graph, there exist a b-closed set U of X and $F \in RO(Y)$ such that $(x, y_2) \in U \times F$ and $f(U) \cap F = \phi$. Since $f(x) \in f(U)$. Hence $y_1 \notin F$. Then $y_2 \notin Y - F \in RC(Y)$ and $y_1 \in Y - F$. This implies that Y is weakly Hausdorff.

Lemma 4.12. *The graph $G(f)$ of $f : X \to Y$ is strongly contra-b- closed graph in $X \times Y$ if and only if for each $(x, y) \in (X \times Y) - G(f)$, there exists $U \in BO(X, x)$ and $V \in RC(Y, y)$ such that $f(U) \cap V = \phi$.*

Theorem 4.13. *If $f : X \to Y$ is almost weakly-b-continuous and Y is Urysohn, then $G(f)$ is strongly contra-b-closed in $X \times Y$.*

Proof. Suppose that $(x, y) \in (X \times Y) - G(f)$. Then $y \neq f(x)$. Since Y is Urysohn, there exist open sets V and W in Y containing y and $f(x)$, respectively, such that $Cl(V) \cap Cl(W) = \phi$. Since f is almost weakly-b-continuous there exists $U \in BO(X, x)$ such that $f(U) \subseteq Cl(W)$. This shows that$f(U)) \cap Cl(V) = f(U) \cap Cl(Int(V)) = \phi$ with $Cl(Int(V)) \in RC(Y, y)$ and hence by Lemma 4.12 we have $G(f)$ is strongly contra-b-closed.

Theorem 4.14. *If $f : X \to Y$ is almost contra-b-continuous, then f is almost weakly-b-continuous.*

Proof. Let $x \in X$ and V be any open set of Y containing $f(x)$. Then $Cl(V)$ is a regular closed set of Y containing $f(x)$. Since f almost contra-b-continuous by Theorem 3.1 (3) in [19] there exists $U \in BO(X, x)$ such that $f(U) \subseteq Cl(V)$. Therefore f is almost weakly-b-continuous.

Corollary 4.15. *If $f : X \to Y$ is almost contra-b-continuous and Y is Urysohn, then $G(f)$ is strongly contra-b-closed.*

Definition 4.16. [7] A function $f : X \to Y$ is called almost-b-continuous if $f^{-1}(V)$ b-open in X for every regular open set V of Y.

The following result can be easily verified.

Lemma 4.17. *[7] A function $f : X \to Y$ is almost b-continuous, if and only if for each $x \in X$ and each regular open set V of Y containing $f(x)$, there exists $U \in BO(X, x)$ such that $f(U) \subseteq V$.*

Theorem 4.18. *If $f : X \to Y$ is almost b-continuous, and Y is Hausdorff, then $G(f)$ is strongly contra-b-closed.*

Proof. Suppose that $(x, y) \in (X \times Y) - G(f)$. Then $y \neq f(x)$. Since Y is Hausdorff, there exist open sets V and W in Y containing y and $f(x)$, respectively, such that $V \cap W = \phi$, hence $Cl(V) \cap Int(Cl(W)) = \phi$. Since f is almost b-continuous, and W is regular open by Lemma 4.17 there exists $U \in BO(X, x)$ such that $f(U) = W \subseteq Int(Cl(W))$. This shows that $f(U) \cap Cl(V) = \phi$ and hence by Lemma 4.12 we have $G(f)$ is strongly contra-b-closed.

We recall that a topological space (X, τ) is said to be extremally disconnected (briefly E.D.) if the closure of every open set of X is open in X.

Theorem 4.19. *Let Y be E.D. Then, a function $f : X \to Y$ is almost contra-b-continuous if and only if it is almost b-continuous.*

Proof. Let $x \in X$ and V be any regular open set of Y containing $f(x)$. Since Y is E.D. then V is clopen. By Theorem 3.1 (3) in [19], there exists $U \in BO(X, x)$ such that $f(U) \subseteq V$. Then Lemma 4.17, implies that f is almost b-continuous. Conversely let F be any regular closed set of Y. Since Y is E.D. Then F is also regular open and $f^{-1}(F)$ is b-open in X. This show that f are almost contra-b-continuous.

The following examples will show that the concepts of almost-b-continuity and almost contra-b-continuity, is independent.

Example 4.20. *Let $X = \{a, b, c\} = Y$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{\phi, Y, \{a\}\}$, then $RO(X, \tau) = \{\phi, X, \{a\}, \{b\}\}$, $BO(Y, \sigma) = \{\phi, X, \{a\}, \{a, b\}$, $\{a, c\}\}$. Then it is clear that (X, τ) is not extremally disconnected. Let function $f : (Y, \sigma) \to (X, \tau)$ as follows: $f(a) = c, f(b) = b$ and $f(c) = a$. Then f is almost-contra-b-continuous and f is not almost-b-continuous, since $\{a\}$ is regular open and $f^{-1}(\{a\}) = \{c\}$ is not b-open. But if we define $g : (Y, \sigma) \to (X, \tau)$ as $g(y) = a$ for all $y \in Y$, then g almost-b-continuous but g is not almost-contra-b-continuous since $f^{-1}(\{a\}) = \{a\}$ is not b-closed.*

Definition 4.21. [11] A space X is said to be b-T_2 if for each pair of distinct points x and y in X, there exist $U \in BO(X, x)$ and $V \in BO(X, y)$ such that $U \cap V = \phi$.

Theorem 4.22. *If $f : X \to Y$ is an injective almost contra-b-continuous function with the strongly contra-b-closed graph, then (X, τ) is b-T_2.*

Proof. Let x and y be distinct points of X. Since f is injective, we have $f(x) \neq f(y)$. Then we have $(x, f(y)) \in (X \times Y) - G(f)$. Since $G(f)$ is strongly contra-b-closed, by Lemma 4.12 there exists $U \in BO(X, x)$ and a regular closed set V containing $f(y)$ such that $f(U) \cap V = \phi$. Since f is almost contra-b-continuous, by Theorem 3.1 (3) in [19] there exists $G \in BO(X, y)$ such that $f(G) \subseteq V$. Therefore we have $f(U) \cap f(G) = \phi$, hence $U \cap G = \phi$. This shows that (X, τ) is b-T_2.

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