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Mapping Properties of Some Classes of Analytic Functions Under New Generalized Integral Operators

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Abstract. In this paper we study the mapping properties with respect to new generalised integral operator which was studied recently.

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1. Introduction

Let $\mathscr{H}(U)$ be the set of functions which are regular in the unit disc U,

$$\mathscr{A} = \{f \in \mathscr{H}(U) : f(0) = f'(0) - 1 = 0\}$$

and $S = \{f \in \mathscr{A} : f \text{ is univalent in } U\}$.

In [10] the subfamily T of S consisting of functions f of the form

$$f(z) = z - \sum_{j=2}^{\infty} a_j z^j, \ a_j \geq 0, j = 2, 3, \ldots, \quad z \in U \tag{1}$$

was introduced.

Thus we have the subfamily $S - T$ consisting of functions f of the form

$$f(z) = z + \sum_{j=2}^{\infty} a_j z^j, \quad a_j \geq 0, j = 2, 3, \ldots, z \in U \tag{2}$$

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A function $f(z) \in \mathscr{A}$ is said to be spiral-like if there exists a real number λ, $|\lambda| < \pi/2$, such that

$$Re\ e^{i\lambda}\frac{zf'(x)}{f(X)}, \quad (z \in U).$$

The class of all spiral-like functions was introduced by L. Spacek [11] and we denote it by $S^{\star}_{\lambda}$. Later, Robertson [9] considered the class C_{λ} of analytic functions in U for which $zf'(z) \in S^{\star}_{\lambda}$.

Let $P^{\lambda}_{k}(\rho)$ be the class of functions $p(z)$ analytic in U with $p(0) = 1$ and

$$\int_{0}^{2\pi}\left|\frac{Re\ e^{i\lambda}p(z) - \rho\cos\lambda}{1-\rho}\right| d\theta \leq k\pi\cos\lambda, \quad z = re^{i\theta} \tag{3}$$

where $k \geq 2$, $0 \leq \rho < 1$, λ is real with $|\lambda| < \frac{\pi}{2}$. In case that $k = 2$, $\lambda = 0$, $\rho = 0$, the class $P^{\lambda}_{k}(\rho)$ reduces to the class P of functions $p(z)$ analytic in U with $p(0) = 1$ and whose real part is positive.

we recall the well-known classes

$$R^{\lambda}_{k}(\rho) = \left\{f(z) : f(z) \in \mathscr{A} \text{ and } \frac{zf'(z)}{f(z)} \in P^{\lambda}_{k}(\rho), \quad 0 \leq \rho < 1\right\},$$

$$V^{\lambda}_{k}(\rho) = \left\{f(z) : f(z) \in \mathscr{A} \text{ and } \frac{(zf'(z))'}{f'(z)} \in P^{\lambda}_{k}(\rho), \quad 0 \leq \rho < 1\right\}.$$

These classes are introduced and studied in [7].

The purpose of this paper is to develop the mapping properties with respect to a new generalized integral operator.

2. Preliminary Results

Prof. Breaz [3] has introduced the following integral operators on univalent function spaces:

$$J(z) = \left\{\beta\int_{0}^{z}\left[f_1'(t^n)\right]^{\gamma_1}\cdot\ldots\cdot\left[f_p'(t^n)\right]^{\gamma_p}dt\right\}^{\frac{1}{\beta}}, \tag{4}$$

$$H(z) = \left\{\beta\int_{0}^{z}t^{\beta-1}\left[f_1'(t)\right]^{\gamma_1}\cdot\ldots\cdot\left[f_p'(t)\right]^{\gamma_p}dt\right\}^{\frac{1}{\beta}}, \tag{5}$$

$$F(z) = \int_{0}^{z}\left(\frac{f_1(t)}{t}\right)^{\gamma_1}\cdot\ldots\cdot\left(\frac{f_p(t)}{t}\right)^{\gamma_p}dt, \tag{6}$$

$$G(z)=\left[\beta\int_0^z\left(\frac{f_1(t)}{t}\right)^{\gamma_1}\cdot\ldots\cdot\left(\frac{f_p(t)}{t}\right)^{\gamma_p}dt\right]^{\frac{1}{\beta}}, \tag{7}$$

$$F_{\gamma,\beta}(z)=\left\{\beta\int_0^z t^{\beta-1}\left(\frac{f_1(t)}{t}\right)^{\frac{1}{\gamma_1}}\cdot\ldots\cdot\left(\frac{f_p(t)}{t}\right)^{\frac{1}{\gamma_p}}dt\right\}^{\frac{1}{\beta}}, \tag{8}$$

and

$$G_{\gamma,p}(z)=\left\{[p(\gamma-1)+1]\int_0^z g_1^{\gamma-1}(t)\cdot\ldots\cdot g_p^{\gamma-1}(t)dt\right\}^{\frac{1}{p(\gamma-1)+1}}, \tag{9}$$

where $\gamma_i,\gamma,\beta\in\mathbb{C}\forall i=\overline{1,p}$, $p\in\mathbb{N}-\{0\}$, $n\in\mathbb{N}-\{0,1\}$.

Let D^n be the Sălăgean differential operator [see 12] $D^n:\mathscr{A}\to\mathscr{A}$, $n\in\mathbb{N}$, defined as:

$$D^0f(z)=f(z), D^1f(z)=Df(z)=zf'(z), D^nf(z)=D(D^{n-1}f(z)) \tag{10}$$

and $D^k, D^k:\mathscr{A}\to\mathscr{A}$, $k\in\mathbb{N}\cup\{0\}$, of form:

$$D^0f(z)=f(z),\ldots,D^kf(z)=D(D^{k-1}f(z))=z+\sum_{n=2}^{\infty}n^ka_nz^n. \tag{11}$$

Definition 1 ([2]). *Let $\beta,\ \lambda\in\mathbb{R}$, $\beta\geq 0$, $\lambda\geq 0$ and $f(z)=z+\sum_{j=2}^{\infty}a_jz^j$. We denote by D_λ^β the linear operator defined by*

$$D_\lambda^\beta:A\to A, D_\lambda^\beta f(z)=z+\sum_{j=n+1}^{\infty}[1+(j-1)\lambda]^\beta a_jz^j. \tag{12}$$

Remark 1. *In [1] we have introduced the following operator concerning the functions of form* (1)*:*

$$D_\lambda^\beta:A\to A, D_\lambda^\beta f(z)=z-\sum_{j=n+1}^{\infty}[1+(j-1)\lambda]^\beta a_jz^j. \tag{13}$$

The neighborhoods concerning the class of functions defined using the operator (13) *is studied in [5].*

Remark 2. *Let consider the following operator concerning the functions $f\in S$, $S=\{f\in\mathscr{A}: f$ is univalent in $U\}$:*

$$D_{\lambda_1,\lambda_2}^{n,\beta}f(z)=(h*\psi_1*f)(z)=z\pm\sum_{k\geq 2}\frac{[1-\lambda_1(k-1))]^{\beta-1}}{[1-\lambda_2(k-1))]^{\beta}}\cdot\frac{1+c}{k+c}\cdot C(n,k)\cdot a_k\cdot z^k, \tag{14}$$

where $C(n,k)=\frac{(n+1)_{k-1}}{(1)_{k-1}}$, $(\cdot)$. is the Pochammer symbol; $k\geq 2$, $c\geq 0$.

The following integral operator is studied in [4], where f_i, $i = 1 \dots n$, $n \in \mathbb{N}$, is considered to be of form (2):

Definition 2. *We define the general integral operator* $I_{k,n,\lambda,\mu} : \mathscr{A}_n \to \mathscr{A}$ *by*

$$I_{k,n,\lambda,\mu}(f_1, \dots, f_n) = F \,, \tag{15}$$

$$D^k F(z) = \int_0^z \left(\frac{D_1^\lambda f_1(t)}{t} \right)^{\mu_1} \cdot \dots \cdot \left(\frac{D_n^\lambda f_n(t)}{t} \right)^{\mu_n} dt,$$

where $f_i \in \mathscr{A}$, $i \in \mathbb{N} - \{0\}$, $\lambda = (\lambda_1, \dots, \lambda_n) \in \mathbb{N}_0^n$, $\mu = (\mu_1, \dots, \mu_n) \in \mathbb{N}^n$, $n \in \mathbb{N}$ *and* $k \in \mathbb{N}_0$.

Theorem 1. *Let* $\alpha,\ \gamma_1,\ \gamma_2,\ \beta \in \mathbb{C}$, *Re* $\alpha = a > 0$ *and* $D^{n,\kappa}_{\lambda_1,\lambda_2} f_j(z) \in \mathscr{A}$, $\lambda_1,\ \lambda_2,\ \kappa \geq 0$, $\sigma \in \mathbb{R}$, $j = \overline{1,p}$, $p \in \mathbb{N}$, $D^{n,\kappa}_{\lambda_1,\lambda_2} f_j(z^n)$ *of form* (14). *If*

$$\left| \frac{(D^{n,\kappa}_{\lambda_1,\lambda_2} f_j(z^n))''}{(D^{n,\kappa}_{\lambda_1,\lambda_2} f_j(z^n))'} \right| \leq \frac{1}{n} \, and \, \left| \frac{(D^{n,\kappa}_{\lambda_1,\lambda_2} f_j(z^n))'}{(D^{n,\kappa}_{\lambda_1,\lambda_2} f_j(z^n))} \right| \leq \frac{1}{n} \quad \forall z \in U, j = \overline{1,p},$$

$$\frac{\sum_{j=1}^{p} [|\delta_j^1| \cdot (|2\gamma_1 - 1| - |\sigma|) + |\delta_j^2| \cdot (|2\gamma_2 - 1| - |\sigma|)]}{|\sigma \cdot (2\gamma_1 - 1) \cdot (2\gamma_2 - 1) \cdot (\prod_{j=1}^{p} \delta_j^1 \cdot \delta_j^2)|} \leq 1,$$

and

$$|\sigma \cdot (2\gamma_1 - 1) \cdot (2\gamma_2 - 1) \cdot (\prod_{j=1}^{p} \delta_j^1 \cdot \delta_j^2)| \leq \frac{n + 2a}{2} \cdot \left(\frac{n + 2a}{n} \right)^{\frac{1}{n+2a}},$$

then $\forall \delta,\ \delta_j^1,\ \delta_j^2 \in \mathbb{C}$, $j = 1 \dots p$, $Re(\beta) \geq a$, $Re(\beta\delta) \geq a$, *the function*

$$I^1(z) = \left\{ \beta \int_0^z t^{\beta\delta - 1} \cdot \prod_{j=1}^{p} \left[\frac{((D^{n,\kappa}_{\lambda_1,\lambda_2} f_j(t^n)')^{2\gamma_1 - 1}}{t^\sigma} \right]^{\delta_j^1} \cdot \left[\frac{(D^{n,\kappa}_{\lambda_1,\lambda_2} f_j(t^n))^{2\gamma_2 - 1}}{t^\sigma} \right]^{\delta_j^2} dt \right\}^{\frac{1}{\beta}} \tag{16}$$

is univalent for all $n \in \mathbb{N} - \{0\}$.

If we consider the operator $D^\beta_\lambda f(z)$ of form (13) we obtain the following Corollary, whose proof is similar with the prove of Theorem 1.

Corollary 1. *Let* $\alpha,\ \gamma_1,\ \gamma_2,\ \chi \in \mathbb{C}$, *Re* $\alpha = a > 0$ *and* $D^\beta_\lambda f_j(z) \in \mathscr{A}$, $\beta \geq 0$, $\lambda \geq 0$, $\sigma \in \mathbb{R}$, $D^\beta_\lambda f(z^n)$ *of form* (13). *If*

$$\left| \frac{(D^\beta_\lambda f_j(z^n))''}{(D^\beta_\lambda f_j(z^n))'} \right| \leq \frac{1}{n} \, and \, \left| \frac{(D^\beta_\lambda f_j(z^n))'}{(D^\beta_\lambda f_j(z^n))} \right| \leq \frac{1}{n}, \quad \forall z \in U, j = \overline{1,p},$$

$$\frac{\sum_{j=1}^{p}[|\delta_j^1|\cdot(|2\gamma_1-1|-|\sigma|)+|\delta_j^2|\cdot(|2\gamma_2-1|-|\sigma|)]}{|\sigma\cdot(2\gamma_1-1)\cdot(2\gamma_2-1)\cdot(\prod_{j=1}^{p}\delta_j^1\cdot\delta_j^2)|}\le 1$$

and

$$|\sigma\cdot(2\gamma_1-1)\cdot(2\gamma_2-1)\cdot(\prod_{j=1}^{p}\delta_j^1\cdot\delta_j^2)|\le\frac{n+2a}{2}\cdot\left(\frac{n+2a}{n}\right)^{\frac{1}{n+2a}},$$

then for all $\delta,\ \delta_j^1,\ \delta_j^2\in\mathbb{C},\ j=1\dots p,\ Re(\chi)\ge a,\ Re(\chi\delta)\ge a,$ *the function*

$$I^2(z)=\left\{\chi\int_0^z t^{\chi\delta-1}\prod_{j=1}^{p}\left[\frac{((D_\lambda^\beta f_j(t^n)')^{2\gamma_1-1}}{t^\sigma}\right]^{\delta_j^1}\left[\frac{(D_\lambda^\beta f_j(t^n))^{2\gamma_2-1}}{t^\sigma}\right]^{\delta_j^2}dt\right\}^{\frac{1}{\chi}} \tag{17}$$

is univalent for $\forall n\in\mathbb{N}-\{0\}$.

Lemma 1 ([6]). *Let* $u=u_1+iu_2,\ v=v_1+iv_2$ *and* $\Psi(u,v)$ *be a complex valued function satisfying the conditions:*

(i) $\Psi(u,v)$ *is continuous in a domain* $D\in\mathbb{C}^2$, *Re*

(ii) $(1,0)\in D$ *and Re* $\Psi(1,0)>0$,

(iii) Re $\Psi(iu_2,v_1)\le 0$, *whenever* $(iu_2,v_1)\in D$ *and* $v_1\le-\frac{1}{2}(1+u_2^2)$.

If $h(z)=1+\sum_{i\ge1}c_iz^i$ is an analytic function in U such that $(h(z),zh'(z))\in D$ and $Re\ \Psi(h(z),zh'(z))>0$ for $z\in U$, then $Re\ h(z)>0$ in U.

Lemma 2 ([8]). *Let* $f(z)\in V_k^\lambda(\rho)$, $0\le\rho<1$ *and* λ *is real with* $|\lambda|<\frac{\pi}{2}$. *Then* $f(z)\in R_k^\lambda(\beta)$, *where* β *is one of the root of*

$$2\beta^3+(1-2\rho)\beta^2+(3\sec^2\lambda-4)\beta-(1+2\rho)\tan^2\lambda=0. \tag{18}$$

Following we present the mapping properties of the general integral operator of form (16), giving also several examples which prove its relevance.

3. Main Results

Theorem 2. *Let* $D_{\lambda_1,\lambda_2}^{n,\kappa}f_j(z^n)\in R_k^\lambda$, $D_{\lambda_1,\lambda_2}^{n,\kappa}f_j(z^n)$ *of form* (14), $n\in\mathbb{N},\ \lambda_1,\ \lambda_2,\ \kappa\ge0,\ \sigma\in\mathbb{R}$, $j=\overline{1,p}\ p\in\mathbb{N}$, *for* $0\le\rho<1$. *Also let* λ *be real,* $|\lambda|<\frac{\phi}{2}$. *If*

$$0\le[\rho-1]\sum_{j=1}^{p}\delta_j^a+\beta\delta<1,$$

then $I^1(z) \in V_k^{\lambda}(\eta)$, $I^1(z)$ *of form* (16), *with*

$$\eta = [\rho - 1]\sum_{j=1}^{p} \delta_j^a + \beta\delta, \tag{19}$$

$\beta,\ \delta,\ \delta_j^a \in \mathbb{C}$, $a \in \{1,2\}$, $j = \overline{1,p}$, $Re(\beta\delta) > 0$.

Proof. Let consider the notations

$$h(z) = \int_0^z t^{\beta\delta-1}\prod_{j=1}^{p}\left[\frac{((D_{\lambda_1,\lambda_2}^{n,\kappa} f_j(t^n)')^{2\gamma_1-1}}{t^{\sigma}}\right]^{\delta_j^1} \cdot \left[\frac{(D_{\lambda_1,\lambda_2}^{n,\kappa} f_j(t^n))^{2\gamma_2-1}}{t^{\sigma}}\right]^{\delta_j^2} dt$$

$$= \int_0^z t^{\beta\delta-1}\prod_{j=1}^{p}\left[h_j^1(t^n)\right]^{\delta_j^1} \cdot \left[h_j^2(t^n)\right]^{\delta_j^2} dt$$

in (16), with $\alpha,\ \gamma_1,\ \gamma_2,\ \beta,\ \delta \in \mathbb{C}$, $Re\ \alpha = a > 0$ and $D_{\lambda_1,\lambda_2}^{n,\kappa} f_j(z) \in \mathscr{A}$, $n \in \mathbb{N}$, $\lambda_1,\ \lambda_2,\ \kappa \geq 0$, $\sigma \in \mathbb{R}$, $j = \overline{1,p}$, $p \in \mathbb{N}$.

From Theorem 1, we obtain

$$\frac{[I^1(z)]''}{[I^1(z)]'} = \left(\frac{1}{\beta} - 1\right) \cdot \frac{h'(z)}{h(z)} + \beta\delta \cdot \frac{1}{z} + \left(\sum_{j=1, a\in\{1,2\}}^{p} \delta_j^a \cdot \frac{[h_j^a(z)]'}{h_j^a(z)} - \frac{1}{z}\right)$$

which is equivalently to

$$e^{i\lambda}\left(1 + \frac{z[I^1(z)]''}{[I^1(z)]'}\right) = e^{i\lambda}\cdot\left[\left(\frac{1}{\beta} - 1\right) \cdot \frac{zh'(z)}{h(z)} + \beta\delta\right] + e^{i\lambda}\cdot\left(\sum_{j=1, a\in\{1,2\}}^{p} \delta_j^a \cdot \frac{z[h_j^a(z)]'}{h_j^a(z)} - 1\right) + e^{i\lambda} \tag{20}$$

Furthermore, we have

$$Re\left[e^{i\lambda}\left(1 + \frac{z[I^1(z)]''}{[I^1(z)]'}\right)\right] \leq (\beta\delta - 1) + Re\left[e^{i\lambda} \cdot \left(\sum_{j=1, a\in\{1,2\}}^{p} \delta_j^a \cdot \frac{z[h_j^a(z)]'}{h_j^a(z)} - 1\right) + e^{i\lambda}\right],$$

which can be written as following

$$Re\left[e^{i\lambda}\left(1 + \frac{z[I^1(z)]''}{[I^1(z)]'}\right)\right] \leq Re\left[e^{i\lambda} \cdot \left(\sum_{j=1, a\in\{1,2\}}^{p} \delta_j^a \cdot \frac{z[h_j^a(z)]'}{h_j^a(z)} - 1\right) + \beta\delta e^{i\lambda}\right].$$

Subtracting and adding $\rho\cos\lambda \sum_{j=1, a\in\{1,2\}}^{p} \delta_j^a$ on the left hand side of (20) and then taking the real part, we have

$$Re\left[e^{i\lambda}\left(1 + \frac{z[I^1(z)]''}{[I^1(z)]'}\right) - \eta\cos\lambda\right] \leq \sum_{j=1, a\in\{1,2\}}^{p} \delta_j^a Re\left[e^{i\lambda} \cdot \frac{[h_j^a(z)]'}{h_j^a(z)} - \rho\cos\lambda\right], \tag{21}$$

where η is given by (19).

Integrating (21) and then using (19), we have

$$\int_0^{2\pi}\left|Re\left[e^{i\lambda}\left(1+\frac{z[I^1(z)]''}{[I^1(z)]'}\right)-\eta\cos\lambda\right]\right|d\theta$$
$$\leq\frac{1-\eta}{1-\rho}\int_0^{2\pi}\left|Re\left[e^{i\lambda}\cdot\frac{[h_j^a(z)]'}{h_j^a(z)}-\rho\cos\lambda\right]\right|d\theta. \qquad (22)$$

Since $f_j(z^n)\in R_k^\lambda(\rho)$, $j=\overline{1,p}$, $p,n\in\mathbb{N}-\{0\}$, we obtain

$$\int_0^{2\pi}\left|Re\left[e^{i\lambda}\cdot\frac{[h_j^a(z)]'}{h_j^a(z)}-\rho\cos\lambda\right]\right|d\theta\leq(1-\rho)k\pi\cos\lambda. \qquad (23)$$

Using (22) and (23), we have

$$\int_0^{2\pi}\left|Re\left[e^{i\lambda}\left(1+\frac{z[I^1(z)]''}{[I^1(z)]'}\right)-\eta\cos\lambda\right]\right|d\theta\leq(1-\eta)k\pi\cos\lambda.$$

Hence $I^1(z)\in V_k^\lambda(\eta)$ with η given by (19).

Remark 3. *If we consider the operator $D_\lambda^\beta f(z)\in R_k^\lambda(\rho)$ of form* (13) *we obtain similar result as in Theorem 2.*

Remark 4. *If we apply the operator* (10) *to the integral operator $F(z)$ of form* (6)*, we obtain the result from [8].*

Next we give few examples of particular cases which can be found in literature.

Let $\beta=0$ in $D_\lambda^\beta f(z)$ of form (12) or (13). So we have that $D_\lambda^0 f(z)=f(z),\forall\lambda\geq 0$. We will use this form of the integral operator, where the function f is of form (2) with respect to the operator (17). For further simplification, we consider that $\gamma_1=\gamma_2=1$, and $\delta=1$ (except of Example 4).

For the first four examples we consider $\delta_j^1=0$, $j=\overline{1,p}$, $p\in\mathbb{N}-\{0\}$, $n=1$.

Example 1. *If $\sigma=1$, $\chi=1$ and we use the notation $\delta_j^2=\gamma_j$, $j=\overline{1,p}$, $p\in\mathbb{N}-\{0\}$, we obtain the operator $F(z)$ of form* (6). *$F(z)\in V_k^\lambda(\eta)$ if $0\leq(\rho-1)\sum_{j=1}^{p}\gamma_j+1<1$ with $\eta=(\rho-1)\sum_{j=1}^{p}\gamma_j+1$.*

Example 2. *If $\sigma=1$ we obtain the operator $G(z)$ of form* (7) *for $\delta_j^2=\gamma_j$, $j=\overline{1,p}$, $p\in\mathbb{N}-\{0\}$. $G(z)\in V_k^\lambda(\eta)$ if $0\leq(\rho-1)\sum_{j=1}^{p}\gamma_j+1<1$ with $\eta=(\rho-1)\sum_{j=1}^{p}\gamma_j+1$.*

Example 3. *If* $\sigma = 1$ *and we use the notation* $\delta_j^2 = 1/\gamma_j$, $j = \overline{1,p}$, $p \in \mathbb{N} - \{0\}$, *we obtain the operator* $F_{\gamma,\beta}(z)$ *of form* (8). $F_{\gamma,\beta}(z) \in V_k^\lambda(\eta)$ *if* $0 \le (\rho - 1)\sum_{j=1}^{p} \frac{1}{\gamma_j} + \beta < 1$ *with*
$\eta = (\rho - 1)\sum_{j=1}^{p} \gamma_j + \beta$.

Example 4. *If* $\sigma = 0$ *we obtain the operator* $G_{\gamma,p}(z)$ *of form* (9) *for* $\chi = [p(\gamma - 1) + 1]$, $\delta = \frac{1}{\chi}$ *and* $\delta_j^2 = \gamma - 1$, $G_{\gamma,p}(z) \in V_k^\lambda(\eta)$ *if* $0 \le (1 - \rho)\sum_{j=1}^{p} \gamma_j + 1 < 1$ *with* $\eta = (\rho - 1)\sum_{j=1}^{p} \gamma_j + 1$.

For the next two examples we consider $\delta_j^2 = 0$, $j = \overline{1,p}$, $p \in \mathbb{N} - \{0\}$, and $\sigma = 0$.

Example 5. *a) If* $\chi = 1$, $\delta = 1$, *we obtain a particular case of the function* $J(z)$ *of form* (4), *in which* $\beta = 1, \forall n \in \mathbb{N} - \{0\}$. $J(z) \in V_k^\lambda(\eta)$ *if* $0 \le (1 - \rho)\sum_{j=1}^{p} \gamma_j + 1 < 1$ *with* $\eta = (\rho - 1)\sum_{j=1}^{p} \gamma_j + 1$.

b) If $\delta = \frac{1}{\chi}$, $\delta_j^1 = \gamma_j$, $j = \overline{1,p}$, $p \in \mathbb{N} - \{0\}$, *we obtain the operator* $J(z)$ *of form* (4), *in which* $\beta = 1, \forall n \in \mathbb{N} - \{0\}$. $J(z) \in V_k^\lambda(\eta)$ *if* $0 \le (1-\rho)\sum_{j=1}^{p} \gamma_j + 1 < 1$ *with* $\eta = (\rho - 1)\sum_{j=1}^{p} \gamma_j + 1$.

Example 6. *If* $n = 1$, $\delta = \frac{1}{\chi}$, *we obtain the operator* $H(z)$ *of form* (5) *for* $\delta_j^1 = \gamma_j$, $j = \overline{1,p}$, $p \in \mathbb{N} - \{0\}$. $F(z) \in V_k^\lambda(\eta)$ *if* $0 \le (1 - \rho)\sum_{j=1}^{p} \gamma_j + \beta < 1$ *with* $\eta = (\rho - 1)\sum_{j=1}^{p} \gamma_j + \beta$.

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