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Γ-semigroups with unities and Morita equivalence for monoids

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Abstract. In this paper we show that the left operator and right operator semigroups of a Γ-semigroup with unities are Morita equivalent monoids. It is also deduced that if L and R are two Morita equivalent monoids then a Γ-semigroup with unities can be constructed such that its left and right operator monoids are isomorphic to L and R respectively. This result is used to obtain some properties of monoids which remain invariant under Morita equivalence.

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1. Introduction

If someone asks "What is the most natural example of ring?", the ring of integers is one possible answer to this question. But a more interesting answer will be the endomorphism ring of an abelian group, i.e., $EndM$ or $Hom(M,M)$ where M is an abelian group. Now if two abelian groups, say A and B instead of one are taken, then $Hom(A,B)$ is no longer a ring in the way as $EndM$ becomes a ring because the composition is no longer defined. However, if one takes an element of $Hom(B,A)$ and put it in between two elements of $Hom(A,B)$ then the composition can be defined.

Taking this as a motivating factor N. Nobusawa [15] generalized the ring theory in the form of Γ-ring in 1964. Later M.K.Sen [18] introduced the notion of Γ-semigroup by taking sets instead of abelian groups. It is well known that with every Γ-structure, there always exist two corresponding operator structures, i.e., for a Γ-ring, there exist two associated operator rings, for a Γ-semigroup, there are operator semigroups. It is natural to ask the reverse question i.e., what relation is to be satisfied by two given rings or semigroups M and N such that one can find a corresponding Γ-structure(i.e. Γ-ring or Γ-semigroup respectively) whose

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associated operator structures are isomorphic to M and N respectively?

This problem for Γ-ring was solved by M. Parvati [17] in 1984. In the same year, N. Nobusawa [16] stated without proof that the problem would have some solution. While answering the question for Γ-rings, M. Parvati adopted the Morita equivalence of rings [6], in the language of category [14], as an important tool. This fact together with the vast literature on Morita equivalence of semigroups and monoids [7, 8, 12, 13, 19, 20] has motivated us to write this paper. In this paper, among other results we deduce that two monoids L and R are Morita equivalent if and only if there exists a Γ-semigroup with unities whose operator monoids are isomorphic to L and R.

This result appears to be very useful to study the Γ-semigroups via Morita theory for monoids by using operator semigroups and conversely the Morita theory for monoids via Γ-semigroups. We will apply this result to obtain some results concerning the Morita invariants of monoids.

2. Preliminaries

We recall here some basic definitions and results on Morita equivalence for monoids as well as some notions of Γ-semigroups with unities for their use in the sequel. Throughout this paper, we assumed the mappings to act from the right.

Definition 1 ([12]). *Let A be a monoid with identity 1. Then a nonempty set M together with a map $A \times M \longrightarrow M$, denoted $(a,x) \mapsto ax$, satisfying $(ab)x = a(bx)$ and $1x = x$ for all $a, b \in A$ and $x \in M$, is called a (left) A-act and is denoted by ${}_AM$.*

Definition 2 ([12]). *Let M and N be two A-acts. Then a mapping $f : M \longrightarrow N$ is called a (left) A-morphism if for all $a \in A$ and $x \in M$, $(ax)f = a(xf)$.*

The notions of $A-B$-biact and $A-B$-morphism are defined in an obvious manner. The category formed by left A-acts together with the A-morphisms is denoted by A-Act. Analogously, the right A-acts (denoted by M_A) and the right A-morphisms can be defined. Their category is denoted by Act-A.

Definition 3 ([12]). *Let A and B be two monoids. Then A and B are said to be Morita equivalent if A-Act and B-Act are two equivalent categories.*

Theorem 1 ([19]). *Let R and S be two Morita equivalent monoids via inverse equivalences $F : R-Act \rightarrow S-Act$ and $G : S-Act \rightarrow R-Act$. Set $P = F(R)$ and $Q = G(S)$. Then P and Q are unitary biacts ${}_SP_R$ and ${}_RQ_S$ such that,*

(1) ${}_SP$, ${}_RQ$ are respectively generators for S-Act and R-Act;

(2) $R \cong End_S(P)$ and $S \cong End_R(Q)$;

(3) $F \cong Hom_R(Q, \_)$ and $G \cong Hom_S(P, \_)$;

(4) ${}_SP_R \cong Hom_R(Q,R)$ *and* ${}_RQ_S \cong Hom_S(P,S)$.

The following theorem characterizes the generators of A-Act.

Theorem 2 ([12]). *An A-act G is a generator for the category A-Act if and only if there exists an epimorphism* $g : G \to A$.

Definition 4 ([12]). *For a right A-act* M_A *and a left A-act* ${}_AN$*, the tensor product of these two acts, denoted by* $M \otimes_A N$*, is the unique solution of the usual universal problem: that is,* $M \otimes_A N = (M \times N)/\sigma$*, where* σ *is the equivalence relation on* $M \times N$ *generated by* $\Sigma = \{((xa, y), (x, ay)) : x \in M, y \in N, a \in A\}$. *We denote the class of* (x, y) *by* $x \otimes y$. *When there is no ambiguity about the monoid A, we write the tensor product as* $M \otimes N$.

Definition 5 ([2]). *Let A and* Γ *be two non-empty sets. Then A is said to be a* Γ*-semigroup if there exist mappings* $A \times \Gamma \times A \to A$*, denoted by* $(a, \gamma, b) \mapsto a\gamma b$*, and* $\Gamma \times A \times \Gamma \to \Gamma$*, denoted by* $(\alpha, a, \beta) \mapsto \alpha a\beta$*, satisfying* $(a\alpha b)\beta c = a(\alpha b\beta)c = a\alpha(b\beta c)$ *for all* $a, b, c \in A$ *and* $\alpha, \beta \in \Gamma$.

Definition 6 ([2]). *Let A be a* Γ*-semigroup and* ρ *be a relation on* $\Gamma \times A$ *defined by* $(\alpha, a)\rho(\beta, b) \Longleftrightarrow x\alpha a = x\beta b$ *and* $\alpha a\gamma = \beta b\gamma$ *for all* $x \in A$ *and* $\gamma \in \Gamma$. *Then* ρ *is an equivalence relation. Let us denote the equivalence class of* (α, a) *by* $[\alpha, a]$. *Then the right operator semigroup of the* Γ*-semigroup A is defined to be* $R = (\Gamma \times A)/\rho = \{[\alpha, a] \mid a \in A, \alpha \in \Gamma\}$*, where the composition is defined as* $[\alpha, a][\beta, b] = [\alpha, a\beta b]$ *and the associativity of this composition comes from the associativity of* Γ*-semigroup A.*
Analogously, the left operator semigroup L is defined and its elements are denoted by $[a, \alpha], \mid a \in A, \alpha \in \Gamma\}$.

Definition 7 ([2]). *If there exists an element* $[\gamma, f]$ *in the right operator semigroup R of a* Γ*-semigroup A such that* $x\gamma f = x$ *for all* $x \in A$*, then that element is called the right unity of A. Similar is the definition of the left unity of A. A is said to be a* Γ*-semigroup with unities if it has both left and right unities. It may be noted that the left unity (right unity) of A becomes the identity of L (respectively, R).*

For more preliminaries on Morita theory of monoids and on Γ-semigroups we refer to [7, 12, 13, 19] and [1, 2, 3, 4, 5], respectively.

3. Relating Γ-semigroups with Morita Equivalence

In this section we are going to explore the relationship between Γ-semigroups with unities and the Morita equivalence for monoids.

Definition 8. *A six-tuple* $\langle A, B, {}_AP_B, {}_BQ_A, \tau, \mu\rangle$ *is said to be a Morita context, where A and B are monoids,* ${}_AP_B$ *and* ${}_BQ_A$ *are biacts,* τ *is an* $A-A$*-morphism of* $P \otimes_B Q$ *into A and* μ *is an* $B-B$*-morphism of* $Q \otimes_A P$ *into B such that if we write* $(p \otimes q)\tau = <p, q>$ *and* $(q \otimes p)\mu = [q, p]$*, then for all* $p, p' \in P$ *and* $q, q' \in Q$ *we have* $<p, q> p' = p[q, p']$ *and* $q <p, q'> = [q, p]q'$.

This definition is analogous to the case of semigroups in [19]. It can be easily seen that the Morita context presented here is "unitary" in the sense given by S.Talwar [19].

The following result is a simple consequence of Theorem 8.3 of S. Talwar in [19] when the semigroups are replaced by monoids.

Theorem 3. *Let* $\langle R, S, {}_R P_S, {}_S Q_R, \tau, \mu\rangle$ *be a Morita context with* τ, μ *surjective. Then the following statements hold:*

(i) The categories R-Act and S-Act are equivalent;

(ii) ${}_S P$ *and* ${}_R Q$ *are respectively generators for S-Act and R-Act;*

(iii) $R \cong End_S(P)$ *and* $S \cong End_R(Q)$ *as semigroups;*

(iv) ${}_S P_R \cong Hom_R(Q, R)$ *and* ${}_R Q_S \cong Hom_S(P, S)$ *as biacts.*

Theorem 4. *Let A be a* Γ*-semigroup with unities and its left and right operator monoids are respectively L and R. Then the following conditions hold:*

(1) L and R are Morita equivalent;

(2) ${}_L A$ *and* ${}_R \Gamma$ *are respectively generators for L-Act and R-Act;*

(3) $R \cong End_L(A)$ *and* $L \cong End_R(\Gamma)$ *as semigroups;*

(4) ${}_L A_R \cong Hom_R(\Gamma, R)$ *and* ${}_R \Gamma_L \cong Hom_L(A, L)$ *as biacts.*

Proof. We first prove that ${}_L A_R$ and ${}_R \Gamma_L$ are biacts. For this we define

$$L \times A \to A \quad \text{and} \quad A \times R \to A$$

respectively as follows

$$[a, \alpha]b := a\alpha b \quad \text{and} \quad a[\beta, b] := a\beta b.$$

Then for all $a, b, c \in A$ and $\alpha, \beta \in \Gamma$, we have

$$a([\alpha, b][\beta, c]) \;\; = a[\alpha, b\beta c] \;\; = a\alpha(b\beta c) \;\; = (a\alpha b)\beta c \;\; = (a[\alpha, b])[\beta, c].$$

Hence A is a right R-act. Similarly we can show that A is a left L-act. That A is a biact follows from the generalized associative property(GAP) of the Γ-semigroup A. Similarly we can show that ${}_R \Gamma_L$ is a biact.
Now consider the mappings

$$\tau : A \otimes \Gamma \to L \quad \text{and} \quad \mu : \Gamma \otimes A \to R$$

respectively defined as follows

$$(a \otimes \alpha)\tau = [a, \alpha] \quad \text{and} \quad (\alpha \otimes a)\mu = [\alpha, a].$$

Now we prove that the mapping τ is well-defined. Let $a \otimes \alpha = b \otimes \beta$. Then either $(a, \alpha) = (b, \beta)$ in which case $[a, \alpha] = [b, \beta]$; or for some positive integer $n \geq 2$ there is a sequence

$$(a, \alpha) = (a_1, \alpha_1) \rightarrow (a_2, \alpha_2) \rightarrow \ldots \rightarrow (a_n, \alpha_n) = (b, \beta)$$

in which for each $i \in \{1, \ldots, n-1\}$

either $((a_i, \alpha_i), (a_{i+1}, \alpha_{i+1})) \in \Sigma$ or $((a_{i+1}, \alpha_{i+1}), (a_i, \alpha_i)) \in \Sigma$.

Here $((a_i, \alpha_i), (a_{i+1}, \alpha_{i+1})) \in \Sigma$ means that for some $r_i \in R$, $a_i = a_{i+1}r_i$, $\alpha_{i+1} = r_i\alpha_i$. Then for all $c \in A$

$$a_i\alpha_i c = (a_{i+1}r_i)\alpha_i c = a_{i+1}(r_i\alpha_i)c = a_{i+1}\alpha_{i+1}c.$$

Also for all $\gamma \in \Gamma$,

$$\gamma a_i\alpha_i = \gamma(a_{i+1}r_i)\alpha_i = \gamma a_{i+1}(r_i\alpha_i) = \gamma a_{i+1}\alpha_{i+1}.$$

So we have $[a_i, \alpha_i] = [a_{i+1}, \alpha_{i+1}]$ for each $i \in \{1, \ldots, n-1\}$. The same happens for the other case also. Using these results we have

$$[a, \alpha] = [a_1, \alpha_1] = \cdots = [a_n, \alpha_n] = [b, \beta].$$

Similarly μ is also well defined. Again, by the definition it is clear that these mappings are surjective.
Now we see that for all $a, b \in A,\ \alpha, \beta \in \Gamma$,

$$(a \otimes \alpha)\tau b = \ [a, \alpha]b = \ a\alpha b = \ a[\alpha, b] = \ a(\alpha \otimes b)\mu \text{and}$$
$$\alpha(a \otimes \beta)\tau = \ \alpha[a, \beta] = \ \alpha a\beta = \ [\alpha, a]\beta = \ (\alpha \otimes a)\mu\beta.$$

Thus $\langle L, R, {}_L A_R, {}_R \Gamma_L, \tau, \mu \rangle$ is a Morita context with τ and μ surjective.
Now from Theorem 3 we can prove the following statements:

(1) L and R are Morita equivalent;

(2) ${}_LA$ and ${}_R\Gamma$ are respectively the generators for the L-Act and the R-Act;

(3) $R \cong End_L(A)$ and $L \cong End_R(\Gamma)$ as semigroups;

(4) ${}_LA_R \cong Hom_R(\Gamma, R)$ and ${}_R\Gamma_L \cong Hom_L(A, L)$ as biacts.

Hence the theorem is proved.

In the following theorem we consider two monoids L and R which are Morita equivalent.

Theorem 5. *Let L and R be two monoids which are Morita equivalent. Then there exists a Γ-semigroup with unities whose left and right operator monoids are respectively isomorphic to L and R respectively.*

Proof. As L and R are Morita equivalent the categories L-Act and R-Act are equivalent categories via functors, say

$$F : R\text{-Act} \to L\text{-Act} \quad \text{and} \quad G : L\text{-Act} \to R\text{-Act}.$$

Now let $A = F(R)$ and $\Gamma = G(L)$. Then by Theorem 1 the following statements follow:

(1) ${}_LA_R$ and ${}_R\Gamma_L$ are biacts;

(2) ${}_LA$ and ${}_R\Gamma$ are generators for L-Act and R-Act;

(3) ${}_LA_R \cong Hom_R(\Gamma, R)$ and ${}_R\Gamma_L \cong Hom_L(A, L)$;

(4) $F \cong Hom_R(\Gamma, \_)$ and $G \cong Hom_L(A, \_)$;

(5) $L \cong End_R\Gamma,\ R \cong End_LA$.

Now considering Γ as $Hom_L(A, L)$ we define the mappings

$$A \times \Gamma \times A \to A \quad \text{and} \quad \Gamma \times A \times \Gamma \to \Gamma$$

such that for $a, b, x \in A$ and $\alpha, \beta, \gamma \in \Gamma$

$$(a, \gamma, b) \mapsto ((a)\gamma)b \quad \text{and} \quad (\alpha, x, \beta) \mapsto \alpha((x)\beta).$$

Now we have

$(a\alpha b)\beta c = (((a)\alpha b)\beta)c = ((a)\alpha(b)\beta)c$, since β is a left L-morphism.

$a\alpha(b\beta c) = ((a)\alpha)(((b)\beta)c) = ((a)\alpha(b)\beta)c$, since A is a left L-act.

$a(\alpha b\beta)c = ((a)(\alpha((b)\beta)))c = ((a)\alpha(b)\beta)c$, this comes from how we have considered $Hom_L(A, L)$ as a right L-act.

Hence A is a Γ-semigroup. Now it remains to prove that the operator semigroups are isomorphic to L and R.
Let L' and R' be the left and right operator semigroups of the Γ-semigroup A. We define

$$f : L' \to L \text{ by } ([a, \alpha])f = (a)\alpha.$$

The mapping f is well-defined, since

$$\begin{aligned}[a, \alpha] = [b, \beta] &\Rightarrow \gamma a\alpha = \gamma b\beta \text{ for all } \gamma \in \Gamma\\ &\Rightarrow \gamma((a)\alpha) = \gamma((b)\beta) \text{ for all } \gamma \in \Gamma\end{aligned}$$

$$\Rightarrow (a)\alpha = (b)\beta.$$

It is also clear that the mapping is injective, since

$$\begin{aligned}(a)\alpha = (b)\beta &\Rightarrow \gamma((a)\alpha) = \gamma((b)\beta) \text{ and } ((a)\alpha)x = ((b)\beta)x \text{ for all } x \in A, \gamma \in \Gamma\\ &\Rightarrow \gamma a\alpha = \gamma b\beta \text{ and } a\alpha x = b\beta x \text{ for all } x \in A,\ \gamma \in \Gamma\\ &\Rightarrow [a,\alpha] = [b,\beta].\end{aligned}$$

Again, since ${}_LA$ is a generator of L-Act so by Theorem 2 there exists a left L-morphism $\psi : A \to L$. Hence for any $l \in L$ there exists $a \in A$ such that $(a)\psi = l$. Then we have $([a,\psi]f = l$. This shows that the mapping f is surjective.
Also f is a semigroup morphism because for all $a, b \in A$ and $\alpha, \beta \in \Gamma$

$$([a,\alpha][b,\beta])f = ([a\alpha b,\beta])f = (((a)\alpha)b)\beta = (a)\alpha(b)\beta = ([a,\alpha])f([b,\beta])f.$$

Hence L and L' are isomorphic as monoids.
Similarly we can prove that R' is isomorphic to R. Hence the proof follows.

We conclude this section by combining the above two results into one theorem.

Theorem 6. *Two monoids L and R are Morita equivalent if and only if there exists a* Γ*-semigroup with unities whose operator monoids are isomorphic to L and R.*

4. Applications

In this section we give some applications of the Theorem 6. By using our result and some well-known theories of Γ-semigroups we are able to deduce some properties of monoids which remain invariant under Morita equivalence. The reader is referred to [1, 2, 3, 4, 5] for the results of Γ-semigroups and to [9, 10, 11] for preliminaries of semigroups used in this section.

Theorem 7. *Let L and R be two Morita equivalent monoids. Then there exists an inclusion preserving bijection between the set of all ideals of R and the set of all ideals of L.*

Proof. By Theorem 6 there exists a Γ-semigroup A with left and right unities whose left and right operator monoids L_1 and R_1 are isomorphic to L and R respectively. Hence it suffices to prove the result for L_1 and R_1. Now for each $P \subseteq L_1$ and $M \subseteq R_1$ we define

$$\begin{aligned}P^+ &= \{x \in A \mid [x,\alpha] \in \text{ for all } \alpha \in \Gamma\} \text{ and}\\ M^* &= \{x \in A \mid [\alpha, x] \in M, \text{ for all } \alpha \in \Gamma\}.\end{aligned}$$

Also for each $Q \subseteq A$ define

$$\begin{aligned}Q^{+'} &= \{[x,\alpha] \in L_1 \mid x\alpha a \in Q, \text{ for all } a \in A\} \text{ and}\\ Q^{*'} &= \{[\alpha, x] \in R_1 \mid a\alpha x \in Q, \text{ for all } a \in A\}.\end{aligned}$$

Now there exists an inclusion preserving bijection between the set of all ideals of A and the set of all ideals of L_1 [see 2] via the mapping

$$Q \mapsto Q^{+'} \text{ with the inverse } P \mapsto P^{+}.$$

And also there exists an inclusion preserving bijection between the set of all ideals of A and the set of all ideals of R_1 via the mapping

$$Q \mapsto Q^{*'} \text{ with the inverse } M \mapsto M^{*}.$$

Thus we find an inclusion preserving bijection between the set of all ideals of R_1 and the set of all ideals of L_1 via the composition mapping

$$J \mapsto J^{*} \mapsto (J^{*})^{+'} \text{ with the inverse } I \mapsto I^{+} \mapsto (I^{+})^{*'}.$$

Hence the proof is completed.

Now from [1, 3, 5] we know that all the mappings $+$, $+'$, $*$ and $*'$ carry the prime ideal to prime ideal; the maximal ideal to maximal ideal; the nilpotent ideal to nilpotent ideal; the nil ideal to nil ideal; the primary ideal to primary ideal; the semiprimary ideal to semiprimary ideal and the semiprime ideal to semiprime ideal. Thus by applying the same arguments as given in the above theorems we see that the composition mappings

$$J \mapsto J^{*} \mapsto (J^{*})^{+'} \text{ and its inverse } I \mapsto I^{+} \mapsto (I^{+})^{*'}.$$

carry the prime ideal to prime ideal; the maximal ideal to maximal ideal; the nilpotent ideal to nilpotent ideal; the nil ideal to nil ideal; the primary ideal to primary ideal; the semiprimary ideal to semiprimary ideal and the semiprime ideal to semiprime ideal from R_1 to L_1 and from L_1 to R_1, respectively. This fact gives rise to the following result for the Morita equivalent monoids.

Theorem 8. *Let L and R be two Morita equivalent monoids. Then there exists an inclusion preserving bijection between the set of all prime (maximal, nilpotent, nil, primary, semiprimary, semiprime) ideals of R and the set of all prime (maximal, nilpotent, nil, primary, semiprimary, semiprime, respectively) ideals of L.*

Also combining the respective results of [3, 5] for left operator L and right operator R of a Γ-semigroup A we find that different types of radicals viz. prime radical, Schwarz radical, Clifford radical are also preserved by the mappings

$$J \mapsto J^{*} \mapsto (J^{*})^{+'} \text{ and its inverse } I \mapsto I^{+} \mapsto (I^{+})^{*'}.$$

Hence, we obtain the following theorem.

Theorem 9. *Let L and R be Morita equivalent monoids. Then the prime (Schwarz, Clifford) radical of $R(L)$ is taken to be the prime (Schwarz, Clifford) radical of L (respectively, R) via the mapping*

$$J \mapsto J^{*} \mapsto (J^{*})^{+'} \text{ (respectively, } I \mapsto I^{+} \mapsto (I^{+})^{*'}).$$

In the following theorem we characterize the Noetherian Morita equivalent monoids.

Theorem 10. *Let L and R be Morita equivalent monoids. Then L is Noetherian if and only if R is Noetherian.*

Proof. By Theorem 6 there exists a Γ-semigroup A with left and right unities whose left and right operator monoids L_1 and R_1 are isomorphic to L and R respectively. Now according to [4]

$$L_1 \text{ is Noetherian } \Leftrightarrow A \text{ is Noetherian } \Leftrightarrow R_1 \text{is Noetherian.}$$

Hence the result follows.

In a similar way by using the result of [1] regarding primary and semiprimary Γ-semigroup we get the following theorem.

Theorem 11. *Let L and R be Morita equivalent monoids. Then L is primary (semiprimary) if and only if R is primary (semiprimary).*

Remark 1. *Since we have deduced that the operator monoids of a Γ-semigroup A with unities are Morita equivalent it is noteworthy that the examples of non-isomorphic operator monoids will also become examples of non-isomorphic Morita equivalent monoids.*

5. Conclusion

It is evident from Theorem 6 that there is a close connection between the Morita equivalence of monoids and Γ-semigroups with unities which supplement the study of the other. In this paper we have already shown how the results of Γ-semigroups can be used to enrich the Morita theory for monoids. It will be nice to see whether similar results will hold when we replace the Morita equivalence by Morita-like equivalence [20].

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