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Representation Number Formulae for Some Mixed Quadratic Forms

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Abstract. We find formulae for the number of representations of a positive integer by some quadratic forms which are sums of squares and the form $x_1^2 + x_1x_2 + x_2^2$.

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1. Introduction

Let N, $\mathbb{N}_0$, $\mathbb{Z}$ and $\mathbb{C}$ denote the set of natural numbers, non-negative integers, integers and complex numbers so that $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. For k, $n \in \mathbb{N}$ we define

$$\sigma_k(n) := \sum_{\substack{d \in \mathbb{N} \\ d|n}} d^k \tag{1}$$

where d runs through the positive integers dividing n. We write $\sigma(n)$ for $\sigma_1(n) = \sum_{\substack{d \in \mathbb{N} \\ d|n}} d$. If $n \notin \mathbb{N}$, we set $\sigma_k(n) = 0$. For $a_1, \ldots, a_6 \in \mathbb{N}$ and $n \in \mathbb{N}_0$ we define

$$M(a_1, a_2, a_3, a_4, a_5, a_6; n) := card \left\{ \begin{array}{l} (x_1, \ldots, x_8) \in \mathbb{Z}^8 : \\ n = a_1x_1^2 + a_2x_2^2 + a_3x_3^2 + a_4x_4^2 \\ +a_5(x_5^2 + x_5x_6 + x_6^2) + a_6(x_7^2 + x_7x_8 + x_8^2) \end{array} \right\}. \tag{2}$$

With our notation, the representation number formulae for

$$\begin{array}{llll} M(1,1,1,1,2,2;n), & M(3,3,3,3,2,2;n), & M(1,1,1,1,1,1;n), & M(3,3,3,3,1,1;n), \\ M(1,1,3,3,1,1;n), & M(1,1,3,3,2,2;n), & M(1,1,3,3,4,4;n), & M(1,1,3,3,1,2;n), \\ M(1,1,3,3,1,4;n), & M(1,1,3,3,2,4;n), & M(1,1,1,1,1,2;n), & M(3,3,3,3,1,2;n), \\ M(1,1,1,1,1,4;n), & M(3,3,3,3,1,4;n), & M(1,1,1,1,2,4;n), & M(3,3,3,3,2,4;n) \end{array}$$

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are given in [5] respectively by the equations from (2.10) to (2.25). In that study the authors firstly uses the $(p-k)$ parametrization of theta functions given by Alaca, Alaca and Williams to establish some new theta function identities and then uses them to determine formulae for the number of representations of positive integers by certain quadratic forms.

The aim of this paper is to determine explicit formulae for

$$\begin{array}{llll} M(1,1,2,2,2,2;n), & M(2,2,2,2,1,1;n), & M(3,3,6,6,4,4;n), & M(1,1,1,1,6,6;n), \\ M(3,3,6,6,2,2;n), & M(1,1,2,2,1,1;n), & M(1,1,2,2,4,4;n), & M(3,3,6,6,1,1;n), \\ M(1,1,3,3,6,6;n), & M(1,2,2,4,2,2;n), & M(1,2,2,4,4,4;n) & \end{array}.$$

We use some known representation number formulae for quaternary quadratic forms and convolutions sums of divisors. The method firstly have been used by Alaca, Alaca and Williams. The quadratic forms considered here have not been considered before. As in [5] the formulae are given in terms of $\sigma_3(n)$ and the numbers $c_{1,6}(n)$, $c_{1,8}(n)$, $c_{1,12}(n)$, $c_{3,4}(n)$, $c_{1,18}(n)$, $c_{2,9}(n)$, $c_{1,24}(n)$, $c_{3,8}(n)$.

The representation numbers for the quaternary quadratic forms

$$f_1 := x_1^2 + x_2^2 + x_3^2 + x_4^2, \tag{3}$$

$$f_2 := x_1^2 + x_2^2 + 2x_3^2 + 2x_4^2, \tag{4}$$

$$f_3 := x_1^2 + x_2^2 + 3x_3^2 + 3x_4^2, \tag{5}$$

$$f_4 := x_1^2 + 2x_2^2 + 2x_3^2 + 4x_4^2, \tag{6}$$

$$f_5 := x_1^2 + x_1x_2 + x_2^2 + x_3^2 + x_3x_4 + x_4^2, \tag{7}$$

have been determined before. For $l \in \mathbb{N}_0$, if we set

$$r_i(l) = card\left\{(x_1,\ldots,x_4) \in \mathbb{Z}^4 : l = f_i(x_1,x_2,x_3,x_4)\right\}$$

then clearly $r_i(0) = 1$ for each $i \in \{1,2,3,4,5\}$. It is known (see for example [4] for the first four and [7] for the last one) that for $l \in \mathbb{N}$,

$$r_1(l) = 8\sigma(l) - 32\sigma(\frac{l}{4}), \tag{8}$$

$$r_2(l) = 4\sigma(l) - 4\sigma(\frac{l}{2}) + 8\sigma(\frac{l}{4}) - 32\sigma(\frac{l}{8}), \tag{9}$$

$$r_3(l) = 4\sigma(l) - 8\sigma(\frac{l}{2}) - 12\sigma(\frac{l}{3}) + 16\sigma(\frac{l}{4}) + 24\sigma(\frac{l}{6}) - 48\sigma(\frac{l}{12}), \tag{10}$$

$$r_4(l) = 2\sigma(l) - 2\sigma(\frac{l}{2}) + 8\sigma(\frac{l}{8}) - 32\sigma(\frac{l}{16}), \tag{11}$$

$$r_5(l) = 12\sigma(l) - 36\sigma(\frac{l}{3}). \tag{12}$$

For $r,s,n \in \mathbb{N}$ with $r \le s$ the convolution sum $W_{r,s}(n)$ is defined by

$$W_{r,s}(n) := \sum_{\substack{(l,m)\in\mathbb{N}_0^2 \\ rl+sm=n}} \sigma(l)\sigma(m).$$

The sum $W_{r,s}(n)$ has been evaluated for certain values, see [6, 7, 10] for $W_{1,1}(n)$, [7] for $W_{1,2}(n)$ and $W_{1,4}(n)$, [7–9, 11] for $W_{1,3}(n)$, [3] for $W_{1,6}(n)$ and $W_{2,3}(n)$, [12] for $W_{1,8}(n)$, [1] for $W_{1,12}(n)$ and $W_{3,4}(n)$, and [2] for $W_{1,24}(n)$ and $W_{3,8}(n)$.

Theorem 1. *Let $n \in N$ then,*

(i)

$$\begin{aligned} M(1,1,2,2,2,2;n) =& \frac{14}{5}\sigma_3(n) - \frac{14}{5}\sigma_3(\frac{n}{2}) - \frac{54}{5}\sigma_3(\frac{n}{3}) + \frac{28}{5}\sigma_3(\frac{n}{4}) + \frac{54}{5}\sigma_3(\frac{n}{6}) \\ & - \frac{448}{5}\sigma_3(\frac{n}{8}) - \frac{108}{5}\sigma_3(\frac{n}{12}) + \frac{1728}{5}\sigma_3(\frac{n}{24}) + \frac{6}{5}c_{1,6}(n) \\ & + \frac{12}{5}c_{1,6}(\frac{n}{2}) - \frac{12}{5}c_{3,4}(\frac{n}{2}), \end{aligned}$$

(ii)

$$\begin{aligned} M(2,2,2,2,1,1;n) =& \frac{21}{5}\sigma_3(n) + \frac{91}{5}\sigma_3(\frac{n}{2}) - \frac{81}{5}\sigma_3(\frac{n}{3}) - \frac{84}{5}\sigma_3(\frac{n}{4}) - \frac{351}{5}\sigma_3(\frac{n}{6}) \\ & - \frac{448}{5}\sigma_3(\frac{n}{8}) + \frac{324}{5}\sigma_3(\frac{n}{12}) + \frac{1728}{5}\sigma_3(\frac{n}{24}) + \frac{12}{5}c_{1,6}(n) \\ & + 6c_{1,8}(n) - \frac{3}{5}c_{3,8}(n), \end{aligned}$$

(iii)

$$\begin{aligned} M(3,3,6,6,4,4;n) =& \frac{1}{10}\sigma_3(n) - \frac{1}{10}\sigma_3(\frac{n}{2}) - \frac{21}{10}\sigma_3(\frac{n}{3}) + \frac{4}{5}\sigma_3(\frac{n}{4}) + \frac{21}{10}\sigma_3(\frac{n}{6}) \\ & - \frac{64}{5}\sigma_3(\frac{n}{8}) - \frac{84}{5}\sigma_3(\frac{n}{12}) + \frac{1344}{5}\sigma_3(\frac{n}{24}) + \frac{2}{5}c_{1,6}(\frac{n}{2}) \\ & + \frac{16}{5}c_{1,6}(\frac{n}{4}) - \frac{1}{10}c_{3,4}(n), \end{aligned}$$

(iv)

$$\begin{aligned} M(1,1,1,1,6,6;n) =& \frac{8}{15}\sigma_3(n) + \frac{76}{15}\sigma_3(\frac{n}{3}) - \frac{128}{15}\sigma_3(\frac{n}{4}) - \frac{108}{5}\sigma_3(\frac{n}{9}) \\ & - \frac{1216}{15}\sigma_3(\frac{n}{12}) + \frac{1728}{5}\sigma_3(\frac{n}{36}) - \frac{4}{5}c_{1,6}(n) + \frac{16}{5}c_{1,6}(\frac{n}{2}) \\ & + \frac{124}{5}c_{1,18}(n) - \frac{16}{15}c_{2,9}(\frac{n}{2}), \end{aligned}$$

(v)

$$\begin{aligned} M(3,3,6,6,2,2;n) =& \frac{2}{5}\sigma_3(n) - \frac{2}{5}\sigma_3(\frac{n}{2}) - \frac{42}{5}\sigma_3(\frac{n}{3}) + \frac{4}{5}\sigma_3(\frac{n}{4}) + \frac{42}{5}\sigma_3(\frac{n}{6}) \\ & - \frac{64}{5}\sigma_3(\frac{n}{8}) - \frac{84}{5}\sigma_3(\frac{n}{12}) + \frac{1344}{5}\sigma_3(\frac{n}{24}) - \frac{2}{5}c_{1,6}(n) \\ & - \frac{4}{5}c_{1,6}(\frac{n}{2}) + \frac{44}{5}c_{1,12}(\frac{n}{2}), \end{aligned}$$

(vi)

$$\begin{aligned} M(1,1,2,2,1,1;n) =& \frac{56}{5}\sigma_3(n) - \frac{56}{5}\sigma_3(\frac{n}{2}) - \frac{216}{5}\sigma_3(\frac{n}{3}) + \frac{28}{5}\sigma_3(\frac{n}{4}) + \frac{216}{5}\sigma_3(\frac{n}{6}) \\ & - \frac{448}{5}\sigma_3(\frac{n}{8}) - \frac{108}{5}\sigma_3(\frac{n}{12}) + \frac{1728}{5}\sigma_3(\frac{n}{24}) - \frac{6}{5}c_{1,6}(n) \\ & + 6c_{1,8}(n) + \frac{3}{5}c_{3,4}(n) - \frac{3}{5}c_{3,8}(n), \end{aligned}$$

(vii)

$$\begin{aligned} M(1,1,2,2,4,4;n) =& \frac{7}{10}\sigma_3(n) - \frac{7}{10}\sigma_3(\frac{n}{2}) - \frac{27}{10}\sigma_3(\frac{n}{3}) + \frac{28}{5}\sigma_3(\frac{n}{4}) + \frac{27}{10}\sigma_3(\frac{n}{6}) \\ & - \frac{448}{5}\sigma_3(\frac{n}{8}) - \frac{108}{5}\sigma_3(\frac{n}{12}) + \frac{1728}{5}\sigma_3(\frac{n}{24}) - \frac{6}{5}c_{1,6}(\frac{n}{2}) \\ & - \frac{48}{5}c_{1,6}(\frac{n}{4}) + \frac{33}{10}c_{1,12}(n), \end{aligned}$$

(viii)

$$\begin{aligned} M(3,3,6,6,1,1;n) =& \frac{8}{5}\sigma_3(n) - \frac{8}{5}\sigma_3(\frac{n}{2}) - \frac{168}{5}\sigma_3(\frac{n}{3}) + \frac{4}{5}\sigma_3(\frac{n}{4}) + \frac{168}{5}\sigma_3(\frac{n}{6}) \\ & - \frac{64}{5}\sigma_3(\frac{n}{8}) - \frac{84}{5}\sigma_3(\frac{n}{12}) + \frac{1344}{5}\sigma_3(\frac{n}{24}) + \frac{2}{5}c_{1,6}(n) \\ & - 18c_{1,8}(\frac{n}{3}) - \frac{11}{5}c_{1,12}(n) + \frac{61}{5}c_{1,24}(n), \end{aligned}$$

(ix)

$$\begin{aligned} M(1,1,3,3,6,6;n) =& \frac{4}{15}\sigma_3(n) - \frac{8}{15}\sigma_3(\frac{n}{2}) - \frac{88}{15}\sigma_3(\frac{n}{3}) + \frac{64}{15}\sigma_3(\frac{n}{4}) + \frac{176}{15}\sigma_3(\frac{n}{6}) \\ & + \frac{108}{5}\sigma_3(\frac{n}{9}) - \frac{1408}{15}\sigma_3(\frac{n}{12}) - \frac{216}{5}\sigma_3(\frac{n}{18}) - \frac{2}{5}c_{1,6}(n) \\ & - \frac{8}{5}c_{1,6}(\frac{n}{2}) - \frac{18}{5}c_{1,6}(\frac{n}{3}) + \frac{1728}{5}\sigma_3(\frac{n}{36}) - \frac{72}{5}c_{1,6}(\frac{n}{6}) \\ & - \frac{16}{3}c_{1,9}(\frac{n}{2}) + \frac{62}{15}c_{1,18}(n) + \frac{8}{15}c_{2,9}(\frac{n}{2}), \end{aligned}$$

(x)

$$\begin{aligned} M(1,2,2,4,2,2;n) =& \frac{7}{5}\sigma_3(n) - \frac{7}{5}\sigma_3(\frac{n}{2}) - \frac{27}{5}\sigma_3(\frac{n}{3}) + \frac{27}{5}\sigma_3(\frac{n}{6}) + \frac{28}{5}\sigma_3(\frac{n}{8}) \\ & - \frac{448}{5}\sigma_3(\frac{n}{16}) - \frac{108}{5}\sigma_3(\frac{n}{24}) + \frac{1728}{5}\sigma_3(\frac{n}{48}) + \frac{3}{5}c_{1,6}(n) \\ & + 6c_{1,8}(\frac{n}{2}) + \frac{3}{5}c_{3,4}(\frac{n}{2}) - \frac{3}{5}c_{3,8}(\frac{n}{2}), \end{aligned}$$

(xi)

$$\begin{aligned} M(1,2,2,4,4,4;n) =& \frac{7}{20}\sigma_3(n) - \frac{7}{20}\sigma_3(\frac{n}{2}) - \frac{27}{20}\sigma_3(\frac{n}{3}) + \frac{27}{20}\sigma_3(\frac{n}{6}) + \frac{28}{5}\sigma_3(\frac{n}{8}) \\ &- \frac{448}{5}\sigma_3(\frac{n}{16}) - \frac{108}{5}\sigma_3(\frac{n}{24}) + \frac{1728}{5}\sigma_3(\frac{n}{48}) - \frac{3}{5}c_{1,6}(\frac{n}{2}) \\ &+ \frac{12}{5}c_{1,6}(\frac{n}{4}) + \frac{33}{20}c_{1,12}(n) - \frac{12}{5}c_{3,4}(\frac{n}{4}), \end{aligned}$$

2. Proof of Theorem 1

In this section we give the proof of Theorem 1. (i). The remaining parts can be proved by a similar way.

Proof. [(i)] The form $f := x_1^2+x_2^2+2x_3^2+2x_4^2+2x_5^2+2x_5x_6+2x_6^2+2x_7^2+2x_7x_8+2x_8^2$ clearly can be obtained from the sum of the quaternary quadratic forms $f_2 := x_1^2 + x_2^2 + 2x_3^2 + 2x_4^2$ and $f_5 := x_1^2 + x_1x_2 + x_2^2 + x_3^2 + x_3x_4 + x_4^2$. It is obvious that

$$\begin{aligned} M(1,1,2,2,2,2;n) =& \sum_{\substack{l,m\in\mathbb{N}_0 \\ l+2m=n}} r_2(l)r_5(m) \\ =& r_1(0)r_5(\frac{n}{2}) + r_1(n)r_5(0) + \sum_{\substack{l,m\in\mathbb{N} \\ l+2m=n}} r_2(l)r_5(m). \end{aligned}$$

Thus by using the equations (9) and (12) we have

$$\begin{aligned} M(1,1,2,2,2,2;n)&-(4\sigma(n) - 4\sigma(\frac{n}{2}) + 8\sigma(\frac{n}{4}) - 32\sigma(\frac{n}{8}) + 12\sigma(n) - 36\sigma(\frac{n}{3})) \\ =& \sum_{\substack{l,m\in\mathbb{N} \\ l+2m=n}} (4\sigma(l) - 4\sigma(\frac{l}{2}) + 8\sigma(\frac{l}{4}) - 32\sigma(\frac{l}{8}))(12\sigma(m) - 36\sigma(\frac{m}{3})) \\ =& 48 \sum_{\substack{l,m\in\mathbb{N} \\ l+2m=n}} \sigma(l)\sigma(m) - 144 \sum_{\substack{l,m\in\mathbb{N} \\ l+2m=n}} \sigma(l)\sigma(\frac{m}{3}) - 48 \sum_{\substack{l,m\in\mathbb{N} \\ l+2m=n}} \sigma(\frac{l}{2})\sigma(m) \\ &+ 144 \sum_{\substack{l,m\in\mathbb{N} \\ l+2m=n}} \sigma(\frac{l}{2})\sigma(\frac{m}{3}) + 96 \sum_{\substack{l,m\in\mathbb{N} \\ l+2m=n}} \sigma(\frac{l}{4})\sigma(m) - 288 \sum_{\substack{l,m\in\mathbb{N} \\ l+2m=n}} \sigma(\frac{l}{4})\sigma(\frac{m}{3}) \\ &- 384 \sum_{\substack{l,m\in\mathbb{N} \\ l+2m=n}} \sigma(\frac{l}{8})\sigma(m) + 1156 \sum_{\substack{l,m\in\mathbb{N} \\ l+2m=n}} \sigma(\frac{l}{8})\sigma(\frac{m}{3}). \end{aligned}$$

So,

$$M(1,1,2,2,2,2;n) = 4\sigma(n) - 4\sigma(\frac{n}{2}) + 8\sigma(\frac{n}{4}) - 32\sigma(\frac{n}{8}) + 12\sigma(n)$$

$$-36\sigma(\frac{n}{3})+48W_{1,2}(n)-144W_{1,6}(n)-48W_{1,1}(\frac{n}{2})$$
$$+144W_{1,3}(\frac{n}{2})+96W_{1,2}(\frac{n}{2})-288W_{2,3}(\frac{n}{2})$$
$$-384W_{1,4}(\frac{n}{2})+1156W_{3,4}(\frac{n}{2}) \tag{13}$$

where

$$W_{1,1}(n)=\frac{5}{12}\sigma_3(n)-\frac{n}{2}\sigma(n)+\frac{1}{12}\sigma(n), \tag{14}$$
$$W_{1,2}(n)=\frac{1}{12}\sigma_3(n)+\frac{1}{3}\sigma_3(\frac{n}{2})-\frac{n}{8}\sigma(n)-\frac{n}{4}\sigma(\frac{n}{2})+\frac{1}{24}\sigma(n)+\frac{1}{24}\sigma(\frac{n}{2}), \tag{15}$$
$$W_{1,3}(n)=\frac{1}{24}\sigma_3(n)+\frac{3}{8}\sigma_3(\frac{n}{3})-\frac{1}{12}n\sigma(n)-\frac{1}{4}n\sigma(\frac{n}{3})+\frac{1}{24}\sigma(n)$$
$$+\frac{1}{24}\sigma(\frac{n}{3}), \tag{16}$$
$$W_{1,4}(n)=\frac{1}{48}\sigma_3(n)+\frac{1}{16}\sigma_3(\frac{n}{2})+\frac{1}{3}\sigma(\frac{n}{4})-\frac{n}{16}\sigma(n)-\frac{n}{4}\sigma(\frac{n}{4})+\frac{1}{24}\sigma(n)$$
$$+\frac{1}{24}\sigma(\frac{n}{4}), \tag{17}$$
$$W_{1,6}(n)=\frac{1}{120}\sigma_3(n)+\frac{1}{30}\sigma_3(\frac{n}{2})+\frac{3}{40}\sigma_3(\frac{n}{3})+\frac{3}{10}\sigma_3(\frac{n}{6})+(\frac{1}{24}-\frac{n}{24})\sigma(n)$$
$$+(\frac{1}{24}-\frac{n}{4})\sigma(\frac{n}{6})-\frac{1}{120}c_{1,6}(n) \tag{18}$$
$$W_{2,3}(n)=\frac{1}{120}\sigma_3(n)+\frac{1}{30}\sigma_3(\frac{n}{2})+\frac{3}{40}\sigma_3(\frac{n}{3})+\frac{3}{10}\sigma_3(\frac{n}{6})+(\frac{1}{24}-\frac{n}{12})\sigma(\frac{n}{2})$$
$$+(\frac{1}{24}-\frac{n}{8})\sigma(\frac{n}{3})-\frac{1}{120}c_{1,6}(n), \tag{19}$$

where

$$\sum_{n=1}^{\infty}c_{1,6}(n)q^n=q\prod_{n=1}^{\infty}(1-q^n)^2(1-q^{2n})^2(1-q^{3n})^2(1-q^{6n})^2, \tag{20}$$

and

$$W_{3,4}(n)=\frac{1}{480}\sigma_3(n)+\frac{1}{160}\sigma_3(\frac{n}{2})+\frac{3}{160}\sigma_3(\frac{n}{3})+\frac{1}{30}\sigma_3(\frac{n}{4})+\frac{9}{160}\sigma_3(\frac{n}{6})$$
$$+\frac{3}{10}\sigma_3(\frac{n}{12})+(\frac{1}{24}-\frac{n}{16})\sigma(\frac{n}{3})+(\frac{1}{24}-\frac{n}{12})\sigma(\frac{n}{4})$$
$$-\frac{1}{480}c_{3,4}(n), \tag{21}$$

where

$$\sum_{n=1}^{\infty}c_{3,4}(n)q^n=10q^2\prod_{n=1}^{\infty}(1-q^n)^3(1-q^{2n})^2(1-q^{3n})^{-1}(1-q^{4n})^{-1}(1-q^{6n})^2(1-q^{12n})^3$$

$$+q\prod_{n=1}^{\infty}(1-q^n)^{-2}(1-q^{2n})^8(1-q^{3n})^{-2}(1-q^{4n})^{-2}(1-q^{6n})^8(1-q^{12n})^{-2}. \tag{22}$$

In any formula $n\in\mathbb{N}$ and $q\in\mathbb{C}$. Substituting (14)-(22) in (13) gives

$$\begin{aligned}M(1,1,2,2,2,2;n)=&\frac{14}{5}\sigma_3(n)-\frac{14}{5}\sigma_3(\frac{n}{2})-\frac{54}{5}\sigma_3(\frac{n}{3})+\frac{28}{5}\sigma_3(\frac{n}{4})+\frac{54}{5}\sigma_3(\frac{n}{6})\\&-\frac{448}{5}\sigma_3(\frac{n}{8})-\frac{108}{5}\sigma_3(\frac{n}{12})+\frac{1728}{5}\sigma_3(\frac{n}{24})+\frac{6}{5}c_{1,6}(n)\\&+\frac{12}{5}c_{1,6}(\frac{n}{2})-\frac{12}{5}c_{3,4}(\frac{n}{2}),\end{aligned} \tag{23}$$

which is the asserted formula. □

Denoting the right hand side of (23) by $F(n)$, we give a list of values of $M(1,1,1,1,2,2;n)$ and $F(n)$ in Table 1 to illustrate the equations.

Table 1: The first ten values of $M(1,1,1,1,2,2;n)$ and $F(n)$

n	$M(1,1,2,2,2,2;n)$	$\sigma_3(n)$	$c_{1,6}(n)$	$c_{1,6}(\frac{n}{2})$	$c_{3,4}(\frac{n}{2})$	$F(n)$
1	4	1	1	0	0	4
2	20	9	−2	1	1	20
3	64	28	−3	0	0	64
4	156	73	4	−2	12	156
5	360	126	6	0	0	360
6	620	252	6	−3	−33	620
7	944	344	−16	0	0	944
8	1452	585	−8	4	−24	1452
9	1828	757	9	0	0	1828
10	2520	1134	−12	6	126	2520

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