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On Some Properties of Weak Soft Axioms

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Abstract. The aim of this paper is to initiate and discuss the properties and characterizations of soft semi-T_i and soft semi-D_i(for $i = 0, 1, 2$) spaces at soft point by analyzing the relationship among them. We also introduce and explore the properties of soft S-continuous functions. These results will be useful to enhance the theoretical framework and to promote further study towards the daily life applications.

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1. Introduction

An application of soft sets in decision making problems that is based on the reduction of parameters to keep the optimal choice objects can be seen. This is also useful in the process to construct models during the modelling process in different fields of life. The real world is inherently uncertain, imprecise and vague. Because of various uncertainties, classical methods are not successful for solving complicated problems in economics, engineering and environment. A soft set is a collection of approximate descriptions of an object and is free from the parameterizations inadequacy syndrome of fuzzy set theory, rough set theory, probability theory and game theory. Soft systems provide a very general framework with the involvement of parameters. Research works on soft set theory, its generalized structures and its applications in various fields are progressing rapidly now a days.

Molodtsov [14, 15] initiated and applied soft sets theory, while modelling the problems in the field of science including engineering physics, computer science, economics, social sciences and medical sciences, to deal with uncertain data and not clear objects without complete information. Maji *et al.* [12, 13] discussed and applied the soft set theory in decision making problems. In [17] and [19], Xiao *et al.* and Pei *et al.* respectively explored the soft sets in information systems. The criteria of measuring the sound quality through the soft sets studied by Kostek [11]. Mushrif *et al.* [16] established the remarkable method for the classification of natural textures by applying the concept of soft set.

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In [18], Shabir and Naz introduced and studied the primary concepts of soft topological spaces. After that Hussain [6, 7], Hussain and Ahmad [8, 9] and [1], Aygunoglu et al. [2], Zorlutana *et al.* [20] continued to add many basic concepts in soft topological spaces. In [3, 4], Chen introduced and explored soft semi-open(closed) sets in soft topological spaces. In [5], Hussain added many concepts toward soft semi-open sets and soft semi-closed sets in soft topological spaces.

Kharral and Ahmad [10] and then Zorlutana [20] discussed the mappings of soft classes and their properties in soft topological spaces. Recently in [7], Hussain presented and discussed basic properties and characterizations of soft pu-continuous functions and soft pu-open(closed) functions.

2. Preliminaries

First we recall some definitions and results which will use in the sequel.

Definition 1 ([14]). *Let X be an initial universe and E be a set of parameters. Let $P(X)$ denotes the power set of X and A be a non-empty subset of E. A pair (F, A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A). Clearly, a soft set is not a set.*

Here we consider only soft sets (F, A) over a universe X in which all the parameters of set A are same. We denote the family of these soft sets by $SS(X)_A$. For soft subsets, soft union, soft intersection, soft complement, their properties and the relations to each other; the interested reader is refer to [12, 13, 14, 15].

Definition 2 ([18]). *Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X, if*

(1) Φ, $\tilde{X}$ belong to τ.

(2) the union of any number of soft sets in τ belongs to τ.

(3) the intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. Every member of τ is called soft open set. A soft set is called soft closed if and only if its complement is soft open.

Definition 3 ([8, 18]). *Let (X, τ, E) be a soft topological space over X and $A \subseteq X$. Then*

(1) soft interior of soft set (F, A) over X denoted by $(F, A)^\circ$ and is defined as the union of all soft open sets contained in (F, A). Thus $(F, A)^\circ$ is the largest soft open set contained in (F, A).

(2) soft closure of (F, A), denoted by $\overline{(F, A)}$ is the intersection of all soft closed super sets of (F, A). Clearly $\overline{(F, A)}$ is the smallest soft closed set over X which contains (F, A).

(3) soft boundary of soft set (F, A) over X denoted by $\underline{(F, A)}$ and is defined as $\underline{(F, A)} = \overline{(F, A)} \cap \overline{((F, A)')}$. Obviously $\underline{(F, A)}$ is a smallest soft closed set over X containing (F, A).

For detailed properties of soft interior, soft closure and soft boundary, we refer to [8].

Definition 4 ([3])**.** *Let (X, τ, E) be a soft topological space over X with $A \subseteq X$ and (F, A) be a soft set over X. Then (F, A) is called soft semi-open set if and only if there exists a soft open set (G, A) such that $(G, A)\tilde{\subseteq}(F, A)\tilde{\subseteq}\overline{(G, A)}$. The set of all soft semi-open sets is denoted by $S.S.O(X)$. Note that every soft open set is soft semi-open set. A soft set (F, A) is said to be soft semi-closed if its soft relative complement is soft semi-open. Equivalently, there exists a soft closed set (G, A) such that $(G, A)^{\circ}\tilde{\subseteq}(F, A)\tilde{\subseteq}(G, A)$. Note that every soft closed set is soft semi-closed set.*

Definition 5 ([5])**.** *Let (X, τ, E) be a soft topological space over X with $A \subseteq X$.*

[(i)]soft semi-interior of soft set (F, A) over X denoted by $int^s(F, A)$ and is defined as the union of all soft semi-open sets contained in (F, A). soft semi-closure of (F, A) over X denoted by $cl^s(F, A)$ is the intersection of all soft semi-closed super sets of (F, A).

For detailed properties of soft semi-open(closed) and soft semi-interior(closure) we refer to [3, 4, 5].

3. Soft Semi-Separation Axioms

Hereafter, $SS(X)_A$ denotes the family of soft sets over X with the set of parameters A.

Definition 6 ([13])**.** *A soft set (F, A) over X is said to be an absolute soft set, denoted by $\tilde{X}_A$, if for all $e \in A$, $F(e) = X$. Clearly, $\tilde{X}_A^c = \Phi_A$ and $\Phi_A^c = \tilde{X}_A$.*

Definition 7 ([13])**.** *A soft set (F, A) over X is said to be null soft set, denoted by $\tilde{\Phi}_A$, if for all $e \in A$, $F(e) = \phi$.*

Proposition 1 ([20])**.** *Let $e_F\tilde{\in}\tilde{X_A}$ and $(G, A)\tilde{\in}SS(X)_A$. If $e_F\tilde{\in}(G, A)$, then $e_F\tilde{\notin}(G, A)^c$.*

Definition 8 ([20])**.** *The soft set $(F, A)\tilde{\in}SS(X)_A$ is called soft point in $\tilde{X_A}$, denoted by e_F, if for the element $e \in A$, $F(e) \neq \phi$ and $F(e') = \phi$, for all $e' \in A - \{e\}$.*

Definition 9 ([20])**.** *The soft point e_F is said to be in the soft set (G, A), denoted by $e_F\tilde{\in}(G, A)$, if for the element $e \in A$, $F(e) \subseteq G(e)$.*

Definition 10 ([6])**.** *Two soft sets (G, A), (H, A) in $SS(X)_A$ are said to be soft disjoint, written $(G, A)\tilde{\cap}(H, A) = \Phi_A$, if $G(e) \cap H(e) = \phi$, for all $e \in A$.*

Definition 11 ([6])**.** *Two soft points e_G, e_H in $\tilde{X_A}$ are distinct, written $e_G \neq e_H$, if there corresponding soft sets (G, A) and (H, A) are soft disjoint.*

Definition 12. *Let (X, τ, A) be a soft topological space over X and $(F, A)\tilde{\in}SS(X)_A$. Then (F, A) is soft semi-D-set, if there exists two soft semi-open sets (G, A) and (H, A) such that $(G, A)\tilde{\neq}\tilde{X}$ and $(F, A)\tilde{=}(G, A)\tilde{\setminus}(H, A)$.*

From the definition, it is clear that every soft semi-open set $(G, A)\tilde{\neq}\tilde{X}$ is soft semi-D-set, if $(F, A)\tilde{=}(G, A)$ and $(H, A)\tilde{=}\tilde{\Phi}$.

Example 1. *Let $X = \{h_1, h_2, h_3\}$, $A = \{e_1, e_2\}$ and*

$$\tau = \{\tilde{\Phi}, \tilde{X}, (K_1, A), (K_2, A), (K_3, A), (K_4, A), (K_5, A), (K_6, A), (K_7, A)\}$$

where (K_1, A), (K_2, A), (K_3, A), (K_4, A), (K_5, A), (K_6, A) and (K_7, A) are soft sets over X, defined as follows:

$K_1(e_1) =\{h_1, h_2\}, K_1(e_2) = \{h_1, h_2\}, K_2(e_1) = \{h_2\}, K_2(e_2) = \{h_1, h_3\}, K_3(e_1) = \{h_2, h_3\},$
$K_3(e_2) =\{h_1\}, K_4(e_1) = \{h_2\}, K_4(e_2) = \{h_1\}, K_5(e_1) = \{h_1, h_2\}, K_5(e_2) = X, K_6(e_1) = X,$
$K_6(e_2) =\{h_1, h_2\}, K_7(e_1) = \{h_2, h_3\}, K_7(e_2) = \{h_1, h_3\}.$

Then τ defines a soft topology on X and hence (X, τ, A) is a soft topological space over X. Clearly $(F, A)\tilde{=}\{\{h_1\}, \{h_2, h_3\}\}$ is a soft semi-D-set, because $(F, A)\tilde{=}(K_5, A)\tilde{\setminus}(K_4, A)$. Similarly,
$(K, A)\tilde{=}\{\{h_1\}, \{h_2\}\}$ is soft semi-D-set, because $(K, A)\tilde{=}(K_1, A)\tilde{\setminus}(K_2, A)$.

Definition 13. *Let (X, τ, A) be a soft topological space over X. Then (X, τ, A) is called soft semi-D_0 space, if for any two distinct soft points e_F and e_G in $\tilde{X_A}$, there exists soft semi-D-set (H, A) in $SS(X)_A$ such that $e_F\tilde{\in}(H, A)$ and $e_G\tilde{\notin}(H, A)$ or soft semi-D-set (K, A) in $SS(X)_A$ such that $e_G\tilde{\in}(K, A)$ and $e_F\tilde{\notin}(K, A)$.*

Definition 14. *Let (X, τ, A) be a soft topological space over X. Then (X, τ, A) is called soft semi-D_1 space, if for any two distinct soft points e_F and e_G in $\tilde{X_A}$, there exists soft semi-D-set (H, A) in $SS(X)_A$ such that $e_F\tilde{\in}(H, A)$ and $e_G\tilde{\notin}(H, A)$ and soft semi-D-set (K, A) in $SS(X)_A$ such that $e_G\tilde{\in}(K, A)$ and $e_F\tilde{\notin}(K, A)$.*

Definition 15. *Let (X, τ, A) be a soft topological space over X. Then (X, τ, A) is called soft semi-D_2 space, if for any two distinct soft points e_F and e_G in $\tilde{X_A}$, there exists disjoint soft semi-D-sets (H, A) and (K, A) in $SS(X)_A$ such that $e_F\tilde{\in}(H, A)$ and $e_G\tilde{\in}(K, A)$.*

Remark 1. *It is clear from the above definitions that*

$$\text{soft semi-}D_2 \Rightarrow \text{soft semi-}D_1 \Rightarrow \text{soft semi-}D_0.$$

The interested reader can easily check that the converse is not true in general.

Definition 16. *Let (X, τ, A) be a soft topological space over X. If for any two distinct soft points e_G, e_H in $\tilde{X_A}$, there exist soft semi-open sets (F_1, A) or (F_2, A) such that $e_G\tilde{\in}(F_1, A)$, $e_H\tilde{\notin}(F_1, A)$, $e_H\tilde{\in}(F_2, A)$, $e_G\tilde{\notin}(F_2, A)$, then (X, τ, A) is called soft semi-T_0-space.*

Example 2. *Let* $X = \{h_1, h_2, h_3\}$, $A = \{e_1, e_2, e_3\}$ *and* $\tau = \{\tilde{\Phi}, \tilde{X}, (K_1, A)\}$, *where* $(K_1, A)\tilde{=}\{(e_1, \{h_1\}), (e_2, \{h_2\}), (e_3, \{h_3\})\}$ *is a soft sets over* X *with soft points:*

$$e_1(K_1) = \{h_1\}, e_2(K_1) = \{h_2\}, e_3(K_1) = \{h_3\}.$$

Then τ *defines a soft topology on* X *and hence* (X, τ, A) *is a soft topological space over* X. *Moreover* (X, τ, A) *is soft semi-*T_0*-space.*

Definition 17. *Let* (X, τ, A) *be a soft topological space over* X. (G, A) *be soft set in* $SS(X)_A$, *and* e_F *be a soft point in* $\tilde{X_A}$. *Then* (G, A) *is said to be soft semi-neighborhood of soft point* e_F, *if there exists a soft open set* (K, A) *such that* $e_F\tilde{\in}(K, A)\tilde{\subset}(G, A)$.

Definition 18. *Let* (X, τ, A) *be a soft topological space over* X. (F, A) *be soft set in* $SS(X)_A$, *and* e_F *be a soft point in* $\tilde{X_A}$. *If every soft neighborhood of* e_F *soft intersects* (F, A) *in some soft points other than* e_F *itself, then* e_F *is called soft semi-limit point of* (F, E). *The set of all soft semi-limit points of* (F, A) *is denoted by* $(F, A)^{ssd}$.

In other words, if (X, τ, A) *is a soft topological space,* (F, A) *be soft set in* $SS(X)_A$, *and* e_F *be soft point in* $\tilde{X_A}$, *then* $e_F\tilde{\in}(F, A)^{ssd}$ *if and only if* $(G, A)\tilde{\cap}((F, A)\tilde{\setminus}\{e_F\})\tilde{\neq}\tilde{\Phi}$, *for all soft semi-open neighborhoods* (G, A) *of* e_F.

Remark 2. *Form the definition, it follows that the soft point* e_F *is a soft semi-limit point of* (F, A) *if and only if* $e_F\tilde{\in}cl^s((F, A)\tilde{\setminus}\{e_F\})$.

Theorem 1. *Let* (X, τ, A) *be a soft topological space over* X. *Then the following are equivalent:*

(1) (X, τ, A) *is soft semi-*T_0 *space.*

(2) For any distinct soft points e_G *and* e_H *in* $\tilde{X_A}$, $cl^s(e_G)\tilde{\neq}cl^s(e_H)$.

Proof. (1) $\Rightarrow$ (2) Suppose that (X, τ, A) is soft semi-T_0 space and e_G and e_H are distinct soft points in $\tilde{X_A}$. Then there exists at least one soft semi-open set (K, A) (say), which contains e_G but not e_H. But then e_G is not soft semi-limit point of e_H. Since e_G is not in e_H, $cl^s(e_G)\tilde{\neq}cl^s(e_H)$.

(2) $\Rightarrow$ (1) Suppose that for any distinct soft points e_G and e_H in X_A, $cl^s(e_G)\tilde{\neq}cl^s(e_H)$. Contrarily suppose that $\tilde{X_A}$ is not soft semi-T_0 space. Then every soft semi-open set which contains e_G also contains e_H. Then by the property of soft semi-limit point, e_G is in $cl^s(e_H)$ so that $cl^s(e_G)\tilde{\subseteq}cl^s(e_H)$. Similarly every soft semi-pen set which contains e_H also contains e_G (otherwise $\tilde{X_A}$ would be a soft semi-T_0 space). So $cl^s(e_H)\tilde{\subseteq}cl^s(e_G)$. Thus $cl^s(e_G)\tilde{=}cl^s(e_H)$. This contradiction proves as required. □

Definition 19. *Let* (X, τ, A) *be a soft topological space over* X *and* e_G, e_H *are two distinct soft points in* $\tilde{X_A}$. *If there exists a soft semi-open set* (F_1, A) *such that* $e_G\tilde{\in}(F_1, A)$, $e_H\tilde{\notin}(F_1, A)$ *and a soft semi-open set* (F_2, A) *such that* $e_H\tilde{\in}(F_2, A)$, $e_G\tilde{\notin}(F_2, A)$, *then* (X, τ, A) *is called soft semi-*T_1*-space.*

Example 3. *In Example 2,* (X, τ, A) *is not soft semi-*T_1 *space.*

Theorem 2. *Let (X, τ, A) be a soft topological space over X. Then the following are equivalent to each other:*

(1) (X, τ, A) is soft semi-T_1.

(2) $\{e_F\}$ is soft semi-closed, for each soft point e_F in $\tilde{X}_A$.

Proof. (1) $\Rightarrow$ (2) Let (X, τ, A) be a soft topological space over X. Let e_F be soft point in $\tilde{X}_A$ and $e_G \in \{e_F\}^c$. Then e_F and e_G are distinct soft points. Since (X, τ, A) is a soft semi-T_1 space, then there exists a soft semi-open set (H, A) with $e_G \tilde{\in} (H, A)$ and $e_F \tilde{\notin} (H, A)$. Thus, $e_G \tilde{\in} (H, A) \tilde{\subseteq} \{e_F\}^c$. This follows that $\{e_F\}^c$ can be written as the soft union of soft semi-open sets (H_i, A) with $e_{H_i} \tilde{\in} \{e_F\}^c$. Hence $\{e_F\}^c$ is soft semi-open which implies that $\{e_F\}$ is soft semi-closed.

(2) $\Rightarrow$ (1) Let $\{e_F\}$ is soft semi-closed, for each soft point e_F in $\tilde{X}_A$. Suppose e_F and e_G be distinct soft points in $\tilde{X}_A$. Then $e_G \tilde{\in} \{e_F\}^c$. Thus $\{e_F\}^c$ is a soft semi-open set with $e_G \tilde{\in} \{e_F\}^c$ and $e_F \tilde{\notin} \{e_F\}^c$. Also $\{e_G\}^c$ is a soft semi-open set with $e_F \tilde{\in} \{e_G\}^c$ and $e_G \tilde{\notin} \{e_G\}^c$. This implies that (X, τ, A) is soft semi-T_1 space. □

Definition 20. *Let (X, τ, A) be a soft topological space over X and e_G, e_H are two distinct soft points in $\tilde{X}_A$. If there exist soft disjoint soft semi-open sets (F_1, A) and (F_2, A) such that $e_G \tilde{\in} (F_1, A)$, $e_H \tilde{\in} (F_2, A)$, then (X, τ, A) is called soft semi-T_2-space.*

Example 4. *Let (X, τ, A) be a soft discrete soft topological space [18]. Then (X, τ, A) is soft semi-T_2 space.*

Theorem 3. *Let (X, τ, A) be a soft topological space over X and e_G, e_H are distinct soft points in $\tilde{X}_A$. Then (X, τ, A) is soft semi-T_2-space, implies that there exist soft semi-closed sets (H, A) and (K, A) such that $e_G \tilde{\in} (H, A)$, $e_H \tilde{\notin} (H, A)$ and $e_G \tilde{\notin} (K, A)$, $e_H \tilde{\in} (K, A)$, and $(H, A) \tilde{\cup} (K, A) = \tilde{X}_A$.*

Proof. Since (X, τ, A) is soft semi-T_2-space and e_G and e_H are distinct soft points in $\tilde{X}_A$, then there exist soft disjoint soft semi-open sets (G_1, A) and (G_2, A) such that $e_G \tilde{\in} (G_1, A)$ and $e_H \tilde{\in} (G_2, A)$. Clearly $(G_1, A) \tilde{\subseteq} (G_2, A)^c$ and $(G_2, A) \tilde{\subseteq} (G_1, A)^c$. Hence $e_G \tilde{\in} (G_2, A)^c$. Put
$(G_2, A)^c = (H, A)$. This gives $e_G \tilde{\in} (H, A)$ and $e_H \tilde{\notin} (H, A)$. Also $e_H \tilde{\in} (G_1, A)^c$. Put $(G_1, A)^c \tilde{=} (K, A)$. Therefore $e_G \tilde{\in} (H, A)$ and $e_H \tilde{\in} (K, A)$. Moreover $(H, A) \tilde{\cup} (K, A) \tilde{=} (G_2, A)^c \tilde{\cup} (G_1, A)^c \tilde{=} \tilde{X}_A$. □

Remark 3. *From the above theorem and by definitions of soft semi-D_i and soft semi-T_i(for
$i = 0, 1, 2$) spaces, clearly we have:*

(1) soft semi-T_2 $\Rightarrow$ soft semi-T_1 $\Rightarrow$ soft semi-T_0

(2) soft semi-T_i $\Rightarrow$ soft semi-D_i (for $i = 0, 1, 2$).

Theorem 4. *Let (X, τ, A) be a soft topological space over X and e_F, e_G are distinct soft points in $\tilde{X_A}$. Then (X, τ, A) is soft semi-D_0 space if and only if (X, τ, A) is soft semi-T_0 space.*

Proof. ($\Rightarrow$) Let (X, τ, A) be a soft semi-D_0 space. Then for each distinct soft point e_F, e_G in $\tilde{X_A}$, there exists a soft semi-D-set (H, A) in $SS(X)_A$ such that $e_F \tilde{\in} (H, A)$ and $e_G \tilde{\notin} (H, A)$. Suppose that $(H, A) \tilde{=} (F, A) \tilde{\setminus} (G, A)$, where (F, A) and (G, A) are soft semi-open sets and $(F, A) \tilde{\neq} \tilde{X}$. This follows that $e_F \tilde{\in} (F, A)$ and for $e_G \tilde{\notin} (H, A)$, we have two possibilities:

[1.]$e_G \tilde{\notin} (F, A)$. Therefore, $e_F \tilde{\in} (F, A)$ and $e_G \tilde{\notin} (F, A)$. $e_G \tilde{\in} (F, A)$ and $e_G \tilde{\in} (G, A)$. Hence $e_G \tilde{\in} (G, A)$ and $e_F \tilde{\notin} (G, A)$.

This follows that (X, τ, A) is soft semi-T_0 space.

($\Leftarrow$) The proof follows from Remark 3(2). □

Theorem 5. *Let (X, τ, A) be a soft topological space over X and e_F, e_G are distinct soft points in $\tilde{X_A}$. Then (X, τ, A) is soft semi-D_1 space if and only if (X, τ, A) is soft semi-D_2 space.*

Proof. ($\Rightarrow$) Let (X, τ, A) be soft semi-D_1 space. Then for any two distinct soft points e_F and e_G in $\tilde{X_A}$, there exists soft semi-D-sets (H, A) and (K, A) in $SS(X)_A$ such that $e_F \tilde{\in} (H, A)$, $e_G \tilde{\notin} (H, A)$, $e_G \tilde{\in} (K, A)$, $e_F \tilde{\notin} (K, A)$. Consider soft sets (F, A), (G, A), (L, A) and (M, A) such that $(H, A) \tilde{=} (F, A) \tilde{\setminus} (G, A)$ and $(K, A) \tilde{=} (L, A) \tilde{\setminus} (M, A)$. $e_F \tilde{\notin} (K, A)$, implies that either $e_F \tilde{\notin} (L, A)$ or
$e_F \tilde{\in} (L, A)$ and $e_F \tilde{\in} (M, A)$. We suppose two cases:

[Case (1).]If $e_F \tilde{\notin} (L, A)$. As $e_G \tilde{\in} (H, A)$ then either $e_G \tilde{\in} (F, A)$ and $e_G \tilde{\in} (G, A)$ or $e_G \tilde{\notin} (F, A)$. If $e_G \tilde{\in} (F, A)$ and $e_G \tilde{\in} (G, A)$. Then $e_F \tilde{\in} (F, A) \tilde{\setminus} (G, A)$, $e_G \tilde{\in} (G, A)$ and $((F, A) \tilde{\setminus} (G, A)) \tilde{\cap} (G, A) \tilde{=} \tilde{\Phi}$. If $e_G \tilde{\notin} (F, A)$. As $e_F \tilde{\in} (F, A) \tilde{\setminus} (G, A)$, we have that $e_F \tilde{\in} (F, A) \tilde{\setminus} ((G, A) \tilde{\cup} (L, A))$ and from $e_G \tilde{\in} (L, A) \tilde{\setminus} (M, A)$, we have $e_G \tilde{\in} (L, A) \tilde{\setminus} ((F, A) \tilde{\cup} (M, A))$. Clearly
$((F, A) \tilde{\setminus} ((G, A) \tilde{\cup} (L, A))) \tilde{\cap} ((L, A) \tilde{\setminus} ((F, A) \tilde{\cup} (M, A))) \tilde{=} \tilde{\Phi}$. If $e_F \tilde{\in} (L, A)$ and $e_F \tilde{\in} (M, A)$. Then $e_G \tilde{\in} (L, A) \tilde{\setminus} (M, A)$, $e_F \tilde{\in} (M, A)$ and
$((L, A) \tilde{\setminus} (M, A)) \tilde{\cap} (M, A) \tilde{=} \tilde{\Phi}$.

Thus in each case, (X, τ, A) is soft semi-D_2 space.

($\Leftarrow$) This follows from Remark 1. Hence the proof. □

Proposition 2. *Let (X, τ, A) be a soft topological space over X. If (X, τ, A) is soft semi-D_1 space, then X is soft semi-T_0 space.*

Proof. The proof follows directly form Remark 1 and Theorem 4. □

Definition 21. *Let (X, τ, A) be a soft topological space over X and e_F, e_G be any soft points in $\tilde{X_A}$. If $e_F \tilde{\in} cl^s(\{e_G\})$ implies $e_G \tilde{\in} cl^s(\{e_F\})$, then (X, τ, A) is called soft semi-symmetric.*

Definition 22. *Let (X, τ, A) be a soft topological space over X and (F, A) be a soft set in $SS(X)_A$. If for any soft semi-open set (H, A) in $SS(X)_A$ and $(F, A)\tilde{\subseteq}(H, A)$ implies $cl^s(F, A)\tilde{\subseteq}(H, A)$, then (F, A) is called soft semi-generalized closed (in short soft sg-closed) set.*

The proof of the following proposition is straightforward form the definition of soft semi-closed and soft sg-closed set.

Proposition 3. *In a soft topological space (X, τ, A) over X, every soft semi closed set (F, A) is soft sg-closed.*

Theorem 6. *Let (X, τ, A) be a soft topological space over X. Then the following statements are equivalent:*

(1) $\{e_F\}$ is soft sg-closed, for any soft point e_F in $\tilde{X_A}$.

(2) (X, τ, A) is soft semi-symmetric.

Proof. (1) $\Rightarrow$ (2) Assume that $e_F\tilde{\in}cl^s(\{e_G\})$. Suppose on the contrarily that $e_G\tilde{\notin}cl^s(\{e_F\})$. Then $e_G\tilde{\in}(cl^s(\{e_G\}))^c$. This follows that $\{e_G\}\tilde{\subseteq}(cl^s(\{e_F\}))^c$. Therefore, $cl^s(\{e_G\})\tilde{\subseteq}(cl^s(\{e_F\}))^c$. Hence $e_G\tilde{\in}(cl^s(\{e_F\}))^c$. This contradiction proves the required result.

(2) $\Rightarrow$ (1) Assume on the contrary that for soft point e_F in $\tilde{X_A}$ and a soft semi-open set (H, A) in $SS(X)_A$ such that $\{e_F\}\tilde{\subseteq}(H, A)$ and $cl^s(\{e_F\})\tilde{\nsubseteq}(G, A)$. This follows that $cl^s(\{e_F\})\tilde{\cap}(H, A)^c\tilde{\neq}\tilde{\Phi}$. Let us take a soft point e_G in $\tilde{X_A}$ and assume that $e_G\tilde{\in}(cl^s(\{e_F\})\tilde{\cap}(H, A)^c)$. Here we have $e_F\tilde{\in}cl^s(\{e_G\})$. This implies that $cl^s(\{e_G\})\tilde{\subseteq}(H, A)^c$ and $e_F\tilde{\notin}(H, A)$. A contradiction. Hence the proof. □

Theorem 7. *Any soft semi-T_1 space is soft semi-symmetric in a soft topological space (X, τ, A) over X.*

Proof. Let (X, τ, A) be soft semi-T_1 space. Then Theorem 2 follows that $\{e_F\}$ is soft semi-closed, for any soft point e_F in $\tilde{X_A}$. Thus $\{e_F\}$ is soft sg-closed, by Proposition 3. Therefore Theorem 6 implies that $\{e_F\}$ is soft semi-symmetric. This completes the proof. □

Theorem 8. *Let (X, τ, A) be a soft topological space over X. Then (X, τ, A) is a soft semi-symmetric and soft semi-T_0 space if and only if it is soft semi-T_1.*

Proof. ($\Rightarrow$) Suppose (X, τ, A) is soft semi-T_0 space. Then for any two distinct soft point e_F and e_G in $\tilde{X_A}$, there exists soft semi-open set (H, A) in $SS(X)_A$ such that $e_F\tilde{\in}(H, A)\tilde{\subseteq}(\{e_G\})^c$. This implies that $e_F\tilde{\notin}cl^s(\{e_G\})$. Therefore, $e_G \notin cl^s(\{e_F\})$. This follows that there exists a soft semi-open set (K, A) such that $e_G\tilde{\in}(K, A)\tilde{\subseteq}(\{e_F\})^c$. Therefore (X, τ, A) is soft semi-T_1 space.

($\Leftarrow$) Using Theorem 6 and Remark 3(1), proof follows directly. Hence the proof. □

The following theorem follows from Remark 3(1), Theorem 5, Proposition 2 and Theorem 8.

Theorem 9. *If a soft topological space (X, τ, A) over X is soft semi-symmetric, then we have: (X, τ, A) is soft semi-T_0 space $\Leftrightarrow$ (X, τ, A) is soft semi-D_1 space $\Leftrightarrow$ (X, τ, A) is soft semi-T_1 space.*

4. Properties of Soft S-Continuous Functions

Definition 23 ([10]). *Let $SS(X)_A$ and $SS(Y)_B$ be two families of soft sets. $u : X \to Y$ and*
$p : A \to B$ be mappings. Then the image and the inverse image of a function $f_{pu} : SS(X)_A \to SS(Y)_B$ is defined as follows:

(1) Let (F, A) be soft set in $SS(X)_A$. The image of (F, A) under f_{pu}, written as $f_{pu}(F, A) = (f_{pu}(F), p(A))$, is a soft set in $SS(Y)_B$ such that

$$f_{pu}(F)(y) = \begin{cases} \bigcup_{x \in p^{-1}(y) \cap A} u(F(x)), & p^{-1}(y) \cap A \neq \phi \\ \phi, & otherwise \end{cases}$$

for all $y \in B$.

(2) Let (G, B) be soft set in $SS(Y)_B$. Then the inverse image of (G, B) under f_{pu}, written as $f_{pu}^{-1}(G, B) = (f_{pu}^{-1}(G), p^{-1}(B))$, is a soft set in $SS(X)_A$ such that

$$f_{pu}^{-1}(G)(x) = \begin{cases} u^{-1}(G(p(x))), & p(x) \in B \\ \phi, & otherwise \end{cases}$$

for all $x \in A$.

The soft function f_{pu} is called soft surjective, if p and u are surjective. The soft function f_{pu} is called soft injective, if p and u are injective.

For detailed properties of soft functions, we refer to [7, 10, 20].

Definition 24. *Let (X, τ, A) and (Y, τ^*, B) be soft topological spaces over X and Y respectively and $u : X \to Y$ and $p : A \to B$ be mappings. Then the soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is soft S-continuous, if for any soft semi-open set (G, B) in $SS(Y)_B$, $f_{pu}^{-1}(G, B))$ is soft semi-open in $SS(X)_A$.*

Theorem 10. *Let (X, τ, A) and (Y, τ^*, B) be soft topological spaces over X and Y respectively. If a soft function $f_{pu} : SS(X)_A \to SS(Y)_B$ is soft surjective soft S-continuous, then for each soft semi-D set (G, B) in $SS(Y)_B$, $f_{pu}^{-1}(G, B)$ is soft semi-D set in $SS(X)_A$.*

Proof. Suppose that soft function f_{pu} is soft surjective soft S-continuous and (G, B) be soft semi-D set in $SS(Y)_B$. Then there exist soft semi-open sets (H, B) and (K, B) in $SS(Y)_B$ such that $(H, B) \tilde{\neq} \tilde{Y}$ and $(G, B) \tilde{=} (H, B) \tilde{\setminus} (K, B)$. Since f_{pu} is soft S-continuous, implies that $f_{pu}^{-1}((H, B))$ and $f_{pu}^{-1}((K, B))$ are soft semi-open in $SS(X)_B$. As f_{pu} is soft surjective, therefore $(H, B) \tilde{\neq} \tilde{Y}$ follows $f_{pu}^{-1}((H, B)) \tilde{\neq} \tilde{X}$. Therefore $f_{pu}^{-1}((G, B)) \tilde{=} f_{pu}^{-1}((H, B)) \tilde{\setminus} f_{pu}^{-1}((K, B))$ is soft semi-D set. Hence the proof. □

Theorem 11. *Let (X,τ,A) and (Y,τ^*,B) be soft topological spaces over X and Y respectively. Then the following statements are equivalent:*

(1) For any two distinct soft points e_F,e_G in $\tilde{X_A}$, there exists soft surjective soft S-continuous function $f_{pu}: SS(X)_A \to SS(Y)_B$, where $\tilde{Y}$ is soft semi-D_1 space with $f_{pu}(e_F)\tilde{\neq}f_{pu}(e_G)$.

(2) $\tilde{X}$ is soft semi-D_1 space.

Proof. (1) ⇒ (2) Since for any two distinct soft points e_F, e_G in $\tilde{X_A}$, there exists soft surjective soft S-continuous $f_{pu}: SS(X)_A \to SS(Y)_B$, where $\tilde{Y}$ is soft semi-D_1 space with $f_{pu}(e_F)\tilde{\neq}f_{pu}(e_G)$. Then there exist soft disjoint soft semi-D sets (G,B) and (H,B) in $\tilde{Y}$ with $f_{pu}(e_F)\tilde{\in}(G,B)$, $f_{pu}(e_G)\tilde{\in}(H,B)$. As f_{pu} is soft surjective soft S-continuous, so Theorem 10 follows that $f_{pu}^{-1}((G,B))$ and $f_{pu}^{-1}((H,B))$ are soft disjoint soft semi-D sets in $\tilde{X}$ with $e_F\tilde{\in}f_{pu}^{-1}((G,B))$, $e_G\tilde{\in}f_{pu}^{-1}((H,B))$. Hence again Theorem 10 implies that $\tilde{X}$ is soft semi-D_1 space.

(2) ⇒ (1) This follows by letting the identity soft function, which fulfills the desired properties. Hence the proof. □

Theorem 12. *Let (X,τ,A) and (Y,τ^*,B) be soft topological spaces over X and Y respectively and soft function $f_{pu}: SS(X)_A \to SS(Y)_B$ is soft bijective soft S-continuous. If $\tilde{Y}$ is soft semi-D_1 space then $\tilde{X}$ is soft semi-D_1 space.*

Proof. Suppose e_F and e_G be two distinct soft points in $\tilde{X_A}$. Since f_{pu} is soft injective and $\tilde{Y}$ is soft semi-D_1, then there exist soft semi-D sets (G,B) and (H,B) in $SS(Y)_B$ such that $f_{pu}(e_F)\tilde{\in}(G,B)$, $f_{pu}(e_G)\tilde{\in}(H,B)$ and $f_{pu}(e_G)\tilde{\notin}(G,B)$, $f_{pu}(e_F)\tilde{\notin}(H,B)$. Therefore, by Theorem 10, $f_{pu}^{-1}((G,B))$ and $f_{pu}^{-1}(H,B)$ are soft semi-D sets in $SS(X)_A$ with $e_F\tilde{\in}f_{pu}^{-1}((G,B))$ and $e_G\tilde{\in}f_{pu}^{-1}(H,B)$. This implies that $\tilde{X}$ is soft semi-D_1 space. This completes the proof. □

Theorem 13. *Let (X,τ,A) and (Y,τ^*,B) be soft topological spaces over X and Y respectively. A soft function $f_{pu}: SS(X)_A \to SS(Y)_B$ is soft S-continuous, if for each soft point e_F in $\tilde{X_A}$ and each soft semi-open set (G,B) in $SS(Y)_B$ such that $f_{pu}(e_F)\tilde{\in}(G,B)$, there exists a soft semi-open set (F,A) in $SS(X)_A$ such that $f_{pu}(F,A)\tilde{\subseteq}(G,B)$.*

Proof. (⇒) Since f_{pu} is soft S-continuous, implies that $f_{pu}^{-1}(G,B)$ is soft semi-open in $SS(X)_A$, for soft semi-open set (G,B) in $SS(Y)_B$. We need to show that there exists a soft semi-open set (F,A) in $SS(X)_A$ such that $f_{pu}(F,A)\tilde{\subseteq}(G,B)$, for each soft point e_F in $\tilde{X_A}$ and each soft semi-open set (G,B) in $SS(Y)_B$ such that $f_{pu}(e_F)\tilde{\in}(G,B)$. Consider the soft point e_F in $\tilde{X_A}$ with $e_F\tilde{\in}f_{pu}^{-1}(G,B)$ and $(F,A) = f_{pu}^{-1}(G,B)$. This implies that for soft semi-open set (G,B), $e_F\tilde{\in}(F,A)$ and $f_{pu}(F,A)\tilde{\subseteq}f_{pu}f_{pu}^{-1}(G,B)\tilde{\subseteq}(G,B)$.

(⇐) Suppose that for each soft point e_F in $\tilde{X_A}$ and each soft semi-open set (G,B) in $SS(Y)_B$ such that $f(e_F)\tilde{\in}(G,B)$, there exists a soft semi-open set (F,A) in $SS(X)_A$ such that $f_{pu}(F,A)\tilde{\subseteq}(G,B)$. To prove that soft function f_{pu} is soft S-continuous. We show that the inverse image of soft semi-open set in $SS(Y)_B$ is soft semi-open set in $SS(X)_A$. Now $e_F \tilde{\in} f_{pu}^{-1}(G,B)$ follows $f_{pu}(e_F) \tilde{\in} (G,B)$. Thus by hypothesis, there exists a soft semi-open set $(F,A)_{e_F}$ such that $e_F \tilde{\in} (F,A)_{e_F}$ and $f_{pu}((F,A)_{e_F}) \tilde{\in} (G,B)$. Thus $e_F \tilde{\in} (F,A)_{e_F} \tilde{\subseteq} f_{pu}^{-1}(G,B)$ and $f_{pu}^{-1}(G,B) \tilde{=} \tilde{\bigcup}_{e_F \tilde{\in} f_{pu}^{-1}(G,B)} (F,A)_{e_F}$, for which $f_{pu}^{-1}(G,B)$ is soft semi-open set in $SS(X)_A$. Hence f_{pu} is soft S-continuous. This completes the proof. □

5. Conclusion

In the present work, we initiated and explored the properties and characterizations of soft semi-T_i (for $i = 0,1,2$) spaces. We also introduced and discussed the concepts of soft semi-D_i (for $i = 0,1,2$) spaces by analyzing the relationship among these spaces. Moreover, we introduced and studied soft S-continuous function and explore the properties of soft semi-D_1 space in soft S-continuous function. There is a wide space to work further in this field using defined concepts, properties and characterizations to enhance the general framework which will be applicable towards daily life to solve the problems having uncertainties.

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