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Some New Regular Generalized Closed Sets in Ideal Topological Spaces

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Abstract. We introduce the notions of saw-I_{rg}-closed sets and weakly-rg_I-closed by using the notion of regular open sets. Further, we study the concept of saw-I_{rg}-closed sets and their relationships in ideal topological spaces by using these new notions. Furthermore, we introduce and examine some properties of α_I-$*$-$normal$ space.

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1. Introduction

In 1990, Jankovic and Hamlett [3], have initiated the application ideal topological spaces. Khan and Noiri [3] have introduced semi-local functions in ideal topological spaces. Firstly the notion of I_g-closed set is given by Dontchev *et al.* [1]. In 2007, Navaneethakrishnan and Joseph [9] have introduced some of properties of I_g-closed sets and I_g-open sets by using local function. Recently Karabiyik [5], has defined concept of rg_I-closed set which is weaker than I_g-closed set and examined some properties. Also, he has given some characterization of this set. In 2013, Ekici and Ozen [2] introduced weakly-I_{rg}-closed sets which is a generalized class of τ^*. In this paper, we define saw-I_{rg}-closed sets and weakly-rg_I-closed by have using the notion of regular open sets. We investigated some of their properties. Also, we generalized many concepts which is defined in ideal topological spaces. The relationships of generalized class and various properties are examined.

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2. Preliminaries

In this paper, (X,τ) symbolize topological spaces on which no separation axioms are assumed unless clearly stated. For a subset A of $X, cl^*(A)$ and $int^*(A)$ will represent the closure of A and the interior of A in (X,τ). A subset A of a topological spaces (X,τ) is said to be regular open [11] (resp. regular closed) if $A = int(cl(A))$ (resp. $A = cl(int(A))$). An ideal I on a nonempty set X is a collection of subsets of X which satisfies the following properties [7]

(i) $A \in I$ and $B \subseteq A$ implies $B \in I$ and

(ii) $A \in I$ and $B \in I$ implies $A \cup B \in I$.

A topological spaces (X,τ) with an ideal I on X is called an ideal topological spaces and is denoted by (X,τ,I). If Y is a subset of X then $I_Y = \{G \cap Y : G \in I\}$ is an ideal on Y and $(Y,\tau_{/Y},I_{/Y})$ denote the ideal topological subspaces. Let $P(X)$ is the set of all subset of X, a set operator $^* : P(X) \to P(X)$, called a local function [6] of A with respect to τ and I, which is defined as: for $A \subset X$, $A^*(I,\tau) = \{x \in X : U \cap A \notin I$ for every $U \in \tau(X,x)\}$. We simply write A^* instead of $A^*(I,\tau)$ in case there is no concision. For every ideal topological spaces (X,τ,I) there exists a topology τ^* finer than τ defined as $\tau^* = \{U \subseteq X : cl^*(X-A) = X-A\}$ generated by the base $\beta(I,\tau) = \{U \subseteq J : U \in \tau$ and $J \in I\}$. A Kuratowski closure operator $cl^*(\cdot)$ for a topology $\tau^*(I,\tau)$ called the $*$-topology, finer than τ is defined by $cl^*(A) = A \cup A^*$ [12]. For a subset A of $X, cl^*(A)$ and $int^*(A)$ will represent the closure of A and the interior of A in (X,τ^*), respectively.

A subset A of an ideal topological spaces (X,τ,I) is said to be a τ^*-closed [3], if $A^* \subset A$. A subset A of an ideal topological spaces (X,τ,I) is said to be a I-open [4], if $A \subset int(A^*)$. A subset A of an ideal topological spaces (X,τ,I) is said to be a I-regular open (resp. I-regular closed) [8], if $A = int^*(cl^*(A))$ (resp. $A = cl^*(int^*(A))$). A subset A of an (X,τ,I) be a ideal topological spaces is said to be a I_{rg}-closed [10], $A^* \subset U$ whenever $A \subseteq U$ and is regular open set in X. A subset A of an (X,τ,I) be a ideal topological spaces is said to be a rg_I-closed [5], if $cl^*(A) \subset U$ whenever $A \subseteq U$ and is regular open set in X. A subset A of an (X,τ,I) be a ideal topological spaces is said to be a weakly-I_{rg}-closed(briefly w-I_{rg}-closed) [2], if $(int(A))^* \subset U$ whenever $A \subseteq U$ and is regular open set in X.

3. Saw-I_{rg}-Closed Sets

In this section, fist of all we introduce the notion called saw-I_{rg}-closed and give some characterizations of this sets.

Definition 1. *A subset A of an (X,τ,I) be a ideal topological spaces is said to be α_I-$*$-closed if $cl^*(int(cl(A))) \subset A$. The complement of α_I-$*$-closed set is said to be α_I-$*$-open.*

Definition 2. *A subset A of an (X,τ,I) be a ideal topological spaces is said to be:*

(1) Strongly almost weakly-I_{rg}-closed(briefly saw-I_{rg}-closed) if $(int(cl(A)))^* \subset U$ whenever $A \subseteq U$ and U is a regular open in X.

(2) *Almost weakly-I_{rg}-closed(briefly aw-I_{rg}-closed) if $(int(A^*))^* \subset U$ whenever $A \subseteq U$ and U is a regular open in X.*

(3) *Almost weakly-rg_I-closed(briefly aw-rg_I-closed) if $((int(cl^*(A))))^* \subset U$ whenever $A \subseteq U$ and U is a regular open in X.*

Theorem 1. *Let (X, τ, I) be a ideal topological spaces and $A \subset X$, the following properties are equivalent:*

(1) *A is a saw-I_{rg}-closed sets*

(2) *$cl^*(int(cl(A))) \subset U$ whenever $A \subseteq U$ and U is a regular open in X.*

Proof. (1) ⇒ (2) Let A is a saw-I_{rg}-closed set. Assume that $A \subseteq U$ and U is a regular open in X.Then we have $(int(cl(A)))^* \subset U$. Since $int(cl(A)) \subset cl(A) \subset A \subset U$. This implies that $int(cl(A)) \cup (int(cl(A)))^* = cl^*(int(cl(A))) \subset U$.

(2) ⇒ (1) Let $cl^*(int(cl(A))) \subset U$ whenever $A \subseteq U$ and U is a regular open in X. Since $(int(cl(A)))^* \cup (int(cl(A))) \subset U$ then $(int(cl(A)))^* \subset U$ whenever $A \subseteq U$ and is U is regular open in X. □

Theorem 2. *For a subset A of X the following properties hold:*

(1) *A is open and saw-I_{rg}-closed then A is I_{rg}-closed,*

(2) *A is open and w-I_{rg}-closed then A is I_{rg}-closed,*

(3) *A is I-open and aw-I_{rg}-closed then A is I_{rg}-closed.*

Proof.

(1) Let A be a open and saw-I_{rg}-closed set in (X, τ, I). Since A is a open, $A \subset int(A) \subset int(cl(A))$. Hence, $A^* \subset (int(cl(A)))^* \subset U$, $A \subseteq U$ and U is regular open in X. So, we have A is a I_{rg}-closed.

(2) Let A be a open and w-I_{rg}-closed set in (X, τ, I). Hence we have $A \subset int(A)$ and $A^* \subset (int(A))^* \subset U$. This implies that A is a I_{rg}-closed.

(3) Let A be a I-open and aw-I_{rg}-closed set in (X, τ, I). Hence we have $A \subset int(A^*)$ and $A^* \subset (int(A^*))^* \subset U$. This shows that A is a I_{rg}-closed.

□

Theorem 3. *Every α_I-∗-closed set is saw-I_{rg}-closed set.*

Proof. Let $A \subseteq U$ and U is a regular open in X. Since A is a α_I-∗-closed, $cl^*(int(cl(A))) \subset cl^*(int(cl(U))) \subset U$. Thus, A is a saw-I_{rg}-closed set in (X, τ, I). □

The following example shows that the reverse of Theorem 3 is not true.

Example 1. *Let* (X, τ, I) *be a ideal topological space such that* $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}, \{c, d\}, \{b, c, d\}, \{a, c, d\}\}$ *and* $I = \{\emptyset\}$. *Then,* $A = \{a, c\} \subset X$ *is saw-*I_{rg}*-closed set but is not* α_I*-*$*$*-closed set.*

Remark 1. *The intersection of two saw-*I_{rg}*-closed set in ideal topological spaces need not be a saw-*I_{rg}*-closed set.*

Example 2. *Let* (X, τ, I) *be a ideal topological space such that* $X = \{a, b, c\}$, $I = \{\emptyset, \{b\}\}$, *and* $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. *Let* $A = \{a, c\}$ *and* $B = \{a, b\}$. *From here A and B are saw-*I_{rg}*-closed set but* $A \cap B = \{a\}$ *is not saw-*I_{rg}*-closed set.*

Theorem 4. *Let* (X, τ, I) *be a ideal topological spaces* $A \subset X$. *If A is a saw-*I_{rg}*-closed set then* $(int(cl(A)))^* - A$ *contains no any nonempty regular closed set.*

Proof. Let A is a saw-I_{rg}-closed set in (X, τ, I). Suppose that U is a closed set. Such that $U \subseteq (int(cl(A)))^* - A$. Since $X - U$ is open and $A \subset X - U$, then $(int(cl(A)))^* \subset X - U$. Then, we get $U \subset X - (int(cl(A)))^*$. Hence $U \subset (int(cl(A)))^*$.

Thus, $U \subset (int(cl(A)))^* \cap X - (int(cl(A)))^* = \emptyset$ and $(int(cl(A)))^* - A$ contains no any nonempty closed set. □

Proposition 1. *Let* (X, τ, I) *be an ideal topological spaces. If* $A \subset B \subset cl^*(int(cl(A)))$ *and A is saw-*I_{rg}*-closed, then B is saw-*I_{rg}*-closed.*

Proof. Let $B \subset U$ and U is a regular open in X. Since $A \subset U$ and A is a saw-I_{rg}-closed set then $cl^*(int(cl(A))) \subset U$. Since, $B \subset cl^*(int(cl(A))) \subset U$, we obtain

$$cl^*(int(cl(B))) \subset cl^*(int(cl(cl^*(int(cl(A)))))) \subset cl^*(int(cl(A))) \subset U.$$

Therefore $cl^*(int(cl(B))) \subset U, B$ is a saw-I_{rg}-closed. □

Corollary 1. *Let* (X, τ, I) *be an ideal topological spaces and* $A \subset X$. *If A is a saw-*I_{rg}*-closed and regular open set, then* $cl^*(A)$ *is a saw-*I_{rg}*-closed.*

Proof. Let A is saw-I_{rg}-closed set and regular open set in (X, τ, I). Then we have $A \subset cl^*(A) \subset cl^*(A) = cl^*(int(cl(A)))$. Hence by Proposition 1 $cl^*(A)$ is a saw-I_{rg}-closed set in (X, τ, I). □

Theorem 5. *Let* (X, τ, I) *be an ideal topological spaces and* $A \subset X$. *Assume that A is a saw-*I_{rg}*-closed set. The following properties are equivalent:*

(1) A is a α_I*-*$*$*-closed,*

(2) $(int(cl(A)))^* - A$ *is a regular closed set.*

Proof. (1) ⇒ (2) Let A be a α_I-$*$-closed set. Which means that $cl^*(int(cl(A))) \subset A$. This implies $(int(cl(A)))^* \subset A$ and $(int(cl(A)))^* - A = \emptyset$. Thus, $cl^*(int(cl(A))) - A$ is a regular closed set.

(2) ⇒ (1) Let $cl^*(int(cl(A))) - A$ be a regular closed set. Since A is a saw-I_{rg}-closed set in (X, τ, I), by Theorem 3 $(int(cl(A)))^* - A = \emptyset$. Hence, we get $cl^*(int(cl(A))) \subset A$. Thus A is a α_I-$*$-closed. □

Corollary 2. *Let (X, τ, I) be an ideal topological spaces and $A \subset X$. If A is a aw-I_{rg}-closed and τ^*-closed set, then $cl^*(int(A^*)) \subset U$ whenever $A \subset U$ and U is a regular open set in X.*

Proof. Let A be a aw-I_{rg}-closed set in X. Suppose that $A \subset U$ and U is a regular open set in X. We have $(int(A^*))^* \subset U$. On the other hand since A is a τ^*-closed, we have $int(A^*) \subset int(A) \subset A \subset U$. Hence $(int(A^*))^* \cup int(A^*) \subset U$. This implies $cl^*(int(A^*)) \subset U$. □

Theorem 6. *If (X, τ, I) is any ideal topological spaces where $I = \{\emptyset\}$, then A is a aw-I_{rg}-closed if and only if $G \subset int^*(cl(A^*))$ whenever $G \subset A$ and G is a regular closed set.*

Proof. Necessity: let G be a regular closed set in X and $G \subset A$. Then it is well-known that $X - G$ is regular open set and $(X - A) \subset (X - G)$. Since $X - A$ is a aw-I_{rg}-closed, then $cl^*(int((X - A)^*)) \subset (X - G)$. From the fact that for $I = \{\emptyset\}$, $A^* = cl(A)$. Therefore $X - int^*(cl(A^*)) \subset (X - G)$. So, we have $G \subset int^*(cl(A^*))$.

Sufficiency: let H be a regular open set in X and $(X - A) \subset H$. Since $(X - H)$ is a regular open set such that $(X - H) \subset A$, then $(X - H) \subset int^*(cl(A^*))$. We have $X - int^*(cl(A^*)) = cl^*(int(A^*)) \subset H$. Thus, $(X - A)$ is a aw-I_{rg}-closed set. Hence, A is a aw-I_{rg}-open set in X. □

4. Weakly-rg_I-Closed Sets

In this section, secondly we introduce weakly-rg_I-closed sets and investigate their basic properties.

Definition 3. *A subset A of an (X, τ, I) be a ideal topological spaces is said to be*

(i) weakly-rg_I-closed(briefly w-rg_I-closed) set if $(int(cl^*(A))) \subset U$ whenever $A \subseteq U$ and U is a regular open in X.

(ii) weakly-Irg-closed(briefly w-Irg-closed) set if $(int(cl^*(A))) \subset U$ whenever $A \subseteq U$ and U is I-regular open in X.

Theorem 7. *Every w-Irg-closed set is a w-rg_I-closed set.*

Proof. Let A be a w-Irg-closed set. Then $(int(cl^*(A))) \subset U$ whenever $A \subseteq U$ and U is a I-regular open in X. Since U is I-regular open, we have $U = int^*(cl^*((U)))$ and $int(cl(U)) \subset int^*(cl^*((U)))$. Therefore, U is a regular open in X. This shows that A is a w-rg_I-closed set. □

The following example shows that the reverse of Theorem 7 is not true.

Example 3. *Let (X,τ,I) be a ideal topological space such that $X=\{a,b,c\}$, $I=\{\emptyset,\{c\}\}$, and $\tau=\{\emptyset,X,\{a\},\{c\},\{a,c\}\}$. Then $A=\{a\}\subset X$ is a w-rg_I-closed set but is not w-Irg-closed set, since $int(cl^*(A))=int(cl^*(\{a\}))=\{a\}$ where A is contained in the regular open set U. But $int^*(cl^*(A))=int^*(cl^*(\{a\}))\neq\{a\}$ which means A is contained there is not I-regular open set U.*

Remark 2. *Let be a (X,τ,I) be a ideal topological spaces. The following diagram holds for a subset $A\subset X$:*

$$\begin{tikzcd}
rg_I-closed \arrow[r] \arrow[d] & w-rg_I-closed \arrow[d] & w-Irg-closed \arrow[l] \\
I_{rg}-closed \arrow[r] & w-I_{rg}-closed & aw-rg_I-closed \arrow[l] \arrow[d] \\
\alpha_I-*-closed \arrow[r] & saw-I_{rg}-closed \arrow[r] \arrow[u] & aw-I_{rg}-closed
\end{tikzcd}$$

Theorem 8. *Let be a (X,τ,I) be a ideal topological spaces. For every subset $A\in I$, A is a rg_I-closed sets.*

Proof. Let $A\subset U$, where U is regular open. Since $A^*=\emptyset$ for every $A\in I$, we obtain $cl^*(A)=A\cup A^*=A\subset U$. Therefore A is a rg_I-closed set. □

Theorem 9. *Let be a (X,τ,I) be a ideal topological spaces. For every subset A of X, A^* is a rg_I-closed sets.*

Proof. Let $A^*\subset U$, where U is a regular open. Since $(A^*)^*\subset A^*$, we have $cl^*(A^*)\subset U$. Hence A^* is a rg_I-closed set. □

Theorem 10. *Let be a (X,τ,I) be a ideal topological spaces. If A is a rg_I-closed set, then $cl^*(A)$—A does not contain any nonempty regular closed set.*

Proof. Let F be a regular closed subset of X, such that $F\subset cl^*(A)-A$ where A is rg_I-closed set. We get $cl^*(A)\subset(X-F)$. This shows that $F\subset(X-cl^*(A))\cap cl^*(A)$. Hence $F=\emptyset$. □

Theorem 11. *(X,τ,I) be a ideal topological spaces and $A\subset X$. If A is w-rg_I-closed set and τ^*-closed, then A is a w-I_{rg}-closed set.*

Proof. Let A be a τ^*-closed and w-rg_I-closed set in (X,τ,I). Then, we have $A^*\subset A$ and we have $int(A^*)\subset int(A)$. On the other hand, $int(cl^*(A))\subset U$ whenever $A\subset U$ and U regular open in X. Hence, $int(A)\subset int(A)\cup int(A^*)\subset int(A\cup A^*)=int(cl^*(A))$. This implies that $(int(A))^*\subset int(cl^*(A))\subset U$ and so A is a w-I_{rg}-closed set. □

Theorem 12. *In an ideal spaces (X,τ,I), the union two rg_I-closed set in an rg_I-closed set.*

Proof. Let A and B rg_I-closed set. Suppose $A \cup B \subset U$ and U is regular open. Then $A \subset U$ and $B \subset U$ by hypothesis, $cl^*(A) \subset U$ and $cl^*(B) \subset U$. Therefore $cl^*(A \cup B) = cl^*(A) \cup cl^*(B) \subset U$. This shows that $A \cup B$ is rg_I-closed set. □

Example 4. *Let $X = \{a, b, c\}, \tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $I = \{\emptyset, \{b\}\}$. For $A = \{a, b\}$ and $B = \{a, c\}$ since X is the only regular open set containing A and B. Therefore A and B are rg_I-closed sets. Now, $A \cap B = \{a\}$ is regular open and $cl^*(A \cap B) = \{a, c\} \nsubseteq \{a\}$. This shows that $A \cap B$ is not an rg_I-closed set.*

Theorem 13. *(X, τ, I) be a ideal topological spaces and $A \subset X$. If A is w-rg_I-closed, B is regular closed and τ^*-closed[3] then, $A \cap B$ is a w-rg_I-closed.*

Proof. Let U be a regular open such that $A \cap B \subset U$. Then we have $A \subset U \cap (X - B)$. Since A is a w-rg_I-closed and B is a τ^*-closed, then $int(cl^*(A)) \subset U \cap (X - B)$. Also, $cl^*(A \cap B) \subset cl^*(A) \cap cl^*(B)$. Therefore $cl^*(A \cap B) \subset U \cap (X - B)$. Since B is τ^*-closed, $cl^*(A \cap B) \subset U$. This shows that $A \cap B$ is a w-rg_I-closed. □

Theorem 14. *Let (X, τ, I) be a ideal topological spaces $A \subset X$. If A is w-rg_I-closed set then $(int(cl^*(A))) - A$ contains no any nonempty regular closed set.*

Proof. This theorem can be proved similar way of Theorem 4. □

Theorem 15. *Let (X, τ, I) be a ideal topological spaces A subset of X. A is w-rg_I-open set if and only if $G \subset cl(int^*(A))$ whenever $G \subset A$ and G is regular closed.*

Proof. Necessity: let $G \subset A$ and G be regular closed. Then $(X - A) \subset (X - G)$ and $(X - G)$ regular open. Since $(X - A)$ is a w-rg_I-closed, $(int(cl^*(x - A))) \subset (X - G)$. Hence, we get $G \subset cl(int^*(A))$.

Sufficiency: suppose that $G \subset cl(int^*(A))$ whenever $G \subset A$ and G is regular closed. Let $(X - A) \subset U$ where U is regular open. Then $(X - U) \subset A$. By hypothesis $(X - U) \subset cl(int^*(A))$ and $cl(int^*(X - A)) \subset U$. Therefore A is w-rg_I-open. □

5. α_I-$*$-$normal$ Spaces

In this section, we talk about the α_I-$*$-$normal$ space and investigate some properties.

Definition 4. *An ideal topological spaces (X, τ, I) is said to be α_I-$*$-$normal$ if for every pair of disjoint regular closed subsets A, B of X, there exist disjoint α_I-$*$-open sets U, V of X such that $A \subseteq U$ and $B \subseteq V$.*

Theorem 16. *Let (X, τ, I) be a ideal topological spaces and $A \subset X$, the following properties are equivalent:*

(1) X is a α_I-$*$-$normal$,

(2) *For any disjoint regular closed sets A and B, there exist disjoint saw-I_{rg}-open sets U and V of X such that $A \subseteq U$ and $B \subseteq V$,*

(3) *For any regular closed set A and any regular open set B containing A, there exists a saw-I_{rg}-open set U such that $A \subseteq U \subseteq cl^*(int(cl(A))) \subseteq B$.*

Proof. (1) ⇒ (2) The proof is obvious.

(2) ⇒ (3) Let A be a regular closed and B be a regular open subset of X, such that $A \subseteq B$. Then A and $(X - B)$ are disjoint regular closed set of X. By the hypothesis, there exist a saw-I_{rg}-open sets U and V of X such that $A \subseteq U$ and $(X - B) \subseteq V$. Since V is saw-I_{rg}-open set, $(X - B) \subseteq int^*(cl(int(V)))$. Hence we have $U \cap int^*(cl(int(V))) = \emptyset$. So, we obtain $cl^*(int(cl(U))) \subseteq cl^*(int(cl(X - V)))$. This shows that $A \subseteq U \subseteq cl^*(int(cl(U))) \subseteq B$.

(3) ⇒ (1) Let A and B any disjoint regular closed set of X. Then, $A \subseteq (X - B)$ and $(X - B)$ is regular open set. Hence there exist a saw-I_{rg}-open set G of X such that
$A \subseteq G \subseteq cl^*(int(cl(G))) \subseteq (X - B)$. □

Definition 5. *A function $f : (X, \tau, I) \to (Y, \varphi)$ is said to be saw-I_{rg}-continuous if for every closed set F in Y, $f^{-1}(F)$ saw-I_{rg}-closed in X.*

Definition 6. *A function $f : (X, \tau, I) \to (Y, \varphi, J)$ is called saw-I_{rg}-irresolute if for every saw-J_{rg}-closed in Y, $f^{-1}(F)$ saw-I_{rg}-closed in X.*

Theorem 17. *Let $f : X \to Y$ be a saw-I_{rg}-continuous regular closed and injective function. If Y is normal, then X is α_I-$*$-$normal$.*

Proof. Let A and B any disjoint regular closed set of X. Since f is regular closed injection, $f(A)$ and $f(B)$ are disjoint regular closed sets of Y. By the normality of Y, there exist disjoint open sets U and V such that $f(A) \subseteq U$ and $f(B) \subseteq V$. Since f is saw-I_{rg}-continuous, then $f^{-1}(U)$ and $f^{-1}(V)$ are saw-I_{rg}-open sets such that $A \subseteq f^{-1}(U)$ and $B \subseteq f^{-1}(V)$. Therefore X is α_I-$*$-$normal$ by Theorem 16. □

Theorem 18. *Let $f : X \to Y$ be a saw-I_{rg}-irresolute regular closed injection. If Y is α_I-$*$-$normal$, then X is α_I-$*$-$normal$.*

Proof. Let A and B any disjoint regular closed set of X. Since f is regular closed injection, $f(A)$ and $f(B)$ are disjoint regular closed sets of Y. Since Y is α_I-$*$-$Normal$, by Theorem 16 there exist disjoint saw-I_{rg}-open U and V such that $f(A) \subseteq U$ and $f(B) \subseteq V$ since f is saw-I_{rg}-irresolute, then $f^{-1}(U)$ and $f^{-1}(V)$ are saw-I_{rg}-open sets such that $A \subseteq f^{-1}(U)$ and
$B \subseteq f^{-1}(V)$. Therefore X is α_I-$*$-$normal$. □

Theorem 19. *Let $f : X \to Y$ be a saw-I_{rg}-irresolute regular closed injection. If X is α_I-$*$-$normal$ and $Y \subset X$ regular closed, then Y is $\alpha_{I_{/Y}}$-$*$-$normal$ spaces.*

Proof. Let A and B any disjoint regular closed set of Y. Since Y is regular closed, we have A and B are disjoint regular closed sets of X. Since X is α_I-$*$-$Normal$, there exist disjoint saw-I_{rg}-open U and V such that $A \subseteq U$ and $B \subseteq V$. If $H \in I$ and $G \in I$, then $A \subset (U \cap H)$ and

$B \subset (V \cap G)$. Since $A \subset Y$ and $B \subset Y$, we have $A \subset Y \cap (U \cap H)$ and $B \subset Y \cap (V \cap G)$. Hence, $A \subset (Y \cap U) \cap H$ and $B \subset (Y \cap V) \cap G$. If we take $(Y \cap U) = U_1$ and $(Y \cap V) = V_1$, then $U_1 \cap V_1 = \emptyset$ such that U_1 and V_1 is saw-I_{rg}-open. This shows that Y is $\alpha_{I_{/Y}}$-$*$-$normal$ spaces. □

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