EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
Vol. 10, No. 2, 2017, 323-334
ISSN 1307-5543 – www.ejpam.com
Published by New York Business Global

bT$^{\mu}$- compactness and bT$^{\mu}$ - connectedness in supra topological spaces

K.Krishnaveni[1,*], M.Vigneshwaran [2]

[1] *Research Scholar, Department of Mathematics, Coimbatore, Tamilnadu, India.*
[2] *Department of Mathematics, Coimbatore, Tamilnadu, India*

Abstract. In this paper we newly originate the notion of bT$^{\mu}$ - compact space and inspected its several effects and characterizations. Also we newly originate and study the concept of bT$^{\mu}$ - Lindelof spaces and Connected spaces.

2010 Mathematics Subject Classifications: 54D05, 54D20, 54D30

Key Words and Phrases: bT^{μ} -open sets; bT^{μ} -compact spaces; bT^{μ} - Lindelof spaces and bT^{μ} - connected spaces.

1. Introduction

The supra topological spaces was introduced by Mashhour.et.al [6] in 1983. They studied S - continuous maps and S*- continuous maps. The supra b- open set and supra b-continuity was brought out by Sayed.et.al [8] in 2010. Recently Krishnaveni and Vigneshwaran [4] came out with supra bT -closed sets and defined their properties. In 2013, Jamal M.Mustafa.et.al[3] came out with the concect of supra b- connected and supra b-Lindelof spaces. Now we bring up with the new concepts of supra bT -compact, supra bT - Lindelof, Countably supra bT - compact and supra bT -connected spaces and reviewed several properties for these concepts.

2. Preliminaries

Definition 1 (6,8). *A subfamily of μ of X is said to be a supra topology on X, if*

(*i*) *X, $\phi \epsilon \mu$*

(*ii*) *if $A_i \epsilon \mu$ for all iϵ J then $\cup A_i \epsilon \mu$.*

The pair (X,μ) is called supra topological space. The elements of μ are called supra open sets in (X,μ) and complement of a supra open set is called a supra closed set.

*Corresponding author.

Email addresses: `krishnavenikaliswami@gmail.com` (K.Krishna), `vignesh.mat@gmail.com` (M.Vignesh)

Definition 2 (6).

(i) *The supra closure of a set A is denoted by* $cl^{\mu}(A)$ *and is defined as* $cl^{\mu}(A) = \cap\{B : B$ *is a supra closed set and* $A \subseteq B\}$.

(ii) *The supra interior of a set A is denoted by* $int^{\mu}(A)$ *and defined as* $int^{\mu}(A) = \cup\{B : B$ *is a supra open set and* $A \supseteq B\}$.

Definition 3 (8). *Let (X,τ) be a topological spaces and μ be a supra topolgy on X. We call μ a supra topology associated with τ if* $\tau \subset \mu$.

Definition 4 (8). *Let (X,μ) be a supra topological space. A set A is called a supra b-open set if* $A \subseteq cl^{\mu}(int^{\mu}(A)) \cup int^{\mu}(cl^{\mu}(A))$. *The complement of a supra b-open set is called a supra b-closed set.*

Definition 5 (4). *A subset A of a supra topological space(X,μ) is called* bT^{μ}*-closed set if* $bcl^{\mu}(A) \subset U$ *whenever* $A \subset U$ *and U is* T^{μ}*- open in (X,μ).*

Definition 6 (4). *Let (X,τ) and (Y,σ) be two topological spaces and μ be an associated supra topology with τ. A function f:(X,τ) →(Y, σ) is called* bT^{μ} *- continuous if* $f^{-1}(V)$ *is* bT^{μ} *- closed in (X,τ) for every supra closed set V of (Y,σ).*

Definition 7 (4). *Let (X,τ) and (Y,σ) be two topological spaces and μ be an associated supra topology with τ. A function f:(X,τ) →(Y, σ) is called* bT^{μ} *- irresolute if* $f^{-1}(V)$ *is* bT^{μ} *- closed in (X,τ) for every* bT^{μ} *- closed set V of (Y,σ).*

Definition 8 (4). *A supra topological space (X,μ) is called* ${}_{bT}T^{\mu}_{c}$*- space,if every* bT^{μ}*-closed set is supra closed set.*

Definition 9 (5). *Let (X,τ) and (Y,σ) be two topological spaces and μ be an associated supra topology with τ. A function f:(X,τ) →(Y, σ) is called strongly* bT^{μ} *- continuous if the inverse image of every* bT^{μ}*-closed in Y is supra closed in X.*

Definition 10 (5). *Let (X,τ) and (Y,σ) be two topological spaces and μ be an associated supra topology with τ. A function f:(X,τ) →(Y,σ) is called perfectly* bT^{μ} *- continuous if the inverse image of every* bT^{μ}*-closed in Y is both supra closed and supra open in X.*

3. supra bT - Compactness

Definition 11. *A collection* $\{A_i : i \in I\}$ *of* bT^{μ} *- open sets in a supra topological space (X,μ) is called a* bT^{μ}*-open cover of a subset B of X if* $B \subset \bigcup\{A_i : i \in I\}$ *holds.*

Definition 12. *A supra topological space (X,μ) is* bT^{μ} *- compact if every* bT^{μ} *- open cover of X has a finite subcover.*

Definition 13. *A subset B of a supra topological space (X,μ) is said to be* bT^{μ} *- compact relative to (X,μ) if, for every collection* $\{A_i : i \in I\}$ *of* bT^{μ} *- open subsets of X such that* $B \subset \bigcup\{A_i : i \in I\}$ *there exist a finite subset* I_o *of I such that* $B \subseteq \bigcup\{A_i : i \in I_o\}$.

Definition 14. *A subset B of a supra topological space (X,μ) is said to be bT^μ - compact if B is bT^μ - compact as a subspace of X.*

Theorem 1. *Every bT^μ - compact space is supra compact.*

Proof. Let $\{A_i : i \in I\}$ be a supra open cover of (X,μ). By [4] $\{A_i : i \in I\}$ is a bT^μ - open cover of (X,μ). Since (X,μ) is bT^μ - compact, bT^μ - open cover $\{A_i : i \in I\}$ of (X,μ) has a finite subcover say $\{A_i : i = 1, 2, \cdots, n\}$ for X. Hence (X,μ) is a supra compact space.

Theorem 2. *Every bT^μ - closed subset of a bT^μ - compact space is bT^μ - compact relative to X.*

Proof. Let A be a bT^μ - closed subset of a supra topological space (X,μ). Then A^c is bT^μ - open in (X,μ). Let S = $\{A_i : i \in I\}$ be an bT^μ - open cover of A by bT^μ - open subset in (X,μ). Let S* = $S \cup A^c$ is a bT^μ - open cover of (X,μ). That is X = $(\bigcup_{i\in I} A_i) \bigcup A^c$. By hypothesis (X,$\mu$) is a bT^μ- compact and hence S* is reducible to a finite sub cover of (X,μ) say X = $A_{i1} \cup A_{i2} \cup, \cdots, \cup A_{in} \cup A^c, A_{ik} \in S^*$. But A and A^c are disjoint. Hence $A \subset A_{i1} \cup A_{i2} \cup, \cdots, \cup A_{in} \in S$. Thus a bT^μ -open cover S of A contains a finite subcover. Hence A is bT^μ- compact relative to (X,μ).

Theorem 3. *A bT^μ - continuous image of a bT^μ - compact space is supra compact.*

Proof. Let f: $X \to Y$ be a bT^μ- continuous map from a bT^μ - compact X onto a supra topological space Y. Let $\{A_i : i \in I\}$ be a supra open cover of Y. Then $f^{-1}\{A_i : i \in I\}$ is a bT^μ - open cover of X, as f is bT^μ - continuous. Since X is bT^μ - compact, the bT^μ - open cover of X, $f^{-1}\{A_i : i \in I\}$ has a finite sub cover say $\{f^{-1}(A_i) : i = 1, 2, \cdots, n\}$. Therefore X = $\bigcup_{i=1}^n f^{-1}(A_i)$, which implies f(X) = $\bigcup_{i=1}^{n} (A_i)$, then Y = $\bigcup_{i=1}^{n} (A_i)$. That is $\{A_1, A_2, \cdots, A_n\}$ is a finite sub cover of $\{A_i : i \in I\}$ for Y. Hence Y is supra compact.

Theorem 4. *If a map $f : (X,\tau) \to (Y, \sigma)$ is bT^μ - irresolute and a subset S of X is bT^μ - compact relative to (X,τ), then the image f(S) is bT^μ - compact relative to (Y,σ).*

Proof. Let $\{A_i : i \in I\}$ be a collection of bT^μ - open cover of (Y,σ), such that f(S)$\subseteq \bigcup_{i\in I} A_i$. Then S $\subseteq \bigcup_{i=1}^{n} f^{-1}(A_i)$, where $\{f^{-1}(A_i) : i \in I)\}$ is bT^μ- open set in (X,τ). Since S is bT^μ -compact relative to (X,τ), there exist finite subcollection $\{A_1, A_2, \cdots, A_n\}$ such that S $\subseteq \bigcup_{i=1}^{n} f^{-1}(A_i)$. That is f(S) $\subseteq \bigcup_{i=1}^{n} A_i$. Hence f(S) is bT^μ - compact relative to (Y,σ).

Theorem 5. *If a map $f : (X,\tau) \to (Y, \sigma)$ is strongly bT^μ - continuous map from a supra compact space (X,τ) onto a supra topological space (Y,σ), then (Y,σ) is bT^μ - compact.*

Proof. Let $\{A_i : i \in I\}$ be a bT^μ - open cover of (Y,σ). Since f is strongly bT^μ - continuous, $\{f^{-1}(A_i : i \in I)\}$ is an supra open cover of (X,τ). Again, since (X,τ) is supra compact, the supra open cover $\{f^{-1}(A_i) : i \in I)\}$ of (X,τ) has a finite sub cover say $\{f^{-1}(A_i) : i = 1, 2, \cdots, n\}$. Therefore X = $\bigcup_{i=1}^{n} f^{-1}(A_i)$, which implies f(X) = $\bigcup_{i=1}^{n}(A_i)$, so that Y = $\bigcup_{i=1}^{n}(A_i)$. That is $A_1, A_2, \cdots, A_n$ is a finite sub cover of $\{A_i : i \in I\}$ for (Y,σ). Hence (Y,σ) is bT^μ -compact.

Theorem 6. *If a map f : (X,τ) $\rightarrow$ (Y, σ) is perfectly bT^μ- continuous map from a compact space (X,τ) onto a supra topological space (Y,σ), then (Y,σ) is bT^μ - compact.*

Proof. Let $\{A_i : i \in I\}$ be a bT^μ - open cover of (Y,σ). Since f is perfectly bT^μ-continuous, $\{f^{-1}(A_i) : i \in I)\}$ is a supra open cover of (X,τ). Again, since (X,τ) is supra compact, the supra open cover $\{f^{-1}(A_i) : i \in I\}$ of (X,τ) has a finite sub cover say $\{f^{-1}(A_i) : i = 1, 2, \cdots, n\}$. Therefore X = $\bigcup_{i=1}^{n} f^{-1}(A_i)$, which implies f(X) = $\bigcup_{i=1}^{n} A_i$, so that Y = $\bigcup_{i=1}^{n} A_i$. That is $A_1, A_2, \cdots, A_n$ is a finite sub cover of $\{A_i : i \in I\}$ for (Y,σ). Hence (Y,σ) is bT^μ -compact.

Theorem 7. *If a map f : (X,τ) $\rightarrow$ (Y, σ) be bT^μ - irresolute map from bT^μ - compact space (X,τ) onto supra topological space (Y,σ) then (Y,σ) bT^μ - compact.*

Proof. If a map f : (X,τ) $\rightarrow$ (Y, σ) is bT^μ - irresolute map from a bT^μ- compact space (X,τ) onto a supra topological space (Y,σ). Let $\{A_i : i \in I\}$ be a bT^μ - open cover of (Y,σ). Then $\{f^{-1}(A_i) : i \in I\}$ is an bT^μ - open cover of (X,τ), since f is bT^μ - irresolute. As (X,τ) is bT^μ - compact, the bT^μ - open cover $\{f^{-1}(A_i) : i \in I\}$ of (X,τ) has a finite sub cover say $\{f^{-1}(A_i) : i = 1, 2, \cdots, n\}$. Therefore X = $\bigcup_{i=1}^{n} f^{-1}(A_i)$, which implies f(X) = $\bigcup_{i=1}^{n} A_i$, so that Y = $\bigcup_{i=1}^{n} A_i$. That is $A_1, A_2, \cdots, A_n$ is a finite sub cover of $\{A_i : i \in I\}$ for (Y,σ). Hence (Y,σ) is bT^μ -compact.

Theorem 8. *If (X,τ) is compact and ${}_{bT}T_c^\mu$ space, then (X,τ) is bT^μ - compact.*

Proof. Let (X,τ) is bT^μ - compact space. Let $\{A_i : i \in I\}$ be a bT^μ - open cover of (X,). Since by ${}_{bT}T_c^\mu$ -space,$\{A_i : i \in I\}$ is a supra open cover of (X,τ). Since (X,τ) is compact, supra open cover $\{A_i : i \in I\}$ of (X,τ) has a finite sub cover say $\{A_i : i = 1, 2, \cdots, n\}$ for X. Hence (X,τ) is a bT^μ - compact space.

Theorem 9. *A supra topological space (X,τ) is bT^{μ} - compact if and only if every family of bT^{μ}-closed sets of (X,τ) having finite intersection property has a non empty intersection.*

Proof. Suppose (X,τ) is bT^{μ}- compact, Let $\{A_i : i \in I\}$ be a family of bT^{μ}- closed sets with finite intersection property. Suppose $\bigcap_{i\in I} A_i = \phi$, then X - $\bigcap_{i\in I} A_i$= X. This implies $\bigcup_{i\in I}(X - A_i)$ = X. Thus the cover $\{X - A_i : i \in I\}$ is a bT^{μ}- open cover of (X,τ). Then, the bT^{μ}-open cover $\{X - A_i : i \in I\}$ has a finite sub cover say X-$\{X - A_i : i = 1, 2, \cdots , n\}$. This implies X = $\bigcup_{i\in I}^{n}(X - A_i)$ which implies X = X - $\bigcap_{i=1}^{n} A_i$, which implies X-X = X - $\left[X - \bigcap_{i=1}^{n} A_i\right]$ which implies $\phi = \bigcap_{i=1}^{n} A_i$. This disproves the assumption. Hence $\bigcap_{i=1}^{n} A_i \neq \phi$

Conversely suppose (X,τ) is not bT^{μ}- compact. Then there exit an bT^{μ}- open cover of (X,τ) say $\{G_i : i \in I\}$ having no finite sub cover. This implies for any finite sub family $G_i : i = 1, 2, \cdots , n$ of $\{G_i : i \in I\}$, we have $\bigcup_{i=1}^{n} G_i \neq X$, which implies X - $\bigcup_{i=1}^{n} G_i \neq X$ - X , therefore $\bigcap_{i\in I}(X - G_i) \neq \phi$. Then the family $\{X - G_i : i \in I\}$ of bT^{μ} - closed sets has a finite intersection property. Also by assumption $\bigcap_{i\in I}(X - G_i) \neq \phi$ which implies X - $\bigcup_{i=1}^{n} G_i \neq \phi$, so that$\bigcup_{i=1}^{n} G_i \neq X$. This implies $\{G_i : i \in I\}$ is not a cover of (X,τ). This disproves the fact that $\{G_i : i \in I\}$ is a cover for (X,τ). Therefore a bT^{μ} - open cover $\{G_i : i \in I\}$ of (X,τ) has a finite sub cover $\{G_i : i = 1, 2, \cdots , n\}$. Hence (X,$\tau$) is bT^{μ} - compact.

Theorem 10. *Let A be a bT^{μ} - compact set relative to a supra topological space X and B be a bT^{μ} -closed subset of X. Then A$\cap$ B is bT^{μ} - compact relative to X.*

Proof. Let A is bT^{μ} - compact relative to X. Suppose that $\{A_i : i \in I\}$ is a cover of A$\cap$ B by bT^{μ} - open sets in X. Then $\{A_i : i \in I\} \cup \{B^c\}$ is a cover of A by bT^{μ} -open sets in X, but A is bT^{μ} - compact relative to X, so there exist $i_1, i_2, \cdots , i_n$ such that A $\subseteq \bigcup\{Aij : j = 1, 2, \cdots , n\} \cup B^c$. Then A$\cap B \subseteq \bigcup\left\{\bigcup A_{ij} \cap B, j = 1, 2, \cdots , n\right\} \subseteq \bigcup\{A_{ij} : j = 1, 2, \cdots , n\}$. Hence A$\cap$ B is bT^{μ} - compact relative to X.

Theorem 11. *If a function $f : (X,\tau) \to (Y, \sigma)$ is bT^{μ} - irresolute and a subset of X is bT^{μ} - compact relative to X, then f(B) is bT^{μ} - compact relative to Y.*

Proof. Let $\{A_i : i \in I\}$ be a cover of f(B) by bT^{μ} -open subsets of Y. Then $\left\{f^{-1}(A_i) : i \in I\right\}$ is a cover of B by bT^{μ} -open subsets of X. Since B is bT^{μ} -compact relative to X, $\left\{f^{-1}(A_i) : i \in I\right\}$ has a finite subcover say $\left\{f^{-1}(A_1), f^{-1}(A_2), \cdots , f^{-1}(A_n)\right\}$ for B. Now

$\{A_1, A_2, \cdots, A_n\}$ is a finite subcover of $\{A_i : i \in I\}$ for f(B). So f(B) is bT^μ -compact relative to Y.

4. Countably supra bT - Compactness in supra topological spaces

In this section,we concentrate on the concept of countably bT$^\mu$- Compactness and their properties.

Definition 15. *A supra topological space (X,τ) is said to be countably bT^μ - compact if every countable bT^μ - open cover of X has a finite subcover.*

Theorem 12. *If (X,τ) is a countably bT^μ - compact space, then (X,τ) is countably supra compact.*

Proof. Let (X,τ) is countably bT^μ - compact space. Let $\{A_i : i \in I\}$ be a countable supra open cover of (X,τ). By [4], $\{A_i : i \in I\}$ is a countable bT^μ - open cover of (X,τ). Since (X,τ) is countably bT^μ - compact, countable bT^μ - open cover $\{A_i : i \in I\}$ of (X,τ) has a finite subcover say $\{A_i : i = 1, 2, \cdots, n\}$ for X. Hence (X,τ) is a countably supra compact space.

Theorem 13. *If (X,τ) is countably supra compact and ${}_{bT}T_c^\mu$ -space, then (X,τ) is countably bT^μ - compact.*

Proof. Let (X,τ) is countably bT^μ - compact space. Let $\{A_i : i \in I\}$ be a countable bT^μ - open cover of (X,τ). Since by ${}_{bT}T_c^\mu$ - space $\{A_i : i \in I\}$ is a countable open cover of (X,τ). Since (X,τ) is countably supra compact, countable supra open cover $\{A_i : i \in I\}$ of (X,τ) has a finite sub cover say $\{A_i : i = 1, 2, \cdots, n\}$ for X. Hence (X,τ) is a countably bT^μ - compact space.

Theorem 14. *Every bT^μ - compact space is countably bT^μ -compact.*

Proof. Let (X,τ) is bT^μ-compact space. Let $\{A_i : i \in I\}$ be a countable bT^μ- pen cover of (X,τ). Since (X,τ) is bT^μ- compact, bT^μ- pen cover$\{A_i : i \in I\}$ of (X,τ) has a finite subcover say $\{A_i : i = 1, 2, \cdots, n\}$ for (X,τ). Hence (X,τ) is a countably bT^μ-compact space.

Theorem 15. *Let f : (X,τ) $\to$ (Y, σ) be a bT^μ - continuous injective mapping. If X is countably bT^μ - compact space then (Y,σ) is countably supra compact.*

Proof. Let f : (X,τ) $\to$ (Y, σ) be a bT^μ - continuous map from a countably bT^μ - compact (X,τ) onto a supra topological space (Y,σ). Let $\{A_i : i \in I\}$ be a countable supra open cover of Y. Then $\left\{f^{-1}(A_i) : i \in I\right\}$is a countable bT^μ - open cover of X, as f is bT^μ - continuous. Since X is countably bT^μ- compact, the countable bT^μ-open cover $\left\{f^{-1}(A_i) : i \in I\right\}$ of X has a finite sub cover say $\left\{f^{-1}(A_i) : i = 1, 2, \cdots, n\right\}$. Therefore X

$= \bigcup_{i=1}^{n} \left\{f^{-1}(A_i)\right\}$, which implies f(X) $= \bigcup_{i=1}^{n} A_i$, then Y $= \bigcup_{i=1}^{n} A_i$. That is $\{A_1, A_2, \cdots, A_n\}$ is a finite sub cover of $\{A_i : i \in I\}$ for Y. Hence Y is countably supra compact.

Theorem 16. *If a map f : (X,τ) $\rightarrow$ (Y, σ) is perfectly bT^{μ} - continuous map from a countably supra compact space (X,τ) onto a supra topological space (Y,σ), then (Y,σ) is countably bT^{μ} - compact.*

Proof. Let $\{A_i : i \in I\}$ be a countable bT^{μ} - open cover of (Y,σ). Since f is perfectly bT^{μ} - continuous, $\left\{f^{-1}(A_i) : i \in I\right\}$ is a countable supra open cover of (X,τ). Again, since (X,τ) is countably supra compact, the countable supra open cover $\left\{f^{-1}(A_i) : i \in I\right\}$ of (X,τ) has a finite sub cover say $\left\{f^{-1}(A_i) : i = 1, 2, \cdots, n\right\}$. Therefore X $= \bigcup_{i=1}^{n} \left\{f^{-1}(A_i)\right\}$, which implies f(X) $= \bigcup_{i=1}^{n} \{(A_i)\}$, so that Y $= \bigcup_{i=1}^{n} \{(A_i)\}$. That is $\{A_1, A_2, \cdots, A_n\}$ is a finite sub cover of $\{A_i : i \in I\}$ for (Y,σ). Hence (Y,σ) is countably bT^{μ} -compact.

Theorem 17. *If a map f : (X,τ) $\rightarrow$ (Y, σ) is strongly bT^{μ}- continuous map from a countably supra compact space (X,τ) onto a supra topological space (Y,σ), then (Y,σ) is countably bT^{μ} - compact.*

Proof. Let $\{A_i : i \in I\}$ be a countable bT^{μ} - open cover of (Y,σ). Since f is strongly bT^{μ} - continuous, $\left\{f^{-1}(A_i) : i = 1, 2, \cdots, n\right\}$ is an countable supra open cover of (X,τ). Again, since (X,τ) is countably supra compact, the countable supra open cover $\left\{f^{-1}(A_i) : i \in I\right\}$ of (X,τ) has a finite sub cover say $\left\{f^{-1}(A_i) : i = 1, 2, \cdots, n\right\}$. Therefore X $= \bigcup_{i=1}^{n} \left\{f^{-1}(A_i)\right\}$, which implies f(X) $= \bigcup_{i=1}^{n} A_i$, so that Y$=\bigcup_{i=1}^{n} A_i$. That is $\{A_1, A_2, \cdots, A_n\}$ is a finite sub cover of $\{A_i : i \in I\}$ for (Y,σ). Hence (Y,σ) is countably bT^{μ} -compact.

Theorem 18. *The image of a countably bT^{μ} - compact space under a bT^{μ}- irresolute map is countably bT^{μ}- compact.*

Proof. If a map f : (X,τ) $\rightarrow$ (Y, σ) is bT^{μ} - irresolute map from a countably bT^{μ} - compact space (X,τ) onto a supra topological space (Y,σ). Let $\{A_i : i \in I\}$ be a countable bT^{μ} - open cover of (Y,σ).Then $\left\{f^{-1}(A_i) : i = 1, 2, \cdots, n\right\}$ is an countable bT^{μ} - open cover of (X,τ), since f is bT^{μ} - irresolute. As (X,τ) is countably bT^{μ} - compact, the countable bT^{μ}- open cover $\left\{f^{-1}(A_i) : i \in I\right\}$ of (X,τ) has a finite sub cover say $\left\{f^{-1}(A_i) : i = 1, 2, \cdots, n\right\}$. Therefore X $= \bigcup_{i=1}^{n} \left\{f^{-1}(A_i)\right\}$, which implies f(X) $= \bigcup_{i=1}^{n} \{(A_i)\}$, so that Y $= \bigcup_{i=1}^{n} \{(A_i)\}$. That is $\{A_1, A_2, \cdots, A_n\}$ is a finite sub cover of $\{A_i : i \in I\}$ for (Y,σ). Hence (Y,σ) is countably bT^{μ} -compact.

5. supra bT-Lindelof space

In this section, we concentrate on the concept of bT^{μ}- Lindelof space and their properties.

Definition 16. *A supra topological space (X,τ) is said to be bT^{μ} - Lindelof space if every bT^{μ} - open cover of X has a countable subcover.*

Theorem 19. *Every bT^{μ} - Lindelof space is supra Lindelof space.*

Proof. Let $\{A_i : i \in I\}$ be a supra open cover of (X,τ). By [4], $\{A_i : i \in I\}$ is a bT^{μ} - open cover of (X,τ). Since (X,τ) is bT^{μ} - Lindelof space, bT^{μ}- open cover $\{A_i : i \in I\}$ of (X,τ) has a countable subcover say $\{A_i : i = 1, 2, \cdots, n\}$ for X. Hence (X,τ) is a supra Lindelof space.

Theorem 20. *If (X,τ) is supra Lindelof space and $_{bT}T_c^{\mu}$ -space, then (X,τ) is bT^{μ} - Lindelof space.*

Proof. Let $\{A_i : i \in I\}$ be a bT^{μ} - open cover of (X,τ). Since by $_{bT}T_c^{\mu}$-space,$\{A_i : i \in I\}$ is a supra open cover of (X,τ). Since (X,τ) is compact, supra open cover $\{A_i : i \in I\}$ of (X,τ) has a countable sub cover say $\{A_i : i = 1, 2, \cdots, n\}$ for X. Hence (X,τ) is a bT^{μ} - Lindelof space.

Theorem 21. *Every bT^{μ} - compact space is bT^{μ} - Lindelof space.*

Proof. Let $\{A_i : i \in I\}$ be a bT^{μ} - open cover of (X,τ). Then $\{A_i : i \in I\}$ has a finite subcover say $\{A_i : i = 1, 2, \cdots, n\}$. Since (X,$\tau$) is bT^{μ} - compact space. Since every finite subcover is always countable subcover and therefore $\{A_i : i = 1, 2, \cdots, n\}$ is countable subcover of $\{A_i : i \in I\}$. Hence (X,τ) is bT^{μ} - Lindelof space.

Theorem 22. *A bT^{μ} - continuous image of a bT^{μ} - Lindelof space is supra Lindelof space.*

Proof. Let f : (X,τ)$\rightarrow$ (Y,σ)be a bT^{μ} - continuous map from a bT^{μ} - Lindelof space X onto a supra topological space Y. Let $\{A_i : i \in I\}$ be a supra open cover of Y. Then $\left\{f^{-1}(A_i) : i \in I\right\}$ is a bT^{μ} - open cover of X, as f is bT^{μ} - continuous. Since X is bT^{μ} - Lindelof space, the bT^{μ} - open cover $\left\{f^{-1}(A_i) : i \in I\right\}$ of X has a countable sub cover say $\left\{f^{-1}(A_i) : i = 1, 2, \cdots, n\right\}$. Therefore X $= \bigcup_{i=1}^{n} \left\{f^{-1}(A_i)\right\}$, which implies f(X) $= \bigcup_{i=1}^{n} A_i$, then Y $= \bigcup_{i=1}^{n} A_i$. That is $\{A_1, A_2, \cdots, A_n\}$ is a countable sub cover of $\{A_i : i \in I\}$ for Y. Hence Y is supra Lindelof space.

Theorem 23. *The image of a bT^{μ} - Lindelof space under a bT^{μ} - irresolute map is bT^{μ} - Lindelof space.*

Proof. If a map f : (X,τ) $\rightarrow (Y,\sigma)$ is bT^{μ} - irresolute map from a bT^{μ} - Lindelof space (X,τ) onto a supra topological space (Y,σ). Let $\{A_i : i \in I\}$ be a bT^{μ} - open cover of (Y,σ).Then $\{f^{-1}(A_i) : i \in I\}$ is an bT^{μ} - open cover of (X,τ). Since f is bT^{μ} - irresolute. As (X,τ) is bT^{μ} - Lindelof space, the bT^{μ} - open cover $\{f^{-1}(A_i) : i \in I\}$ of (X,τ) has a countable sub cover say $\{f^{-1}(A_i) : i = 1, 2, \cdots, n\}$. Therefore X = $\bigcup_{i=1}^{n} \{f^{-1}(A_i)\}$, which implies f(X) = $\bigcup_{i=1}^{n} A_i$, so that Y=$\bigcup_{i=1}^{n} A_i$. That is $\{A_1, A_2, \cdots, A_n\}$ is a countable sub cover of $\{A_i : i \in I\}$ for (Y,σ). Hence (Y,σ) is bT^{μ} - Lindelof space.

Theorem 24. *If (X,τ) is bT^{μ} - Lindelof space and countably bT^{μ} - compact space then (X,τ) is bT^{μ} - compact space.*

Proof. Suppose (X,τ) is bT^{μ} - Lindelof space and countably bT^{μ} - compact space. Let $\{A_i : i \in I\}$ be a bT^{μ} - open cover of (X,τ). Since (X,τ) is bT^{μ} - Lindelof space, $\{A_i : i \in I\}$ has a countable subcover say $\{A_{in} : i \in I, n \in N\}$, therefore $\{A_{in} : i \in I, n \in N\}$ is a countable subcover of (X,τ) and $\{A_{in} : i \in I, n \in N\}$ is subfamily of $\{A_i : i \in I\}$ and so $\{A_{in} : i \in I, n \in N\}$ is a countable bT^{μ} - open cover of (X,τ). Again, since (X,τ) is countably bT^{μ} - compact, $\{A_{in} : i \in I, n \in N\}$ has a finite subcover and $\{A_{ik} : i \in I, k = 1, 2n\}$. Therefore $\{A_{ik} : i \in I, k = 1, 2, \cdots, n\}$ is a finite subcover of $\{A_i : i \in I\}$ for (X,τ). Hence (X,τ) is bT^{μ} - compact space.

Theorem 25. *If a function f : (X,τ) $\rightarrow (Y,\sigma)$ is bT^{μ} - irresolute and a subset of X is bT^{μ} - Lindelof relative to X, then f(B) is bT^{μ} - Lindelof relative to Y.*

Proof. Let $\{A_i : i \in I\}$ be a cover of f(B) by bT^{μ} -open subsets of Y. Then $\{f^{-1}(A_i) : i \in I\}$ is a cover of B by bT^{μ} -open subsets of X. Since B is bT^{μ} -Lindelof relative to X, $\{f^{-1}(A_i) : i \in I\}$ has a countable subcover say $\{f^{-1}(A_1), f^{-1}(A_2), \cdots, f^{-1}(A_n)\}$ for B. Now $\{A_1, A_2, \cdots, A_n\}$ is a countable subcover of $\{A_i : i \in I\}$ for f(B). So f(B) is bT^{μ} -Lindelof relative to Y.

6. supra bT-Connectedness in Supra Topological space

Definition 17. *A supra topological space (X,μ) is said to be bT^{μ} - Connected if X cannot be written as a disjoint union of two non empty bT^{μ} -open sets. A subsets of (X,μ) is bT^{μ} -connected if it is bT^{μ} -connected as a subspace.*

Theorem 26. *Every bT^{μ} -connected space is supra connected.*

Proof. Let A and B are supra open sets in X. Since every supra open sets is bT^{μ} -open set. Therefore A and B are bT^{μ} -open and X is bT^{μ} - connected space. Therefore $X \neq A \cup B$. Therefore X is supra connected.

Example 1. *Let X=$\{a,b,c\}$ and τ = $\{X,\phi,\{a\}\}$. Then it is bT^{μ} -connected.*

Remark 1. *The converse of the above theorem need not be true in general, which follows from the following example.*

Example 2. *Let X=$\{a,b,c\}$ and τ = $\{X,\phi,\{a\},\{b\},\{a,b\}\}$.Clearly (X,$\tau$) is supra connected . The bT^{μ} - open sets of X are $\{X,\phi,\{b,c\},\{a,c\},\{a,b\},\{b\},\{a\}\}$. Therefore (X,$\tau$) is not a bT^{μ} -connected space, since $X = \{b,c\} \cup \{a\}$ where $\{b,c\}$ and $\{a\}$ are non empty bT^{μ} -open sets.*

Theorem 27. *For a supra topological space (X,τ) the following are equivalent*

(i) *(X,τ) is bT^{μ} -connected.*

(ii) *The only subset of (X,τ) which are both bT^{μ} - open and bT^{μ} -closed are the empty set X and ϕ.*

(iii) *Each bT^{μ} -continuous map of (X,τ) into a discrete space (Y,σ) with atleast two points is a constant map.*

Proof. (1)⇒(2) Let G be a bT^{μ}-open and bT^{μ}- closed subset of (X,τ).Then X-G is also both bT^{μ}-open and bT^{μ} -closed. Then X = G∪(X-G) a disjoint union of two non empty bT^{μ} -open sets which contradicts the fact that (X,τ) is bT^{μ}-connected. Hence G = ϕ (or) X.
(2)⇒(1)Suppose that X = A∪ B where A and B are disjoint non empty bT^{μ}-open subsets of (X,τ). Since A = X-B, then A is both bT^{μ} -open and bT^{μ}- closed. By assumption A =ϕ or X, which is a contradiction. Hence (X,τ) is bT^{μ}-connected.
(2)⇒(3) Let f : (X,τ) → (Y,σ) be a bT^{μ}-continuous map, where (Y,σ) is discrete space with atleast two points. Then $f^{-1}(y)$ is bT^{μ} -closed and bT^{μ} -open for each y∈ Y. That is (X,τ) is covered by bT^{μ}-closed and bT^{μ}-open covering $\left\{f^{-1}\{y\} : y \in Y\right\}$.By assumption,$\left\{f^{-1}\{y\}\right\} =$ ϕ or X for each y∈ Y. If $f^{-1}\{y\} = \phi$ for each y ∈ Y, then f fails to be a map. Therefore their exist atleast one point say $f^{-1}\{y_1\} \neq \phi, y_1 \in$ Y such that $f^{-1}(\{y_1\})$ = X. This shows that f is a constant map.
(3)⇒(2) Let G be both bT^{μ} -open and bT^{μ} -closed in (X,τ). Suppose G≠ ϕ. Let f : (X,τ) → (Y,σ) be a bT^{μ} -continuous map defined by f(G) = $\{a\}$ and f(X-G) = $\{b\}$ where a≠ b and a,b∈ Y. By assumption , f is constant so G = X.

Theorem 28. *Let f : (X,τ) → (Y,σ) be a bT^{μ} -continuous surjection and (X,τ) is bT^{μ} -connected, then (Y,σ) is supra connected .*

Proof. Suppose (Y,σ) is not supra connected. Let Y = A∪ B, where A and B are disjoint non empty supra open subsets in (Y,σ). Since f is bT^{μ} -continuous, X = $f^{-1}(A)\bigcup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non empty bT^{μ} -open subsets in (X,τ). This disproves the fact that (X,τ) is bT^{μ} -connected. Hence (Y,σ) is supra connected.

Theorem 29. *If f : (X,τ) $\rightarrow$ (Y, σ) is a bT^{μ}-irresolute surjection and X is bT^{μ}-connected, then Y is bT^{μ}-connected.*

Proof. Suppose that Y is bT^{μ}-connected. Let Y = A∪B, where A and B are non empty bT^{μ}-open set in Y. Since f is bT^{μ} -irresolute and onto, X = $f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non empty bT^{μ}-open sets in (X,τ). This contradicts the fact that (X,τ) is bT^{μ}- connected. Hence (Y,σ) is bT^{μ} -connected.

Theorem 30. *Suppose that X is a $_{bT}T_c^{\mu}$ -space and X is supra connected then bT^{μ} -connected.*

Proof. Suppose that X is supra connected. Then X cannot be expressed as disjoint union of two non empty proper subset of X. Suppose X is not bT^{μ} -connected space. Let A and B be any two bT^{μ}-open subsets of X such that X = A∪ B, where A$\cap B = \phi$ and A⊂X, B⊂ X. Since X is $_{bT}T_c^{\mu}$ -space and A, B are bT^{μ} -open. A,B are open subsets of X, which contradicts that X is supra connected. Therefore X is bT^{μ} -connected.

Theorem 31. *If the bT^{μ} -open sets C and D form a separation of X and if Y is bT^{μ}-connected subspace of X, then Y lies entirely within C or D.*

Proof. Since C and D are both bT^{μ} -open in X. The set C∩Y and D∩Y are bT^{μ} -open in Y, these two sets are disjoint and their union is Y. If they were both non empty, they would constitute a separation of Y. Therefore, one of them is empty. Hence Y must lie entirely in C or D.

Theorem 32. *Let A be a bT^{μ}-connected subspace of X. If $A \subset B \subset bT^{\mu}cl(A)$,then B is also bT^{μ}-connected.*

Proof. Let A be bT^{μ} -connected.Let $A \subset B \subset bT^{\mu}cl(A)$. Suppose that B = $C \cup D$ is a separation of B by bT^{μ} -open sets.Thus by previous theorem above A must lie entirely in C or D.Suppose that A⊂ C,then $bT^{\mu}cl(A) \subseteq bT^{\mu}cl(C)$.Since $bT^{\mu}cl(C)$ and D are disjoint, B cannot intersect D.This disproves the fact that D is non empty subset of B.So D=ϕwhich implies B is bT^{μ}-connected.

7. References

1 D.Andrijevic, On b- open sets, *Mat. Vesnik*, 48(1996), no.1-2,59-64.

2 R.Devi, S.Sampathkumar and M.Caldas, On supra α open sets and S-continuous maps, *General Mathematics*, 16(2), (2008), 77-84.

3 Jamal M.Mustafa, Supra b-compact and supra b-Lindelof spaces, *J.Mat.App.* 36,(2013),79-83.

4 K.Krishnaveni and M.Vigneshwaran,On bT^{μ} -Closed sets in supra topological Spaces, *Int.J. Mat.Arc.*, 4(2),(2013),1-6.

5 K.Krishnaveni and M.Vigneshwaran,Some Stronger forms of supra bT^{μ} - continuous function, *Int.J.Mat. Stat.Inv.*,1(2),(2013), 84-87.

6 A.S.Mashhour,A.A.Allam,F.S.Mohamoud and F.H.Khedr, On supra topological spaces, *Indian J.Pure and Appl.Math.*, No.4,14(1983), 502- 510.

7 P.G.Patil,w -compactness and w-connectedness in topological spaces, *Thai. J. Mat.*, (12),(2014),499-507.

8 O.R. Sayed and Takashi Noiri, On b-open sets and supra b-Continuity on topological spaces, *Eur.J.Pure and App. Mat.*,3(2)(2010), 295-302.