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Contra δgb-Continuous Functions in Topological Spaces

S.S.Benchalli[1], P.G.Patil [2,*], J.B.Toranagatti[3], S.R.Vighneshi[4]

[1,2] *Department of Mathematics, Karnatak University, Dharwad, India*
[3] *Department of Mathematics, Karnatak University's Karnatak College, Dharwad, India*
[4] *Department of Mathematics, R.L.S College, Dharwad, India*

Abstract. In this paper,the notion of δgb-open sets in topological spaces is applied to study a new class of functions called contra δgb - continuous functions as a new generalization of contra continuity and obtain their characterizations and properties.

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Key Words and Phrases: δgb-open, δgb-closed, δgb-connected, contra δgb-continuous, δgb-continuous.

1. Introduction and Preliminaries

In 1996, Dontchev[6] introduced contra continuous functions. Nasef [10] introduced and studied contra b-continuous functions.Al-Omari and Noorani[1] introduced the concept of contra gb-continuous functions. Recently Benchalli et.al.[5] introduced and studied δgb-continuous functions.These concepts motivated us to define a new class of functions called contra δgb-continuous functions.

Throughout this paper,(X, τ),(Y,σ) and (Z,η)(or simply X,Y and Z) represent topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, the closure of A, interior of A and complement of A are denoted by cl(A), int(A) and A^c respectively.

Definition 1. *A subset A of a topological space X is called a*

(i) pre-closed [9] *if cl(int(A))$\subseteq$A*

(ii) b-closed [2] *if cl(int(A))$\cap$int(cl(A))$\subseteq$A*

(iii) regular-closed [14] *if A=cl(int(A))*

(iv) δ-closed [17] *if $A=cl_\delta(A)$ where $cl_\delta(A)=\{x\in X$: int(cl(U))$\cap A\neq\phi$, $U\in\tau$ and $x\in U\}$*

*Corresponding author.

Email addresses: benchalliss@gmail.com(S.S.Benchalli),pgpatil01@gmail.com(P.G.Patil), jagadeeshbt2000@gmail.com(J.B.Toranagatti),vighneshisr@gmail.com (S.R.Vighneshi)

(v) delta generalized b-closed (briefly,δgb-closed) [4] *if bcl(A)$\subseteq$G whenever A$\subseteq$G and G is δ-open in X.*

The complements of the above mentioned closed sets are their respective open sets. The b-closure of a subset A of X is the intersection of all b-closed sets containing A and is denoted by bcl(A).

Definition 2. *A function f:X$\rightarrow$Y from a topological space X into a topological space Y is called a,*

(i) contra continuous [6] *if f^{-1}(G) is closed in X for every open set G of Y.*

(ii) contra b-continuous [10] *if f^{-1}(G) is b-closed in X for every open set G of Y.*

(iii) contra rgb-continuous [13] *if f^{-1}(G) is rgb-closed in X for every open set G of Y.*

(iv) δgb-continuous [5] *if f^{-1}(G) is δgb-open in X for every open set G of Y.*

(v) completely-continuous [3] *if f^{-1}(G) is regular-open in X for every open set G of Y.*

(vi) perfectly-continuous [12] *if f^{-1}(G) is clopen in X for every open set G of Y.*

(vii) δ^*-continuous if f^{-1}(G) is δ-open in X for every open set G of Y.

(viii) contra gb-continuous [1] *if f^{-1}(G) is gb-closed in X for every open set G of Y.*

(ix) pre-closed [7]*if for every closed subset A of X f(A) is pre-closed in Y.*

Definition 3. [5] *A topological space X is said to be a,*

(i) $T_{\delta gb}$-space if every δgb-closed subset of X is closed.

(ii) $\delta gbT_{\frac{1}{2}}$-space if every δgb-closed subset of X is b-closed.

2. Contra δgb-Continuous Functions.

Definition 4. *A function f:X$\rightarrow$Y is called contra δgb-continuous if f^{-1}(V) is δgb-closed in X for each open set V of Y.*
Clearly, f:X$\rightarrow$Y is contra δgb-continuous if and only if f^{-1}(G) is δgb-open in X for every closed set G in Y.

Theorem 1. *If f:X$\rightarrow$Y is contra gb-continuous then it is contra δgb-continuous.*
Proof: Follows from the fact that every gb-closed set is δgb-closed.

Theorem 2. *If f:X$\rightarrow$Y is contra b-continuous then it is contra δgb-continuous.*
Proof:Follows from the fact that every contra b-continuous function is contra gb-continuous and Theorem 1.

Remark 1. *The converse of Theorem 1 and Theorem 2 need not be true as seen from the following example.*

Example 1. *Let X=Y={a,b,c}. Let* τ=*{X,*ϕ*,{a}} and* σ=*{X,*ϕ*,{a},{b},{a,b}} be topologies on X and Y respectively. Then the identity function f:X*$\rightarrow$*Y is contra* δ*gb-continuous but neither contra b-continuous and nor contra gb-continuous, since {a} is open in Y but* f^{-1}*({a})={a} is not gb-closed in X and hence not b-closed in X.*

Theorem 3. *If f:X*$\rightarrow$*Y is contra* δ*gb-continuous then it is contra rgb-continuous.*
Proof: Follows from the fact that every δ*gb-closed set is rgb-closed.*

Remark 2. *The converse of Theorem 3 need not be true as seen from the following example.*

Example 2. *Let X=Y={a,b,c}. Let* τ=*{X,*ϕ*,{a},{b},{a,b}} and* σ=*{X,*ϕ*,{a}} be topologies on X and Y respectively.Let f:X*$\rightarrow$*Y be a function defined by f(a)=a=f(b) and f(c)=c. Then f is contra rgb-continuous but not contra* δ*gb-continuous,since {a} is open in Y but* f^{-1}*({a})={a,b} is not* δ*gb-closed in X .*

Theorem 4. *Let f:X*$\rightarrow$*Y be a function.*

(i) If X is $T_{\delta gb}$*-space then f is contra* δ*gb-continuous if and only if it is contra continuous.*

(ii) If X is $\delta gbT_{\frac{1}{2}}$*-space then f is contra* δ*gb-continuous if and only if it is contra b-continuous.*

Proof:(i) Suppose X is $T_{\delta gb}$*-space and f is contra* δ*gb-continuous. Let G be an open set in Y. Then by hypothesis* f^{-1}*(G) is* δ*gb-closed in X and hence* f^{-1}*(G) is closed in X.Therefore f is contra continuous.*
Converse is obvious .
(ii)Suppose X is $\delta gbT_{\frac{1}{2}}$*-space and f is contra* δ*gb-continuous. Let G be an open set in Y then* f^{-1}*(G) is* δ*gb-closed in X and hence* f^{-1}*(G) is b-closed in X.Therefore f is contra b-continuous.*
Converse is follows from the Theorem 2.

Theorem 5. [5] *Let A*$\subseteq$*X.Then x* $\in$ δ*gbcl(A) if and only if U*$\cap$*A*$\neq$ Φ*, for every* δ*gb-open set U containing x.*

Lemma 1. [8] *The following properties are hold for subsets A and B of a space X :*

(i) x$\in$*ker(A) if and only if A*$\cap$*F=*ϕ *for any closed set F of X containing x.*

(ii) A$\subseteq$*ker(A) and A = ker(A) if A is open in X.*

(iii) If A$\subseteq$*B then ker(A)*$\subseteq$*ker(B).*

Theorem 6. *Suppose that* δ*GBC(X) is closed under arbitrary intersections.Then the following are equivalent for a function f:X*$\rightarrow$*Y:*

(i) f is contra δ*gb-continuous*

(ii) *For each* $x \in X$ *and each closed set B of Y containing f(x) there exists an* δgb*-open set A of X containing x such that f(A)*$\subseteq$*B*

(iii) *For each* $x \in X$ *and each open set G of Y not containing f(x) there exists an* δgb*-closed set H in X not containing x such that* $f^{-1}(G) \subseteq H$

(iv) $f(\delta gbcl(A)) \subseteq ker(f(A))$ *for every subset A of X*

(v) $\delta gbcl(f^{-1}(B)) \subseteq f^{-1}(ker(B))$ *for every subset B of Y.*

Proof:(i)→(ii) Let B be a closed set in Y containing f(x) then $x \in f^{-1}(B)$*. By (i),*$f^{-1}(B)$ *is* δgb*-open set in X containing x. Let A =*$f^{-1}(F)$ *then* $f(A)=f(f^{-1}(B)) \subseteq B$*.*
(ii)→(i) Let F be a closed set in Y containing f(x) then $x \in f^{-1}(F)$*. From (ii), there exists* δgb*-open set* G_x *in X containing x such that* $f(G_x) \subset F$ *which implies* $G_x \subseteq f^{-1}(F)$*.Thus* $f^{-1}(F)=\cup\{Ux: x \in f^{-1}(F)\}$ *which is* δgb*-open. Hence* $f^{-1}(F)$ *is* δgb*-open set in X.*
(ii)→(iii) Let G be an open set in Y not containing f(x). Then Y-G is closed set in Y containing f(x). From (ii), there exists a δgb*-open set F in X containing x such that* $f(F) \subseteq Y\text{-}G$ *. This implies* $F \subseteq f^{-1}(Y\text{-}G)=X\text{-}f^{-1}(G)$*. Hence* $f^{-1}(G) \subseteq X\text{-}F$*. Set H=X-F,then H is* δgb*-closed set not containing x in X such that* $f^{-1}(G) \subseteq H$*.*
(iii)→(ii) Let F be a closed set in Y containing f(x). Then Y-F is an open set in Y not containing f(x). From (iii), there exists δgb*-closed set K in X not containing x such that* $f^{-1}(Y\text{-}F) \subseteq K$*.This implies* $X\text{-}K \subseteq f^{-1}(F)$ *that is* $f(X\text{-}K) \subseteq F$*.Set U=X-K then U is* δgb*-open set containing x in X such that* $f(U) \subseteq F$*.*
(i)→(iv) Let A be any subset of X. Suppose $y \notin ker(f(A))$*. Then by Lemma 1, there exists a closed set F in Y containing y such that* $f(A) \cap F=\phi$*. Hence we have* $A \cap f^{-1}(F)=\phi$ *and* $\delta gb\text{-}cl(A) \cap f^{-1}(F)=\phi$ *which implies* $f(\delta gbcl(A)) \cap F=\phi$ *and hence* $y \notin \delta gbcl(A)$*. Therefore* $f(\delta gbcl(A)) \subset ker(f(A))$
(iv)→(v) Let $B \subseteq Y$ *then* $f^{-1}(B) \subseteq X$*. By (iv),* $f(\delta gbcl(f^{-1}(B))) \subseteq ker(f(f^{-1}(B))) \subseteq Ker(B)$*. Thus* $\delta gbcl(f^{-1}(B)) \subseteq f^{-1}(ker(B))$ *.*
(v)→(i) Let V be any open subset of Y. Then by (v) and Lemma 1,
$\delta gbcl(f^{-1}(V) \subseteq f^{-1}(ker(V))=f^{-1}(V)$ *and* $\delta gbcl(f^{-1}(V))=f^{-1}(V)$*.*
Therefore $f^{-1}(V)$ *is* δgb*-closed set in X*

Lemma 2. [16] *For a subset A of a space X,the following are equivalent:*

(i) *A is open and gb-closed*

(ii) *A is regular open.*

Theorem 7. [4]*If* $A \subseteq X$ *is both* δ*-open and* δgb*-closed then it is b-closed.*

Theorem 8. *If* $A \subseteq X$ *is regular open then it is b-closed.*

Lemma 3. *For a subset A of a space X the following are equivalent:*

(i) *A is* δ*-open and* δgb*-closed*

(ii) *A is regular open*

(iii) A is open and b-closed.

Proof:(i)→(ii):Let A be an δ-open and δgb-closed set.Then by Theorem 7, A is b-closed that is bcl(A)⊆A and so int(cl(A))⊆A. Since A is δ-open then A is pre-open and thus A⊆int(cl(A)).Hence A is regular open.
(ii)→(i): Follows from the fact that every regular open set is δ-open and by Theorem 8.
(ii)→(iii): Follows from the fact that every regular open set is open and Theorem 8.
(iii)→(ii): Let A be an open and b-closed set then bcl(A)⊆A and so int(cl(A))⊆A. since A is open,then A is pre-open and thus A⊆int(cl(A)),which implies A=int(cl(A)).

As a consequence of the above lemma, we have the following result:

Theorem 9. *The following statements are equivalent for a function f:X→Y:*

(i) f is completely continuous

(ii) f is contra δgb-continuous and δ^*-continuous

(iii) f is contra b-continuous and continuous.

Definition 5. [16] *A subset A of X is said to be Q-set if int(cl(A))=cl(int(A)).*

Definition 6. [16] *A function f:X→Y is Q-continuous if $f^{-1}(V)$ is Q-set in X for every open set V of Y.*

Theorem 10. *For a subset A of a space X the following are equivalent:*

(i) A is clopen

(ii) A is δ-open and δ-closed

(iii) A is regular-open and regular-closed.

Theorem 11. *For a subset A of a space X the following are equivalent:*

(i) A is clopen

(ii) A is δ-open,Q-set and δgb-closed

(iii) A is open,Q-set and b-closed.

Proof:(i)→(ii):Let A be clopen then by Theorem 10 we have A=int(cl(A))=cl(int(A)). Hence A is Q-set.Again by Theorem 10, A is δ-open and δ-closed. Since every δ-closed set is δgb-closed. Therefore (ii) holds.
(ii)→(iii): Follows from the Theorem 7.
(iii)→(i):Let A be an open,Q-set and b-closed set then by Lemma 3, A is regular open. Since A is Q-set,then A=int(cl(A))=cl(int(A)) which implies A is regular closed. Hence by Theorem 10, A is clopen.

Theorem 12. *The following statements are equivalent for a function f:X→Y:*

(i) f is perfectly continuous

(ii) f is δ^*-continuous,Q-continuous and contra δgb-continuous

(iii) f is continuous,Q-continuous and contra b-continuous.

Definition 7. *A space X is called locally δgb-indiscrete if every δgb-open set is closed in X.*

Theorem 13. *If f:X→Y is a contra δgb-continuous and X is locally δgb-indiscrete space then f is continuous.*
Proof: Let G be a closed set in Y.Since f is contra δgb-continuous and X is locally δgb-indiscrete space then f^{-1}(G)is a closed set in X. Hence f is continuous

Definition 8. [11] *A space X is called locally indiscrete if every open set is closed in X.*

Theorem 14. *If f:X→Y is a contra δgb-continuous preclosed surjection and X is $T_{\delta gb}$-space then Y is locally indiscrete .*
Proof: Let V be an open set in Y. Since f is contra δgb-continuous and X is $T_{\delta gb}$-space then f^{-1}(G)is closed in X. Also f is preclosed then V is preclosed in Y. Now we have cl(V)=cl(int(V))⊆V. This means V is closed in Y and hence Y is indiscrete.

Theorem 15. *Suppose that δGBC(X) is closed under arbitrary intersections. If f:X→Y is contra δgb-continuous and Y is regular then f is δgb-continuous.*
Proof: Let x∈X and V be an open set of Y containing f(x). Since Y is regular,there exists an open set G in Y containing f(x) such that cl(G)⊆V. Since f is contra δgb-continuous,there exists an δgb-open set U in X containing x such that f(U)⊆cl(G).Then f(U)⊆cl(G)⊆V. Hence f is δgb-continuous.

Recall that for a function f:X→Y the subset {(x,f(x)): x∈X}⊆X×Y is called the graph of f and is denoted by G(f).

Definition 9. *The graph G(f) of a function f:X→Y is said to be contra δgb-closed if for each (x,y)∈(X×Y)-G(f) there exists U∈δgbO(X,x) and V∈C(Y,y) such that (U×V)∩G(f)=ϕ.*

Theorem 16. *The graph G(f) of a function f:X→Y is contra δgb-closed in X×Y if and only for each (x,y)∈(X×Y)-G(f) there exists U∈δgbO(X,x) and V∈C(Y,y) such that f(U)∩V=ϕ.*

Theorem 17. *If f:X→Y is contra δgb-continuous and Y is Urysohn then G(f) is contra δgb-closed in the product space X×Y.*
Proof: Let (x,y)∈(X×Y)-G(f), then y≠f(x) and there exist open sets A and B such that f(x)∈A, y∈B and cl(A)∩cl(B)=ϕ. Since f is contra δgb-continuous then there exists U∈δgbO(X,x) such that f(U)⊆cl(A). Therefore we obtain f(U)∩cl(B)=ϕ.
This shows that G(f) is contra δgb-closed.

Theorem 18. *If f:X→Y is δgb-continuous and Y is T_1 then G(f) is contra δgb-closed in X× Y.*
Proof: Let (x,y)∈(X× Y)-G(f) then y≠f(x) and there exists open set U such that f(x)∈U and y∉U. Since f is δgb-continuous, then there exists V∈δgbO(X,x) such that f(V)⊆U.Therefore we obtain f(V)∩(Y-U)=ϕ and Y-U∈C(Y,y).This shows that G(f) is contra δgb-closed.

Theorem 19. *Let f:X→Y be a function and g:X→X× Y be the graph function of f defined by g(x)=(x,f(x)) for each x∈X. If g is contra δgb-continuous then f is contra δgb-continuous.*
Proof: Let U be an open set in Y then X× U is an open set in X× Y. Since g is contra δgb-continuous. It follows that f^{-1}(U)=g^{-1}(X× U) is δgb-closed in X. Thus f is contra δgb-continuous.

Theorem 20. *If f:X→Y is contra δgb-continuous then for each x∈X and for each closed set V in Y with f(x)∈V there exists a δgb-open set U in X containing x such that f(U)⊆V.*
Proof: Let x∈X and V is a closed set in Y with f(x)∈V then x∈f^{-1}(V). Since f is contra δgb-continuous,f^{-1}(V) is δgb-open in X. Put U=f^{-1}(V) then x∈U and f(U)=f(f^{-1}(V))⊆V.

Definition 10. [16] *A space X is submaximal and extremally disconnected if every b-open set is open.*

Theorem 21. *If A and B are δgb-closed sets in submaximal and extremally disconnected space X then A∪B is δgb-closed in X.*
Proof: Let A∪B⊆G where G is δ-open in X. Since A⊆G,B⊆G, A and B are δgb-closed sets then bcl(A)⊆G and bcl(B)⊆G. As X is submaximal and extremally disconnected,bcl(M)=cl(M) for any M⊆X.Therefore bcl(A∪B)=bcl(A)∪bcl(B)⊆G and hence A∪B is δgb-closed.

Corollary 1. *If A and B are δgb-open sets in submaximal and extremally disconnected space X then A∩B is δgb-open in X.*

Theorem 22. *:Suppose that δGBC(X) is closed under arbitrary intersections then A⊆X is δgb-closed if and only if A=δgbcl(A)*

Theorem 23. *Suppose that δGBC(X) is closed under arbitrary intersections. If f:X→Y and g:X→Y are contra δgb-continuous ,Y is Urysohn and X is submaximal and extremally disconnected, then K={x∈X:f(x)=g(x)} δ is gb-closed in X.*
Proof: Let x∈X-K.Then f(x)≠g(x).Since Y is Urysohn there exist open sets U and V such that f(x)∈U,g(x)∈V and cl(U)∩cl(V)=ϕ. Since f and g are contra δgb-continuous,f^{-1}(cl(U)) and g^{-1}(cl(V)) are δgb-open sets in X. Let A=f^{-1}(cl(U)) and B=g^{-1}(cl(V)). Then A and B are δgb-open sets containing x.Set C=A∩B, then C is δgb-open set in X. Hence f(C)∩g(C)=f(A∩B)∩g(A∩B)⊆f(A)∩g(B)=cl(U)∩cl(V)=ϕ. Therefore C∩K=ϕ.
By Theorem 5, x∉δgbcl(K). Hence K is δgb-closed in X.

Definition 11. *A space X is called δgb-connected provided that X is not the union of two disjoint nonempty δgb-open sets.*

Theorem 24. *If f:X→Y is a contra δgb-continuous function from a δgb- connected space X onto any space Y then Y is not a discrete space.*

Proof: *Since f is contra δgb-continuous and X is δgb-connected space. Suppose Y is a discrete space. Let V be a proper non empty open and closed subset of Y. Then $f^{-1}(V)$ is proper nonempty δgb-open and δgb-closed subset of X, which contradicts the fact that X is δgb-connected space. Hence Y is not a discrete space.*

Theorem 25. *If f:X→Y is a contra δgb-continuous surjection and X is δgb-connected space then Y is connected.*
Proof: *Let f:X→Y is a contra δgb-continuous and X is δgb-connected space. Suppose Y is not connected. Then there exist disjoint open sets U and V in Y such that Y=U∪V. Therefore U and V are clopen in Y. Since f is contra δgb-continuous $f^{-1}(U)$ and $f^{-1}(V)$ are δgb-open sets in X. Further f is surjective implies, $f^{-1}(U)$ and $f^{-1}(V)$ are non empty disjoint and X $=f^{-1}(U)\cup f^{-1}(V)$. This contradicts the fact that X is δgb-connected space. Therefore Y is connected.*

Theorem 26. *Let X be a δgb-connected and Y be T_1-space. If f:X→Y is contra δgb-continuous then f is constant.*
Proof: *Since Y is T_1-space, U=$\{f^{-1}(y)$: y∈Y$\}$ is a disjoint δgb-open partition of X. If $|U|\geq 2$ then X is the union of two nonempty δgb-open sets.This contradicts the fact that X is δgb-connected. Therefore $|U|=1$ and hence f is constant.*

Definition 12. [4] *A topological space X is said to be δgb-T_2 space if for any pair of distinct points x and y there exist disjoint δgb-open sets G and H such that x∈G and y∈H.*

Theorem 27. *Let X and Y be topological spaces. If*

(i) for each pair of distinct points x and y in X there exists a function f:X→Y such that f(x)≠f(y),

(ii) Y is Urysohn space and

(iii) f is contra δgb-continuous at x and y. Then X is δgb-T_2 .

Proof: *Let x and y be any distinct points in X and f is a function such that f(x)≠f(y). Let a=f(x) and b=f(y) then a≠b. Since Y is an Urysohn space there exist open sets V and W in Y containing a and b respectively such that cl(V)∩cl(W)=ϕ. Since f is contra δgb-continuous at x and y then there exist δgb-open sets A and B in X containing x and y respectively such that f(A)⊆cl(V) and f(B)⊆cl(W). We have $A\cap B\subseteq f^{-1}(cl(V))\cap f^{-1}(cl(W))=f^{-1}(\phi)=\phi$. Hence X is δgb-T_2.*

Corollary 2. *Let f:X→Y be a contra δgb-continuous injective function from a space X into Urysohn space Y then X is δgb-T_2.*

Definition 13. [14] *A topological space X is called Ultra Hausdorff space if for every pair of distinct points x and y in X there exist disjoint clopen sets U and V in X containing x and y respectively.*

Theorem 28. *If f:X→Y be contra δgb-continuous injective function from space X into a Ultra Hausdorff space Y then X is δgb-T_2 .*
Proof: Let x and y be any two distinct points in X . Since f is injective f(x)≠f(y) and Y is Ultra Hausdorff space implies there exist disjoint clopen sets U and V of Y containing f(x) and f(y) respectively. Then x∈f^{-1}(U) and y∈f^{-1}(V) where f^{-1}(U) and f^{-1}(V) are disjoint δgb-open sets in X . Therefore X is δgb-T_2 .

Definition 14. [14] *A space X is called Ultra normal space if each pair of disjoint closed sets can be separated by disjoint clopen sets.*

Definition 15. [4] *A topological space X is said to be δgb-normal if each pair of disjoint closed sets can be separated by disjoint δgb-open sets.*

Theorem 29. *If f:X→Y be contra δgb-continuous closed injection and Y is ultra normal then X is δgb-normal.*
Proof: Let E and F be disjoint closed subsets of X. Since f is closed and injective f(E) and f(F) are disjoint closed sets in Y. Since Y is ultra normal there exists disjoint clopen sets U and V in Y such that f(E)⊆U and f(F)⊆V. This implies E⊆f^{-1}(U) and F⊆f^{-1}(V). Since f is contra δgb-continuous injection,f^{-1}(U) and f^{-1}(V) are disjoint δgb-open sets in X . This shows X is δgb-normal.

Remark 3. *The composition of two contra-δgb-continuous functions need not be contra-δgb-continuous as seen from the following example.*

Example 3. *Let X=Y=Z={a,b,c}, τ={X,ϕ,{a},{b},{a,b}},σ={Y,ϕ,{a}} and η={Z,ϕ,{b,c}} be topologies on X,Y and Z respectively.Then the identity function f:X→Y and a function g:Y→Z defined by g(a)=b,g(b)=c and g(c)=a are contra δgb-continuous but g∘f:X→Z is not contra δgb-continuous, since there exists a open set {b,c} in Z such that $(g\circ f)^{-1}$\{b,c\}={a,b} is not δgb-closed in X.*

Theorem 30. *Let f:X→Y and g:Y→Z be any two functions.*

(i) If f is contra δgb-continuous and g is continuous then g∘f is contra δgb-continuous.

(ii) If f is contra δgb-continuous and g is contra continuous then g∘f is δgb-continuous.

(iii) If f is δgb-continuous and g is contra continuous then g∘f is contra δgb-continuous.

(iv) If f is δgb-irresolute and g is contra δgb-continuous then g∘f is contra δgb-continuous.

Proof:(i) Let h=g∘f and V be an open set in Z. Since g is continuous, g^{-1}(V) is open in Y. Therefore f^{-1}[g^{-1}(V)]=h^{-1}(V) is δgb-closed in X because f is contra δgb-continuous. Hence g∘f is contra δgb-continuous.
The proofs of (ii),(iii) and (iv) are similar to (i).

Theorem 31. *Let f:X→Y be contra δgb-continuous and g:Y→Z be δgb-continuous. If Y is $T_{\delta gb}$-space, then g∘f:X→Z is contra δgb- continuous.*
Proof: Let V be any open set in Z . Since g is δgb-continuous g^{-1}(V) is δgb-open in Y and since Y is $T_{\delta gb}$-space, g^{-1}(V) open in Y. Since f is contra δgb-continuous, then $f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ is δgb-closed set in X. Therefore g∘f is contra δgb-continuous.

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References

[1] A.Al-Omari and M.S.Noorani, *Decomposition of continuity via b-open set*, Bol.Soc.Paran.Mat., 26(2008),53-64.

[2] D. Andrijivic, *On b-open sets, Mat.Vesnic*, 48(1996), 59-64.

[3] S.P.Arya and R.Gupta, *On strongly continuous mappings, Kyungpook Math.*, 14(1974),131-143.

[4] S.S.Benchalli,P.G.Patil,J,B.Toranagatti and S.R.Vighneshi, *δgb-separation axioms in topological spaces*, International Mathematical Forum, 23(2016),1117-1131.

[5] S.S.Benchalli,P.G.Patil and J.B.Toranagatti, *Delta generalized b-continuous functions in topological spaces*, International Journal of Scientific and Innovative Mathematical Research, 3(2015), 440-446.

[6] J.Dontchev, *Contra continuous functions and strongly S-closed mappings*, Int.J.Math.Sci.,19(1996),303-310.

[7] N.El-Deeb,I.A.Hasanein,A.S.Mashhour and T.Noiri, *On p-regular spaces*, Bull. Math. Soc.Sci.Math.R.S.Roumanie, 27(1983),311-315.

[8] S.Jafari and T.Noiri, *Contra-super-continuous functions.* Ann.Univ.Sci. Budapest. Eotvos Sect.Math.,42(1999),27-34.

[9] A.S.Mashhour,M.E.Abd El-Monsef and S. N. EL-Deeb,*On pre-continuous and weak pre continuous mappings*, Proc. Math and Phys.Soc. Egypt,53(1982),47-53.

[10] A.A.Nasef, *Some properties of contra-γ-continuous functions*, Chaos Solitons and Fractals, 24(2005),471-477.

[11] T.Nieminen, *On ultrapseudocompact and related spaces*, Ann.Acad.Sci.Fenn.Ser.A.I.Math.,3(1977),185-205.

[12] T.Noiri, *Super-continuity and some strong forms of continuity*, Indian J.Pure Appl.Math.,15(1984),241-250.

[13] S.Sekar and K.Mariappa, *Almost contra regular generalized b-continuous functions*,International Journal of Pure and Applied Mathematics, 97(2014),161-176.

[14] R.Staum, *The algebra of bounded continuous functions into a non-archimedean field*, Pacific J.Math.,50(1974),169-185.

[15] M.Stone,*Application of the theory of Boolean rings to general topology*, Trans.Amer.Math.Soc.,41(1937),371-381.

[16] K.V.Tamilselvi, P.Thangaraj and O.Ravi, *On contra $g\gamma$-continuous functions*, Recent and Innovation Trends in Computing and Communication,4(2016),260-269.

[17] N.V.Veliko, *H-closed topological spaces*,Amer.Math.Soc.Transl.,78(1968),103-118.