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Project Duration Performance Measurement By Fuzzy Approach Under Uncertainty

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Abstract. In recent years, a various of novel techniques for measuring project performance have been proposed. The Earned Duration Management (EDM) is the most recent technique which uses time-based data exclusively. In this paper, as the uncertainty is inherent in real-life activities, linguistic terms are used to describe progress of activities rather to evaluate it deterministically and a fuzzy approach is applied on EDM methodology. The proposed approach derived development of new fuzzy indices which are capable of measuring project duration performance under uncertainty. A small example illustrates how the proposed model can be employed in reality.

2010 Mathematics Subject Classifications: 03E72, 68T37,90B50, 62F07

Key Words and Phrases: Earned Duration Management, Project Performance Measurement, The fuzzy approach, Uncertain Condition

1. Introduction

Project management is the application of knowledge, skills, tools and techniques to project activities to meet project requirements.

The earned value management is a project management technique used to measure the project's schedule and cost performance within a single integrated methodology [15].

This technique assists managers in estimating the final cost and time of the projects and is a routine project control technique that has been applied successfully for managing various types of projects since 1960 [8] [10] [12][16]. Also, EVM has been known to follow-up both time and cost, the majority of the research has been focused on the cost aspect [6][19].

The EVM schedule indicators are, contrary to expectation, reported in units of cost rather than time. Also, Because EVM schedule indicators are expressed in units of cost; comparison with the time-based network schedule indicators is very difficult. The much more serious issue whereby the EVM schedule indicators always return to unity at project completion. The EV always equals the final PV, the BAC. Therefore, the SV always

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returns to zero and SPI always returns to one irrespective of duration based project delay. The schedule indicators also fail for projects which continue to execute beyond the planned completion date[7].so Fleming and Koppelman[6] Suggest that schedule Performance Index should be apply just as a warning mechanism and not as a real tool to analyze how the project is performing with regard to schedule.

Walt Lipke [11] in order to eliminate the shortcomings of EVM introduced the concept of Earned Schedule (ES) technique which leads to computing change of EVM schedule indicators. Also ES and presented index known as SPI(t) is a better schedule performance measure compared to SPI, the use of cost data in their calculation causes the obtained information to not always be reliable [19]. Khamooshi and Golafshani[9] published a new approach known as Earned Duration Management (EDM) for project schedule Performance Management which eliminates the use of cost data in the schedule context.

Its foundation lies in the exclusive usage of time-based data for the generation of Physical Progress indicators [19].

EDM, EVM and ES techniques activities are considered deterministic, however nature of some activities are uncertain, mostly because the data regarding the activities come from people judgments which they carry some degree of uncertainty. For this reason, interpreting and calculating this uncertainty would cause better performance measurement and extend EDM applicability in real-life and uncertain conditions.

In this regard, there are studies which paid attention to uncertainty in project management. Naeni et al [13] developed a new fuzzy-based EVM technique to measure and evaluate the performance and the progress of a project and its activities under uncertainty. Dehabadi et al [4], in order to deal with the vagueness and impreciseness of real data in project, proposed a theoretical framework to estimate future performance of project regarding the past relative information which benefits from fuzzy regression (FR) models. Ponz-tienda et al [14], considering duration, cost and production, and alternatives in the scheduling between the earliest and latest times, present a proposal for project scheduling and control by applying fuzzy earned values and their findings suggest that: "different possible schedules and the fuzzy arithmetic provide more objective results in uncertain environments than the traditional methodology."

In this paper, project duration performance using Earned Duration Management is measured by fuzzy approach under uncertainty.

2. The Earned Duration Management

The Earned Duration Management, in contrast to Earned Value and Earned Schedule, decoupled schedule and cost performance measures and developed a number of indices to measure progress and performance of schedule and cost.

This technique uses time-based data exclusively and decouples duration and cost for the purpose of performance management and measurement, so it is the counterpart or complement of EVM and takes care of duration and schedule management of any project [9].

Batselier and Vanhoucke [1] concluded that EDM(t) as proposed by Khamooshi and

Golafshani[9] certainly proves to be a valid methodology for forecasting project duration, as it can compete with - and potentially improve - the currently most recommended methodology of ESM.

3. Utilization of fuzzy set theory

Fuzzy sets theory first introduced by Zadeh [20] applied to many areas which need to manage uncertain and vagueness in reality, so using linguistic term with the fuzzy theory can model and treat the uncertainty in such areas. With the aim of reaching this goal, fuzzy theory employs the different types of numbers with certain membership function [18].

In general, the related membership function of a trapezoidal fuzzy number, for instance $\tilde{A} = [a, b, c, d]$ is defined as below:

$$\mu_{\tilde{A}} x = \begin{cases} 0 & x \leq a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & b \leq x \leq c \\ \frac{x-c}{d-c} & c \leq x \leq d \\ 0 & d \leq x \end{cases} \tag{1}$$

The trapezoidal fuzzy number can be changed into a triangular fuzzy number if $a_2=a_3$. In this paper due to easiness in calculation, the fuzzy variables are chosen as trapezoidal and triangular fuzzy numbers are also represented as a trapezoidal fuzzy number [a, b, b, d] or [a, c, c, d] .

The basic operations of these two fuzzy numbers are presented as follows [20].

Assume $r \geq 0$ is a real number and $\tilde{A}$ and $\tilde{B}$ are two trapezoidal fuzzy numbers with four numbers:

$$\tilde{A} + \tilde{B} = (a_1 + b_1 \ , a_2 + b_2, \ a_3 + b_3, a_4 + b_4 \ )$$
$$\tilde{A} - \tilde{B} = (a_1 - b_1 \ , a_2 - b_2, \ a_3 - b_3, a_4 - b_4 \ )$$
$$\tilde{A} \times \tilde{B} = (a_1 \times b_1 \ , a_2 \times b_2, \ a_3 \times b_3, a_4 \times b_4 \ )$$
$$\tilde{A} \div \tilde{B} = (a_1/b_4, a_2/b_3, a_3/b_2, a_4/b_1 \ )$$
$$\tilde{A} \times r = (a_1 \times r \ , a_2 \times r, \ a_3 \times r \ , a_4 \times r \ )$$

The proposed method, apply where the amount of the work required to perform the activities are unknown or uncertain, and is out of control. For instance, in Road construction projects, in many cases, the exact amount of excavation is unknown and out of control.

Assume that an activity progress can't be stated deterministic.in this regard, Linguistic terms can help and it may be stated as "Low", "Less than half", etc. linguistic terms, make it easier to figure out the activity progress by answering question "what fraction of the activity is completed?". It is obvious that this linguistic term first should transformed to a fuzzy number t then applied on the EDM technique. For this purpose, we assign a membership function to these linguistic terms.

Fig.1 and Table.1 show this process.

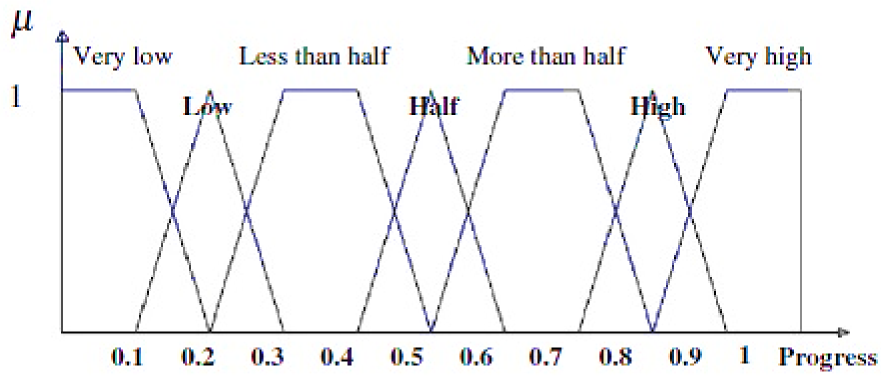


Figure 1: fuzzy membership and their linguistic terms

Table 1: The relationship between a fuzzy membership and linguistic terms

Linguistic Term	Fuzzy number
Very low	[0, 0, 0.1, 0.2]
Low	[0.1, 0.2, 0.2, 0.3]
Less than half	[0.2, 0.3, 0.4, 0.5]
Half	[0.4, 0.5, 0.5, 0.6]
More than Half	[0.5, 0.6, 0.7, 0.8]
High	[0.7, 0.8, 0.8, 0.9]
Very high	[0.8, 0.9, 1, 1]

For example, the linguistic term "Less than half" equals to the fuzzy number [0.2, 0.3, 0.4, 0.5].

4. Fuzzy Performance Indices

According to Fig.1 and Table.1, activity progress can express in linguistic terms in case of uncertainty, these linguistic terms should transform to mutual fuzzy numbers to apply on EDM technique.

If $\widetilde{P}_i$ is progress percent of the activity i, then:

$$\widetilde{P}_i = [a_{1i}, a_{2i}, a_{3i}, a_{4i}] \tag{2}$$

Earned duration of the activity i is:

$$\widetilde{ED}_i = API_i \times BPD_i = \widetilde{P}_i \times BPD_i = [E_{1i}, E_{2i}, E_{3i}, E_{4i}] \tag{3}$$

Note that BPDi, is the authorized duration assigned to the scheduled work to be accomplished for activity i.to calculate the Total Earned Duration we should sum up all $\widetilde{\mathrm{ED}}_{\mathrm{i}}$ for i = 1, . . ., n (n is the total number of project activities)

$$\widetilde{\mathrm{TED}} = \sum_{\mathrm{i}=1}^{\mathrm{n}} \widetilde{\mathrm{ED}}_{\mathrm{i}} = [\mathrm{ED}_1, \mathrm{ED}_2, \mathrm{ED}_3, \mathrm{ED}_4] \tag{4}$$

Note that in this equation some $\widetilde{\mathrm{ED}}_{\mathrm{i}}$ can be both deterministic numbers and fuzzy numbers.

ED (t), according to Table.1, for the project, at any point in time, is the duration corresponding to Total Earned Duration (TED) on Total Planned Duration S-curve.

Note that the calendar unit represents the unit in which time instant t is measured.

Find t such that TED TPD_t and TED< $\mathrm{TPD}_{t+1(Calendarunit)}$

$$\widetilde{ED\,(t)}_i = t + \frac{\mathrm{ED}_i - TPD_t}{TPD_{t+1(Calender\ unit)} - TPD_t} \times 1\ ,\ i = 1;2;3;4$$

$$\widetilde{ED\,(t)} = \left[\widetilde{ED(t)}_1, \widetilde{ED(t)}_2, \widetilde{ED(t)}_3, \widetilde{ED(t)}_4\right] \tag{5}$$

Duration Performance Index (DPI) and Earned Duration Index (EDI) are the two commonly performance indices use in EDM technique.in this section these 2 indices are developed to fuzzy indices which can measure duration performance under uncertainty:

DPI or Duration Performance Index shows how well the project is doing in achieving the target completion date in consideration of the critical path and compares the progress made with the time passed.

$\widetilde{DPI}$ Is calculated as follow:

$$\widetilde{DPI} = \frac{\widetilde{ED\,(t)}_i}{AD} = \left[\frac{\widetilde{ED}_{t1}}{AD}, \frac{\widetilde{ED}_{t2}}{AD}, \frac{\widetilde{ED}_{t3}}{AD}, \frac{\widetilde{ED}_{t4}}{AD}\right] \tag{6}$$

Earned Duration Index (EDI) at any point in time, is a duration-based measure of overall work performed in terms of Earned Duration, in comparison with the work planned up to that point in time.

In other word, EDI simply compares the overall actual achievements and planned achievements.

$$\widetilde{EDI} = \frac{\widetilde{TED}}{TPD} = \left[\frac{\widetilde{ED}_1}{TPD}, \frac{\widetilde{ED}_2}{TPD}, \frac{\widetilde{ED}_3}{TPD}, \frac{\widetilde{ED}_4}{TPD}\right] \tag{7}$$

5. Interpretation of fuzzy Duration Performance Indices

The fuzzy-based indices are developed. Now in order to compare the fuzzy values of these indices against the value 1, they should interpret to have an inference regarding the project progress.

In this regard, we can use methods proposed for comparing the fuzzy numbers.

A well-known fuzzy ranking method proposed by Dubois and Prade [5] out of many different methods proposed to rank fuzzy numbers in the literature [2] [3] [5] [17] is presented is this paper.

According to Dubois and Prade[5], $T\ \tilde{A}$ is the degree of possibility of fuzzy number , thus $T\ \tilde{A} = \mu_{\tilde{A}}\ x$ where x∈. Given two fuzzy numbers $\tilde{A}$ and $\tilde{B}$, the degree of possibility that $\tilde{A} > \tilde{B}$ is:

$$T\ \tilde{A} \geq \tilde{B} = sup_{x \geq y} \min(\mu_{\tilde{A}}x\ ,\ \ \mu_{\tilde{B}}y) \tag{8}$$

The proposed equation leads to certain conclusion almost in all cases, so to apply Eq.8 to $\widetilde{DPI}$ (Eq.6) and $\widetilde{EDI}$ (Eq.7), however the comparison is made against 1, thus:

$$T\ \widetilde{DPI} \geq 1 = sup_{x \geq 1} \min\left(\mu_{\widetilde{DPI}}x, 1\right)\ = sup_{x \geq 1}\ \mu_{\widetilde{DPI}}x \tag{9}$$

$$T\ \widetilde{EDI} \geq 1 = sup_{x \geq 1} \min\left(\mu_{\widetilde{EDI}}x, 1\right)\ = sup_{x \geq 1}\ \mu_{\widetilde{EDI}}x \tag{10}$$

In this regard, both $T\ \widetilde{DPI} \geq 1$ and $T\ \widetilde{EDI} \geq 1$ results in five situations as Table.2 and Table.3 are demonstrated.

Note that [13] used these tables to perform a fuzzy approach for Earned Value Management (EVM). the vertical line in these tables show the position of value 1 that should compare to $\widetilde{DPI}$ and $\widetilde{EDI}$.

Table 2: The degree of possibility of $\widetilde{DPI} \geq 1$ and $\widetilde{DPI} \leq 1$

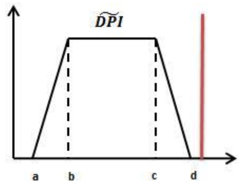
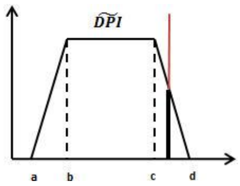
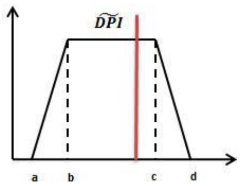
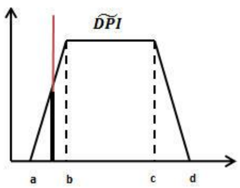
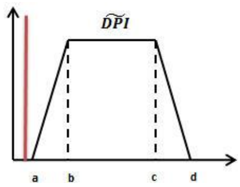
State of $\widetilde{DPI}$ against 1	Degree of possibility of $\widetilde{DPI} \geq 1$	Degree of possibility of $\widetilde{DPI} \leq 1$	Graphical Description	Decision Making
$d < 1$	$T\widetilde{DPI} \geq 1 \quad = 0$	$T\widetilde{DPI} \leq 1 \quad = 1$		Behind the Schedule
$c < l < d$	$T\widetilde{DPI} \geq 1 \quad = \frac{d-1}{d-c}$	$T\widetilde{DPI} \leq 1 \quad = 1$		Approximately Behind the Schedule
$b < l < c$	$T\widetilde{DPI} \geq 1 \quad = 1$	$T\widetilde{DPI} \leq 1 \quad = 1$		On the Schedule
$a < l < b$	$T\widetilde{DPI} \geq 1 \quad = 1$	$T\widetilde{DPI} \leq 1 \quad = \frac{1-a}{b-a}$		Approximately Ahead of the schedule
$a > 1$	$T\widetilde{DPI} \geq 1 \quad = 1$	$T\widetilde{DPI} \leq 1 \quad = 0$		Ahead of the schedule

Table 3: The degree of possibility of $\widetilde{EDI} \geq 1$ and $\widetilde{EDI} \leq 1$

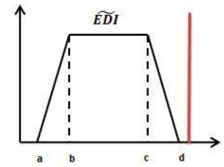
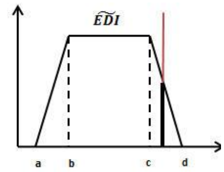
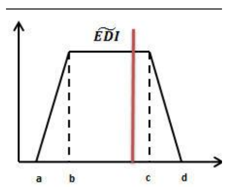
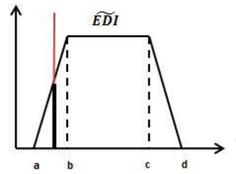
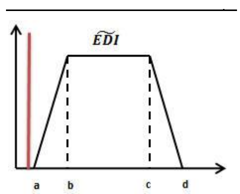
State of $\widetilde{EDI}$ against 1	Degree of possibility of $\widetilde{EDI} \geq 1$	Degree of possibility of $\widetilde{EDI} \leq 1$	Graphical Description	Decision Making **(In Comparison With Planned)**
$d < 1$	$T\widetilde{EDI} \geq 1 \quad = 0$	$T\widetilde{EDI} \leq 1 \quad = 1$		Less amount of work
$c < l < d$	$T\widetilde{EDI} \geq 1 \quad = \frac{d-1}{d-c}$	$T\widetilde{EDI} \leq 1 \quad = 1$		Approximately less amount of work
$b < l < c$	$T\widetilde{EDI} \geq 1 \quad = 1$	$T\widetilde{EDI} \leq 1 \quad = 1$		The same amount of work
$a < l < b$	$T\widetilde{EDI} \geq 1 \quad = 1$	$T\widetilde{EDI} \leq 1 \quad = \frac{1-a}{b-a}$		Approximately more amount of work
$a > 1$	$T\widetilde{EDI} \geq 1 \quad = 1$	$T\widetilde{EDI} \leq 1 \quad = 0$		More amount of work

Table 4: TPD and TAD of the example

Month	1	2	3	4	5	6	7	8	9	10
TPD	30	65	90	135	155	190	240	270	290	325
TAD	46	81	115	142	-	-	-	-	-	-

6. Example

In this section, a small example is brought to illustrate the developed approach. The case consists of 4 activities and he baseline planned duration is 10 months. Table 4 shows the Total Planned Duration (TPD) and Total Actual Duration (TAD) up to month 4.

The data regarding activity progress and activity duration are brought in Table 5.

Table 5: Activities information of the example

Activity Name	**Duration (Days)**	**Progress**
Activity 1	90	Very high
Activity 2	70	High
Activity 3	80	Less than half
Activity 4	85	Not started

Now, the progress of each activity should transform to fuzzy numbers ($\tilde{P}_i$) using linguistic term mentioned in Table. 1, then the $\widetilde{ED}_i$ of each activity and $\widetilde{TED}$ is calculated using Eq.3 and Eq.4:

$$\widetilde{ED}_1 = \tilde{P}_1 \times BPD_1 = [0.8,\ 0.9,\ 1,\ 1]{\times}90 \approx [72, 81, 90, 90]$$
$$\widetilde{ED}_2 = \tilde{P}_2 \times BPD_2 = [0.4,\ 0.5,\ 0.5,\ 0.6]{\times}70 \approx [28,\ 35,\ 35,\ 42]$$
$$\widetilde{ED}_3 = \tilde{P}_3 \times BPD_3 = [0.1,\ 0.2,\ 0.2,\ 0.3]{\times}80 \approx [8,\ 16,\ 16,\ 24]$$
$$\widetilde{ED}_4 = \tilde{P}_4 \times \text{BPD}_4 = [0, 0, 0, 0]{\times}85 \approx [0\ , 0,\ 0,\ 0]$$

Table 6: The activities Progress and $\widetilde{\mathbf{E}D}_\mathbf{i}$ of the example

Activity Name	**Progress**	$\widetilde{\mathbf{P}}_\mathbf{i}$	$\widetilde{\mathbf{E}D}_\mathbf{i}$
Activity 1	Very high	[0.8, 0.9, 1, 1]	$[72, 81, 90, 90]$
Activity 2	half	[0.4, 0.5, 0.5, 0.6]	$[28, 35, 35, 42]$
Activity 3	Low	[0.1, 0.2, 0.2, 0.3]	$[8, 16, 16, 24]$
Activity 4	Not Started	[0, 0, 0, 0]	$[0, 0, 0, 0]$

$\widetilde{\mathrm{ED}}_\mathrm{i}$ And $\widetilde{\mathrm{P}}_\mathrm{i}$ of each activity is presented in Table.6.

According to Eq.4 and Table.6, Total Earned Duration (TED) for all activities up to week 6 equals to:

$$\widetilde{TED} = \sum_{i=1}^{8} \widetilde{ED}_i = [108, 132, 141, 174]$$

In order to calculate fuzzy performance indices, $\widetilde{ED\,(t)}$ is needed:

According to Eq.5:

$$(\widetilde{ED}_1 = 108 \; and \; TPD_3 < 108 < TPD_4) \ggg\gg t_1 = 3$$

$$\widetilde{ED\,(t_1)} = t_1 + \frac{108-90}{135-90} \approx 3 + 0.4 = 3.4$$

Applying Eq.5 for $\widetilde{ED\,(t_2)}$, $\widetilde{ED\,(t_3)}$ & $\widetilde{ED\,(t_4)}$ will result in:

$$\widetilde{ED\,(t_2)} = 3.93 \quad , \; \widetilde{ED\,(t_3)} = 4.41 \; , \; \widetilde{ED\,(t_4)} = 5.54$$

$$\widetilde{ED\,(t)} = [\widetilde{ED(t_1)}, \widetilde{ED(t)}_2, \widetilde{ED(t)}_3, \widetilde{ED(t)}_4] = [3.4, \; 3.93, \; 4.41, \; 5.54]$$

So $\widetilde{DPI}$ and $\widetilde{EDI}$ of this project up to month 4 are calculated as follow:

$$\widetilde{DPI} = \frac{\widetilde{ED\,(t)}_i}{AD} = \frac{[3.4, 3.93, 4.41, 5.63]}{4} = [0.78, \; 0.98, \; 1.1, \; 1.38]$$

According to Table.2, The project is on the schedule and graphical description is as below (Fig.2):

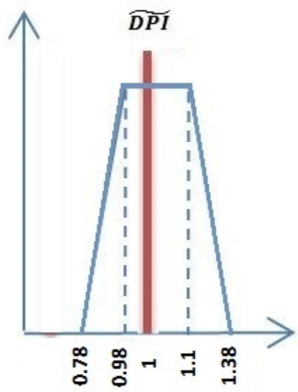


Figure 2: Graphical Description of $\widetilde{\mathbf{D}PI}$ of the example

$$\widetilde{EDI} = \frac{\widetilde{TED}}{TPD} = \frac{[108, 132, 141, 174]}{135} = [0.8, \; 0.9, \; 1.04, \; 1.3]$$

Also, according to Table.3, The project achieves the same amount of work in comparison with planned and graphical description is as below (Fig.3):

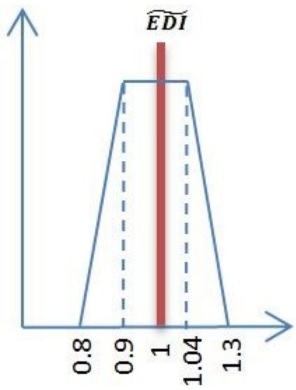


Figure 3: Graphical Description of $\widetilde{\mathbf{E}DI}$ of the example

7. Conclusion

The Earned Duration Management (EDM) provides early indications of project duration performance to highlight the need for eventual corrective action. This technique is originally developed for duration management but despite the uncertain nature of activities' progress in projects, they are considered deterministic in EDM technique.

In this paper, in order to solve this problem, a new fuzzy approach is presented to measure project duration performance. Where the completion percent of activities include uncertainty, linguistic terms are used to describe activities progress rather to evaluate it deterministically. The percent complete of activities are transformed to fuzzy numbers and EDM performance indices are developed to fuzzy duration performance indices.

In the interpretation of the fuzzy indices, a well-known method based on evaluating the degree of possibility of a fuzzy number in taking different values is used.

The proposed approach is employed in a small example which results illustrated that the sample project is ahead of the schedule and achieves the same amount of work in comparison with planned.

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