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Biharmonic Maps into S-Space Forms

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Abstract. We study in this paper the condition on second fundamental form for biharmonicity of submanifolds in S-space forms.

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1. Introduction

Harmonic maps between Riemannian manifolds have been discussed for last decades, initiated with the paper of J. Eells and J.H. Sampson [7]. Since harmonic maps have both properties analytic and geometric, they have become an important and interesting research field. The study of harmonic maps on Riemannian manifolds with some structures started from the paper of Lichnerowicz [13]. After that, Rawnsley [14] studied structure preserving harmonic maps between f-manifolds. Later on many authors studied harmonic maps (see [6] [10], [11], [12], [15] [16]).

The biharmonic maps theory is an old and attractive subject. They have been studied since 1862 by Maxwell and Airy to describe a mathematical model of elasticity. The Euler-Lagrange equation for bienergy functional was first derived by Jiange in 1986 [8]. After this biharmonic maps were studied by many authors see [2], [3], [5]. In [5], authors have studied the biharmonic submanifolds in complex space form. The objective of this paper is to find condition on second fundamental form for biharmonicity of a map from submanifolds of S-space form to S-space forms. After we recall some well known facts about biharmonic maps and S-manifolds , we prove the main results in third section.

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2. Preliminaries

In this section, we recall some well known facts concerning harmonic maps, biharmonic maps and S-manifolds.
Let $F : (M, g) \longrightarrow (N, h)$ be a smooth map between two Riemannian manifolds of dimensions m and n respectively. The energy density of F is a smooth function $e(F) : M \longrightarrow [0, \infty)$ given by [7],

$$e(F)_p = \frac{1}{2} Tr_g(F^* h)(p) = \frac{1}{2} \sum_{i=1}^{m} h(F_{*p} u_i, F_{*p} u_i),$$

for any $p \in M$ and any orthonormal basis $\{u_1, \ldots, u_m\}$ of $T_p M$. If M is a compact Riemannian manifold, the energy $E(F)$ of F is the integral of its energy density:

$$E(F) = \int_M e(F) \upsilon_g,$$

where υ_g is the volume measure associated with the metric g on M. A map $F \in C^\infty(M, N)$ is said to be harmonic if it is a cricital point of the energy functional E on the set of all maps between (M, g) and (N, h). Now, let (M, g) be a compact Riemannian manifold. If we look at the Euler-Lagrange equations for the corresponding variational problem, a map $F : M \longrightarrow N$ is harmonic if and only if $\tau(F) \equiv 0$, where $\tau(F)$ is the tension field which is defined by

$$\tau(F) = Tr_g \widetilde{\nabla} dF,$$

where $\widetilde{\nabla}$ is the connection induced by the Levi-Civita connection on M and the F-pullback connection of the Levi Civita connection on N.

We take now a smooth variation $F_{s,t}$ with two parameters $s, t \in (-\epsilon, \epsilon)$ such that $F_{0,0} = F$. The corresponding variation vector fields are denoted by V and W.

The second variation formula of E is:

$$H_F(V, W) = \frac{\partial^2}{\partial s \partial t} (E(F_{s,t}))\big|_{(s,t)=(0,0)} = \int_M h(J_F(V), W) \upsilon_g,$$

where J_F is a second order self-adjoint elliptic operator acting on the space of variation vector fields along F (which can be identified with $\Gamma(F^{-1}(TN))$) and is defined by

$$J_F(V) = -\sum_{i=1}^{m} (\widetilde{\nabla}_{u_i} \widetilde{\nabla}_{u_i} - \widetilde{\nabla}_{\nabla_{u_i} u_i}) V - \sum_{i=1}^{m} R^N(V, dF(u_i)) dF(u_i), \tag{1}$$

for any $V \in \Gamma(F^{-1}(TN))$ and any local orthonormal frame $\{u_1, \ldots, u_m\}$ on M. Here R^N is the curvature tensor of (N, h) (see [9] for more details on harmonic maps).
J. Eells and L. Lemaire [9] proposed polyharmonic (k-harmonic) maps, and Jiang [8]

studied the first and second variation formulas of biharmonic maps. Let us consider the bienergy functional defined by:

$$E_2(F) = \frac{1}{2}\int_M \mid \tau(F) \mid^2 \nu_g, \tag{2}$$

where $\mid V \mid^2 = h(V,V)$, $V \in \Gamma(F^{-1}TN)$.
Then, the first variation formula of the bienergy functional is given by:

$$\frac{d}{dt} \mid_{t=0} E_2(F_t) = -\int_M h(\tau_2(F), V)\nu_g, \tag{3}$$

here

$$\tau_2(F) := J(\tau(F)) = \bar{\triangle}(\tau(F)) - R(\tau(F)), \tag{4}$$

which is called the bitension field of F and J is given by (1).
A smooth map F of (M, g) into (N, h) is said to be biharmonic if $\tau_2(F) = 0$.

As a generalization of both almost complex (in even dimension) and almost contact (in odd dimension) structures, Yano introduced in [17] the notion of f-structure on a smooth manifold of dimension $2n+s$, i.e. a tensor field of type (1,1) and rank $2n$ satisfying $f^3 + f = 0$. The existence of such a structure is equivalent to a reduction of the structural group of the tangent bundle to $U(n) \times O(s)$. Let N be a $(2n+s)$-dimensional manifold with an f-structure of rank $2n$. If there exist s global vector fields $\xi_1, \xi_2, \ldots, \xi_s$ on N such that:

$$f\xi_\alpha = 0, \qquad \eta_\alpha \circ f = 0, \qquad f^2 = -I + \sum \xi_\alpha \otimes \eta_\alpha, \tag{5}$$

where η_α are the dual 1-forms of ξ_α, we say that the f-structure has complemented frames. For such a manifold there exists a Riemannian metric g such that

$$g(X,Y) = g(fX, fY) + \sum \eta_\alpha(X)\eta_\alpha(Y)$$

for any vector fields X and Y on N. See [1].

An f-structure f is normal, if it has complemented frames and

$$[f,f] + 2\sum \xi_\alpha \otimes d\eta_\alpha = 0,$$

where $[f,f]$ is Nijenhuis torsion of f.

Let Ω be the fundamental 2-form defined by $\Omega(X,Y) = g(X, fY)$, X, $Y \in T(N)$. A normal f-structure for which the fundamental form Ω is closed, $\eta_1 \wedge \cdots \wedge \eta_s \wedge (d\eta_\alpha)^n \neq 0$ for any α, and $d\eta_1 = \cdots = d\eta_s = \Omega$ is called to be an S-structure. A smooth manifold endowed with an S-structure will be called an S-manifold. These manifolds were introduced by Blair in [1].

We have to remark that if we take $s = 1$, S-manifolds are natural generalizations of Sasakian manifolds. In the case $s \geq 2$ some interesting examples are given in [1].

If N is an S-manifold, then the following formulas are true (see [1]):

$$\overline{\nabla}_X \xi_\alpha = -fX, \qquad X \in T(N), \quad \alpha = 1, \ldots, s, \tag{6}$$

$$(\overline{\nabla}_X f)Y = \sum \{g(fX, fY)\xi_\alpha + \eta_\alpha(Y)f^2 X\}, \quad X, Y \in T(N), \tag{7}$$

where $\overline{\nabla}$ is the Riemannian connection of g. Let L be the distribution determined by the projection tensor $-f^2$ and let K be the complementary distribution which is determined by $f^2 + I$ and spanned by $\xi_1, \ldots, \xi_s$. It is clear that if $X \in L$ then $\eta_\alpha(X) = 0$ for any α, and if $X \in K$, then $fX = 0$. A plane section π on N is called an invariant f-section if it is determined by a vector $X \in L(x)$, $x \in N$, such that $\{X, fX\}$ is an orthonormal pair spanning the section. The sectional curvature of π is called the f-sectional curvature. If N is an S-manifold of constant f-sectional curvature k, then its curvature tensor has the form [1]

$$\begin{aligned}\overline{R}(S,T,V,W) = \sum_{\alpha,\beta} \{ & g(fS, fW)\eta_\alpha(T)\eta_\beta(V) - g(fS, fV)\eta_\alpha(T)\eta_\beta(W) + \\ & + g(fT, fV)\eta_\alpha(S)\eta_\beta(W) - g(fT, fW)\eta_\alpha(S)\eta_\beta(V)\} + \\ & + \frac{1}{4}(k + 3s)\{g(fS, fW)g(fT, fV) - g(fS, fV)g(fT, fW)\} + \\ & + \frac{1}{4}(k - s)\{\Omega(S, W)\Omega(T, V) - \Omega(S, V)\Omega(T, W) - 2\Omega(S, T)\Omega(V, W)\},\end{aligned} \tag{8}$$

S, T, V, W $\in T(N)$. Ω is fundamental 2-form. Such a manifold $N(k)$ will be called an S-space form. The Euclidean space E^{2n+s} and the hyperbolic space H^{2n+s} are examples of S-space forms.

Let M be an m-dimensional submanifold immersed in N. Then M is an invariant submnaifold if $\xi_\alpha \in TM$ for any α and $fV \in TM$ for any $V \in TM$. It is said to be anti-invariant submanifold if $fV \in TM^\perp$ for any $V \in TM$. For a vector field $X \in TM^\perp$, it can be written as $fX = tX + nX$, where tX is tangent component of fX, nX is normal component of f X. If n does not vanishes, then its an f-structure [4].

Consider the structure vector fields $\xi_1, \xi_2, \ldots, \xi_s$ are tangent to M, $dim(M) \geq s$. Then M is CR-submaifold of N if there are two differentiable distributions D and $D^\perp$ on M, $TM = D + D^\perp$ such that

- D and $D^\perp$ are mutually orthogonal to each other.
- The distribution D is invariant under f, i.e. $fD_p = D_p$, for any $p \in M$
- The distribution $D^\perp$ is anti invariant under f, i.e. $fD_p^\perp \subseteq T_p M^\perp$ for any $p \in M$.

It can be proved that each hypersurface of N which is tangent to $\xi_1, \xi_2, \ldots, \xi_s$, has the structure of CR-submnaifold of N, for detail see [4].

3. Biharmonic Maps into S-space form

Before the main results recall the following results by Jiang:

Lemma 1. *[8] Let $f : (M^m, g) \to (N^n, h)$ be an isometric immersion whose mean curvature vector field $H = \frac{1}{m}\tau(f)$ is parallel; $\nabla^{\perp} H = 0$, where $\nabla^{\perp}$ is the induced connection of the normal bundle $T^{\perp}M$ by f. Then,*

$$\overline{\triangle}\tau(f) = \sum_{i=1}^{m} h(\overline{\triangle}\tau(f), df(e_i))df(e_i) - \sum_{i,j=1}^{m} h(\widetilde{\nabla}_{e_i}\tau(f), df(e_j))(\widetilde{\nabla}_{e_i} df)(e_j),$$

where $\{e_i\}$ is a locally defined orthonormal frame field of (M, g).

Lemma 2. *[8] Let $f : (M^m, g) \to (N^n, h)$ be an isometric immersion whose mean curvature vector field $H = \frac{1}{m}\tau(f)$ is parallel; $\nabla^{\perp} H = 0$, where $\nabla^{\perp}$ is the induced connection of the normal bundle $T^{\perp}M$ by f. Then,*

$$\overline{\triangle}\tau(f) = - \sum_{j,k=1}^{m} h(\tau(f), R^N(df(e_j), df(e_k))df(e_j)df(e_k) - \\ - \sum_{i,j=1}^{m} h(\tau(f), (\widetilde{\nabla}_{e_i} df)(e_j))(\widetilde{\nabla}_{e_i} df)(e_j),$$

where $\{e_i\}$ is a locally defined orthonormal frame field of (M, g).

Lemma 3. *[8] Let $f : (M^m, g) \to (N^{m+1}, h)$ be an isometric immersion which is not harmonic. Then, the condition that $\| \tau(f) \|$ is constant is equivalent to the one that*

$$\overline{\nabla}_X \tau(f) \in \Gamma(f_* TM), \qquad \textit{for all} \qquad X \in TM,$$

that is the mean curvature tensor is parallel with respect to $\bigtriangledown^{\perp}$.

For details and proof of these Lemmas, see [5], [8]. Now the main result of this article;

3.1. Main results

Theorem 1. *Let (M, g) be a 2m+s-dimensional submanifold of S-space form N of dimension (2n+s), and $\Phi : (M, g) \to (N, h)$ be an isometric immersion with non zero constant parallel mean curvature with respect to connection on normal bundle, then necessary and sufficient conditions for Φ to be biharmonic is*

- $\| B(\Phi) \|^2 = \frac{k+3s}{4}(2n - 1 + s) + \frac{3(k-s)}{4}$, *for M^{2m+s} to be a hypersurface, 2m=2n-1*
- $\| B(\Phi) \|^2 = \frac{k+3s}{4}(2m + s)$, *for M^{2m+s} to be an inavriant submanifold, $m < n$*

Proof. Consider an s-manifold with constant f-sectional curvature k. Let $\{v_i\}_{i=1}^{2m+s}$ be orthonormal basis on M. Then from equation (8) we have

$$\begin{aligned} R^N(d\Phi(v_j), d\Phi(v_k))d\Phi(v_k) &= \sum_{\alpha,\beta} \{-f^2 d\Phi(v_j)\eta_\alpha(d\Phi v_k)\eta_\beta(d\Phi v_k) - h(fd\Phi v_j, d\Phi v_k). \\ &\quad .\eta_\alpha(d\Phi v_k)\xi_\beta + h(fd\Phi(v_k), fd\Phi(v_k))\eta_\alpha(d\Phi v_j)\xi_\beta + \\ &\quad +f^2 d\Phi(v_k)\eta_\alpha(d\Phi v_j)\eta_\beta(d\Phi v_k)\} + \frac{1}{4}(k+3s)\{-f^2 d\Phi(v_j). \\ &\quad .h(fd\Phi v_k, fd\Phi v_k) + h(fd\Phi v_j, fd\Phi v_k)f^2 d\Phi(v_k)\} \\ &\quad +\frac{1}{4}(k-s)\{-fd\Phi(v_j)h(d\Phi v_k, d\Phi v_k) + fd\Phi(v_k) \\ &\quad .h(d\Phi v_j, fd\Phi v_k) + 2fd\Phi(v_k)h(d\Phi v_j, fd\Phi v_k)\}, \end{aligned}$$

and

$$\begin{aligned} R^N(d\Phi(v_j), d\Phi(v_k))d\Phi(v_k) &= \frac{1}{4}(k+3s)\{d\Phi(v_j) + \delta_{jk}(-d\Phi v_k)\} \\ &+ \frac{3}{4}(k-s)h(d\Phi v_j, fd\Phi v_k)fd\Phi(v_k). \end{aligned}$$

Then we have

$$\begin{aligned} &\sum_{j,k=1}^{m} h(\tau(\Phi), R^N(dF(v_j), dF(v_k))dF(v_k))dF(v_j) \\ = &\frac{1}{4}(k+3s)\{h(\tau(\Phi), d\Phi(v_j)) - \delta_{jk}h(\tau(\Phi), d\Phi v_k)\} \\ + &\frac{3}{4}(k-s)\{h(d\Phi v_j, fd\Phi v_k)h(\tau, fd\Phi(e_k))\}. \end{aligned} \tag{9}$$

Let $\tau(\Phi) \in TM^\perp$, then

$$\sum_{j,k=1}^{m} h\left(\tau(\Phi), R^N(d\Phi(v_j), d\Phi(v_k)\right) d\Phi(v_k))d\Phi(v_j) = 0.$$

Let M^{2m+s} be a hyperspace of s-manifold N. Each hypersurface of s-manifolds has the structure of CR-submanifold. For $\tau(\Phi) \in TM^\perp$, we can take $f\tau(\Phi) \in \Gamma TM$. Now $dim(M) = 2m+s = 2n-1+s$. For orthonormal basis $\{d\Phi(v_k)\}_{k=1}^{m}$ of $d\Phi(T_xM)$ at all points on M, we can write

$$f\tau(\Phi) = \sum_{k=1}^{2n-1+s} h(f\tau(\Phi), d\Phi(v_k))d\Phi(v_k).$$

By Lemma 2,

$$\overline{\Delta}\tau(\Phi) = \sum_{i,j=1}^{2n-1+s} h\left(\tau(F), (\widetilde{\nabla}_{v_i} d\Phi)(v_j)\right)(\widetilde{\nabla}_{v_i} d\Phi)(v_j)$$

Furthermore, we have

$$R(\tau(\Phi)) = \sum_{k=1}^{2n-1+s} R^N(\tau(\Phi), d\Phi(e_k))d\Phi = \frac{k+3s}{4}(2n-1+s)\tau(\Phi) + \frac{3(k-s)}{4}\tau(\Phi). \quad (10)$$

Now the necessary and sufficient conditions F to be biharmonic is that

$$\tau_2(F) = \bar{\triangle}\tau(F) - R(\tau(F)) = 0, \quad (11)$$

this becomes

$$\sum_{i,j=1}^{2n-1+s} h\left(\tau(F), \widetilde{\nabla}_{v_j} dF(v_k)\right) \widetilde{\nabla}_{v_j} dF(v_k) - \left[\frac{k+3s}{4}(2n-1+s)\tau(\Phi) + \frac{3(k-s)}{4}\tau(\Phi)\right] = 0 (12)$$

Now let

$$B(\Phi)(e_j, e_k) = (\widetilde{\nabla}_{v_j} d\Phi)v_k = h(d\Phi(v_j), d\Phi(v_k))V = h_{jk}U,$$

where U is the unit normal vector along $F(M)$. then

$$\tau(F) = \sum_{r=1}^{2n-1+s} (\widetilde{\nabla}_{v_r} dF)(v_r) = \sum_{r=1}^{2n-1+s} h_{rr}U,$$

where U is the unit normal vector along $F(M)$. Thus, the left hand side of (15) becomes as:

$$\sum_{j,k,r=1}^{2n-1+s} \left\{ h_{rr}h_{jk}h_{jk}U - \left[\frac{k+3s}{4}(2n-1+s)\tau(\Phi) + \frac{3(k-s)}{4}\tau(\Phi)\right]\right\} = 0,$$

$$\left(\sum_{r=1}^{2n-1+s} h_{rr}\right) \left\{ \sum_{j,k=1}^{m} h_{jk}h_{jk}U - \left[\frac{k+3s}{4}(2n-1+s)U + \frac{3(k-s)}{4}U\right]\right\} = 0,$$

$$\| \tau(\Phi) \| \left\{ \| B(\Phi) \|^2 U - \left[\frac{k+3s}{4}(2n-1+s)U + \frac{3(k-s)}{4}U\right]\right\} = 0.$$

since $\tau(\Phi) \neq 0$, by assumption, then we have

$$\| B(\Phi) \|^2 - \left[\frac{k+3s}{4}(2n-1+s) + \frac{3(k-s)}{4}\right] = 0,$$

and

$$\| B(\Phi) \|^2 = \left[\frac{k+3s}{4}(2n-1+s) + \frac{3(k-s)}{4}\right]. \quad (13)$$

Next let M^{2m+s} be an invariant submanifold of s-manifold N. Then for $\tau(\Phi) \in \Gamma TM^{\perp}$, $f\tau(\Phi) \in \Gamma TM^{\perp}$. For orthonormal basis $\{d\Phi(v_k)\}_{k=1}^{m}$ of $d\Phi(T_xM)$ at all points $x \in M$, we have $h(f\tau(\Phi), d\Phi(v_k)) = 0$, then in this case

$$R(\tau(\Phi)) = \sum_{k=1}^{2n-1+s} R^N(\tau(\Phi), d\Phi(e_k))d\Phi = \frac{k+3s}{4}(2m+s)\tau(\Phi). \tag{14}$$

Then by equation (11),

$$\sum_{i,j=1}^{2m+s} h\left(\tau(F), \widetilde{\nabla}_{v_j} dF(v_k)\right) \widetilde{\nabla}_{v_j} dF(v_k) - \frac{k+3s}{4}(2m+s)\tau(\Phi) = 0 \tag{15}$$

With similar computations as above we have

$$\| \tau(\Phi) \| \left\{ \| B(\Phi) \|^2 U - \left[\frac{k+3s}{4}(2m+s)U \right] \right\} = 0.$$

This implies

$$\| B(\Phi) \|^2 = \frac{k+3s}{4}(2m+s)$$

Corollary 1. *Let (M, g) be a $2n-1$-dimensional submanifold of complex space form N of dimension $2n$, and $\Phi : (M, g) \to (N, h)$ be an isometric immersion with non zero constant parallel mean curvature with respect to connection on normal bundle, then necessary and sufficient conditions for Φ to be biharmonic is*

$$\| B(\Phi) \|^2 = \frac{k}{2}(n+1)$$

Proof. In equation (13), for s=0 we get result.

Corollary 2. *Let (M, g) be a $(2n-1)+1$-dimensional submanifold of Sasakian space form N of dimension $2n+1$, and $\Phi : (M, g) \to (N, h)$ be an isometric immersion with non zero constant parallel mean curvature with respect to connection on normal bundle, then necessary and sufficient conditions for Φ to be biharmonic is*

$$\| B(\Phi) \|^2 = \frac{k}{4}(2n+3) + \frac{3}{4}(2n-1)$$

Proof. In equation (13), for s=1 we get result.

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