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Almost prime ideal in gamma near ring

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Abstract. In this manuscript we introduce the notion of almost prime ideals in Γ-near-rings along with few of their characterizations. We also present the interesting relations among almost prime, prime and primary ideal in Γ-nearrings.

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1. Introduction and Preliminaries

Recently, the generalization of prime ideal i.e., almost prime ideal in commutative rings has been introduced and discussed by Srikant M. Bhatwadekar and Pramod K. Sharma (See [3]). Following [3], an ideal I of a ring R is said to be an almost prime if for all $a, b \in R$ implies $ab \in I - I^2$ either $a \in I$ or $b \in I$. All prime and idempotent ideals are almost prime [3]. It has been proved that every almost prime ideal in a noetherian domain R is primary [3]. Further to this, almost primary ideals in rings have been introduced by A. K. Jabbar and C. A. Ahmed in [12], a proper ideal A of a ring R is an almost primary ideal if for $a, b \in R$ such that $ab \in A - A^2$, then $a \in A$ or $b \in A$, for some positive integer n [12]. In [12], authors have also discussed several characterizations of almost primary ideals. It is evident that primary ideals, almost prime ideals and idempotent ideals of a ring R are almost primary ideals, but the converse is not true in each case. Notion of weakly prime element (author called it a prime) was introduced by Steven Galovich while studying the property of unique factorization of rings with zero divisors [10]. Following [10], let $r \neq 0$ be in R than r is prime if, whenever r divides ab where $ab \neq 0$, then r divides a or r divides b. Author established the fundamental results: (i) In [10], author also

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showed that every irreducible is a prime, (ii) every irreducible in R is a zero divisor [10], (iii) every irreducible element of R is nilpotent, and (iv) every nonunit in R is nilpotent. Consequently the author declared the unique maximal ideal consists of nonunit elements [10]. In [1], authors declare that (which was named prime by Galovich in [10]) a nonzero nonunit $p \in R$ is weakly prime if $p|ab \neq 0$ implies $p|a$ or $p|b$. Consequently, an ideal I of a commutative ring R is called a weakly prime if $0 \neq ab \in I$ implies $a \in I$ or $b \in I$, and also p is weakly prime iff (p) is weakly prime [1]. Following [2], P is weakly prime ideal if and only if $0 \neq AB \subseteq P$, A and B ideals of R, implies $A \subseteq P$ or $B \subseteq P$. Further to this, every weakly prime ideal is an almost prime ideal.
We call an algebraic system N with two binary operation " + " and "." (right) near-ring if it is a group (not necessarily abelian) under addition, and N is associative group under multiplication and distribution of multiplication over addition on the right holds i.e., for any $x, y, z \in N$, it satisfies that $(x + y)z = (xz) + (yz)$[15]. Likewise, a left near-ring can be defined by replacing the right distributive law by the equivalent left distributive law. Suppose N is a left near-ring with binary operation " + " and "." then a subset I is said to be an ideal if (i) $(I, +)$ is a normal subgroup of a $(N, +)$, (ii) For each $n \in N$, $i \in I$, $n_i \in I$ i.e., $NI \subseteq I$, and (iii) $(n_1 + i)n_2\ n_1n_2 \in I$ for each $n_1, n_2 \in N$ and $i \in I$. But A. Frohlich [9] showed that for d.g. near-rings the third condition is equivalent to $in \in I$ i.e., $IN \subseteq I$. Hence a subset I is a right (left) ideal if I satisfies the first and third (second) conditions. A proper ideal P of a near ring N is prime if for ideals A and B of N, $AB \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$. An ideal P of a near-ring N is a completely prime (prime ideal of type-2) if for all $x, y \in N$, $xy \in P$ implies $x \in P$ or $y \in P$. Almost prime ideals in near rings have been endorsed by B. Elavarasan (see [8]). A proper ideal P of a near ring N is said to be almost prime if for any ideals A and B of N such that $AB \subseteq P$ and $AB \nsubseteq P^2$, we have $A \subseteq P$ or $B \subseteq P$[8]. The author established few relationships between almost prime and prime ideals [8]. Weakly prime ideals in near rings have been introduced by P. Dheena and B. Elavarasan [6], a proper ideal P of near ring N is said to be weakly prime if $0 \neq AB \subseteq P$, A and B are ideals of N, implies $A \subseteq P$ or $B \subseteq P$. Clearly, every prime ideal is weakly prime and $\{0\}$ is always weakly prime ideal of a near ring N. Also every prime ideal is a weakly prime, and a weakly prime ideal is an almost prime ideal. An ideal I of a near ring N is said to be a completely prime ideal if $x, y \in N$, $xy \in I$ implies $x \in I$ or $y \in I$ [11]. Similarly, an ideal of a near ring N is said to be primary ideal of N if $x, y \in N$, $xy \in I$ implies $x \in I$ or $y^m \in I$ for some $m \in Z$. An ideal I of a near ring N is called a completely semiprime ideal of a near ring N if $y^2 \in I$ implies $y \in I$ for all $y \in N$ [11]. Further to this, almost prime ideals in near rings have been endorsed by B. Elavarasan (see [8]). A proper ideal P of a near ring N is said to be almost prime if for any ideals A and B of N such that $AB \subseteq P$ and $AB \nsubseteq P^2$, we have $A \subseteq P$ or $B \subseteq P$ [8]. The author established few relationships between almost prime and prime ideals [8]. Number of ideals in near ring have been introduced and discussed such as completely prime, primary, completely primary and so on. Following [11], an ideal I of a near ring N is said to be a completely prime ideal if $x, y \in N$, $xy \in I$ implies $x \in I$ or $y \in I$ [11]. Similarly, an ideal of a near ring N is said to be primary ideal of N if $x, y \in N$, $xy \in I$ implies $x \in I$ or $y^m \in I$ for some $m \in Z$. An ideal I of a near ring N is called a

completely semiprime ideal of a near ring N if $y^2 \in I$ implies $y \in I$ for all $y \in N$ [11].
The ideal theory is the most important part of algebra, different types of ideals in rings have been discussed in the literature. A right (left) ideal of a Γ-ring M is an additive subgroup I of M such that $I\Gamma M \subseteq I$ ($M\Gamma I \subseteq I$). If I is both a right and a left ideal, then we say that I is an ideal or a two-sided ideal of M. In rings, an ideal P is prime ideal if and only if A and B are ideals in M such that $AB \subseteq P$, then $A \subseteq P$ or $B \subseteq P$ [13]. The prime ideals of the $\Gamma_{n,m}$ -ring $M_{m,n}$ are the sets $P_{m,n}$ corresponding to the prime ideals P of the Γ-ring M [13]. If P is an ideal in a Γ-ring M then, (i) Ideal P is a prime ideal of M, (ii) If $a, b \in M$ and $a\Gamma M\Gamma b \subseteq P$ then either $a \in P$ or $b \in P$, (iii) If ideal generated by $< a >$ and $< b >$ are called principal ideals in M and $< a > \Gamma < b > \subseteq P$, then $a \in P$ or $b \in P$, (iv) If U and V are right ideals in M with $U\Gamma V \subseteq P$, then $U \subseteq P$ or $V \subseteq P$, (v) If U and V are left ideals in M with $U\Gamma V \subseteq P$, either $U \subseteq P$ or $V \subseteq P$ [16].
Γ-near rings were introduced by Satyanarayana Bhavanari (see [14], [15]). A subset A of a Γ-near-ring M is called a left (resp. right) ideal of M if $(A, +)$ is a normal divisor of $(M, +)$, $u\alpha(x + v) - u\alpha v \in A$ (resp. $x\alpha u \in A$) for all $x \in A$, $\alpha \in \Gamma$ and $u, v \in M$. An ideal P of Γ-near ring $(M, +, (.)_\Gamma)$ is called prime, if for every two ideals I, J of M, $I\Gamma J \subseteq P$ implies $I \subseteq P$ or $J \subseteq P$. An ideal P of a Γ-near-ring N is called a completely primary ideal if for $a, b \in N$ and $\gamma \in \Gamma$ such that $a\gamma b \in P$ implies that $a \in P$ or $b \in P$, for some positive integer n [17]. If an ideal I of Γ-near-ring M is maximal, then it is prime or $M\Gamma M = I$ [7]. If $(M, +, (.)_\Gamma)$ is a Γ-near-ring such that for any $\gamma \in \Gamma$ there is an element which is Γ-unit, then every maximal ideal I of M is prime [7]. For every ideal I of Γ-near-ring M exists prime minimal ideal of I [7]. In this note first we introduce the notion of almost prime ideals in Γ-near-rings along with few of their characterizations. Finally, we present the interesting relations of an almost prime with the prime and primary ideal in Γ-near-rings.

2. Almost prime ideal in Γ-near-ring

In this section we introduce almost prime ideal in Γ-near-rings. Furthermore, we also present its implications with the some ideals, we start with the following definition.
Definition 1. Let M be Γ-near-ring and P be a prime ideal of M then P is almost prime ideal if $a, b \in R$, $ab \in P - P\Gamma P$, either $a \in P$ or $b \in P$.
Example 1. Suppose $Z_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$ and $\Gamma = \{0, 2, 4\}$. Let $P = 2Z_8 = \{0, 2, 4\}$ be a prime ideal in Z_8 and consider $P\Gamma P = \{0, 6\}$, $P - P\Gamma P = \{2, 4\}$. Here $2, 3 \in Z_8$ and $2.2.3 = 4 \in P - P\Gamma P$ where $2 \in P$ and $3 \notin P$. Similarly we can check for other elements as well. Hence P is an almost prime ideal in Γ-near ring.
Example 2. Suppose R is a Γ-near ring of algebraic integers such that the integral closure of Z in C. Suppose that I be a radical ideal of R say $I\Gamma I = I$, if $\alpha \in I$ then $\beta \in R$ exist such that $\beta\Gamma\beta = \alpha$. Since $\beta\Gamma\beta = \alpha \in I$, $\beta \in I$ implies $I = I\Gamma I$.
Example 3. Consider the near ring $N = \{0, 1, 2, 3\}$ and $\Gamma = \{0, 2\}$ such that addition and multiplication defined as follow.

$$\begin{pmatrix} + & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 & 3 \\ 1 & 1 & 0 & 3 & 2 \\ 2 & 2 & 3 & 0 & 1 \\ 3 & 3 & 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} \cdot & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 2 & 3 \\ 2 & 0 & 2 & 0 & 2 \\ 3 & 0 & 3 & 2 & 1 \end{pmatrix}$$

Suppose $P = \{0, 2\} = 2N$ be a prime ideal of N because for all $a, b \in N$ and $a\gamma b \in P$ implies $a \in P$ or $b \in P$. As $P\Gamma P = \{0\}$ then $P - P\Gamma P = \{2\}$, then for all $a, b \in N$ such that $a\gamma b \in P - P\Gamma P$ either $a \in P$ or $b \in P$ which is almost prime ideal.

Preposition 1. Every prime ideal in a Γ-near ring is almost prime ideal. Proof. Suppose P be a prime ideal of Γ-near ring but not an almost prime. Assume $a\gamma b \in P - P\Gamma P$, implies $a\gamma b \in P$. If $a\gamma b \notin P\Gamma P$ implies $a \in P$ or $b \in P$ then contradiction arise to our supposition. Hence P must be a prime.

Remark 1. If I is a maximal ideal of Γ-near-ring M then it is prime or $M\Gamma M = I$.
Supporting the above remark 1, we present the below example.

Example 4. Let $M = \{0, 1, 2, 3\}$ is a Γ-near-ring where $\Gamma = \{0, 2\}$ and ideal $I = 2M = \{0, 2\}$ that is maximal in M. Obviously I is prime ideal in M also $M\Gamma M = I$.

Lemma 1. Suppose N is a Γ-near-ring and for any $\gamma \in \Gamma$ there is an element which is Γ-unit then every maximal ideal I of M is prime.

Proof. If for one $\gamma \in \Gamma$ the element e is γ-one of M then $M\gamma M = \{m_1\gamma m_2 : m_1; m_2 \in M\} = M$ since for any $m \in M$, $m = m\gamma e$. Because $M \neq I$ the equation is not true $M\Gamma M = I$. When $M = I$ or $M = 0$ then equation is true so M is simple and $M\Gamma M \neq 0$, as a result M is prime.

Preposition 2. Suppose I be a P-primary ideal of a Γ-near ring such that $P\Gamma P = I\Gamma I$ implies I is an almost prime.

Proof. Suppose $a, b \in R$, $a\gamma b \in I - I\Gamma I$, $a \notin I$ and $b \notin I$. As $a \notin I$ and I is a P-primary ideal it implies that $b \in P$. Also $a \in P$ thus $a\gamma b \in P\Gamma P = I\Gamma I$, which is a contradiction.

Lemma 2. Suppose that R be a near integral domain and c be a nonzero nonunit element of R. If element c is other than prime element then there exist $a \notin R\Gamma c$, $b \notin R\Gamma c$ such that $a\gamma b \in R\Gamma c$ but $a\gamma b \notin R\Gamma c^2$.

Proof. Suppose an ideal Rc is not prime then there exist $a \notin R\Gamma c$, $b \notin R\Gamma c$ such that $a\gamma b \in R\Gamma c$. If the case $a\gamma b \in R\Gamma c^2$ then for $d = (b + c)\gamma \notin R\Gamma c$ and $a\gamma d \in R\Gamma c$. If $a\gamma d \in R\Gamma c^2$, implies $a\gamma c \in R\Gamma c^2$ as $a\gamma b \in R\Gamma c^2$ implies $a \in R\Gamma c$, a contradiction to our supposition. Hence the result follows.

Example 5. Let Z be a Γ-near ring and $\Gamma = \{0, 1, 2, 3\}$ consider $c = 6$ be an non prime element of Z then $Z\Gamma 6$ is non prime ideal because $3 \notin Z\Gamma 6$ and $4 \notin Z\Gamma 6$ but $12 \in Z\Gamma 6$ and $12 \notin Z\Gamma 6^2$.

In the below proposition, we reverse the situation occurring in lemma 2.

Preposition 3. Suppose that R be Γ-near integral domain and c be a nonzero nonunit element of R. If c is not a prime element then there exists $a \in R\Gamma c$ and $b \in R\Gamma c$ such that $a\gamma b \in R\Gamma c$ and $a\gamma b \in R\Gamma c^2$.

Proof. Suppose an ideal $R\Gamma c$ is not prime and consider $a \in R\Gamma c$, $b \in R\Gamma c$ such that $a\gamma b \in R\Gamma c$. If the case, $a\gamma b \notin R\Gamma c^2$ then for $d = (b + c) \in R\Gamma c$ and $a\gamma d \in R\Gamma c$. Consider $a\gamma d \notin R\Gamma c^2$) implies $ac \notin R\Gamma c^2$ and because $a\gamma b \notin R\Gamma c^2$ implies $a \notin R\Gamma c$, a contradiction to our hypothesis. Hence the result is valid. Supporting the above lemma3 we present the below example.

Example 6. Let $Z_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$ and $\Gamma = \{0, 2, 4\}$ consider a non-prime element of Z_8 i.e., $c = 6$ implies $6Z_8 = \{0, 2, 4\}$. Consider $6, 4 \in 6Z_8$ such that $6.2.4 = 0 \in 6Z_8$ and $c^2 = 6^2$ and $6^2Z_8 = \{0, 4\}$, hence $6.2.4 = 0 \in 6^2Z_8$. Further we consider $6.4.4 = 4 \in 6^2Z$ and take $4, 2 \in 6Z^8$ then $4.2.2 = 0 \in 6Z_8$, and again we get $4.2.2 = 0 \in 6^2Z_8$, similarly $4.4.2 = 0 \in 6Z_8$ and $4.4.2 = 0 \in 6^2Z_8$.

Theorem 1. Suppose N be a Γ-near-ring with identity and P be an almost prime ideal of N. If P is not prime then $P\Gamma P = P$.

Proof. Let us assume that $P \subseteq P\Gamma P$. We have to prove that P is prime. Let us suppose that two ideals A and B contained in N such that $A\Gamma B \subseteq P$. If $A\Gamma B \nsubseteq P\Gamma P$ then $A \nsubseteq P$ or $B \nsubseteq P$. We assume that $A\Gamma B \nsubseteq P\Gamma P$. Since $P \nsubseteq P\Gamma P$ as a result $p \in P$ such that $< p >\nsubseteq P\Gamma P$ hence $(A+ < p >)\Gamma(B + N) \nsubseteq P\Gamma P$. Consider $(A+ < p >)\Gamma(B + N) \nsubseteq P$, there exist an element $a \in A$, $b \in B$, $p_0 \in< p >$ and $q_0 \in N$ such that $(a+p_0)\gamma(b+q_0) \notin P$ implies $a\gamma(b + q_0) \notin P$, but $a\gamma(b + q_0) = a\gamma(b + q_0) - a\gamma b + a\gamma b \in P$ as $A\Gamma B \subseteq P$, a contradiction. Hence $(A+ < p >)\Gamma(B + N) \subseteq P$ implies $A \subseteq P$.

Corollary 1. Consider N a Γ-near-ring having identity and containing an ideal P. If $P\Gamma P \neq P$ then P is prime if and only if P is almost prime.

Proposition 4. If $P \neq 0$ be a proper ideal of a Γ-near-ring N such that P is almost prime and $(P\Gamma P : P) \subseteq P$ then P is prime.

Proof. We suppose that P is not a prime ideal of N. Then there exist $x/P\Gamma P$ and $y \notin P$ such that $< x > \Gamma < y >\subseteq P$. If $< x > \Gamma < y >\nsubseteq P\Gamma P$, then the result holds. Hence $< x > \Gamma < y >\subseteq P\Gamma P$. Suppose $< x > \Gamma(< y > +P) \subseteq P$. If $< x > \Gamma(< y > +P) \nsubseteq P$ then we have $x \in P$ or $y \in P$, a contradiction to our assumption, or else $< x > \Gamma(< y > +P) \subseteq P\Gamma P$. Thus $< x > \Gamma P \subseteq P\Gamma P$ implies $x \in (P\Gamma P : \Gamma : P) \subseteq P$.

Theorem 2. Suppose N be a Γ-near-ring and let P be an ideal of N. Then the following statements are equivalent:

i) If elements $a, b, c \in N$ with $a\gamma(< b > + < c >) \in P$ and $a\gamma(< b > + < c >) \nsubseteq P\Gamma P$ then $a \in P$ or b, c in P.

ii) If $x \in N - P$, then $(P : \Gamma :< x > + < y >) = P \cup (P\Gamma P : \Gamma :< x > + < y >)$ for some $y \in N$.

iii) If $x \in NP$, then $(P : \Gamma :< x > + < y >) = P$ or $(P : \Gamma :< x > + < y >) = (P\gamma P : \Gamma :< x > + < y >)$ for some $y \in N$.

iv) P is an almost prime.

Proof. (i) implies (ii) Consider $t \in (P : \Gamma :< x > + < y >)$ for some $x \in N - P$, $\gamma \in \Gamma$ and $y \in N$. After that $t\Gamma(< x > + < y >) \subseteq P$. If $t\Gamma(< x > + < y >) \subseteq P\Gamma P$ subsequently $t^2\Gamma(P\Gamma P : \Gamma :< x > + < y >)$. If $t\Gamma(< x > + < y >\nsubseteq P\Gamma P$, then $t \in P$ by assumption. (ii) implies (iii) holds from the truth that if union of two ideal is an ideal then it is equal to one of them.(iii) implies (iv) Imagine A and B be ideals of N such that $A\Gamma B \subseteq P$. Assume $A \nsubseteq P$ and $B \nsubseteq P$ implies $a \in A$ and $b \in B$ exist with $a, b \notin P$. Now we say that $A\Gamma B \nsubseteq P\Gamma P$ and consider $b_1 \in B$. In that case $A\Gamma(< b > + < b_1 >) \nsubseteq P$ which implies $A \subseteq (P : \Gamma :< b > + < b_1 >)$. Then by supposition $A \subseteq (< b > + < b_1 >)\Gamma P\Gamma P$ implies $A\Gamma b_1 \subseteq P\Gamma P$. Consequently $AB \subseteq P\Gamma P$ and therefore P is an almost prime ideal of N.

(iv) implies (i) is obvious.

Theorem 3. Suppose N_1, N_2 be any two Γ-near-rings with identity and let P be a proper ideal of N_1. Then P is almost prime if and only if $(P \times N_2)$ is an almost prime ideal of $N_1 \times N_2$.

Proof. Suppose P be an almost prime ideal of N_1 and consider $(A_1 \times B_1)$ and $(A_2 \times B_2)$ be ideals of $N_1 \times N_2$ such that $(A_1 \times B_1)\Gamma(A_2 \times B_2) \subseteq (P \times N_2)$ and $(A_1 \times B_1)\Gamma(A_2 \times B_2) \nsubseteq (P \times N_2)\Gamma(P \times N_2)$. In this case $(A_1\Gamma A_2 \times B_1\Gamma B_2) \subseteq (P \times N_2)$ and $(A_1\Gamma A_2 \times B_1\Gamma B_2) \nsubseteq (P\Gamma P \times N\Gamma N)$,therefore $A_1\Gamma A_2 \times P$ and $A_1\Gamma A_2 \nsubseteq P\Gamma P$ implies $A_1 \subseteq P$ or $A_2 \subseteq P$. Conversely, assume that $(P \times N_2)$ is an almost prime ideal of $N_1 \times N_2$ and consider I and J be ideals of N_1 such that $I\Gamma J \subseteq P$ and $I\Gamma J \nsubseteq P\Gamma P$. Then $(I \times N_2)\Gamma(J \times N_2) \subseteq (P \times N_2)$ and $(I \times N_2)\Gamma(J \times N_2) \nsubseteq (P \times N_2)\Gamma(P \times N_2)$. By hypothesis, we have $(I \times N_2) \subseteq (P \times N_2)$ or $(J \times N_2) \subseteq (P \times N_2)$. Thus $I \subseteq P$ or $J \subseteq P$.

Lemma 3. If $c \neq 0$ is a nonunit element in Γ-near integral domain R then ideal $R\Gamma c$ is prime if and only if $R\Gamma c$ is an almost prime.

Proof. Let $c \neq 0$ is a nonunit element in an Γ-near integral domain R. Assume that ideal $R\Gamma c$ is an almost prime we need to prove that $R\Gamma c$ is prime. As we know that ideal $R\Gamma c$ is an almost prime for some $a, b \in R$ and $a\gamma b \in R\Gamma c - R\Gamma c\Gamma R\Gamma c$ implies either $a \in R\Gamma c$ or $b \in R\Gamma c$ where $a\gamma b \notin R\Gamma c\Gamma R\Gamma c$ implies $a\gamma b \in R\Gamma c$. Hence $R\Gamma c$ is a prime ideal. Conversely, suppose that ideal $R\Gamma c$ is prime and we use a result that every prime ideal is almost prime then $R\Gamma c$ is almost prime ideal which is immediate from Lemma 2.

Lemma 4. Suppose I be an almost prime ideal in a Γ-near integral domain R. Then the below statements hold.

(i) If element b is a zero divisor in R/I, in that case $b\Gamma I \subseteq I\Gamma I$.

(ii) If for any ideal J of R such that $I \subseteq J$ where J consists of zero divisors on R/I then $J\Gamma I = I\Gamma I$.

(iii) If I is an invertible ideal then I is prime.

Proof. (i) Let us suppose that there is an element $c \in I$ such that $b\gamma c \in I$. If $b \in I$ then obviously $b\Gamma I \subseteq I\Gamma I$, so let $b \in I$. Since we have $b \notin I$, $c \notin I$ and $b\gamma c \in I$. Furthermore I is an almost prime and $b\gamma c \in I\Gamma I$. Also, for any $x \in I$, $x + c \notin I$ and $b\gamma(x + c) \in I$. Thus, as I is almost prime, $b\gamma(x + c) \in I\Gamma I$. As a result $b\gamma c \in I\Gamma I$, $b\gamma x \in I\Gamma I$. Therefore $b\Gamma I \subseteq I\Gamma I$. (ii) This is obvious from (i). (iii) Let $x\gamma y \in I$ and $x \in I$. Then from (i) $y\Gamma I \subseteq I\Gamma I$. Since I is invertible it is immediate that $y \in I$. Thus I is a prime ideal.

Lemma 5. Let $S^{-1}I$ is an almost prime in the ring $S^{-1}R$, where R be a Γ-near integral domain. Then I be an almost prime ideal in R and S be a multiplicatively closed subset of R disjoint from I.

Proof. Suppose for $x, y \in R$ and $s, t \in S$, $x\gamma y/s\gamma t \in S^{-1}(I - I\Gamma I)$. Then there exists $u, w \in S$ such that $u\gamma x\gamma y \in I$ and $w\gamma x\gamma y \notin I\Gamma I$. Therefore, $u\gamma x\gamma y \in I - I\Gamma I$. Since I is almost prime so $u\gamma x \in I$ or $y \in I$. Therefore, either $x/s \in S^{-1}I$ or $y/t \in S^{-1}I$ implies $S^{-1}I$ is an almost prime ideal.

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