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Lie algebras with BCL algebras

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Abstract. The subject matter of this work is hoping for a new relationship between the Lie algebras and the algebra of logic, which will constitute an important part of our study of "pure" algebra theory. BCL algebras as a class of logical algebras can be generated by a Lie algebra. The opposite is also true that when special conditions occur. The aim of this paper is to prove several theorems on Lie algebras with BCL algebras. I introduce the notion of a "pseudo-association" which I propose as the adjoint notion of BCL algebra in the abelian group.

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1. Introduction

Lie algebras comprise a significant part of Lie group theory (see [1]) and are being vibrantly studied. On the other hand, Lie algebras and their representations are used extensively in physics, notably in quantum mechanics and particle physics. But it is significant that our results show that the Lie algebra and logical algebra are closely linked. Sure, BCL algebras as a class of logical algebras were introduced by Liu in 2011 [2]. The last results was discovered and developed in [3-15]. From set theory perspective, BCL algebras are the algebraic formulations of the set difference together with its properties.

In the paper, I just want to prove that the connectivity theorems but that I have suspected for a long time, which is the relationship between the Lie algebras and the BCL algebras. More importantly, we developed the theory that Lie algebras do have a preferred direction that causes us to the study of logic issues so we can capture new method. Meanwhile, let the theory of BCL algebras becomes strong enough.

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2. Basic definitions

In this section, we list two definitions from the literature that will be used in the sequel.

Definition 2.1 A Lie algebra over a field k is a vector space $\mathbf{g}$ over k together with a k-bilinear map

$$[\ ,\ ]\colon \mathbf{g} \times \mathbf{g} \to \mathbf{g}$$

(called the bracket) such that
(*Lie* 1) $[x, x] = 0$ for all $x \in \mathbf{g}$
(*Lie* 2) $[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0$ for all $x, y, z \in \mathbf{g}$.
A homomorphism of Lie algebras is a k-linear map $\alpha\colon \mathbf{g} \to \mathbf{g}'$
such that

$$\alpha([x, y]) = [\alpha(x), \alpha(y)] \text{ for all } x, y, z \in \mathbf{g}.$$

Condition (*Lie* 2) is called the Jacobi identity. Note that (*Lie* 1) applied to $[x + y, x + y]$ shows that the Lie bracket is skew-symmetric.

$$[x, y] = -\ [y, x] \text{ for all } x, y, z \in \mathbf{g}.$$

Definition 2.2 ([2], Definition 2.1.) A BCL algebra is a triple $(A; \to, 0)$, where A is a nonempty set, $\to$ is a binary operation on A, the following three axioms hold for any $x, y, z \in A$.
(BCL 1) $x \to x = 0$.
(BCL 2) $x \to y = 0$ and $y \to x = 0$ imply $x = y$.
(BCL 3) $(((x \to y) \to z) \to ((x \to z) \to y)) \to ((z \to y) \to x) = 0$.

3. Results

Theorem 3.1 *Let L be Lie algebras. Define*

$$x \to y = [x, y] - [y, x],$$

$$(x \to y) \to z = [[x, y], z],$$

and

$$0 \to x = 0 = [x, x].$$

Then L be BCL algebras.

Proof. Let $x, y, z \in L$. Then

(1) $$x \to x = [x,\, x] - [x,\, x] = 0.$$

(2) $$[x,\, x] = 0 = x \to y \text{ and } [y,\, y] = 0 = y \to x \text{ imply } x = y.$$

(3) $$[[[[x,\, y],\, z],\, [[x,\, z],\, y]],\, [[z,\, y],\, x]] = 0.$$

Clearly, proving (1) and (2).
Now we need to prove (3), we define

$$[x,\, y] = x + y.$$

Then

$$\begin{aligned} [[z,\, y],\, x] &= [x,\, [z,\, y]] \\ &\subseteq [z,\, [y,\, x]] + [y,\, [x,\, z]] \\ &= [[x,\, y],\, z] + [[x,\, z],\, y] \\ &\subseteq [[x,\, y],\, z],\, [[x,\, z],\, y]], \end{aligned}$$

and (3) is proved. We see that L be BCL algebras. □

Theorem 3.2 *Let $x,\, y,\, z \in P$ be BCL algrbras. Then P is abelian Lie algebra iff*

$$x = y = z.$$

Proof. Assume that P is abelian Lie algebra, sine $x,\, y,\, z \in P$, we have

$$[x,\, y] = 0 = [y,\, z].$$

Therefore, $x = y = z$.

Conversely, assume $x = y = z$. To prove that this algebra is a BCL algebra. Let $x,\, y,\, z \in P$. By Theorem 2.1. Then

(4) $$[x,\, x] = 0 = x \to x.$$

(5) $$[x,\, x] = 0 = x \to y \text{ and } [y,\, y] = 0 = y \to x \text{ imply } x = y.$$

(6) $$[0,\, [0,\, x]] = [[0,\, x],\, 0] = (0 \to x) \to 0 = 0 \to 0 = 0.$$

This completes the proof. □

Definition 3.1 Let $(G,\, +)$ be an abelian, $(G;\, -,\, 0)$ be an adjoint BCL algrbras and

$(G; \to, 0)$ be a pseudo-association BCL algrbras. Suppose the following conditions hold:

$(GPA\ 1)$ $\quad x - (0 - y) = x + y.$

$(GPA\ 2)$ $\quad x - y = x \to y.$

Then adjoint group of $(G; -, 0)$ is abelian $(G, +)$, and adjoint BCL algrbras of abelian $(G, +)$ is $(G; \to, 0)$.

Theorem 3.3 *Let P be a pseudo-association BCL algrbra. The bracket*

$$[x, y] = x \to y - (y \to x), \ for\ all\ x, y \in P.$$

Then P be a Lie algebras about the bracket $[\ ,\ ]$ and we use notation P_L, for the Lie algebra is generated by the pseudo-association BCL algrbra.

Proof. By definition of the bracket, $[x, x] = 0$ trivially hoid. To prove bilinear, sine $x_1, x_2, y \in P_L$, and $\lambda_1, \lambda_2 \in P_L$, we have

$$[\lambda_1\ x_1 + \lambda_2\ x_2, y]$$

$$= ((\lambda_1\ x_1 + \lambda_2\ x_2) \to y) - (y \to (\lambda_1\ x_1 + \lambda_2\ x_2))$$

$$= (\lambda_1(x_1 \to y) + \lambda_2(x_2 \to y)) - (\lambda_1(y \to x_1) + \lambda_2(y \to \lambda_2\ x_2))$$

$$= \lambda_1[x_1, y] + \lambda_2[x_2, y]$$

To prove Jacobi identity, sine $x, y, z \in P_L$, we have

$$(7) \qquad [x, [y, z]] = [x, y \to z - (z \to y)]$$

$$= x \to (y \to z - (z \to y)) - ((y \to z - (z \to y)) \to x)$$

$$= x \to y \to z - x \to z \to y - y \to z \to x + z \to y \to x.$$

$$(8) \qquad [y, [z, x]] = y \to z \to x - y \to x \to z - z \to x \to y + x \to z \to y.$$

$$(9) \qquad [z, [x, y]] = z \to x \to y - z \to y \to x - x \to y \to z + y \to x \to z.$$

Therefore, the sum of three brackets, i.e., (7), (8) and (9) satisfying the Jacobi identity

$$(10) \qquad [x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0.$$

□

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