

Interaction of Pulsatile Flow on the Peristaltic Motion of Couple Stress Fluid Through Porous Medium in a Flexible Channel

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Abstract. In this paper, we discuss the peristaltic and pulsatile flow of a couple stress fluid through porous medium in a channel bounded by flexible walls. The non-linear equations governing the flow through porous medium are solved under perturbation scheme. The flow separation, the formation of bolus and phenomenon of reflux and the velocity field and the wall stress are investigated analytically and their behavior is discussed computationally.

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1. Introduction

The peristaltic motion in porous media is of interest in analyzing the reflux conditions which is responsible for understanding the complexity of diseases like interstitial cystitis, bladder stones, bacterial stones, and bacterial affection of kidneys and so on. Rudraiah et al [18] in their work on flows through porous media discussed the problems of peristaltic flow through a porous medium in a channel in order to explain some of the above mentioned pathological phenomenon. Recently, physiologists observed that the intra-uterine fluid flow due to myometrial contractions is peristaltic-type motion and the myometrial contractions may occur in both symmetric and asymmetric directions, DeVries et al. [14], Eytan et al. [12] have developed that the characterization of non-pregnant women uterine constrictions is very complicated as they are composed of variable amplitudes, a range of frequencies and different wavelengths. The interaction of purely periodic mean flow with a peristaltic induced flow is investigated within the framework of a two-dimensional analogue has been studied by N.A.S.Afifi and N.S.Gad [1], Eytan and Elad [13] have developed a mathematical modal of wall-induced peristaltic fluid flow in two-dimensional channel with wave trains having a

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phase difference moving independently on the upper and lower walls to simulate intra-uterine fluid motion in a sagittal cross-section of the uterus. They have used the lubrication theory to obtain a time dependent flow solution in a fixed frame. The results obtained by Eytan and Elad [13] have been used to evaluate the fluid flow pattern in a non-pregnant uterus. The problem of peristaltic transport of an incompressible viscous fluid in an asymmetric channel through a porous medium is analyzed. The flow is investigated in a wave frame of reference moving with velocity of the wave under the assumptions of long wave length has been studied by E.F.Elshehawey et al [9] The possible particle trajectories were also calculated by Eytan and Elad [13] and they have used the results to understand the embryo transport within the uterine cavity before it gets implanted at the uterine wall. El-Shehed [8] studied the pulsatile flow of Newtonian fluid through a stenosed porous medium under the influence of periodic body acceleration. The first study of peristaltic flow through a porous medium is presented by Elshehawey et al [9].

Lukashev [15] has formulated a modal for the peristaltic transport of liquid motion caused by the auto-wave process of mass transport through a porous capillarity wall. Recently El-Shehawey and Husseny [10] studied the effect of porous boundaries on peristaltic transport by using two modals of boundaries porosity. More recently El-Shehawey and Husseny [11] studied the effect of porous boundaries on the peristaltic transport through porous medium. Some authors [5,17 and 18] have studied the steady flow of couple stress fluid, without paying any attention to the pulsatile nature of the blood flow. The rheological studies of steady flow of blood are useful in providing reference information on the rheological characteristics of blood, for clinical purpose, in viscometers. On the other hand, in reality, blood flow in arterial system is pulsatile, with time varying characteristics, which even extends into the capillarity bed. Some authors [2] have studied pulsatile flow of blood assuming different modals. Pulsatile flow [6] of blood with or without body acceleration [16] through stenosed arteries (porous or non-porous) [8] has been studied extensively using various non-Newtonian fluid modals [3, 19, 20]. They have obtained algebraic expression for mass flow rate and velocity profile in terms of unsteadiness parameter α and wall vibration parameter β . Their velocity profiles are in good agreement with the experimental results for only small values of α whereas the mass flow rate results are satisfactory even for large values of α . Using the spin at the boundary, Ariman et al. [2] have studied the steady and pulsatile flow of micro polar fluid and have obtained the exact solution for velocity and cell rotation velocity in the form of Bessel-Fourier series. Since the couple stresses are caused by the presence of suspended particles, the clear fluid cannot support couple stresses near the boundary. Based on this assumption Valanis and Sun [21] have formulated a boundary condition to be satisfied by the velocity at the boundary. Because the theoretical results obtained by Valanis and Sun [21], the study of pulsatile flow of a couple stress fluid with boundary conditions proposed by Valanis and Sun [21] is of interest. Since blood is a suspension of red cells in plasma; it behaves as a non-Newtonian fluid at low shear rate. Chaturvani and Upadhya [3] have developed a method for the study of the pulsatile flow of couple stress fluid through circular tubes. The Poiseuille flow of couple stress fluid has been critically examined by Chaturvani and Rathod [4].

2. Formulation of Problem

We consider a peristaltic flow of a couple stress fluids in a symmetric channel with flexible boundary, existed by an imposed traveling wave along the boundary walls. An oscillatory time dependent flux is being imposed on the peristaltic flow. We make use of long wave length approximation in analyzing the flow. Choosing the cartesian coordinate system $O(x, y)$, the flexible walls are represented by

$$y = \pm a_0 s\left(\frac{z - ct}{\lambda}\right). \quad (1)$$

where a_0 is the wave amplitude, c is the wave velocity, ' λ ' is the wave length and ' s ' is an arbitrary function of the normalized axial coordinate

$$x^* = \left(\frac{X - ct}{\lambda}\right). \quad (2)$$

The governing equation for couple stress incompressible fluid flow through porous medium in vector form is

$$\rho \left[\frac{1}{\delta} \frac{\partial q}{\partial t} + \frac{1}{\delta^2} (q \cdot \nabla) q \right] = -\nabla p + \rho g - \frac{\mu_f}{k} q + \mu_e (\nabla^2 q) - \eta \nabla^4 q \quad (3)$$

The above equations of motion for two-dimensional flow incompressible couple stress fluid in component form are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (4)$$

$$\rho \left[\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = -\frac{\partial p}{\partial x} + \mu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] - \eta \left[\frac{\partial^4 u}{\partial x^4} + \frac{\partial^4 u}{\partial y^4} + 2 \frac{\partial^4 u}{\partial x^2 \partial y^2} \right] - \left[\frac{\mu}{\rho k} u \right] \quad (5)$$

$$\rho \left[\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = -\frac{\partial p}{\partial y} + \mu \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] - \eta \left[\frac{\partial^4 v}{\partial x^4} + \frac{\partial^4 v}{\partial y^4} + 2 \frac{\partial^4 v}{\partial x^2 \partial y^2} \right] - \left[\frac{\mu}{\rho k} v \right] \quad (6)$$

(Suffixes 't', 'x', 'y' denote differentiation with respect to the respective variable).

(u, v) are the velocity components along $O(x, y)$ directions respectively, ' p ' is the fluid pressure, ' ρ ' is the density of the fluid, ' μ ' is the coefficient of the viscosity, ' η ' is the coefficient of couple. The flow being the two dimensional in view of the incompressibility of the flow using (1) we introduce a stream function ' ψ ' such that

$$u = -\psi_y \text{ and } v = \psi_x \quad (7)$$

Substituting (4) in (2) and (3) and eliminating p , the governing equations in terms of ' ψ ' reduces to

$$\frac{\partial}{\partial t} [\nabla^2 \psi] - [\psi_y \nabla^2 \psi_x] + [\psi_x \nabla^2 \psi_y] = \left[\frac{\mu}{\rho} \nabla^4 \psi \right] - \left[\frac{\eta}{\rho} \nabla^6 \psi \right] - \left[\frac{\mu}{\rho k} \nabla^2 \psi \right] \quad (8)$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

The relevant conditions on ψ are

$$\psi_x = 0, \psi_{yy} = 0 \quad \text{on } y = 0 \tag{9}$$

$$\psi = \psi_f [1 + ke^{i\omega t}] - a_0cs \quad \text{on } y = \pm a_0s[x] \tag{10}$$

$$\psi_{yyyy} = 0 \quad \text{on } y = \pm a_0s[x] \tag{11}$$

(6) guarantees the vanishing of the transverse flow on the axis of channel in view the of the symmetry. (7) corresponds to the no slip of the axial velocity on the channel and also guarantees the assumption of the imposed oscillatory flux across the channel. (8) is the boundary condition related to couple stress fluid. We define the following non-dimensional variables.

$$x^* = [x - \frac{ct}{\lambda}], y^* = [\frac{Y}{a_0}], t^* = [\omega t], \psi^* = [\frac{\psi}{a_0c}], \varepsilon = [\frac{a_0}{\lambda}]$$

Introducing these non- dimensional variables in (5) the governing equation in terms of ψ reduces to (on dropping the asterisks)

$$\begin{aligned} & \left[-R\varepsilon^3\psi_{xxx} - R\varepsilon\psi_y\psi_{xxx} - R\varepsilon\psi_y\psi_{xyy} + R\varepsilon^3\psi_y\psi_{xxy} + R\varepsilon\psi_x\psi_{yyy} \right] = \\ & [\varepsilon^4\psi_{xxxx} + \psi_{yyyy} + 2\varepsilon^2\psi_{xxxxx} - RS\psi_{yyyyyy}] - [3R\varepsilon^4S\psi_{xxxxyy} - 3R\varepsilon^2S\psi_{xxyyyy} - D^{-1}\varepsilon^2\psi_{xx}] \end{aligned} \tag{12}$$

where $R = [\frac{\rho ca_0}{\mu}]$, Reynolds number, $S = [\frac{\eta}{\rho ca_0^3}]$, Couple stress parameter, $D^{-1} = [\frac{a_0^2}{k}]$, Inverse Darcy parameter.

The relevant conditions on ψ are

$$\psi_x = 0, \psi_{yy} = 0 \quad \text{on } y = 0 \tag{13}$$

$$\psi = \psi_f [1 + ke^{i\omega t}] - a_0cs \quad \text{on } y = \pm a_0s[x] \tag{14}$$

$$\psi_{yyy} = 0 \quad \text{on } \pm a_0s[x] \tag{15}$$

3. Method of Solution

Under long wave length assumption ($\varepsilon \ll 1$) keeping in view of the condition (11) ψ may be assumed in the form

$$\psi = [\psi_0 + ke^i t \bar{\psi}_0 + \varepsilon[\psi_1 + ke^i t \bar{\psi}_1]] \tag{16}$$

Substituting (16) in (9) and equating the like powers of ε , the equations corresponding to the zeroth and first order steady components are

$$RS\psi_{0yyyyy} - \psi_{0yyy} + \sigma^2\psi_{0yy} = 0 \tag{17}$$

The conditions to be satisfied by ψ_0 and ψ_1 are

$$\psi_0 = 1 - S[x] \text{ on } y = \pm S[x] \quad (18)$$

$$\psi_{0x} = 0 \text{ on } y = 0 \quad (19)$$

$$\psi_{0yy} = 0 \text{ on } y = 0 \quad (20)$$

$$\psi_{0yyy} = 0 \text{ on } y \pm S[x] \quad (21)$$

The equations related to zeroth and first order oscillatory terms are

$$RS\bar{\psi}_{0y y y y} - \bar{\psi}_{0y y y} + \sigma^2 \bar{\psi}_{0y y} = 0 \quad (22)$$

The conditions to be satisfied by $\bar{\psi}$ are

$$\bar{\psi}_0 = 1 \text{ on } y = \pm S[x] \quad (23)$$

$$\bar{\psi}_{0x} = 0 \text{ on } y = 0 \quad (24)$$

$$\bar{\psi}_{0yy} = 0 \text{ on } y = 0 \quad (25)$$

$$\bar{\psi}_{0yyy} = 0 \text{ on } y \pm S[x] \quad (26)$$

Solving (17) and subject to the conditions (18)-(21), we obtain

$$\begin{aligned} \psi_0 &= N_1 + N_2 y + N_3 \cos(\alpha_1 y) \exp(\alpha_2 y) + N_4 \sin(\alpha_1 y) \exp(\alpha_2 y) \\ &+ N_5 \cos(\alpha_1 y) \exp(-\alpha_2 y) + N_6 \sin(\alpha_1 y) \exp(-\alpha_2 y) \end{aligned} \quad (27)$$

Similarly solving (22) and subject to the boundary conditions (23-26), we get

$$\begin{aligned} \bar{\psi}_0 &= N_7 + N_8 y + N_9 \cos(\alpha_1 y) \exp(\alpha_2 y) + N_{10} \sin(\alpha_1 y) \exp(\alpha_2 y) \\ &+ N_{11} \cos(\alpha_1 y) \exp(-\alpha_2 y) + N_{12} \sin(\alpha_1 y) \exp(-\alpha_2 y) \end{aligned} \quad (28)$$

4. Shear Stress and Flux

The shear stress at the upper wall $y = s(x)$, in the dimensional form is given by

$$T = \frac{\frac{1}{2} \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right] \left[1 - \left(\frac{ds}{dx} \right)^2 \right] + \left[\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x} \right] \left(\frac{ds}{dx} \right)}{\left[1 + \left(\frac{ds}{dx} \right)^2 \right]} \quad (29)$$

and is given by

$$\tau = \frac{\frac{1}{2}[A+B][1-m^2] + [C-D]m}{[1+m^2]} \quad (30)$$

The volume flux of the fluid Q is given by the formula $\int_0^s U(y) dy$, and is given by

$$\begin{aligned} Q &= [-e^{y\alpha_2}] \left[e^{y\alpha_2} y N_2 + e^{2y\alpha_2} \cos(\alpha_1 y) N_3 + e^{2y\alpha_2} \sin(\alpha_1 y) N_4 + \cos(\alpha_1 y) N_5 \right] \\ &+ [-e^{y\alpha_2}] \left[\sin(\alpha_1 y) N_6 + e^{(t+y)\alpha_2} k y N_8 + e^{it+2y\alpha_2} k \cos(\alpha_1 y) N_9 + e^{(it+2y\alpha_2)} k \sin(\alpha_1 y) N_{10} \right] \\ &+ [-e^{y\alpha_2}] \left[e^{it} k \cos(\alpha_1 y) N_{11} + e^{it} k \sin(\alpha_1 y) N_{12} \right] \end{aligned}$$

5. Discussion of the Problem and Numerical Results

In this paper, an attempt has been made to study analytically a mathematical model for the peristaltic flow of a bio-fluid through a porous medium under the influence of a pulsatile pressure gradient, considering the bio-fluid to be a couple stress fluids. Such a study possibly explains the pathological situations when a distribution of fatty cholesterol and artery-clogging, blood clots are formed in the lumen of the coronary artery, which can be considered as equivalent to a fictitious porous medium. To begin with, we discuss the phenomenon of the flow separation in this peristaltic flow, observing the behavior of the shear stress on the flexible wall throughout the cycle of oscillations at different points with in a wavelength. From fig1 to 6 corresponds to the behavior of the shear stress in a cycle of oscillations at different points of the wavelength for various in the governing parameters R , S and D^{-1} . We notice that for $R \geq 20$, $s \geq 0.2$ and $D^{-1} < 8 * 10^3$, separation occurs in the flow field (fig 1 to 4). However, for $D^{-1} \geq 8 * 10^3$ no such separation occurs in the flow field irrespective of values R and S . Thus, we may conclude that at sufficiently low permeable medium flow does not experience any separation (fig 5 and 6).

Fig 7-11 corresponds to variation of the axial velocity ' u ' with governing parameters R , D^{-1} and S and fig 12-16 represent the corresponding profiles for transverse velocity ' v ', whenever separation takes place in the flow filed with in the flexible channel, the resulting velocity in the converging (constricted) part of the channel is directed towards the boundary with the fluid moving in clockwise direction while in the dilated part, it moves in the anti-clockwise sense with resulting velocity directed towards the axis of the channel. The axial velocity gradually grows in its magnitude with its minimum of the axis of the channel to the maximum of the flexible wall. In contrast, the transverse velocity attains maximum on the boundary and attaining zero on the mid axis, in accordance with symmetry of the flow. The magnitude of ' u ' enhances with R and S . In other words lesser the permeability lower the axial velocity in the flow field. From 12 -16, we find that the transverse velocity ' v ' also enhances in magnitude with R and S in either of constricted and dilated Channel while reducing with D^{-1} . We also notice from profile of ' v ', that the rate of growth of the magnitude of ' v ' with R and S , is high in comparison to its depreciation with reference to the variation in permeability. The stress on the upper wall and fluid flux are evaluated for variation in the governing parameters and tabulated 1 and 2. We notice from table in both constricted and dilated parts whereas reduces with increase in D^{-1} , fixing the remaining parameters, the stress reduces, whereas it enhances with increase in S keeping R and D^{-1} fixed. An increase in R for any fixed value of S and D^{-1} rapidly increases the stress although its magnitude reduces with increase in D^{-1} . The fluid flux enhances with R or S reduces with increase in D^{-1} . As the permeability of the medium reduces the fluid flux also drops rapidly.

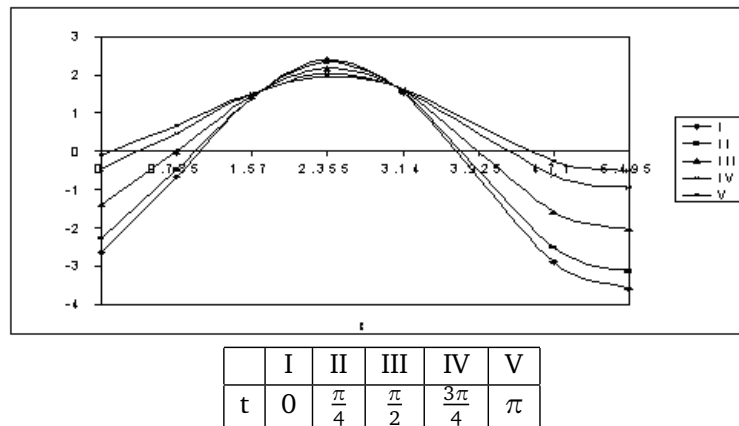


Figure 1: Shear stresses t for $R = 20, S = 0.2, D^{-1} = 7000, \beta = 0.005, P = 1, k = 0.1, y = 1.0043$

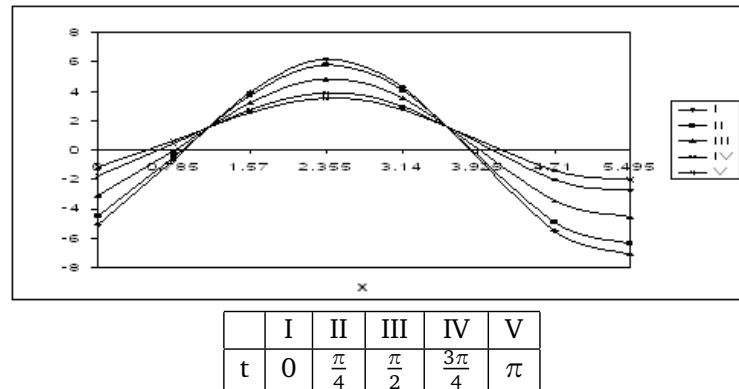


Figure 2: Shear stresses t for $R = 30, S = 0.2, D^{-1} = 7000, \beta = 0.005, P = 1, k = 0.1, y = 1.0043$

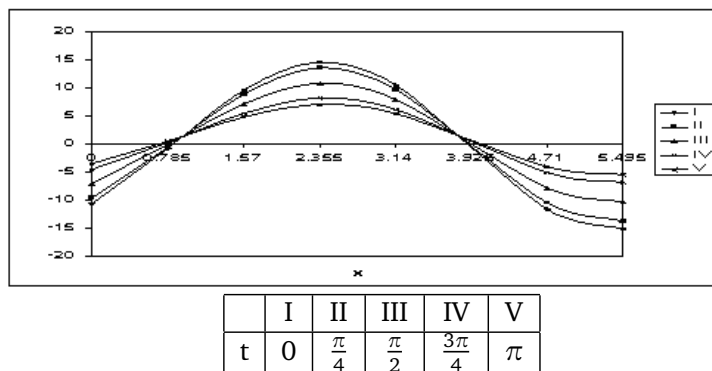


Figure 3: Shear stresses t for $R = 30, S = 0.3, D^{-1} = 7000, \beta = 0.005, P = 1, k = 0.1, y = 1.0043$

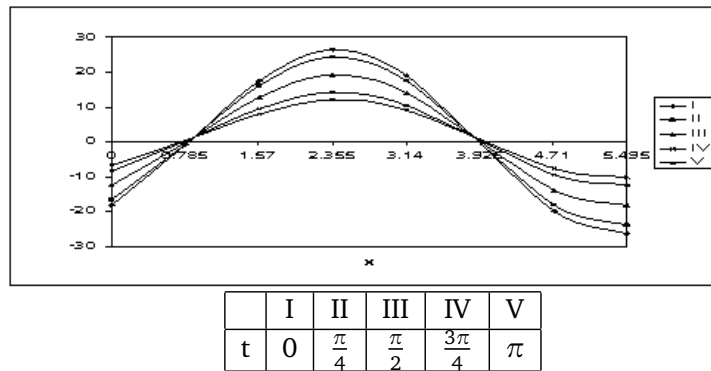


Figure 4: Shear stresses t for $R = 30, S = 0.4, D^{-1} = 7000, \beta = 0.005, P = 1, k = 0.1, \gamma = 1.0043$

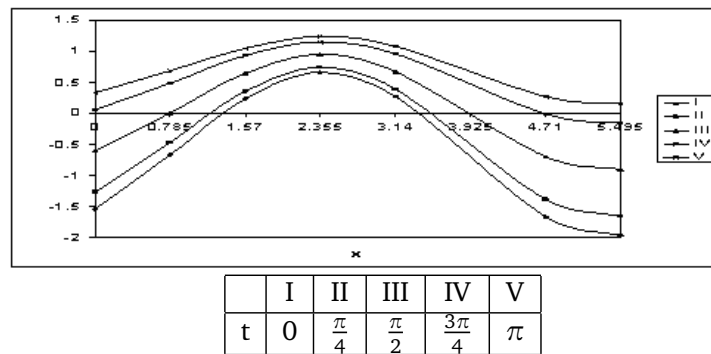


Figure 5: Shear stresses t for $R = 20, S = 0.2, D^{-1} = 8000, \beta = 0.005, P = 1, k = 0.1, \gamma = 1.0043$

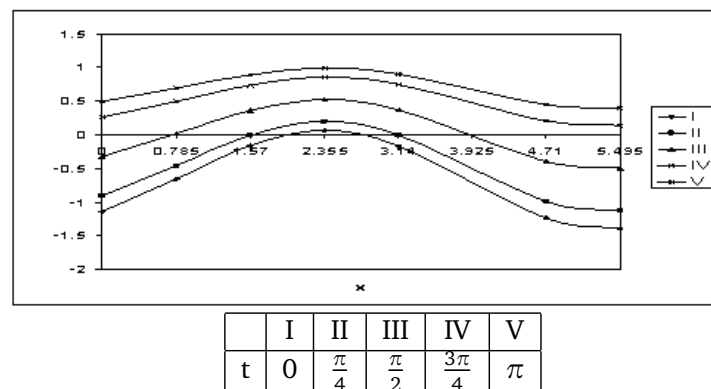


Figure 6: Shear stresses t for $R = 20, S = 0.2, D^{-1} = 9000, \beta = 0.005, P = 1, k = 0.1, \gamma = 1.0043$

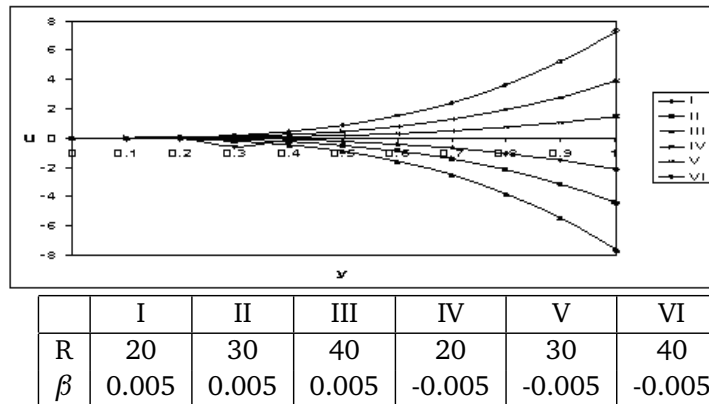


Figure 7: U with R when $R = 20, S = 0.2, D^{-1} = 6000, k = 0.1, x = t = \frac{\pi}{6}$

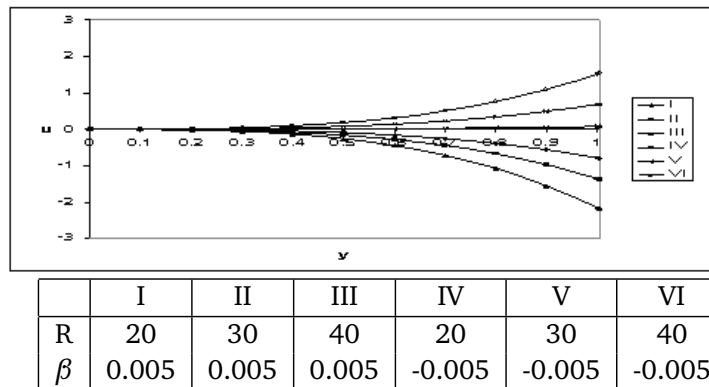


Figure 8: U with R when $R = 20, S = 0.2, D^{-1} = 7000, k = 0.1, x = t = \frac{\pi}{6}$

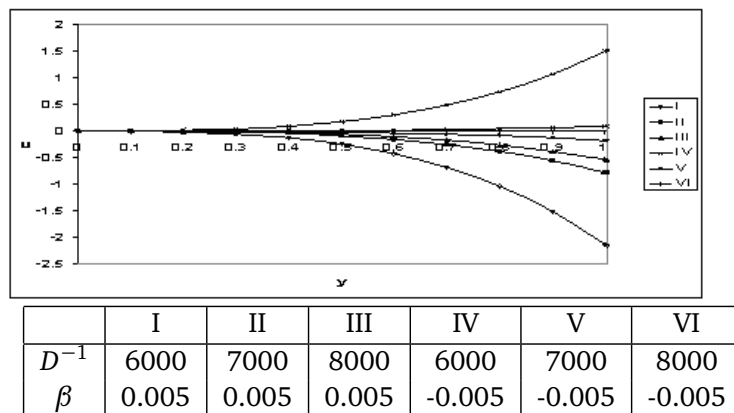
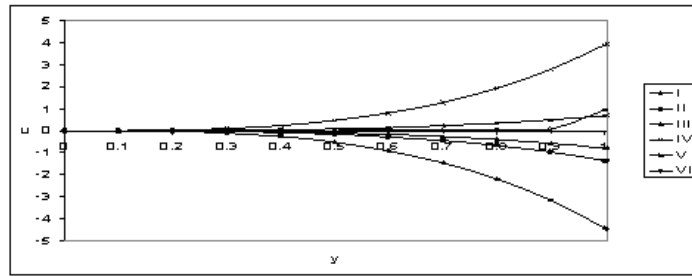
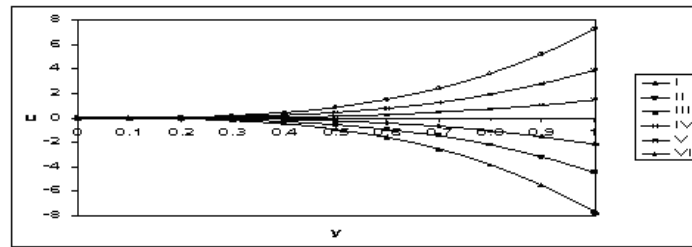


Figure 9: U with D^{-1} when $R = 20, S = 0.2, k = 0.1, x = t = \frac{\pi}{6}$



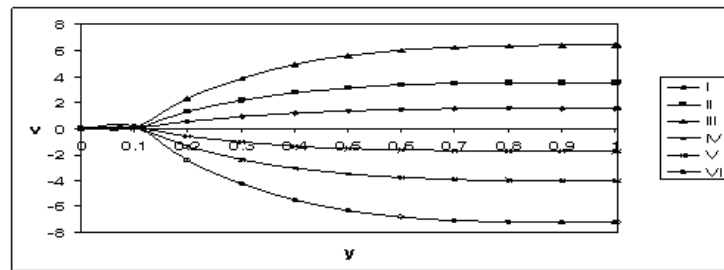
	I	II	III	IV	V	VI
D^{-1}	6000	7000	8000	6000	7000	8000
β	0.005	0.005	0.005	-0.005	-0.005	-0.005

Figure 10: U with D^{-1} when $R = 30, S = 0.2, k = 0.1, x = t = \frac{\pi}{6}$



	I	II	III	IV	V	VI
S	0.2	0.3	0.4	0.2	0.3	0.4
β	0.005	0.005	0.005	-0.005	-0.005	-0.005

Figure 11: U with S when $R = 20, D^{-1} = 6000, k = 0.1, x = t = \frac{\pi}{6}$



	I	II	III	IV	V	VI
R	20	30	40	20	30	40
β	0.005	0.005	0.005	-0.005	-0.005	-0.005

Figure 12: v with R when $D^{-1} = 6000, S = 0.2, k = 0.1, x = t = \frac{\pi}{6}$

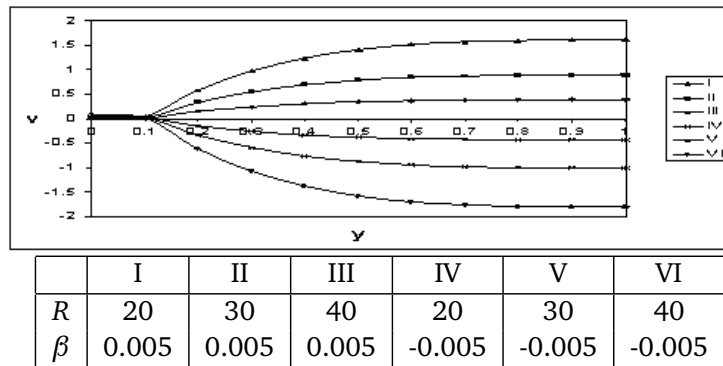


Figure 13: v with R when $D^{-1} = 7000, S = 0.2, k = 0.1, x = t = \frac{\pi}{6}$

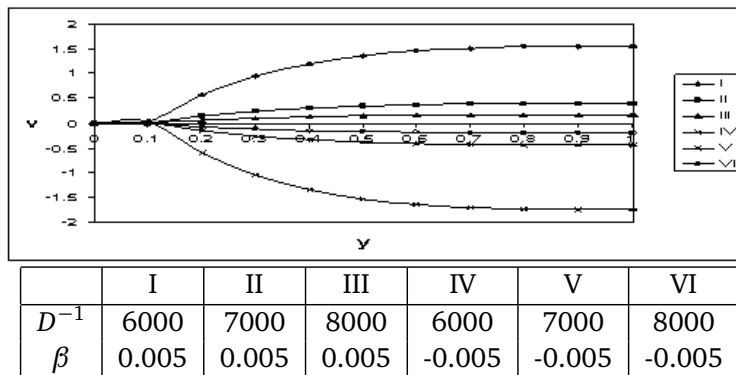


Figure 14: V with D^{-1} when $R = 20, S = 0.2, k = 0.1, x = t = \frac{\pi}{6}$

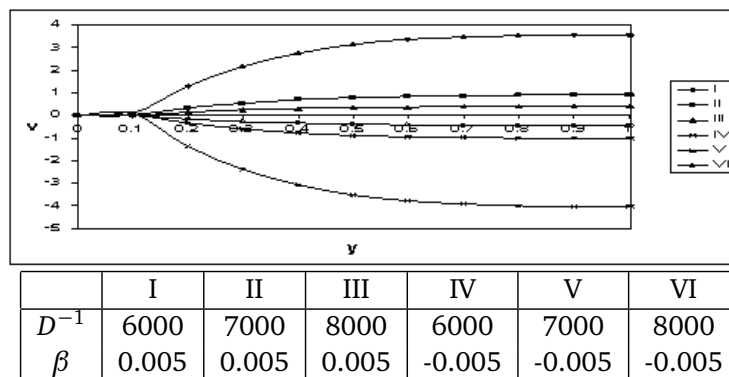
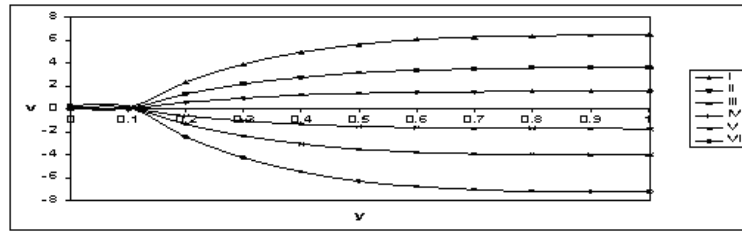


Figure 15: V with D^{-1} when $R = 30, S = 0.2, k = 0.1, x = t = \frac{\pi}{6}$



	I	II	III	IV	V	VI
S	0.2	0.3	0.4	0.2	0.3	0.4
β	0.005	0.005	0.005	-0.005	-0.005	-0.005

Figure 16: V with S when $R = 20, D^{-1} = 6000, k = 0.1, x = t = \frac{\pi}{6}$

Table 1: STRESS AT THE UPPER WALL(τ), $y = 1.0043301, x = 2.355, \beta = 0.005, t = \frac{\pi}{2}$

D^{-1}	I	II	III	IV	V	VI	VII	VIII	IX
6000	8.77	19.5	34.5	19.5	43.58	77.1	34.51	77.15	136.7
7000	2.17	4.86	8.61	4.86	10.87	19.5	18.61	19.27	34.15
8000	0.95	2.14	3.81	2.14	4.82	8.55	3.81	8.55	15.16
8000	0.52	1.19	2.13	1.19	2.702	4.80	2.13	4.80	8.52

	I	II	III	IV	V	VI	VII	VIII	IX
S	0.2	0.2	0.2	0.3	0.3	0.3	0.4	0.4	0.4
R	2	3	4	2	3	4	2	3	4

Table 2: FLUID FLUX(Q), $x = 2.355, \beta = 0.005, t = \frac{\pi}{2}$

D^{-1}	I	II	III	IV	V	VI	VII	VIII	IX
6000	0.84	1.95	3.53	1.95	4.49	8.06	3.53	8.06	14.4
7000	0.21	0.48	0.88	0.48	1.12	2.01	0.88	2.01	3.60
8000	0.09	0.21	0.39	0.21	0.49	0.89	0.39	0.89	1.60
9000	0.05	0.12	0.22	0.12	0.28	0.50	0.22	0.50	0.90

	I	II	III	IV	V	VI	VII	VIII	IX
hline S	0.2	0.2	0.2	0.3	0.3	0.3	0.4	0.4	0.4
R	2	3	4	2	3	4	2	3	4

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