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Soft hypervector spaces and fuzzy soft hypervector spaces

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Abstract. In this paper, the notions of soft hypervector space and fuzzy soft hypervector space are introduced, and several basic properties are provided. Also, we define and analyze the concept of image and pre-image of fuzzy soft hypervector space.

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1. Introduction

The concept of soft set has been introduced in 1999 by Molodtsov [15] as a general mathematical tool for dealing with uncertainties and imprecision. After Molodtsovs work, some research papers have appeared on the algebraic structures of soft set theory. Then, Maji et al. [17] introduced and analyzed several operations on soft sets. In 2007, Aktas and Cagman [2] studied the basic concepts of soft sets theory and soft groups, providing examples to clarify their differences. Moreover, in [1] Acar et al. and in [18] Sun et al. defined the concepts of soft rings and soft modules, respectively. The notion of a fuzzy subset introduced by Zadeh in 1965 [22]. In [12] Maji et al. presented the concept of fuzzy soft sets. In particular, fuzzy soft set theory has been investigated by some researchers, for examples, see [6], [12], [13] and [20].

The hyperstructure theory was introduced by Marty [14] at the 8th Congress of Scandinavian Mathematicians in 1934. As a generalization of hypervector spaces, the fuzzy hypervector spaces are studied by Ameri and et. (see [3, 4]). Jun et al. [8] discussed the applications of fuzzy soft set in BCK/BCI-algebras. Fuzzy soft hypergroups were defined and analysed by Leoreanu-Fotea et al. [10]. In [5] Ameri et al. extended the study application of fuzzy sets and fuzzy soft sets in hypermodules.

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In this paper, applying the notions of soft sets and fuzzy soft sets to the theory of hypervector spaces, we introduce soft hypervector spaces and fuzzy soft set to hypervector spaces, and study some properties of them.

2. Preliminaries

In this section, some definitions and various results of hyperstructure, fuzzy sets and soft sets are presented.

A *hyperstructure* is a non-empty set H together with a mapping $\circ : H \times H \to P^*(H)$, where $P^*(H)$ is the set of all the non-empty subsets of H. If $x \in H$ and $A, B \in P^*(H)$, then by $A \circ B$, $A \circ x$ and $x \circ B$, we mean $A \circ B = \bigcup_{a \in A, b \in B} a \circ b$, $A \circ x = A \circ \{x\}$ and $x \circ B = \{x\} \circ B$, respectively.

Definition 1. *[21] Let K be a field and $(V, +)$ be an abelian group. A* hypervector space *over K is defined to be the quadraple $(V, +, \circ, K)$, where "$\circ$" is a mapping $\circ : K \times V \to P^*(V)$ such that for all $a, b \in K$ and $x, y \in V$ the following conditions hold:*

(*i*) $a \circ (x + y) \subseteq a \circ x + a \circ y$

(*ii*) $(a + b) \circ x \subseteq a \circ x + b \circ y$

(*iii*) $a \circ (b \circ x) = (ab) \circ x$

(*iv*) $a \circ (-x) = (-a) \circ x = -(a \circ x)$

(*v*) $x \in 1 \circ x$.

V is said to be anti-left distributive *if it satisfies the following condition:*
$\forall a, b \in K,\ \forall x \in V,\ (a + b) \circ x \supseteq a \circ x + b \circ x$, *and* strongly left distributive, *if*

$$\forall a, b \in K,\ \forall x \in V,\ (a + b) \circ x = a \circ x + b \circ x$$

In similar way, the *anti right distributive* and *strongly right distributive* hypervector space are defined, respectively. V is called strongly distributive if it is both strongly left and strongly right distributive.

Example 1. *Let $V = M_{2\times 2}(\mathbb{R})$ and $W = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : a, b \in \mathbb{R} \right\}$ and*

$$\circ : \mathbb{R} \times V \to P^*(V)$$

$$\forall r \in \mathbb{R},\ \forall \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in V :\ r \circ \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & rb \\ rc & 0 \end{bmatrix} + W$$

Then $(V, +, \circ, \mathbb{R})$ is a strongly distributive hypervector space.

Definition 2. *[19] A non-empty subset W of V is a* sub-hypervector space *if W is itself a hypervector space with the hyperoperation on V, i.e,*

(i) $W \neq \emptyset$

(ii) $x, y \in W \Rightarrow x - y \in W$

(iii) $a \in K, x \in W \Rightarrow a \circ x \subseteq W$

In this case, we write $W \leq V$. Where $FS(X)$ is the set of all fuzzy subset of X.

Definition 3. *[19] Let $f : X \to Y$ be a mapping and let $\mu \in FS(X)$ and $\nu \in FS(Y)$. Then, $f(\mu) \in FS(Y)$ and $f^{-1}(\nu) \in FS(X)$ respectively, are defined as follows:*

$$f(\mu)(y) = \begin{cases} \bigvee_{x \in f^{-1}(y)} \mu(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise.} \end{cases}$$

For all $y \in Y$ and $f^{-1}(\nu)(x) = \nu(f(x))$, for all $x \in X$.

Let U and E be an initial universe set and a set of parameters, respectively. Let $P(U)$ denote the power set of U and $A \subseteq E$ unless otherwise specified. Molodtsov defined the notion of a soft set in the following way.

Definition 4. *[15] A pair (f, A) is called a* soft set *over U where f is a mapping given by $f : A \to P(U)$.*
In fact, a soft set over U is a parameterized family of subsets of the universe U. For all $a \in A$, $f(a)$ may be considered as the set of a-approximate elements of the soft set (f, A).

Definition 5. *[11] Let (f, A) and (g, B) be two soft sets over common universe U. We say that (f, A) is a* soft subset *of (g, B) if*

(i) $A \subseteq B$

(ii) $a \in A \Rightarrow f(a) \subseteq g(a)$.

Definition 6. *[11] Let (f, A) and (g, B) be two soft sets over common universe U. The* union *of two soft sets (f, A) and (g, B) is the soft set (h, C), where $C = A \cup B$ and h is defined as follows:*

$$h(c) = \begin{cases} f(c) & \text{if } c \in A - B \\ g(c) & \text{if } c \in B - A \\ f(c) \cup g(c) & \text{if } c \in A \cap B. \end{cases}$$

Definition 7. *[11] Let (f, A) and (g, B) be two soft sets over common universe U, such that $A \cap B \neq \emptyset$. The* intersection *of (f, A) and (g, B) is the soft set (h, C), where $C = A \cap B$ and $h(c) = f(c) \cap g(c)$ for all $c \in C$.*

Definition 8. *[11] Let (f, A) and (g, B) be two soft sets over common universe U. Then (f, A)* AND *(g, B) denoted by $(f, A) \wedge (g, B)$ and is defined by $(f, A) \wedge (g, B) = (h, A \times B)$, where $h((a, b)) = f(a) \cap g(b)$, for all $(a, b) \in A \times B$.*

Definition 9. *[12] Let $I = [0, 1]$ and I^U denoted the set of all fuzzy sets on U and $A \subseteq E$. A pair (f, A) is called a* fuzzy soft set *over U, where f is a mapping from A into I^U. That is, for each $a \in A$, $f(a) = f_a : U \to I$, is a fuzzy set on U.*

Definition 10. *[12] Let (f, A) and (g, B) be two fuzzy soft sets over common universe U. We say that (f, A) is a* fuzzy soft subset *of (g, B) and write $(f, A) \sqsubseteq (g, B)$ if*

(i) $A \subseteq B$

(ii) $a \in A \Rightarrow f_a \leq g_a$, *that is f_a is a fuzzy subset of g_a.*

Definition 11. *[12]* Union of two fuzzy soft sets (f, A) *and* (g, B) *over common universe U, denoted by $(f, A) \sqcup (g, B)$ is the fuzzy soft sets (h, C), where $C = A \cup B$ and for all $c \in C$,*

$$h(c) = \begin{cases} f_c & \textit{if } c \in A - B \\ g_c & \textit{if } c \in B - A \\ f_c \vee g_c & \textit{if } c \in A \cap B. \end{cases}$$

Definition 12. *[12]* Intersection of two fuzzy soft sets (f, A) *and* (g, B) *over a common universe U, denoted by $(f, A) \sqcap (g, B)$ is the fuzzy soft set (h, C), where $C = A \cap B \neq \emptyset$ and $h_c = f_c \wedge g_c$ for all $c \in C$.*

Definition 13. *[12] If (f, A) and (g, B) are two fuzzy soft sets, then (f, A)* AND *(g, B) is denoted by $(f, A) \wedge (g, B) = (h, A \times B)$, where $h((a, b)) = h_{a,b} = f_a \wedge g_b$, for all $(a, b) \in A \times B$.*

3. Soft hypervector space

In this section, we introduce the notions of soft hypervector space. Also several basic properties are provided.

Definition 14. *Let V be a hypervector space over a filed K and (f, A) be a soft set over V. Then (f, A) is called a* soft hypervector space *over V if and only if $f(a)$ be a sub-hypervector space of V, for all $a \in A$.*

Example 2. *Let $V = \{P(x) : \ deg(P(x)) \leq 2\}$ and $K = R$ with the following hyperoperation:*

$$\circ : R \times V \to P(V)$$

$$\circ(r, P(x)) = \{r \cdot P(x)\}.$$

Then $(V, +, \circ, R)$ is a strongly distributive hypervector space. If $A = \{\circ, x, x^2\}$ and define the set-valued function $f : A \to P(V)$ by $f(a) = \langle a \rangle = \{r \circ a : r \in R\}$. Then $f(0) = \{\{0\}\}$, $f(x) = \{\{r \cdot x\} : r \in R\}$ and $f(x^2) = \{\{r \cdot x^2\} : r \in R\}$ that all of these are sub-hypervector space of V. Hence, (f, A) is a soft hypervector space over V.

Proposition 1. *Let V be a hypervector space over a field K and let (f, A) and (g, B) be two soft hypervector space over V. If $A \cap B \neq \emptyset$ and for all $x \in A \cap B$, $f(x) \cap g(x) \neq \emptyset$, then their intersection $(f, A) \cap (g, B)$ is a soft hypervector space over V.*

Proof. Straightforward.

Corollary 1. *Let V be a hypervector space over field K and $\{(f_i, A_i) : i \in I\}$ be a non-empty family of soft hypervector space over common universe V. If $\bigcap_{i\in I} A_i \neq \emptyset$ and for all $x \in \bigcap_{i\in I} A_i$, $\bigcap_{i\in I} f_i(x) \neq \emptyset$, then $\bigcap_{i\in I}(f_i, A_i)$ is a soft hypervector space over V.*

Definition 15. *Let (f, A) be a soft hypervector space over V. Then $L(f, A)$ is defined by:*

$$L(f, A) = \bigcap\{(g, B) : (f, A) \subseteq (g, B),\ (g, B) \text{ is a soft set of } V\}.$$

Lemma 1. *$L(f, A)$ is the smallest soft hypervector space over V containing (f, A).*

Proof. Obvious.

Lemma 2. (i) *If (f, A) is a soft hypervector space over V, then $L(f, A) = (f, A)$.*

(ii) *If (f, A) is a soft set over V, then $L(L(f, A)) = L(f, A)$.*

(iii) *Let (f, A) and (g, B) be two soft sets over V and $(f, A) \subseteq (g, B)$. Then $L(f, A) \subseteq L(g, B)$.*

Proof. Obvious.

Lemma 3. *Let $(f_1, A_1), (f_2, A_2)$ and (f_3, A_3) be three soft sets over V. If $(f_1, A_1) \subseteq L(f_3, A_3)$ and $(f_2, A_2) \subseteq L(f_3, A_3)$, then $L((f_1, A_1)\bigcup(f_2, A_2)) \subseteq L(f_3, A_3)$.*

Proof. Since $(f_1, A_1) \subseteq (f_3, A_3)$ and $(f_2, A_2) \subseteq (f_3, A_3)$, therefore we have $A_1 \cup A_2 \subseteq A_3$.
Let $x \in A_1 \cup A_2$ and $(f, A) = (f_1, A_1) \cup (f_2, A_2)$.
If $x \in A_1 - A_2$, then $f(x) = f_1(x) \subseteq f_3(x)$, If $x \in A_2 - A_1$, then $f(x) = f_2(x) \subseteq f_3(x)$, If $x \in A_1 \cap A_2$, then $f(x) = f_1(x) \cap f_2(x) \subseteq f_3(x)$. Thus $(f_1, A_1) \cup (f_2, A_2) \subseteq (f_3, A_3)$. So, we have

$$L((f_1, A_1) \cup (f_2, A_2)) \subseteq L(f_3, A_3).$$

Lemma 4. *Let (f, A) and (g, B) be two soft sets over V. Then*

$$L((f, A) \cup (g, B)) = L(L(f, A) \cup L(g, B)).$$

Proof. Clearly, $L((f, A) \cup (g, B)) \subseteq L(L(f, A) \cup L(g, B))$. Moreover, since $L(f, A) \subseteq L((f, A)\cup(g, B))$ and $L(g, B) \subseteq L((f, A)\cup(g, B))$, thus $L(L(f, A)\cup L(g, B)) \subseteq L((f, A)\cup (g, B))$.

Example 3. *Consider $V = \{0, 1, 2\}$ and $K = \{0, 1, 2\}$ with the following operation:*

$+$	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

$\cdot$	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

We set:

$$\circ : K \times V \to P(V)$$

$$\circ(a,x) = \begin{cases} \{1\} & \text{if } a = x = 2 \\ \{a \cdot x\} & \text{otherwise.} \end{cases}$$

Then $(V, +, \circ, K)$ *is a strongly distributive hypervector space over* K. *Let* (f, A) *and* (g, B) *be two soft sets over* V, *where* $A = \{0, 1, 2\}$, $B = \{0, 1, 2\}$, $f(0) = \{0\}$, $f(1) = \{0, 1\}$, $g(0) = \{0\}$, $g(1) = \{0, 2\}$ *and* $g(2) = \{2\}$. *Suppose* $C = A \cap B$ *and* $(h, c) = (f, A) \cap (g, B)$, *then* $h(0) = f(0) \cap g(0) = \{0\}$ *and* $h(1) = f(1) \cap g(1) = \{0\}$. *Therefore*

$$L((f, A) \cap (g, B)) = L((h, c)) = (h, c).$$

Let $L((f, A)) = (\alpha, A_0)$ *and* $L((g, B)) = (\beta, B_0)$. *Then* $\alpha(0) = \{0\}$, $\alpha(1) = V$, $\beta(0) = \{0\}$ *and* $\beta(1) = \beta(2) = V$. *Assume* $L((f, A)) \cap L((g, B)) = (\gamma, C_0)$, *where* $C_0 = A_0 \cap B_0$ *and* $\gamma(0) = \alpha(0) \cap \beta(0) = \{0\}$ *and* $\gamma(1) = \alpha(1) \cap \beta(1) = V$.
So,

$$L(L((f, A)) \cap L((g, B))) = L((\gamma, C_0)) = (\gamma, C_0).$$

Since $\gamma(1) \neq h(1)$, *thus*

$$L(L((f, A)) \cap L((g, B))) \neq L((f, A) \cap (g, B)).$$

Remark 1. *Let* (f, A) *and* (g, B) *be two soft sets over* V. *Then in general,*

$$L((f, A) \cap (g, B)) \neq L(L((f, A)) \cap L((g, B))).$$

Theorem 1. *Let* (f, A) *and* (g, B) *be two soft hypervector space over* V. *If* $(f, A) \subseteq (g, B)$, *then* $(f, A) \cup (g, B)$ *is a soft hypervector space over* V.

Proof. Let $(f, A) \cup (g, B) = (h, C)$. Since $(f, A) \subseteq (g, B)$, thus $A \subseteq B$ and $f(a) \subseteq g(a)$ for all $a \in A$. Hence $C = A \cup B = B$ and for all $c \in C$, we have

$$h(c) = \begin{cases} g(c) & \text{if } c \in B - A \\ f(c) \cup g(c) & \text{if } c \in A \cap B = A \end{cases} = g(c)$$

Since $g(c)$ is a sub-hypervector space of V. Therefore, $(f, A) \cup (g, B)$ is a soft hypervector space over V.

It is clear that we have the followings:

Proposition 2. *Let* (f, A) *and* (g, B) *be two soft hypervector space over* V. *Then* (f, A) *AND* (g, B) *is a soft hypervector space over* V.

4. Fuzzy soft hypervector space

In this section, the notions of a fuzzy soft hypervector space is introduced, and several basic properties of fuzzy soft hypervector space are provided.

Definition 16. *A fuzzy subset μ of a hypervector space V over a field K is said to be a* fuzzy sub-hypervector space *of V if and only if for all $x, y \in V$ and $r \in K$,*

(*i*) $\mu(x+y) \geq \mu(x) \wedge \mu(y)$

(*ii*) $\mu(-x) \geq \mu(x)$

(*iii*) $\inf\limits_{z \in r \circ x} \mu(z) \geq \mu(x)$.

Example 4. *In example 1, if fuzzy subset μ of V is defined by*

$$\mu(x) = \begin{cases} 1 & \text{if } x \in W \\ t & \text{otherwise,} \end{cases}$$

where $t \in [0, 1)$, then μ is a fuzzy sub-hypervector space of V.

Definition 17. *Let V be a hypervector space over a field K and (f, A) be a fuzzy soft set over V. Then (f, A) is said to be a* fuzzy soft hypervector space *over V if and only if for all $a \in A$, f_a is a fuzzy sub-hypervector space of V.*

Example 5. *In Example 3, let $A = \{0, 1, 2\}$ and defined the set-valued function $f : A \to I^v$ by*

$$f_0(x) = \begin{cases} \frac{1}{2} & \text{if } x = 0 \\ 0 & \text{otherwise,} \end{cases} \qquad f_1(x) = f_2(x) = \begin{cases} \frac{1}{3} & \text{if } x = 1 \text{ or } x = 2 \\ \frac{1}{2} & \text{if } x = 0 \end{cases}$$

Since f_0, f_1 and f_2 are fuzzy sub-hypervector space of V, then (f, A) is a fuzzy soft hypervector space over V.

Lemma 5. *Let (f, A) be a fuzzy soft set over hypervector space V. If (f, A) is a fuzzy soft hypervector space, then for all $x, y \in V$ and $a \in A$, we obtain*

(*i*) $f_a(x-y) \geq f_a(x) \wedge f_a(y)$

(*ii*) $f_a(-x) = f_a(x)$

(*iii*) $f_a(x) = \inf\limits_{z \in 1 \circ x} f_a(z)$.

Proof. (i) For all $x, y \in V,\ a \in A$, we obtain

$$f_a(x-y) = f_a(x+(-y)) \geq f_a(x) \wedge f_a(-y) \geq f_a(x) \wedge f_a(y)$$

(ii) For every $x \in V,\ a \in A$, we have

$$f_a(x) = f_a(-(-x)) \geq f_a(-x) \geq f_a(x)$$

Thus $f_a(-x) = f_a(x)$.
(iii) Since $x \in 1 \circ x$, therefore for all $x \in V$ and $a \in A$, we obtain

$$f_a(x) \geq \underset{z \in 1 \circ x}{inf} f_a(z) \geq f_a(x).$$

Hence $f_a(x) = \underset{z \in 1 \circ x}{inf} f_a(z)$.

Theorem 2. *Let V be a hypervector space over a field K. If (f, A) and (g, B) are two fuzzy soft hypervector space over V, then $(f, A) \sqcap (g, B)$ is a fuzzy soft hypervector space over V.*

Proof. Let $(f, A) \sqcap (g, B) = (h, C)$, where $C = A \cap B \neq \emptyset$ and for all $x \in V,\ c \in C,\ h_c(x) = f_c(x) \wedge g_c(x)$.
If $y \in V$, then $h_c(x+y) = f_c(x+y) \wedge g_c(x+y) \geq (f_c(x) \wedge f_c(y)) \wedge (g_c(x) \wedge g_c(y)) = (f_c(x) \wedge g_c(x)) \wedge (f_c(y) \wedge g_c(y)) = h_c(x) \wedge h_c(y)$.
Moreover,

$$h_c(-x) = f_c(-x) \wedge g_c(-x) \geq f_c(x) \wedge g_c(x) = h_c(x)$$

Also for every $r \in K$,

$$\underset{z \in r \circ x}{inf} h_c(z) = \underset{z \in r \circ x}{inf} (f_c(z) \wedge g_c(z)) = \underset{z \in r \circ x}{inf} f_c(z) \wedge \underset{z \in r \circ x}{inf} g_c(z) \geq f_c(x) \wedge g_c(x) = h_c(x).$$

Therefore, $(h, C) = (f, A) \sqcap (g, B)$ is a fuzzy soft hypervector space over V.

Theorem 3. *Let (f, A) and (g, B) be two fuzzy soft hypervector space over V. If $A \cap B = \emptyset$, then $(f, A) \sqcup (g, B)$ is a fuzzy soft hypervector space over V.*

Proof. Let $(f, A) \sqcup (g, B) = (h, C)$. Since $A \cap B = \emptyset$, thus for all $c \in C = A \cup B$,

$$h(c) = h_c = \begin{cases} f_c & \text{if } c \in A - B \\ g_c & \text{if } c \in B - A. \end{cases}$$

Since f_c and g_c are fuzzy sub-hypervector space of V, therefore, $(f, A) \sqcup (g, B)$ is a fuzzy soft hypervector space over V.

Theorem 4. *If (f, A) and (g, B) be two fuzzy soft hypervector space over V, then $(f, A) \wedge (g, B)$ is a fuzzy soft hypervector space over V.*

Proof. Let $(f,A)\wedge(g,B)=(h,A\times B)$. Since for all $a\in A,\ b\in B,\ f_a$ and g_b are fuzzy sub-hypervector space of V, so is $h(a,b)=h_{a,b}=f_a\wedge g_b$ for every $(a,b)\in A\times B$. Thus $(f,A)\wedge(g,B)$ is a fuzzy soft hypervector space over V.

Definition 18. Sum *of two fuzzy soft sets (f,A) and (g,B) over a common universe U, denoted by $(f,A)+(g,B)$ is the fuzzy soft set (h,C), where $C=A\cup B$ and for all $c\in C$,*

$$h(c)=\begin{cases} f_c+g_c & \text{if } c\in A\cap B\\ f_c & \text{if } c\in A-B\\ g_c & \text{if } c\in B-A.\end{cases}$$

And for every $x\in V$,

$$(f_c+g_c)(x)=\bigvee\{f_c(y)\wedge g_c(z):\ y,z\in V,\ y+z=x\}.$$

Theorem 5. *Let (f,A) and (g,B) be two fuzzy soft hypervector space over V. Then $(f,A)+(g,B)$ is a fuzzy soft hypervector space over V.*

Proof. Let $(f,A)+(g,B)=(h,C)$, where $C=A\cup B$ and

$$h_c(x)=\begin{cases} (f_c+g_c)(x) & \text{if } c\in A\cap B\\ f_c(x) & \text{if } c\in A-B\\ g_c(x) & \text{if } c\in B-A\end{cases}$$

for all $c\in C$ and $x\in V$.
If $c\in A-B$ or $c\in B-A$, the proof is straightforward.
Let $c\in A\cap B$, $h_c(u+v)=a,\ h_c(u)=a'$ and $h_c(v)=a''$ for all $u,v\in V$. Then

$$\exists y_0,z_0\in V: y_0+z_0=u+v,\ h_c(u+v)=(f_c+g_c)(u+v)=f_c(y_0)\wedge g_c(z_0)=a,$$

$$\exists y_0',z_0'\in V: y_0'+z_0'=u,\ h_c(u)=(f_c+g_c)(u)=f_c(y_0')\wedge g_c(z_0')=a',$$

and

$$\exists y_0'',z_0''\in V: y_0''+z_0''=v,\ h_c(v)=(f_c+g_c)(u+v)=f_c(y_0'')\wedge g_c(z_0'')=a''.$$

Since
$a=f_c(y_0)\wedge g_c(z_0)\geq f_c(y_0'+y_0'')\wedge g_c(z_0'+z_0'')\geq f_c(y_0')\wedge f_c(y_0'')\wedge g_c(z_0')\wedge g_c(z_0'')=(f_c(y_0')\wedge g_c(z_0'))\wedge(f_c(y_0'')\wedge g_c(z_0''))=a'\wedge a''$. Therefore, $h_c(u+v)\geq h_c(u)\wedge h_c(v)$.
On the other hand, since f_c and g_c are fuzzy soft hypervector space, thus $f_c(-y)\geq f_c(y)$ and $g_c(-z)\geq g_c(z)$ for all $y,z\in V$. So

$$\bigvee\{f_c(-y)\wedge g_c(-z):\ (-y)+(-z)=-v\}\geq\bigvee\{f_c(y)\wedge g_c(z):\ y+z=v\}.$$

Hence

$$\bigvee\{f_c(a)\wedge g_c(b):\ a+b=-v\}\geq\bigvee\{f_c(y)\wedge g_c(z):\ y+z=v\}.$$

Therefore

$$h_c(-v) \geq h_c(v) \quad \forall v \in V, \; c \in A \cap B.$$

Now, we show that

$$\underset{z \in r \circ v}{inf} \, (f+g)_c(z) \geq (f+g)_c(v).$$

Suppose

$$\underset{z \in r \circ v}{inf} \, (f+g)_c(z) = \underset{z \in r \circ v}{inf} \, (\bigvee \{f_c(y) \wedge g_c(w) : \; y + w = z\}),$$

and

$$(f+g)_c(v) = \bigvee \{f_c(y') \wedge g_c(w') : \; y' + w' = v\}.$$

Since $y' + w' = v$, this implies that $r \circ v = r \circ (y' + w') \subseteq r \circ y' + r \circ w'$. Thus, since $z \in r \circ v$, then there exist $a, b \in V$ such that $a \in r \circ y', \; b \in r \circ w'$ and $z = a + b$. Also, since

$$\underset{a \in r \circ y'}{inf} \, f_c(a) \geq f_c(y'), \quad \underset{b \in r \circ w'}{inf} \, g_c(b) \geq g_c(w').$$

We conclude that, $f_c(a) \geq f_c(y')$ and $g_c(b) \geq g_c(w')$ for all $a \in r \circ y'$, and for all $b \in r \circ w'$. Therefore, $f_c(a) \wedge g_c(b) \geq f_c(y') \wedge g_c(w') \;\; \forall y', w' \in V$, where $y' + w' = v$ for all $z = a + b \in r \circ v$. Hence

$$\bigvee \{f_c(a) \wedge g_c(b) : a + b = z\} \geq \bigvee \{f_c(y') \wedge g_c(w') : y' + w' = v\}.$$

So

$$\underset{z \in r \circ v}{inf} \, (\bigvee \{f_c(a) \wedge g_c(b) : a + b = z\}) \geq \bigvee \{f_c(y') \wedge g_c(w') : y' + w' = v\}.$$

This completes the proof.

Definition 19. *[5] Let (f, A) be a fuzzy soft set over hypervector space V. The soft set*

$$(f, A)_\alpha = \{(f_a)_\alpha : a \in A\} \text{ where } (f_a)_\alpha = \{x \in V : f_a(x) \geq \alpha\},$$

for all $\alpha \in (0, 1]$, is called an α-level soft set *of the fuzzy soft set (f, A), where $(f_a)_\alpha$ is an α-level subset of the fuzzy set f_a.*

Theorem 6. *Let (f, A) be a fuzzy soft set over hypervector space V of field K. Then (f, A) is a fuzzy soft hypervector space over V if and only if for all $a \in A$ and for arbitrary $\alpha \in (0, 1]$ with $(f_a)_\alpha \neq \emptyset$, the α-level soft set $(f, A)_\alpha$ is a soft hypervector space over V.*

Proof. Let (f, A) be a fuzzy soft hypervector space over V, suppose $\alpha \in (0, 1]$ with $(f_a)_\alpha \neq \emptyset$ and $x, y \in (f_a)_\alpha$. Since $f_a(x) \geq \alpha$ and $f_a(y) \geq \alpha$. Therefore, $f_a(x - y) \geq \alpha$. Thus, $x - y \in f_a$. Furthermore, $\underset{z \in r \circ x}{inf} \, f_a(z) \geq f_a(x) \geq \alpha \;\; \forall r \in K, \; \forall z \in r \circ x$. Hence $f_a(z) \geq \alpha$, so $z \in (f_a)_\alpha$. Thus, $r \circ x \subseteq (f_a)_\alpha$ for all $r \in K$. We obtain that $(f_a)_\alpha$ is a sub-hypervector space of V for all $a \in A$. Consequently, $(f, A)_\alpha$ is a soft hypervector space over V.

Conversely, let $(f, A)_\alpha$ be a fuzzy soft hypervector space over V for all $\alpha \in (0, 1]$. Let there

exist $x_0, y_0 \in V$ such that $f_a(x_0+y_0) < f_a(x_0) \wedge f_a(y_0)$. Let $f_a(x_0+y_0) = \delta$, $f_a(x_0) = \beta$ and $f_a(y_0) = \gamma$. We have $\delta < min\{\beta, \gamma\}$. Let $\alpha = \dfrac{\delta + min\{\beta, \gamma\}}{2}$, then $\delta < \alpha < min\{\beta, \gamma\}$. Since $\beta > min\{\beta, \gamma\} > \alpha$ and $\gamma > min\{\beta, \gamma\} > \alpha$, we obtain $f_a(x_0) \geq \alpha$ and $f_a(y_0) \geq \alpha$, that is $x_0, y_0 \in (f_a)_\alpha$. This contradicts with the fact $(f, A)_\alpha$ is a soft hypervector space over V. Therefore, $f_a(x+y) \geq f_a(x) \wedge f_a(y)$ for all $x, y \in V$.
Moreover, let there exists $x_0 \in V$ such that $f_a(-x_0) < f_a(x_0)$. Let $f_a(-x_0) = \beta$, $f_a(x_0) = \gamma$ and $\alpha = \dfrac{\beta + \gamma}{2}$. Since $\beta < \alpha < \gamma$ and $f_\alpha(-x_0) = \beta < \alpha$, therefore $-x_0 \notin (f_a)_\alpha$. But, $f_a(x_0) = \gamma \geq \alpha$, that is, $x_0 \in (f_a)_\alpha$. This is a contradiction. Hence for all $x \in V$, $f_a(-x) \geq f_a(x)$.
Now, let there exists $v_0 \in V$ such that $\underset{z \in r o v_0}{inf} f_a(z) < f_a(v_0)$. Let $f_a(v_0) = \gamma$, $\underset{z \in r o v_0}{inf} f_a(z) = \beta$ and $\alpha = \dfrac{\beta + \gamma}{2}$.
Since $\beta < \alpha < \gamma$ and $f_a(v_0) = \gamma > \alpha$, that is $v_0 \in (f_a)_\alpha$, so $r \circ v_0 \subseteq (f_a)_\alpha$, $\forall r \in K$. Therefore, $z \in r \circ v_0 \Rightarrow \underset{z \in r o v_0}{inf} f_a(z) = \beta > \alpha$. This is a contradiction. Hence $\underset{z \in r o v}{inf} f_a(z) \geq f_a(v) \quad \forall v \in V$, for all $r \in K$. Therefore, (f, A) is a fuzzy soft hypervector space over V.

Definition 20. *[5] Let (f, A) be a fuzzy soft hypervector space over V. Then, the soft set $(f, A)_0$ is defined by:*

$$(f, A)_0 = \{(f_a)_0 : a \in A\} \text{ where } (f_a)_0 = \{x \in V : f_a(x) = f_a(0)\}.$$

Theorem 7. *Let (f, A) be a fuzzy soft hypervector space of V over field K. Then $(f, A)_0$ is a soft hypervector space over V.*

Proof. For every $a \in A$ and for all $x, y \in (f_a)_0$, we have $f_a(x) = f_a(0)$, $f_a(y) = f_a(0)$, therefore $f_a(x-y) \geq f_a(x) \wedge f_a(y) = f_a(0)$. Since $f_a(0) \geq f_a(x-y)$, thus $f_a(x-y) = f_a(0)$. Hence $x-y \in (f_a)_0$. Now, let $x \in (f_a)_0$ and $r \in K$. Since $\underset{z \in r o x}{inf} f_a(z) \geq f_a(x) = f_a(0)$. Thus $f_a(z) = f_a(0)$ for all $z \in r \circ x$. Hence $r \circ x \subseteq (f_a)_0$. Therefore $(f_a)_0$ is a sub-hypervector space of V, consequently $(f, A)_0$ is a soft hypervector space over V.

Definition 21. *Let (f, A) be a fuzzy soft hypervector space over V. Then, the soft set $(f, A)^0$ is defined by:*

$$(f, A)^0 = \{(f_a)^0 : a \in A\} \text{ where } (f_a)^0 = \{x \in V : f_a(x) > 0\}.$$

It is clear that we have the followings:

Theorem 8. *Let (f, A) be a fuzzy soft set over hypervector space V of field K. Then $(f, A)^0$ is a fuzzy soft set over hypervector space V.*

It is clear that we have the followings:

Theorem 9. *Let (f, A) and (g, B) be two fuzzy soft set over hypervector space V. Then*

(i) $(f, A)_0 \sqcap (g, B)_0 \sqsubseteq (h, c)_0$

(ii) $f_c(0) = g_c(0) \Rightarrow (f, A)_0 \sqcap (g, B)_0 = (h, C)_0, \quad \forall c \in A \cap B.$

5. Image and pre-image of fuzzy soft hypervector space

In this section, the theorem of homomorphic image and homomorphic pre-image of fuzzy soft hypervector spaces are studied.

Definition 22. *Let*

$$F_s^s(V, E) = \{(f, A) : A \subseteq E \text{ and } (f, A) \text{ is a fuzzy soft hypervector space over } V\},$$

then $F_s^s(V, E)$ is called a fuzzy soft hypervector space set class over V.

Definition 23. *[5] Let $\phi : X \to Y$ and $\Psi : A \to B$ be two functions, where A and B are parameter sets for the crisp sets X and Y, respectively. Then the pair (φ, ψ) is called a* fuzzy soft function from X to Y.

Definition 24. *[5] Let $F_s^s(V, E)$ and $F_s^s(V', E')$ be two fuzzy soft hypervector space set classes. Let $\phi : V \to V'$ and $\psi : E \to E'$ be mappings. If $(f, A) \in F_s^s(V, E)$, the* image *of (f, A) under the function (ϕ, ψ), denoted by $(\phi, \psi)(f, A) = (\phi(f), \psi(A))$, where*

$$\phi(f)_b(v') = \begin{cases} \bigvee_{x \in \phi^{-1}(v')} \bigvee_{a \in A \cap \phi^{-1}(b)} f_a(x) & \text{if } \phi^{-1}(v') \neq \emptyset \\ 0 & \text{otherwise,} \end{cases}$$

for all $b \in \psi(A)$ and $v' \in V'$.

Definition 25. *[5] Let $(g, B) \in F_s^s(V', E')$, then the* pre-image *of (g, B) under the fuzzy soft function (ϕ, ψ), denoted $(\phi, \psi)^{-1}(g, B) = (\phi^{-1}(g), \psi^{-1}(B))$, where*

$$\phi^{-1}(g)_\alpha(v) = g_{\psi(\alpha)}(\phi(v)), \text{ for all } \alpha \in \psi^{-1}(B) \text{ and } v \in V.$$

Theorem 10. *Let V and V' be two hypervector space over a field K. If $(g, B) \in F_s^s(V', E')$ and (ϕ, ψ) is a fuzzy soft homomorphism from V to V', then $(\phi, \psi)^{-1}(g, B) \in F_s^s(V, E)$.*

Proof. If $\alpha \in \psi^{-1}(B), \ u, v \in V$, then

$$\phi^{-1}(g)_\alpha(v + u) = g_{\psi(\alpha)}(\phi(v + u)) = g_{\psi(\alpha)}(\varphi(v) + \psi(u))$$

$$\geq g_{\psi(\alpha)}(\phi(v)) \wedge g_{\psi(\alpha)}(\phi(u)) = \phi^{-1}(g)_\alpha(u) \wedge \phi^{-1}(g)_\alpha(v).$$

Moreover, for all $v \in V$,

$$\phi^{-1}(g)_\alpha(-v) = g_{\psi(\alpha)}(\phi(-v)) = g_{\psi(\alpha)}(-\phi(v)) \geq g_{\psi(\alpha)}(\phi(v)) = \phi^{-1}(g)_\alpha(v).$$

Also, for all $r \in K, v \in V$ and $z \in r \circ v$, we have

$$\underset{z\in r\circ v}{inf}\, \phi^{-1}(g)_\alpha(z) = \underset{\phi(z)\in r\circ\phi(v)}{inf} g_{\psi(\alpha)}(\phi(z)) \geq g_{\psi(\alpha)}(\phi(v)) = \phi^{-1}(g)_\alpha(v).$$

Therefore, $(\phi, \psi)^{-1}(g, B) \in F_s^s(V, E)$.

Theorem 11. *Let V, V' and V'' be three hypervector space over a field K. If $(h, C) \in F_s^s(V'', E'')$ and $(\phi_1, \psi_1), (\phi_2, \psi_2)$ are two fuzzy soft homomorphisms from V to V' and from V' to V'', respectively. Then,*

$$(\phi_2 \circ \phi_1, \psi_2 \circ \psi_1)^{-1}(h, C) \in F_s^s(V, E).$$

Proof. Show that, for all $a \in \psi_1^{-1}(\psi_2^{-1}(c))$ and $v \in V$,

$$\begin{aligned}(\phi_2 \circ \phi_1)^{-1}(h) &= \phi_1^{-1}(\phi_2^{-1}(h))_a(v)\\ &= \phi_2^{-1}(h)_{\psi_1(a)}(\phi_1(v))\\ &= h_{\psi_2(\psi_1(a))}(\phi_2(\phi_1(v)))\\ &= h_{(\psi_2\circ\psi_1)(a)}((\phi_2 \circ \phi_1)(v)),\end{aligned}$$

is a fuzzy soft hypervector space over V.

Let $a \in \psi_1^{-1}(\psi_2^{-1}(c)),\ u, v \in V$. Then

$$\begin{aligned}&(\phi_2 \circ \phi_1)^{-1}(h)_a(u + v)\\ &= h_{(\psi_2\circ\psi_1)(a)}(\phi_2 \circ \phi_1)(v + u)\\ &= h_{(\psi_2\circ\psi_1)(a)}(\phi_2(\phi_1(v)) + \phi_2(\phi_1(u)))\\ &\geq h_{(\psi_2\circ\psi_1)(a)}(\phi_2 \circ \phi_1(v)) \wedge h_{(\psi_2\circ\psi_1)(a)}(\phi_1 \circ \phi_2(u)).\end{aligned}$$

Moreover, for all $v \in V$, we have

$$\begin{aligned}(\phi_2 \circ \phi_1)^{-1}(h)_a(-v) &= h_{(\psi_2\circ\psi_1)(a)}(\phi_2 \circ \phi_1)(-v)\\ &= h_{(\psi_2\circ\psi_1)(a)}(-(\phi_2 \circ \phi_1(v))\\ &\geq h_{(\psi_2\circ\psi_1)(a)}(\phi_2 \circ \phi_1)(v)\\ &= (\psi_2 \circ \psi_1)^{-1}(h)_a(v),\end{aligned}$$

and also, for all $r \in K, v \in V$ and $z \in r \circ v$, we have

$$\begin{aligned}&\underset{z\in r\circ v}{inf}\, (\phi_2 \circ \phi_1)^{-1}(h)_a(z)\\ &= \underset{z\in r\circ v}{inf}\, h_{(\psi_2\circ\psi_1)(a)}(\phi_2 \circ \phi_1)(z)\\ &= \underset{\phi_2(\phi_1(z))\in r\circ\phi_2(\phi_1(v))}{inf} h_{(\psi_2\circ\psi_1)(a)}(\phi_2 \circ \phi_1(z))\\ &\geq h_{(\psi_2\circ\psi_1)(a)}(\phi_2(\phi_1(v)))\\ &= (\phi_2 \circ \phi_1)^{-1}(h)_a(v).\end{aligned}$$

Consequently, $(\phi_2 \circ \phi_1, \psi_2 \circ \psi_1)^{-1}(h, C) \in F_s^s(V, E)$.

Theorem 12. *Let V and V' be two hypervector space over a field K. If $(f, A) \in F_s^s(V, E)$ and (ϕ, ψ) is a fuzzy soft homomorphism from V to V', then*

$$(\phi, \psi)(f, A) \in F_s^s(V', E').$$

Proof. Let $b \in \psi(A), v', u' \in V'$. If $\phi^{-1}(v') = \emptyset$ or $\phi^{-1}(u') = \emptyset$, the proof is straightforward. Assume that there exist $v, u \in V$, such that $\phi(v) = v'$ and $\phi(u) = u'$. Then

$$\begin{aligned}
\varphi(f)_b(u'+v') &= \bigvee_{\substack{u_i \in \phi^{-1}(u') \\ v_j \in \phi^{-1}(v') \\ i \in I, j \in J}} \bigvee_{a \in A \cap \psi^{-1}(b)} f_a(u_i + v_j) \\
&\geq \bigvee_{\substack{u_i \in \phi^{-1}(u') \\ v_j \in \phi^{-1}(v') \\ i \in I, j \in J}} \bigvee_{a \in A \cap \psi^{-1}(b)} (f_a(u_i) \wedge f_a(v_j)) \\
&= \bigvee_{a \in A \cap \psi^{-1}(b)} \Big[\Big(\bigvee_{\substack{v_j \in \phi^{-1}(v') \\ j \in J}} (f_a(u_1) \wedge f_a(v_j)) \Big) \vee \Big(\bigvee_{\substack{v_j \in \phi^{-1}(v') \\ j \in J}} (f_a(u_2) \wedge f_a(v_j)) \Big) \vee \cdots \Big] \\
&= \bigvee_{a \in A \cap \psi^{-1}(b)} \Big(\Big[f_a(u_1) \wedge \big(\bigvee_{\substack{v_j \in \phi^{-1}(v') \\ j \in J}} f_a(v_j) \big) \Big] \vee \Big[f_a(u_2) \wedge \big(\bigvee_{\substack{v_j \in \phi^{-1}(v') \\ j \in J}} f_a(v_j) \big) \Big] \vee \cdots \Big) \\
&= \bigvee_{a \in A \cap \psi^{-1}(b)} \Big(\bigvee_{\substack{u_i \in \phi^{-1}(u') \\ i \in I}} f_a(u_i) \Big) \wedge \Big(\bigvee_{\substack{v_i \in \phi^{-1}(v') \\ j \in J}} f_a(v_j) \Big) \\
&= \Big(\bigvee_{\substack{u_i \in \phi^{-1}(u') \\ i \in I}} \bigvee_{a \in A \cap \psi^{-1}(b)} f_a(u_i) \Big) \wedge \Big(\bigvee_{\substack{v_i \in \phi^{-1}(v') \\ j \in J}} \bigvee_{a \in A \cap \psi^{-1}(b)} f_a(v_j) \\
&= \phi(f)_b(u') \wedge \phi(f)_b(v').
\end{aligned}$$

Moreover, for all $v' \in V'$, where $\phi(v) = v'$ and $v \in V$, we have

$$\varphi(f)_b(-v') = \bigvee_{-v \in \phi^{-1}(-v')} \bigvee_{a \in A \cap \psi^{-1}(b)} f_a(-v) \geq \bigvee_{v \in \phi^{-1}(v')} \bigvee_{a \in A \cap \psi^{-1}(b)} f_a(v) = \varphi(f)_b(v).$$

Also, let $r \in K, v' \in V'$. If $\phi^{-1}(v') = \emptyset$, the proof is straightforward. Assume that there exists $v \in V$, such that $\phi(v) = v'$. Hence, for all $z' \in r \circ v'$, such that $\varphi(z) = z'$, we have

$$\begin{aligned}
&\mathop{inf}_{z' \in r \circ v'} \phi(f)_b(z') = \mathop{inf}_{z \in r \circ v} \phi(f)_b(z) \\
&= \mathop{inf}_{z \in r \circ v} \bigvee_{z \in \psi^{-1}(z')} \bigvee_{a \in A \cap \psi^{-1}(b)} f_a(z) \geq \bigvee_{z \in \psi^{-1}(z')} \bigvee_{a \in A \cap \psi^{-1}(b)} \mathop{inf}_{z \in r \circ v} f_a(z) \\
&= \bigvee_{z \in \phi^{-1}(z')} \bigvee_{a \in A \cap \psi^{-1}(b)} f_a(v) = \bigvee_{v \in \phi^{-1}(v')} \bigvee_{a \in A \cap \psi^{-1}(b)} f_a(v) = \phi(f)_b(v').
\end{aligned}$$

Therefore, $(\phi, \psi)(f, A) \in F_s^s(V', E')$.

Lemma 6. *Let V and V' be two hypervector space over a field K. If $(f, A) \in F_s^s(V, E)$, then $(f, A) \sqsubseteq (\phi, \psi)^{-1}((\phi, \psi)(f, A))$.*

Proof. It is obviously that, $A \subseteq \psi^{-1}(\psi(A))$. Let $v \in V$ and $a \in A$. Since $\phi^{-1}(\phi(v)) \neq \emptyset$, thus

$$\phi^{-1}(\phi(f))_a(v) = \phi(f)_{\psi(a)}(\phi(v)) = \bigvee_{x\in\phi^{-1}(\phi(v))} \bigvee_{a\in A\cap\psi^{-1}(b)} f_a(x) \geq f_a(v).$$

Consequently, $f_a \leq \phi^{-1}(\phi(f))_a, \quad \forall\, a \in A$.
Therefore, $(f, A) \sqsubseteq (\phi, \psi)^{-1}((\phi, \psi)(f, A))$.
In particular, if ϕ and ψ are injection, then

$$(\phi, \psi)^{-1}((\phi, \psi)(f, A)) = (f, A).$$

Lemma 7. *Let V and V' be two hypervector space over a field K. If $(g, B) \in F_s^s(V', E')$, then $(\phi, \psi)((\phi, \psi)^{-1}(g, B)) \sqsubseteq (g, B)$.*

Proof. For all $v' \in V'$ and $b \in \psi(\psi^{-1}(B))$, we have

$$\phi(\phi^{-1}(g))_b(v') = \begin{cases} \bigvee_{x\in\phi^{-1}(v')} \bigvee_{a\in\psi^{-1}(B)\cap\phi^{-1}(b)} \phi^{-1}(g)_a(x) & \text{if } \phi^{-1}(v') \neq \emptyset \\ 0 & \text{otherwise.} \end{cases}$$

$$= \begin{cases} \bigvee_{\phi(x)=v'} \bigvee_{a\in\psi^{-1}(B)\cap\phi^{-1}(b)} g_{\psi(a)}(\phi(x)) & \text{if } \phi^{-1}(v') \neq \emptyset \\ 0 & \text{otherwise.} \end{cases} \leq g_{\psi(a)}(v') = g_b(v').$$

Hence, for all $b \in \psi(\psi^{-1}(B))$, we have $\phi(\phi^{-1}(g))_b \leq g_b$. Therefore, $(\phi, \psi)((\phi, \psi)^{-1}(g, B)) \sqsubseteq (g, B)$. In particular, if ϕ and ψ are surjection, then

$$(\phi, \psi)((\phi, \psi)^{-1}(g, B)) = (g, B).$$

Corollary 2. *Let V and V' be two hypervector space over a field K. If $(f, A) \in F_s^s(V, E)$ and $(g, B) \in F_s^s(V', E')$, then*

$$(\phi, \psi)(f, A) \sqsubseteq (g, B) \Leftrightarrow (f, A) \sqsubseteq (\phi^{-1}, \psi^{-1})(g, B).$$

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