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On Slightly Compressible-Injective Modules

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Abstract. In this paper, we introduce the concept of slightly compressible-injective modules, following this, a right R-module N is called an M-slightly compressible-injective module, if every R-homomorphism from a non-zero M-slightly compressible submodule of M to N can be extended to M. We give some characterizations and properties of slightly compressible-injective modules.

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1. Introduction

Throughout all rings are associative with identity and modules are unitary right R-modules. Let M be a right R-module and $S = End_R(M)$, its endomorphism ring. We denote $\sigma[M]$ the full subcategory of Mod-R whose objects are submodules of M-generated modules. A right R-module M is called a *subgenerator*, if it generates $\sigma[M]$ and a *self-generator*, if it generates all its submodules. We denote the socle and radical of the right R-module M by $Soc(M)$ and $Rad(M)$, respectively. The Jacobson radical of a ring R is denoted by $J(R)$. We use the notations l and r to denote left and right annihilator, respectively.

The Baer Criterion has been generalized by many authors. For example, in 1989, Camillo introduced the notion of principally injective modules for commutative rings in [3]. A right R-module M is called *principally injective* (or *p-injective*), if every R-homomorphism from a principal right ideal of R to M can be extended to one from R

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to M. Next in 1999, Sanh and his group introduced M-principal injectivity for a given right R-module M in [7]. Let M be a right R-module. A right R-module N is called *M-principally injective*, if every R-homomorphism from an M-cyclic submodule of M to N can be extended to one from M to N. Slightly compressible modules were studied by Smith in [9]. In 2006, essentially compressible modules and rings were introduced and studied by Smith and Vedadi in [10]. Essentially compressible modules is a one of generalization of compressible modules. Next, the concepts of essentially slightly compressible modules and rings were introduced, and related properties were investigated by Singh in [8]. Essentially slightly compressible modules and rings are a generalization of essentially compressible modules and rings. Celik introduced and investigated completely slightly compressible modules as a one of generalization of compressible modules. Recently, Baupradist et al. studied a general form of slightly compressible modules in [2]. That is, for a right R-module M and N, N is called an *M-slightly compressible module*, if every non-zero submodule A of N, there exists a non-zero R-homomorphism from M to A. In that paper, they provided conditions for right R-module to be an M-slightly compressible module and examples of M-slightly compressible modules.

In this paper, we introduce the concept of M-slightly compressible-injective modules, which extended from the Baer Criterion. Moreover, we study some properties of M-slightly compressible-injective modules and relationship between M-principally injective modules and M-slightly compressible-injective modules. For the some examples of M-slightly compressible-injective modules are provided. For definitions and terminologies not given in this paper, the reader is refereed to [1, 5, 6].

2. Slightly compressible injectivity

For a ring R, we see that every right ideal of R is an R_R-slightly compressible submodule of R_R and every R_R-slightly compressible submodule of R_R is a right ideal of R because every submodule of R_R is a right ideal of R. We use this fact to generalize the notion of injectivity to an M-slightly compressible-injective module for a given right R-module M. By an M-cyclic submodule, we mean the submodule of M of the form $s(M)$ with $s \in S = End_R(M)$.

Definition 1. ([2]) Let M be a right R-module. A submodule A of M is called an *M-slightly compressible submodule of M*, if every non-zero submodule A of N, there exists a non-zero R-homomorphism s from M to N such that $s(M) \hookrightarrow A$. A right R-module N is called *quasi-slightly compressible*, if N is an N-slight compressible module.

Definition 2. Let M be a right R-module. A right R-module N is called an *M-slightly compressible-injective module* (or *M-sc-injective module* for short), if every R-homomorphism from an M-slightly compressible submodule of M to N can be extended to an R-homomorphism from M to N. A right R-module N is called *quasi-slightly compressible-injective* (or *quasi-sc-injective* for short), if N is an N-slightly compressible-injective module.

Example 1.

(1) Every simple right R-module is a quasi-slightly compressible-injective module.

(2) The following example [see [5], Exercise(2), p.361], let F be a field and $R = \begin{pmatrix} F & F \\ 0 & F \end{pmatrix}$ be the ring of all matrices of the form $\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$ with $a, b, c \in F$. Let $M = \begin{pmatrix} F & F \\ 0 & 0 \end{pmatrix}$ be a right R-module of all matrices of the form $\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}$ with $a, b \in F$ and $N = \begin{pmatrix} 0 & 0 \\ 0 & F \end{pmatrix}$ be a right R-module of all matrices of the form $\begin{pmatrix} 0 & 0 \\ 0 & c \end{pmatrix}$ with $c \in F$. Then N is an M-slightly compressible-injective module.

Proof. Let A be a non-zero M-slightly compressible submodule of M and α an R-homomorphism from A to N. Then A has the form $\begin{pmatrix} 0 & F_1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} F_2 & 0 \\ 0 & 0 \end{pmatrix}$ or $\begin{pmatrix} F_3 & F_4 \\ 0 & 0 \end{pmatrix}$ where F_1, F_2, F_3 and F_4 are subfields of F.

Let $\alpha \in Hom_R(A, N)$ such that any element $x \in A$, $\alpha(x) = \begin{pmatrix} 0 & 0 \\ 0 & \alpha_x \end{pmatrix} \in N$. It is easy to define $\overline{\alpha}$ from M to N by $\overline{\alpha}(\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}) = \begin{pmatrix} 0 & 0 \\ 0 & \alpha_x \end{pmatrix}$. It is clear that $\bar{\alpha}|_A = \alpha$. Therefore N is an M-slightly compressible-injective module.

Proposition 1. *Let M be a right R-module and A be a non-zero submodule of M. If A is an M-slightly compressible-injective module, then A is a direct summand of M.*

Proof. Assume that A is an M-slightly compressible-injective module. Then there exists $\alpha : M \to A$ such that $\alpha i_A = I_A$ where i_A is the inclusion map from A to M and I_A is the identity map on A. So A is a direct summand of M.

Proposition 2. *Let M be a quasi-slightly compressible-injective module and $f, g \in S = End_R(M)$. Then $f \in Sg$ if and only if $Ker(g) \subseteq Ker(f)$.*

Proof. ($\Rightarrow$) Obviously.

($\Leftarrow$) Assume that $Ker(g) \subseteq Ker(f)$. By the Factor's Theorem, there exists $g' : g(M) \to M$ such that $g'g = f$. Since M is a quasi-slightly compressible-injective module, there exists $h \in S$ such that $hi_{g(M)} = g'$ where $i_{g(M)} : g(M) \to M$ is an embedding. So $hg = hi_{g(M)}g = g'g = f$. Therefore $f \in Sg$.

Proposition 3. *Let M and N be right R-modules. If N is an M-slightly compressible-injective module, then any R-monomorphism from N to M splits.*

Proof. Assume that N is an M-slightly compressible-injective module. Let $f : N \to M$ be an R-monomorphism. Thus $f^{-1} : f(N) \to M$ is well defined and is an R-homomorphism. Since $f(N)$ is an M-slightly compressible submodule of M, f^{-1} can be extended to an R-homomorphism $\alpha : M \to N$ such that $\alpha i_{f(N)} = f^{-1}$ where $i_{f(N)} : f(N) \to M$ is an embedding. Therefore $\alpha f = I_N$ where I_N is an identity map on N and hence f splits.

Proposition 4. *Let M and N be right R-modules. If N is an M-slightly compressible injective module and $A \subset^{\oplus}_{>} N$, then A is an M-slightly compressible injective module.*

Proof. Assume that N is an M-slightly compressible injective module and $A \subset^{\oplus}_{>} N$. Let B be a non-zero M-slightly compressible submodule of M and $\alpha : B \to A$ be an R-homomorphism. Since $A \subset^{\oplus}_{>} N$, there exists $A' \hookrightarrow N$ such that $N = A \oplus A'$. Let $i_A : A \to N$ be the canonical injection map. Since N is an M-slightly compressible injective module, there exists $f : M \to N$ such that $f i_B = i_A \alpha$ where $i_B : B \to M$ is an embedding. Let $\pi_A : N \to A$ be the canonical projection map. We can choose $\bar{\alpha} = \pi_A f$. Then $\bar{\alpha} i_B = \pi_A f i_B = \pi_A i_A \alpha = I_A \alpha = \alpha$ where I_A is the identity on A. Hence A is an M-slightly compressible-injective module.

Proposition 5. *Let M be a right R-module. If M is a quasi-slightly compressible-injective module, then every submodule of M which is isomorphic to a direct summand of M is a direct summand of M.*

Proof. Let M be a quasi-slightly compressible-injective module, $A \subset^{\oplus}_{>} M$ and $B \hookrightarrow M$ such that $A \cong B$. By Proposition 4, A is an M-slightly compressible-injective module. Since $A \cong B$, B is an M-slightly compressible-injective module. By Proposition 3, we have $i_B : B \to M$ splits where i_B is a monomorphism from B to M. Therefore B is a direct summand of M.

Proposition 6. *Let M, N be right R-modules and N be an M-slightly compressible-injective module. Then*

(1) *N is a K-slightly compressible-injective module for all $K \subset^{\oplus}_{>} M$.*

(2) *H is a K-slightly compressible-injective module for all $H \subset^{\oplus}_{>} N$ and $K \subset^{\oplus}_{>} M$.*

Proof. (1) Let $K \subset^{\oplus}_{>} M$ and $0 \neq A$ be an K-slightly compressible submodule and α be an R-homomorphism from A to N. Then there exists $0 \neq s \in End_R(K)$ such that $s(M) \hookrightarrow A$, so $s\pi_K \in End_R(M)$ and $s\pi_K(M) \hookrightarrow A$ where $\pi_K : M \to K$ is the canonical map. Since N is an M-slightly compressible-injective module, α extends to an R-homomorphism $\bar{\alpha}$ from M to N such that $\bar{\alpha} i_A = \alpha$ where $i_A : A \to M$ is an embedding. Thus $\bar{\alpha}|_K : K \to N$ and $\bar{\alpha}|_K i_A = \alpha$. Therefore N is an K-slightly compressible-injective module.

(2) Let $H \subset^{\oplus}_{>} N$ and $K \subset^{\oplus}_{>} M$. From (1), N is K-slightly compressible injective. By Proposition 4, H is an K-slightly compressible injective module.

Recall that a right R-module M is said to be *direct-projective*, if given any summand N of M with projection map $p : M \to N$ and any epimorphism $f : M \to N$, there exists

$g \in S = End_R(M)$ such that $fg = p$. For more details of direct-projective, we refer to [12].

Theorem 1. *Let M be a right R-module and $S = End_R(M)$ be the endomorphism ring of M. If M is a direct-projective and every submodule of M is an M-slightly compressible-injective module, then S is a von Neumann regular.*

Proof. Assume that M is a direct-projective and every submodule of M is an M-slightly compressible injective module. Let $s \in S$. By assumption, $s(M)$ is an M-slightly compressible injective module. Let $i_{s(M)} : s(M) \to M$ be an embedding. By Proposition 3, $i_{s(M)} : s(M) \to M$ splits. Then $s(M)$ is a direct summand of M. We can construct epimorphism $s' : M \to Im(s)$ by $s'(m) = s(m)$ for all $m \in M$. Since M is a direct-projective, then the short exact sequence $0 \to Ker(s') \hookrightarrow M \xrightarrow{s'} s(M) \to 0$ split and we have $Ker(s')$ is a direct summand M. But $Ker(s') = Ker(s)$. So $Ker(s)$ is a direct summand of M. From Proposition 37.7(1) in [11], there exists $g \in S$ such that $s = sgs$. Therefore S is a von Neumann regular.

Theorem 2. *Let M be a right R-module and $S = End_R(M)$ be the endomorphism ring of M.*

(1) *If M is a quasi-slightly compressible-injective module, then $l_S(Ker(s)) = Ss$ for all $s \in S$.*

(2) *If M is a quasi-slightly compressible-injective module, then $Ker(t) \subseteq Ker(s)$ implies $Ss \subseteq St$ for any $s, t \in S$.*

(3) *If M is a quasi-slightly compressible-injective module, then $l_S(Im(t) \cap Ker(s)) = l_S(Im(t)) + Ss$ for all $s, t \in S$.*

Proof. (1) Assume that M is a quasi-slightly compressible-injective module. It is easy to show that $Ss \subseteq l_S(Ker(s))$. Let $s \in S$ and $u \in l_S(Ker(s))$. We have $u(Ker(s)) = 0$. Then $Ker(s) \subseteq Ker(u)$. By the Factor's Theorem, there exists an R-homomorphism $\alpha : s(M) \to M$ such that $\alpha s = u$. Since M is a quasi-slightly compressible-injective module, there exists an R-homomorphism $\bar{\alpha} : M \to M$ such that $\bar{\alpha}|_{s(M)} = \alpha$. Then $u = \alpha s = \bar{\alpha} s \in Ss$. Hence $l_S(Ker(s)) \subseteq Ss$. Therefore $l_S(Ker(s)) = Ss$.

(2) Assume that M is a quasi-slightly compressible-injective module. Let $s, t \in S$. Suppose $Ker(t) \subseteq Ker(s)$. By Proposition 2, $s \in St$. Therefore $Ss \subseteq St$.

(3) Assume that M is a quasi-slightly compressible-injective module. Let $s, t \in S$. Suppose $u \in l_S(Im(t) \cap Ker(s))$. Then $u(Im(t) \cap Ker(s))) = 0$ and we have $Ker(st) \subseteq Ker(ut)$. By Factor's Theorem, there exists a map $g' : st(M) \to M$ such that $g'st = ut$. Since M is a quasi-slightly compressible-injective module, there exists an R-homomorphism $g : M \to M$ such that $g|_{st(M)} = g'$. Thus $ut = gst$. It follows that $(u - gs)t = 0$ and hence $u - gs \in l_S(Im(t))$. Thus $u \in l_S(Im(t)) + Ss$. We have $l_S(Im(t) \cap Ker(s)) \subseteq l_S(Im(t)) + Ss$. But it is clear that $l_S(Im(t)) + Ss \hookrightarrow l_S(Im(t) \cap Ker(s))$. Therefore $l_S(Im(t) \cap Ker(s)) = l_S(Im(t)) + Ss$ for all $s, t \in S$.

Theorem 3. *Let M be a quasi-slightly compressible module, $S = End_R(M)$, Δ be the set of all $s \in S$ such that $Ker(s)$ is an essential in M and $J(S)$ be the Jacobson radical of S. If M is a quasi-slightly compressible-injective module and every M-cyclic submodule of M is an injective, then $J(S) = \Delta$.*

Proof. Assume that M is a quasi-slightly compressible-injective module and every M-cyclic submodule of M is an injective. Let $s \in \Delta$. Then $Ker(s)$ is an essential in M. Since $Ker(s) \cap Ker(1-s) = 0$, $Ker(1-s) = 0$, $l_S(Ker(1-s)) = S$. By Theorem 2(2), $l_S(Ker(1-s)) = S(1-s)$. Then $S(1-s) = S$. Hence $1-s$ has left inverse in S. By Theorem 9.3.1 in [5], $\Delta \subseteq J(S)$. Next, let $s \in J(S)$. We want to show that $Ker(s)$ is an essential in M. First, we claim that if $Im(t) \cap Ker(s) = 0$ for all $t \in S$, then $t = 0$. Let $t \in S$ such that $Im(t) \cap Ker(s) = 0$. By Theorem 2(3), $l_S(Im(t) \cap Ker(s)) = l_S(Im(t)) + Ss$ but $l_S(Im(t) \cap Ker(s)) = S$. Then $l_S(Im(t)) + Ss = S$. Since $s \in J(S)$, Ss is a small in S, $l_S(Im(t)) = S$, $Im(t) = 0$. Then $t = 0$. Let $A \hookrightarrow M$ such that $Ker(s) \cap A = 0$. Since M is a quasi-slightly compressible module and every M-cyclic submodule is an injective, from Corollary 2.14 in [2], M is a self-generator, $A = \sum_{t \in I} t(M)$ where $I \subseteq S = End_R(M)$, $\sum_{t \in I} t(M) \cap Ker(s) = 0$, $t(M) \cap Ker(s) = 0$ for all $t \in I$. We have $t = 0$ for all $t \in I$. Then $A = \sum_{t \in I} t(M) = 0$, so $Ker(s)$ is an essential in M. Hence $s \in \Delta$, $J(S) \subseteq \Delta$. Therefore $J(S) = \Delta$.

Theorem 4. *Let M be a quasi-slightly compressible-injective module and $s, t \in S = End_R(M)$. If $s(M) \cong t(M)$, then $Ss \cong St$.*

Proof. Assume that $s(M) \cong t(M)$. Then there exists an isomorphism f from $s(M)$ to $t(M)$. Since M is a quasi-slightly compressible-injective module, $s(M)$ is an M-slightly compressible submodule, $i_{t(M)} f : s(M) \to M$ is an R-homomorphism, so $i_{t(M)} f$ can be extended to $\bar{f} : M \to M$ such that $\bar{f} i_{s(M)} = i_{t(M)} f$ where $i_{s(M)} : s(M) \to M$ and $i_{t(M)} : t(M) \to M$ are embedding. Define $\beta : St \to Ss$ by $\beta(ut) = u\bar{f}s$ for all $u \in S$. Since $Im(\bar{f}s) = Im(t)$, we can show that β is an well-defined. Moreover, β is a left S-homomorphism. For any $v \in S$, $vi_{s(M)} : s(M) \to M$ can be extended to an R-homomorphism $\varphi : M \to M$ such that $\varphi i_{t(M)} f = v i_{s(M)}$ and we can construct the map $s' : M \to s(M)$ such that $s'(m) = s(m)$ for all $m \in M$, so $i_{s(M)} s' = s$ where $i_{s(M)} : s(M) \to M$ and $i_{t(M)} : t(M) \to M$ are embedding. We have $\beta(\varphi t) = \varphi \bar{f} s = \varphi \bar{f} i_{s(M)} s' = \varphi i_{t(M)} f s' = v i_{s(M)} s' = vs$. This shows that β is an epimorphism. It is clear that β is a left S-monomorphism. Therefore $Ss \cong St$.

Theorem 5. *Let M be a quasi-slightly compressible-injective module and $s_1, \ldots, s_n \in S = End_R(M)$ such that the sum $\sum_{i=1}^{n} Ss_i$ is direct. Then any R-homomorphism from $\sum_{i=1}^{n} s_i(M)$ to M can be extended to an R-homomorphism from M to M.*

Proof. Since $\left(\sum_{i=1}^{n} s_i\right)(M) \subseteq \sum_{i=1}^{n} s_i(M)$ and M is a quasi-slightly compressible-injective module, so any R-homomorphism from $\sum_{i=1}^{n} s_i(M)$ to M can be extended to an R-homomorphism from M to M.

Theorem 6. *Let M be a quasi-slightly compressible-injective module, $s_1, \ldots, s_n \in S = End_R(M)$ such that the sum $\sum_{i=1}^{n} Ss_i$ is direct, $A = s_1(M) + \ldots + s_k(M)$ and $B = s_{k+1}(M) + \ldots + s_n(M)$ where $1 \leq k \leq n$. Then $l_S(A \cap B) = l_S(A) + l_S(B)$.*

Proof. Clearly, $l_S(A \cap B) \supseteq l_S(A) + l_S(B)$. Let $u \in l_S(A \cap B)$. Consider the map $\alpha : A + B \to M$ by $\alpha(a + b) = u(a)$ for all $a \in A, b \in B$. Since $u(A \cap B) = 0$, α is well-defined and is an R-homomorphism. By Theorem 5, $\alpha : A + B \to M$ can be extended to an R-homomorphism $\varphi : M \to M$. Clearly, $\varphi(b) = 0$ for all $b \in B$ and hence $\varphi \in l_S(B)$ and $u - \varphi \in l_S(A)$. Therefore $u = (u - \varphi) + \varphi \in l_S(A) + l_S(B)$.

Theorem 7. *Let M and M_i be right R-modules for all $i \in I = \{1, 2, ..., n\}$ where n is a positive integer. Then M_i is an M-slightly compressible-injective module for all $i \in I$ if and only if $\oplus_{i=1}^{n} M_i$ is an M-slightly compressible-injective module.*

Proof. ($\Rightarrow$) Assume that M_i is an M-slightly compressible-injective module for all $i \in I$. Let $j \in I$, $P = \oplus_{i=1}^{n} M_i$, A be a non-zero M-slightly compressible submodule of M and α be an R-homomorphism from A to P. Since $\pi_i\alpha$ is an R-homomorphism from A to M_i where π_j the $j^{\underline{th}}$ canonical projection map from P to M_j and M_j is an M-slightly compressible-injective module, there exists $\bar{\alpha}_j : M \to M_j$ such that $\bar{\alpha}_j i_A = \pi_j\alpha$ where $i_A : A \to M$ is an embedding. We can choose $\bar{\alpha} = \sum_{j=1}^{n} i_j\bar{\alpha}_j$ where $i_j : M_j \to M$ be the canonical injection map. Then $\bar{\alpha} i_A = \sum_{j=1}^{n} i_j\bar{\alpha}_j i_A = (\sum_{j=1}^{n} i_j\pi_j)\alpha = I_P\alpha = \alpha$ where I_P is the identity map on P. Hence $P = \oplus_{i=1}^{n} M_i$ is an M-sligthly compressible-injective module.

($\Leftarrow$) Assume that $P = \oplus_{i=1}^{n} M_i$ is an M-slightly compressible-injective module. Let $j \in I$, A be a non-zero M-slightly compressible submodule of M and α_j be an R-homomorphism from A to M_j. Since $i_j\alpha_j$ is an R-homomorphism from A to P where $i_j : M_j \to M$ is the canonical injection map and P is an M-slightly compressible-injective module, there exists $\bar{\alpha} : M \to P$ such that $\bar{\alpha} i_A = i_j\alpha_j$ where $i_A : A \to M$ is an embedding. We can choose $\bar{\alpha}_j = \pi_j\bar{\alpha}$ where π_j is the $j^{\underline{th}}$ canonical projection map. Then $\bar{\alpha}_j i_A = \pi_j\bar{\alpha} i_A = \pi_j i_j\alpha_j = I_{M_j}\alpha_j = \alpha_j$ where I_{M_j} is an identity map on M_j. Hence M_i is an M-slightly compressible-injective module for all $i \in I$.

Theorem 8. *Let M be a quasi-slightly compressible module. Then M is a semisimple module if and only if every non-zero submodule of M is M-slightly compressible-injective.*

Proof. ($\Rightarrow$) It is easy.

($\Leftarrow$) Assume that every non-zero submodule of M is an M-slightly compressible-injective module. Let A be a submodule of M. If $A = 0$, then we are done. Suppose that $0 \neq A \hookrightarrow M$. By assumption, A is an M-slightly compressible-injective module. Since M is a quasi-slightly compressible module, A is an M-slightly compressible submodule of M. Then there exists $\alpha : M \to A$ such that $\alpha i_A = I_A$ where $i_A : A \to M$ is the embedding and I_A is an identity map of A. Then the short exact sequence $0 \to A \xrightarrow{i_A} M \xrightarrow{\pi} M/A \to 0$ splits. Thus A is a direct summand of M. Therefore M is a semisimple module.

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