EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
Vol. 12, No. 1, 2019, 176-193
ISSN 1307-5543 – www.ejpam.com
Published by New York Business Global

Other kinds of soft β mappings via soft topological ordered spaces

Tareq M. Al-shami[1,2*], Mohammed E. El-Shafei[1], Baravan A. Asaad[3,4]

[1] *Department of Mathematics, Faculty of Science, Mansoura University, Mansoura, Egypt*
[2] *Department of Mathematics, Sana'a University, Sana'a, Yemen*
[3] *Department of Computer Science, College of Science, Cihan University-Duhok, Kurdistan Region, Iraq*
[4] *Department of Mathematics, Faculty of Science, University of Zakho, Kurdistan Region, Iraq*

Abstract. The authors of [13] formulated a soft topological ordered spaces concept and then they established and studied some ordered mappings [14]. In the present work, we define new ordered mappings via soft topological ordered spaces based on soft β-open sets, namely soft $x\beta$-continuous, soft $x\beta$-open, soft $x\beta$-closed and soft $x\beta$-homeomorphism mappings, for $x \in \{I, D, B\}$. We give various characterizations of each one of the introduced soft mappings. One of the most important obtained results is that an extended soft topologies notion guarantees the equivalent between the soft mappings initiated herein and their counterparts of mappings on topological ordered spaces. We provide several interesting examples to examine the relationships among these soft mappings.

2010 Mathematics Subject Classifications: 54F05, 54F15

Key Words and Phrases: Soft $I(D,B)\beta$-continuous mapping; Soft $I(D,B)\beta$-open mapping; Soft $I(D,B)\beta$-homeomorphism mapping and soft ordered separation axioms.

1. Introduction

In the year 1965, Nachbin [37] started studying the topological ordered spaces concept by defining two independent mathematical structures, on a non-empty set X, namely a topology τ and a partial order relation $\preceq$. Depending on these two structures, he redefine and reinvestigate some topological concepts such as normal, regular and and completely regular spaces to be normally ordered, regularly ordered and completely regular ordered spaces, respectively, on topological ordered spaces. Later on, McCartan [32] presented the notions of T_i-ordered and strong T_i-ordered spaces ($i = 0, 1, 2, 3, 4$) and compared them with T_i-spaces. Also, he completely descried T_i-ordered and supplied interesting examples

*Corresponding author.
DOI: https://doi.org/10.29020/nybg.ejpam.v12i1.3312

Email addresses: tareqalshami83@gmail.com (T. M. Al-shami), meshafei@hotmail.com (M. E. El-Shafei), baravan.asaad@uoz.edu.krd (B. A. Asaad)

to illustrate the concepts introduced and findings obtained therein. Based on β-open sets [2], Leela and Balasubramanian [28] in 2002, probed new ordered axioms; and Rao and Chudamani [42] in 2012, defined new kinds of continuous and homeomorphism mappings on topological ordered spaces. With regard to the generalizations of topological ordered spaces, we observe that this topic takes two directions, the first one is formulated by generalizing a partial order relation (see, for example, [25], [33], [34], [40]) and the second one is formulated by generalizing a topology (see, for example, [3], [8], [10], [12], [18], [20], [21]).

To handle problems and phenomena which suffering from uncertainties and incomplete of data, Molotdsov [36] in 1999, proposed a new mathematical tool, namely soft sets. He pointed out that the previous theories such as probability and fuzzy set theory have difficulties which attributed to the inadequacies of their parameterizations tools and show that soft set theory is more suitable for dealing with uncertainties with adequate parameterizations. Maji et al. [31] introduced some soft operators such as soft equality relation, soft union and intersection between two soft sets. These soft operators were generalized and studied in several directions in [17], [24], [29], [30] and [41]. Aktas and Cağman [6] were the first who studied soft algebraic structure. They introduced the soft group and soft subgroup notions and concluded their basic properties. In 2010, Acar et al. [4] presented a concept of soft rings and investigated its main features; and in 2013, Shah and Shaheen [45] established the notions of a soft topological group and a soft topological ring over a group and a ring, respectively. Hida [27] adopted a differen view to define soft topological group which help to make it a natural extension of the usual topological group notion.

In the year 2011, Shabir and Naz [44] initiated the concept of soft topological spaces and gave its fundamental notions such as soft open and soft closed sets, soft neighborhoods, soft interior and soft closure points. They also probed soft separation axioms and examined their properties. Min [35] gave deeper explanation for soft regular spaces and corrected some errors in [44]. Later on, desire of obtaining a deeper understanding of soft topology prompted interested researchers to carry out many studies on soft topological notions and their features. In 2012, Rong [43] investigated the countability axioms of soft topological spaces and and studied the possibility of carry over the results of countability axioms via general topology to the soft topology setting. Aygünoğlu and Aygün [16] introduced and studied a soft compactness concept; and Hida [26] gave two types of soft compactness and pointed out the relationships between them. The authors of [5] and [1] introduced the notions of soft β-open sets and soft β-separations axioms, respectively. They examined which results related to β-open sets and βT_i-spaces from the topological spaces remain valid in the context of soft topological spaces.

Recently, Al-shami et al. [13] introduced a concept of soft topological ordered spaces and established the notions of p-soft T_i-ordered spaces ($i = 0, 1, 2, 3, 4$) depending on totally non belong relations, which introduced in [23], and monotone soft neighborhoods. Also, they [14] defined newly ordered mappings via topological ordered spaces and obtained interesting results. Al-shami and Kočinac [15] verified the equivalence between the enriched and extended soft topologies and concluded many findings related to soft mappings and soft axioms.

We aim in this study to propose and investigate newly ordered mappings on soft topological ordered spaces, namely soft $x\beta$-continuous, soft $x\beta$-open, soft $x\beta$-closed and soft $x\beta$-homeomorphism mappings, for $x \in \{I, D, B\}$. The examples which illustrate the relationships among these soft mappings are given and the conditions which guarantee the equivalent between soft $x\beta$-open and soft $x\beta$-closed mappings are discussed, for $x \in \{I, D, B\}$. Also, the various characterizations of each one of the initiated soft mappings are investigated and the interrelations between these soft mappings and their counterparts of mappings in topological ordered spaces are studied amply.

2. Preliminaries

In what follows, we mention the definitions and results related to soft set, soft topological spaces and ordered spaces that will be needed in investigating the concepts introduced and results obtained herein.

Definition 1. *[36] A notation G_E is said to be a soft set over X if G is a mapping of a set of parameters E into 2^X and it is written as a set of ordered pairs $G_E = \{(e, G(e)) : e \in E$ and $G(e) \in 2^X\}$.*
For $x \in X$ and a soft set G_E over X, we say that $x \in G_E$ if $x \in G(e)$, for each $e \in E$ and $x \notin G_E$ if $x \notin G(e)$, for some $e \in E$.

Definition 2. *[31] A soft set G_E over X is called a null soft set, denoting by $\widetilde{\Phi}$, if $G(e) = \emptyset$, for each $e \in E$; and it is called an absolute soft set, denoting by $\widetilde{X}$, if $G(e) = X$, for each $e \in E$.*

Definition 3. *[7] The relative complement of a soft set G_E is denoted by G^c_E, where $G^c : E \to 2^X$ is a mapping defined by $G^c(e) = X \setminus G(e)$, for each $e \in E$.*
In this connection, it is worth noting that $x \notin G_E$ does not imply that $x \in G^c_E$.

Definition 4. *[44] A soft topology on a non-empty set X is a collection τ of soft sets over X under a parameters set E satisfying the following axioms:*

(i) *$\widetilde{X}$ and $\widetilde{\emptyset}$ belong to τ.*

(ii) *τ is closed under finite soft intersection.*

(iii) *τ is closed under arbitrary soft union.*

The triple (X, τ, E) is called a soft topological space. Every member of τ is called a soft open set and its relative complement is called soft closed.

Proposition 1. *[44] Let (X, τ, E) be a soft topological space. Then $\tau_e = \{G(e) : G_E \in \tau\}$ defines a topology on X, for each $e \in E$.*

Definition 5. *[38] Consider (X, τ, E) is a soft topological space and τ_e is a topology on X as in the above proposition. Then $\tau^\star = \{G_E : G(e) \in \tau_e$, for each $e \in E\}$ is a soft topology on X finer than τ.*
In [15], the authors termed $\tau^\star$ an extended soft topology.

Definition 6. *[46] Consider $f : X \to Y$ and $\phi : A \to B$ are two mappings and let $f_\phi : S(X_A) \to S(Y_B)$ be a soft mapping. Let G_K and H_L be soft subsets of $S(X_A)$ and $S(Y_B)$, respectively. Then*

(i) *$f_\phi(G_K) = (f_\phi(G))_B$ is a soft subset of $S(Y_B)$ such that*

$$f_\phi(G)(b) = \begin{cases} \bigcup_{a\in\phi^{-1}(b)\bigcap K} f(G(a)) & : \ \phi^{-1}(b)\bigcap K \neq \emptyset \\ \emptyset & : \ \phi^{-1}(b)\bigcap K = \emptyset \end{cases}$$

for each $b \in B$.

(ii) *$f_\phi^{-1}(H_L) = (f_\phi^{-1}(H))_A$ is a soft subset of $S(X_A)$ such that*

$$f_\phi^{-1}(H)(a) = \begin{cases} f^{-1}(H(\phi(a))) & : \ \phi(a) \in L \\ \emptyset & : \ \phi(a) \notin L \end{cases}$$

for each $a \in A$.

Remark 1. *Henceforth, a soft mapping $f_\phi : S(X_A) \to S(Y_B)$ implies that a mapping f of the universe set X into the universe set Y and a mapping ϕ of the set of parameters A into the set of parameters B*

Definition 7. *[46] A soft mapping $f_\phi : S(X_A) \to S(Y_B)$ is said to be injective (resp. surjective, bijective) if f and ϕ are injective (resp. surjective, bijective).*

Proposition 2. *[46] Consider $f_\phi : S(X_A) \to S(Y_B)$ is a soft mapping and let G_A and H_B be two soft subsets of $S(X_A)$ and $S(Y_B)$, respectively. Then we have the following results:*

(i) *$G_A \widetilde{\subseteq} f_\phi^{-1} f_\phi(G_A)$ and the equality relation holds if f_ϕ is injective.*

(ii) *$f_\phi f_\phi^{-1}(H_B) \widetilde{\subseteq} H_B$ and the equality relation holds if f_ϕ is surjective.*

Definition 8. *[5] A soft subset H_E of (X, τ, E) is said to be soft β-open if $H_E \widetilde{\subseteq} cl(int(cl(H_E)))$. And its relative complement is said to be soft β-closed.*

Definition 9. *([5], [44]) For a soft subset H_E of (X, τ, E), we define the following four operators:*

(i) *$int(H_E)$ (resp. $int_\beta(H_E)$) is the largest soft open (resp. soft β-open) set contained in H_E.*

(ii) *$cl(H_E)$ (resp. $cl_\beta(H_E)$) is the smallest soft closed (resp. soft β-closed) set containing H_E.*

Definition 10. *[5] A soft mapping $f_\phi : (X, \tau, A) \to (Y, \theta, B)$ is said to be:*

(i) *Soft β-continuous if the inverse image of each soft open subset of (Y, θ, B) is a soft β-open subset of (X, τ, A).*

(ii) *Soft β-open (resp. soft β-closed) if the image of each soft open (resp. soft closed) subset of (X, τ, A) is a soft β-open (resp. soft β-closed) subset of (Y, θ, B).*

(iii) *Soft β-homeomorphism if it is bijective, soft β-continuous and soft β-open.*

Definition 11. *[19, 38] A soft set P_E over X is called soft point if there exists $e \in E$ and there exists $x \in X$ such that $P(e) = \{x\}$ and $P(a) = \emptyset$, for each $a \in E \setminus \{e\}$.*
A soft point will be shortly denoted by P_e^x and we say that $P_e^x \in G_E$, if $x \in G(e)$.

Definition 12. *[13] Let $\preceq$ be a partial order relation on a non-empty set X and let E be a set of parameters. A triple $(X, E, \preceq)$ is said to be a partially ordered soft set.*

Definition 13. *[13] We define an increasing soft operator $i : (SS(X_E), \preceq) \to (SS(X_E), \preceq)$ and a decreasing soft operator $d : (SS(X_E), \preceq) \to (SS(X_E), \preceq)$ as follows, for each soft subset G_E of $SS(X_E)$*

(i) *$i(G_E) = (iG)_E$, where iG is a mapping of E into X given by $iG(e) = i(G(e)) = \{x \in X : y \preceq x$, for some $y \in G(e)\}$.*

(ii) *$d(G_E) = (dG)_E$, where dG is a mapping of E into X given by $dG(e) = d(G(e)) = \{x \in X : x \preceq y$, for some $y \in G(e)\}$.*

Definition 14. *[13] A soft subset G_E of a partially ordered soft set $(X, E, \preceq)$ is said to be increasing (resp. decreasing) if $G_E = i(G_E)$(resp. $G_E = d(G_E)$).*

Theorem 1. *[13] If a soft mapping $f_\phi : (S(X_A), \preceq_1) \to (S(Y_B), \preceq_2)$ is increasing, then the inverse image of each increasing (resp. decreasing) soft subset of $\widetilde{Y}$ is an increasing (resp. a decreasing) soft subset of $\widetilde{X}$.*

Definition 15. *[13] A quadrable system $(X, \tau, E, \preceq)$ is said to be a soft topological ordered space, where (X, τ, E) is a soft topological space and $(X, E, \preceq)$ is a partially ordered soft set. Henceforth, the two notations $(X, \tau, E, \preceq_1)$ and $(Y, \theta, F, \preceq_2)$ stand for soft topological ordered spaces.*

Definition 16. *[42] A mapping $(X, \tau, \preceq_1) \to (Y, \theta, \preceq_2)$ is said to be:*

(i) *I (resp. D, B) β-continuous if the inverse image of each open set is I (resp. D, B) β-open.*

(ii) *I (resp. D, B) β-open if the image of each open set is I (resp. D, B) β-open.*

(iii) *I (resp. D, B) β-closed if the image of each open set is I (resp. D, B) β-closed.*

(iv) *I (resp. D, B) β-homeomorphism if it is bijective, I (resp. D, B) β-continuous and I (resp. D, B) β-open.*

Definition 17. *[14] The composition of two soft mappings $f_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta, F, \preceq_2)$ and $g_\lambda : (Y, \theta, F, \preceq_2) \to (Z, \upsilon, K, \preceq_3)$ is a soft mapping $f_\phi \circ g_\lambda : (X, \tau, E, \preceq_1) \to (Z, \upsilon, K, \preceq_3)$ and is given by $(f_\phi \circ g_\lambda)(P_e^x) = f_\phi(g_\lambda(P_e^x))$.*

3. Soft $I(D,B)\beta$-continuity

In this section, the notions of $I(D,B)\beta$-continuity at soft point, ordinary point and on the universe set are given and studied. Each one of the introduced soft mappings are characterized and some examples are provided to show the relationships among them.

Definition 18. *A soft subset H_E of $(X,\tau,E,\preceq_1)$ is said to be:*

(i) *Soft I (resp. Soft D, Soft B) β-open if it is soft β-open and increasing (resp. decreasing, balancing).*

(ii) *Soft I (resp. Soft D, Soft B) β-closed if it is soft β-closed and increasing (resp. decreasing, balancing).*

Definition 19. *A soft mapping $f_\phi : (X,\tau,E,\preceq_1) \to (Y,\theta,F,\preceq_2)$ is called:*

(i) *Soft I (resp. Soft D, Soft B) β-continuous at $P_e^x \in \widetilde{X}$ if for each soft open set H_F containing $f_\phi(P_e^x)$, there exists a soft I (resp. soft D, soft B) β-open set G_E containing P_e^x such that $f_\phi(G_E)\widetilde{\subseteq}H_F$.*

(ii) *Soft I (resp. Soft D, Soft B) β-continuous at $x \in X$ if it is soft I (resp. soft D, soft B) β-continuous at each P_e^x.*

(iii) *Soft I (resp. Soft D, Soft B) β-continuous if it is soft I (resp. soft D, soft B) β-continuous at each $x \in X$.*

Theorem 2. *A soft mapping $f_\phi : (X,\tau,E,\preceq_1) \to (Y,\theta,F,\preceq_2)$ is soft I (resp. soft D, soft B) β-continuous if and only if the inverse image of each soft open subset of $\widetilde{Y}$ is a soft I (resp. soft D, soft B) β-open subset of $\widetilde{X}$.*

Proof. We prove the theorem in the case of f_ϕ is soft Dβ-continuous and the other cases can be achieved similarly.
Necessity: Let G_F be a soft open subset of $\widetilde{Y}$, Then we have the following two cases:

(i) Either $f_\phi^{-1}(G_F) = \widetilde{\emptyset}$.

(ii) Or $f^{-1}(G_F) \neq \widetilde{\emptyset}$. By choosing $P_e^x \in X$ such that $P_e^x \in f_\phi^{-1}(G_F)$, we obtain $f_\phi(P_e^x) \in G_F$. So there exists a soft Dβ-open set H_E containing P_e^x such that $f_\phi(H_E)\widetilde{\subseteq}G_F$. Since P_e^x is chosen arbitrary, then $f_\phi^{-1}(G_F) = \widetilde{\bigcup}_{P_e^x \in f_\phi^{-1}(G_F)} H_E$.

From the two cases above, we conclude that $f_\phi^{-1}(G_F)$ is a soft Dβ-open subset of $\widetilde{X}$.
Sufficiency: Let G_F be a soft open subset of $\widetilde{Y}$ containing $f_\phi(P_e^x)$. Then $P_e^x \in f_\phi^{-1}(G_F)$. By hypothesis, $f_\phi^{-1}(G_F)$ is a soft Dβ-open set. Since $f_\phi(f_\phi^{-1}(G_F))\widetilde{\subseteq}G_F$, then f_ϕ is a soft Dβ-continuous mapping at $P_e^x \in X$ and since P_e^x is chosen arbitrary, then f_ϕ is a soft Dβ-continuous mapping.

Remark 2. *From Definition* (19)*, we can note the following:*

(i) *Every soft I (D, B) β-continuous mapping is always soft β-continuous.*

(ii) *Every soft Bβ-continuous mapping is soft Iβ-continuous or soft Dβ-continuous.*

The two examples below elucidates that the converse of the two results of the remark above need not be true in general.

Example 1. *Let the two parameters sets $A = \{\frac{1}{2}, \frac{1}{4}\}$, $B = \{\frac{1}{3}, \frac{1}{5}\}$ and the two universe sets $X = \{m, n, r, s\}$, $Y = \{u, v, w\}$. Consider a mapping $\phi : A \to B$ is defined as, $\phi(\frac{1}{2}) = \frac{1}{3}$ and $\phi(\frac{1}{4}) = \frac{1}{5}$, and a mapping $f : X \to Y$ is defined as, $f(m) = u$, $f(n) = v$ and $f(r) = f(s) = w$. We define a partial order relation on X as $\preceq = \triangle \bigcup \{(m, n), (n, r), (m, r)\}$ and we define two soft topologies τ and θ on X and Y, respectively, as $\tau = \{\widetilde{\emptyset}, \widetilde{X}, F_A, G_A\}$ and $\theta = \{\widetilde{\emptyset}, \widetilde{Y}, H_B\}$, where $F_A = \{(\frac{1}{2}, \{m, n, s\}), (\frac{1}{4}, \{m, r\})\}$, $G_A = \{(\frac{1}{2}, \emptyset), (\frac{1}{4}, \{r\})\}$ and $H_B = \{(\frac{1}{3}, \{u\}), (\frac{1}{5}, \{w\})\}$. Since $f_\phi^{-1}(H_B) = \{(\frac{1}{2}, \{m\}), (\frac{1}{4}, \{r, s\})\}$ is a soft β-open set, then $f_\phi : S(X_A) \to S(Y_B)$ is a soft β-continuous mapping. On the other hand, $f_\phi^{-1}(H_B)$ is neither a soft Dβ-open nor a soft Iβ-open set. Hence f_ϕ is not soft I (soft D, soft B) β-continuous.*

Example 2. *In Example above, if we only replace the partial order relation by $\preceq = \triangle \bigcup \{(m, n)\}$*(*resp.* $\preceq = \triangle \bigcup \{(n, r)\}$)*, then the soft mapping f_ϕ is soft D-continuous* (*resp. soft I-continuous*)*, but is not soft B-continuous.*

Definition 20. *For a soft subset H_E of $(X, \tau, E, \preceq)$, we define the following six operators:*

(i) *$H_E^{i\beta o}$(resp. $H_E^{d\beta o}, H_E^{b\beta o}$) is the largest soft I (resp. soft D, soft B) β-open set contained in H_E.*

(ii) *$H_E^{i\beta cl}$(resp. $H_E^{d\beta cl}, H_E^{b\beta cl}$) is the smallest soft I (resp. soft D, soft B) β-closed set containing H_E.*

Lemma 1. *For any soft subset H_E of $(X, \tau, E, \preceq)$, the following statements hold:*

(i) $(H_E^{d\beta cl})^c = (H_E^c)^{i\beta o}$.

(ii) $(H_E^{i\beta cl})^c = (H_E^c)^{d\beta o}$.

(iii) $(H_E^{b\beta cl})^c = (H_E^c)^{b\beta o}$.

Proof.

(i) $(H_E^{d\beta cl})^c = \{\widetilde{\bigcup} F_E : F_E$ is a soft Dβ-closed set containing $H_E\}^c$
$= \widetilde{\bigcap}\{F_E^c : F_E^c$ is a soft Iβ-open set contained in $H_E^c\} = (H_E^c)^{i\beta o}$.

By analogy with **(i)**, one can prove **(ii)** and **(iii)**.

Theorem 3. *The following five properties of a soft mapping* $f_\phi : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$ *are equivalent:*

(i) f_ϕ *is soft I*β*-continuous;*

(ii) $f_\phi^{-1}(L_F)$ *is a soft D*β*-closed subset of* $\widetilde{X}$*, for each soft closed subset* L_F *of* $\widetilde{Y}$*;*

(iii) $(f_\phi^{-1}(M_F))^{d\beta cl} \widetilde{\subseteq} f_\phi^{-1}(cl(M_F))$*, for every* $M_F \widetilde{\subseteq} \widetilde{Y}$*;*

(iv) $f_\phi(N_E^{d\beta cl}) \widetilde{\subseteq} cl(f_\phi(N_E))$*, for every* $N_E \widetilde{\subseteq} \widetilde{X}$*;*

(v) $f_\phi^{-1}(int(M_F)) \widetilde{\subseteq} (f_\phi^{-1}(M_F))^{i\beta o}$*, for every* $M_F \widetilde{\subseteq} \widetilde{Y}$*.*

Proof. **(i)** $\Rightarrow$ **(ii)** : Consider L_F is a soft closed subset of $\widetilde{Y}$. By hypothesis, $f_\phi^{-1}(L_F^c)$ is a soft Iβ-open subset of $\widetilde{X}$ and by the fact that $f_\phi^{-1}(L_F^c) = (f_\phi^{-1}(L_F))^c$, we obtain $f_\phi^{-1}(L_F)$ is soft Dβ-closed as required.
(ii) $\Rightarrow$ **(iii)** : It follows from **(ii)** that $f_\phi^{-1}(cl(M_E))$ is a soft Dβ-closed subset of $\widetilde{X}$, for every $M_F \widetilde{\subseteq} \widetilde{Y}$. So $(f_\phi^{-1}(M_F))^{d\beta cl} \widetilde{\subseteq} (f_\phi^{-1}(cl(M_F))^{d\beta cl} = f_\phi^{-1}(cl(M_F))$.
(iii) $\Rightarrow$ **(iv)** : From the fact that $N_E^{d\beta cl} \widetilde{\subseteq} (f_\phi^{-1}(f_\phi(N_E))^{d\beta cl}$ and from **(iii)**, we have $(f_\phi^{-1}(f_\phi(N_E))^{d\beta cl} \widetilde{\subseteq} f_\phi^{-1}(cl(f_\phi(N_E))$. This implies that $f_\phi(N_E^{d\beta cl}) \widetilde{\subseteq} cl(f_\phi(N_E))$.
(iv) $\Rightarrow$ **(v)** : For any soft subset M_F of $\widetilde{Y}$, we obtain from Lemma (1) that $f_\phi(\widetilde{X} - (f_\phi^{-1}(N_E))^{i\beta o}) = f_\phi(((f_\phi^{-1}(N_E))^c)^{d\beta cl})$. It follows from **(iv)**, that $f_\phi(((f_\phi^{-1}(N_E))^c)^{d\beta cl})$ $\widetilde{\subseteq} cl(f_\phi(f_\phi^{-1}(N_E))^c) = cl(f_\phi(f_\phi^{-1}(N_E^c))) \widetilde{\subseteq} cl(\widetilde{Y} - N_E) = \widetilde{Y} - int(N_E)$. Therefore $(\widetilde{X} - (f_\phi^{-1}(N_E))^{i\beta o}) \widetilde{\subseteq} f_\phi^{-1}(\widetilde{Y} - int(N_E)) = \widetilde{X} - f_\phi^{-1}(int(N_E))$. Thus $f_\phi^{-1}(int(N_E)) \widetilde{\subseteq} (f_\phi^{-1}(N_E))^{i\beta o}$.
(v) $\Rightarrow$ **(i)**: Consider M_F is a soft open subset of $\widetilde{Y}$. Then $f_\phi^{-1}(M_F) = f_\phi^{-1}(int(M_F)) \widetilde{\subseteq}$ $(f_\phi^{-1}(M_F))^{i\beta o}$. So $(f_\phi^{-1}(M_F))^{i\beta o} = f_\phi^{-1}(M_F)$ and this means that $f_\phi^{-1}(M_F)$ is a soft Iβ-open subset of $\widetilde{X}$. Hence the desired result is proved.

Theorem 4. *The following five properties of a soft mapping* $f_\phi : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$ *are equivalent:*

(i) f_ϕ *is soft D*β*-continuous (resp. soft B*β*-continuous);*

(ii) $f_\phi^{-1}(L_F)$ *is a soft I*β*-closed (resp. soft B*β*-closed) subset of* $\widetilde{X}$*, for each soft closed subset* L_F *of* $\widetilde{Y}$*;*

(iii) $(f_\phi^{-1}(M_F))^{i\beta cl} \widetilde{\subseteq} f_\phi^{-1}(cl(M_F))($ *resp.* $(f_\phi^{-1}(M_F))^{b\beta cl} \widetilde{\subseteq} f_\phi^{-1}(cl(M_F))$*, for every* $M_F \widetilde{\subseteq} \widetilde{Y}$*;*

(iv) $f_\phi(N_E^{i\beta cl}) \widetilde{\subseteq} cl(f_\phi(N_E))($ *resp.* $f_\phi(N_E^{b\beta cl}) \widetilde{\subseteq} cl(f_\phi(N_E))$*, for every* $N_E \widetilde{\subseteq} \widetilde{X}$*;*

(v) $f_\phi^{-1}(int(M_F)) \widetilde{\subseteq} (f_\phi^{-1}(M_F))^{d\beta o}($ *resp.* $f_\phi^{-1}(int(M_F)) \widetilde{\subseteq} (f_\phi^{-1}(M_F))^{b\beta o}$*, for every* $M_F \widetilde{\subseteq} \widetilde{Y}$*.*

Proof. The proof is similar to that of Theorem (3).

Theorem 5. *Let $\tau^\star$ be an extended soft topology on X. Then a soft mapping g_ϕ : $(X, \tau^\star, E, \preceq_1) \rightarrow (Y, \theta, F, \preceq_2)$ is soft I (resp. soft D, soft B) β-continuous If and only if a mapping $g : (X, \tau_e^\star, \preceq_1) \rightarrow (Y, \theta_{\phi(e)}, \preceq_2)$ is I (resp. D, B) β-continuous.*

Proof. Necessity: Let U be an open subset of $(Y, \theta_{\phi(e)}, \preceq_2)$. Then there exists a soft open subset G_F of $(Y, \theta, F, \preceq_2)$ such that $G(\phi(e)) = U$. Since g_ϕ is a soft I (resp. soft D, soft B) β-continuous mapping, then $g_\phi^{-1}(G_F)$ is a soft I (resp. soft D, soft B) β-open set. From Definition (6), it follows that a soft subset $g_\phi^{-1}(G_F) = (g_\phi^{-1}(G))_E$ of $(X, \tau, E, \preceq_1)$ is given by $g_\phi^{-1}(G)(e) = g^{-1}(G(\phi(e)))$, for each $e \in E$. By hypothesis, $\tau^\star$ is an extended soft topology on X, we obtain a subset $g^{-1}(G(\phi(e))) = g^{-1}(U)$ of $(X, \tau_e, \preceq_1)$ is I (resp. D, B) β-open. Hence a mapping g is I (resp. D, B) β-continuous.
Sufficiency: Let G_F be a soft open subset of $(Y, \theta, F, \preceq_2)$. Then from Definition (6), it follows that a soft subset $g_\phi^{-1}(G_F) = (g_\phi^{-1}(G))_E$ of $(X, \tau^\star, E, \preceq_1)$ is given by $g_\phi^{-1}(G)(e) = g^{-1}(G(\phi(e)))$, for each $e \in E$. Since a mapping g is I (resp. D, B) β-continuous, then a subset $g^{-1}(G(\phi(e)))$ of $(X, \tau_e^\star, \preceq_1)$ is I (resp. D, B) β-open. By hypothesis, $\tau^\star$ is an extended soft topology on X, we obtain $g_\phi^{-1}(G_F)$ is a soft I (resp. soft D, soft B) β-open subset of $(X, \tau^\star, E, \preceq_1)$. Hence a soft mapping g_ϕ is soft I (resp. soft D, soft B) β-continuous.

Proposition 3. *Let a surjective soft mapping $f_\phi : (X, \tau, E \preceq_1) \rightarrow (Y, \theta, F, \preceq_2)$ be soft $B\beta$-continuous. Then:*

(i) *If $\preceq_1$ is linearly order, then θ is the soft indiscrete topology.*

(ii) *If θ is the soft discrete topology, then $\preceq_1$ is an equality relation.*

4. Soft $I(D, B)\beta$-openness and soft $I(D, B)\beta$-closedness

In this section, the concepts of soft $I(D, B)$-open and soft $I(D, B)$-closed mappings are introduced and two examples are provided to elucidate the relationships among them. Then the equivalent conditions for each one of these soft mappings are discussed and some results related to them are initiated.

Definition 21. *A soft mapping $f_\phi : (X, \tau, E, \preceq_1) \rightarrow (Y, \tau, F, \preceq_2)$ is called:*

(i) *Soft I (resp. Soft D, Soft B) β-open if the image of every soft open subset of $\widetilde{X}$ is a soft I (resp. soft D, soft B) β-open subset of $\widetilde{Y}$.*

(ii) *Soft I (resp. Soft D, Soft B) β-closed if the image of every soft closed subset of $\widetilde{X}$ is a soft I (resp. soft D, soft B) β-closed subset of $\widetilde{Y}$.*

Remark 3. *From Definition (21), we can note the following:*

(i) *Every soft I (D, B) β-open mapping is soft β-open.*

(ii) *Every soft I (D, B) β-closed mapping is soft β-closed.*

(iii) *Every soft Bβ-open (resp. soft Bβ-closed) mapping is soft Iβ-open or soft Dβ-open (resp. soft Iβ-closed or soft Dβ-closed).*

We construct the following two examples to show that the converse of the three statements of remark above fails.

Example 3. *Let the two soft topological spaces* (X, τ, A), (Y, θ, B) *and the two mappings* $f : X \to Y$, $\phi : A \to B$ *be the same as in Example* (1). *Consider a partial order relation on* Y *as* $\preceq = \triangle \bigcup\{(u, w), (w, v), (u, v)\}$. *Then one can easily noted that* $f_\phi : S(X_A) \to S(Y_B)$ *is soft β-open and soft β-closed mapping. Because* $f_\phi(G_A) = \{(\frac{1}{3}, \emptyset), (\frac{1}{5}, \{w\})\}$ *is neither a soft Dβ-open nor a soft Iβ-open set, then* f_ϕ *is not a soft I (soft D, soft B) β-open mapping and because* $f_\phi(F_A^c) = \{(\frac{1}{3}, \{w\}), (\frac{1}{5}, \{v, w\})\}$ *is neither a soft Dβ-closed nor a soft Iβ-closed set, then* f_ϕ *is not a soft I (soft D, soft B) β-closed mapping.*

Example 4. *In Example above, if we only replace the partial order relation by* $\preceq = \triangle \bigcup\{(u, w)\}$ *(resp.* $\preceq = \triangle \bigcup\{(w, v)\}$*), then the soft mapping* f_ϕ *is soft Iβ-open and soft Iβ-closed (resp. soft Dβ-open and soft Dβ-closed), but is not soft Bβ-open and soft Bβ-closed.*

Theorem 6. *The following three properties of a soft mapping* $f_\phi : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$ *are equivalent:*

(i) f_ϕ *is soft Iβ-open;*

(ii) $int(f_\phi^{-1}(M_F)) \widetilde{\subseteq} f_\phi^{-1}(M_F^{i\beta o})$, *for every* $M_F \widetilde{\subseteq} \widetilde{Y}$;

(iii) $f_\phi(int(N_E)) \widetilde{\subseteq} (f_\phi(N_E))^{i\beta o}$, *for every* $N_E \widetilde{\subseteq} \widetilde{X}$.

Proof. **(i)** ⇒ **(ii)**: Given a soft subset M_F of $\widetilde{Y}$, it is obvious that $int(f_\phi^{-1}(M_F))$ is a soft open subset of $\widetilde{X}$. Then, by hypothesis, it follows that $f_\phi(int(f_\phi^{-1}(M_F)))$ is a soft Iβ-open subset of $\widetilde{Y}$. Since $f_\phi(int(f_\phi^{-1}(M_F))) \widetilde{\subseteq} f_\phi(f_\phi^{-1}(M_F)) \widetilde{\subseteq} M_F$, then $int(f_\phi^{-1}(M_F)) \widetilde{\subseteq} f_\phi^{-1}(M_F^{i\beta o})$.
(ii) ⇒ **(iii)**: Given a soft subset N_E of $\widetilde{X}$, from **(ii)**, we obtain $int(f_\phi^{-1}(f_\phi(N_E))) \widetilde{\subseteq} f_\phi^{-1}((f_\phi(N_E))^{i\beta o})$. Since $int(N_E) \widetilde{\subseteq} f_\phi^{-1}(f_\phi(int(f_\phi^{-1}(f_\phi(N_E))))) \widetilde{\subseteq} f_\phi^{-1}((f_\phi(N_E))^{i\beta o})$, then $f_\phi(int(N_E)) \widetilde{\subseteq} (f_\phi(N_E))^{i\beta o}$ as required.
(iii) ⇒ **(i)**: Let G_E be a soft open subset of $\widetilde{X}$. Then $f_\phi(int(G_E)) = f_\phi(G_E) \widetilde{\subseteq} (f_\phi(G_E))^{i\beta o}$. Hence f_ϕ is a soft Iβ-open mapping.

In a similar manner, one can prove the following theorem.

Theorem 7. *The following three properties of a soft mapping* $f_\phi : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$ *are equivalent:*

(i) f_ϕ *is soft Dβ-open (resp. soft Bβ-open);*

(ii) $int(f_\phi^{-1}(M_F)) \widetilde{\subseteq} f_\phi^{-1}(M_F^{d\beta o})$*(resp.* $int(f_\phi^{-1}(M_F)) \widetilde{\subseteq} f_\phi^{-1}(M_F^{b\beta o})$*), for every* $M_F \widetilde{\subseteq} \widetilde{Y}$;

(iii) $f_\phi(int(N_E)) \widetilde{\subseteq} (f_\phi(N_E))^{d\beta o}$*(resp.* $f_\phi(int(N_E)) \widetilde{\subseteq} (f_\phi(N_E))^{b\beta o}$*), for every* $N_E \widetilde{\subseteq} \widetilde{X}$.

Theorem 8. *The following three statements hold for a soft mapping* $f_\phi : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$*:*

(i) f_ϕ *is soft I*β*-closed if and only if* $(f_\phi(G_E))^{i\beta cl} \widetilde{\subseteq} f_\phi(cl(G_E))$*, for every* $G_E \widetilde{\subseteq} \widetilde{X}$*.*

(ii) f_ϕ *is soft D*β*-closed if and only if* $(f_\phi(G_E))^{d\beta cl} \widetilde{\subseteq} f_\phi(cl(G_E))$*, for every* $G_E \widetilde{\subseteq} \widetilde{X}$*.*

(iii) f_ϕ *is soft B*β*-closed if and only if* $(f_\phi(G_E))^{b\beta cl} \widetilde{\subseteq} f_\phi(cl(G_E))$*, for every* $G_E \widetilde{\subseteq} \widetilde{X}$*.*

Proof. We only prove the first statement and the others follow similar lines.
Necessity: Since f_ϕ is soft Iβ-closed, then $f_\phi(cl(G_E))$ is a soft Iβ-closed subset of $\widetilde{Y}$ and since $f_\phi(G_E) \widetilde{\subseteq} f_\phi(cl(G_E))$, then $(f_\phi(G_E))^{i\beta cl} \widetilde{\subseteq} f_\phi(cl(G_E))$.
Sufficiency: Consider H_E is a soft closed subset of $\widetilde{X}$. Then $f_\phi(H_E) \widetilde{\subseteq} (f_\phi(H_E))^{i\beta cl} \widetilde{\subseteq} f_\phi(cl(H_E)) = f_\phi(H_E)$. Therefore $f_\phi(H_E) = (f_\phi(H_E))^{i\beta cl}$. This means that $f_\phi(H_E)$ is a soft Iβ-closed set. Hence the proof is complete.

Theorem 9. *The following three statements hold for a bijective soft mapping* $f_\phi : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$*:*

(i) f_ϕ *is soft I (resp. soft D, soft B)* β*-open if and only if* f_ϕ *is soft D (resp. soft D, soft B)* β*-closed.*

(ii) f_ϕ *is soft I (resp. soft D, soft B)* β*-open if and only if* f_ϕ^{-1} *is soft I (resp. soft D, soft B)* β*-continuous.*

(iii) f_ϕ *is soft D (resp. soft I, soft B)* β*-closed if and only if* f_ϕ^{-1} *is soft I (resp. soft D, soft B)* β*-continuous.*

Proof. For the sake of brevity, we only give proofs of cases outside the parenthesis for the three statements above and the cases between parenthesis can be made similarly.

(i) To prove the necessary condition, let H_E be a soft closed subset of $\widetilde{X}$ and consider f_ϕ is a soft Iβ-open mapping. Then H_E^c is soft open and $f_\phi(H_E^c)$ is soft Iβ-open. It follows from the bijectiveness of f_ϕ, that $f_\phi(H_E^c) = [f_\phi(H_E)]^c$. This automatically implies that $f_\phi(H_E)$ is soft Dβ-closed. Thus f_ϕ is a soft Dβ-closed mapping. In a similar manner, we can prove the sufficiency condition.

(ii) Necessity: Let G_E be a soft open subset of $\widetilde{X}$ and consider f_ϕ is a soft Iβ-open mapping. Then $f_\phi(G_E)$ is soft Iβ-open. It follows from the bijectiveness of f_ϕ, that $f_\phi(G_E) = (f_\phi^{-1})^{-1}(G_E)$. This automatically implies that $(f_\phi^{-1})^{-1}(G_E)$ is soft Iβ-open. Thus f_ϕ^{-1} is a soft Iβ-continuous mapping. In a similar manner, we can prove the sufficiency condition.

(iii) The proof of this statement comes immediately from **(i)** and **(ii)** above.

Theorem 10. *Let $\theta^\star$ be an extended soft topology on Y and ϕ is an injective mapping. Then a soft mapping $g_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta^\star, F, \preceq_2)$ is soft I (resp. soft D, soft B) β-open if and only if a mapping $g : (X, \tau_e, \preceq_1) \to (Y, \theta^\star_{\phi(e)}, \preceq_2)$ is I (resp. D, B) β-open.*

Proof. To prove the necessary part, let U be an open subset of $(X, \tau_e, \preceq_1)$ and $\phi(e) = f$. Then there exists a soft open subset G_E of $(X, \tau, E, \preceq_1)$ such that $G(e) = U$. Since g_ϕ is a soft I (resp. soft D, soft B) β-open mapping, then $g_\phi(G_E)$ is a soft I (resp. soft D, soft B) β-open set. From Definition (6), it follows that a soft subset $g_\phi(G_E) = (g_\phi(G))_F$ of $(Y, \theta, F, \preceq_2)$ is given by $g_\phi(G)(f) = \bigcup_{e\in\phi^{-1}(f)} g(G(e))$, for each $f \in F$. By hypothesis, $\theta^\star$ is an extended soft topology on Y, a subset $\bigcup_{e\in\phi^{-1}(f)} g(G(e)) = g(U)$ of $(Y, \theta_{\phi(e)}, \preceq_2)$ is I (resp. D, B) β-open. Hence a mapping g is I (resp. D, B) β-open.
To prove the sufficient part, let G_E be a soft open subset of $(X, \tau, E, \preceq_1)$. Then from Definition (6), it follows that a soft subset $g_\phi(G_E) = (g_\phi(G))_F$ of $(Y, \theta^\star, F, \preceq_2)$ is given by $g_\phi(G)(f) = \bigcup_{e\in\phi^{-1}(f)} g(G(e))$, for each $f \in F$. Since a mapping g is I (resp. D, B) β-open, then a subset $\bigcup_{e\in\phi^{-1}(f)} g(G(e))$ of $(Y, \theta^\star_{\phi(e)}, \preceq_2)$ is I (resp. D, B) β-open. By hypothesis, $\theta^\star$ is an extended soft topology on Y, $g_\phi(G_E)$ is a soft I (resp. soft D, soft B) β-open subset of $(Y, \theta^\star, F, \preceq_2)$. Hence a soft mapping g_ϕ is soft I (resp. soft D, soft B) β-open.

The result above is restated in the case of a soft I (resp. soft D, soft B) β-closed mapping and one can prove them similarly. So the proof will be omitted.

Theorem 11. *Let $\theta^\star$ be an extended soft topology on Y and ϕ is an injective mapping. Then a soft mapping $g_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta^\star, F, \preceq_2)$ is soft I (resp. soft D, soft B) β-closed if and only if a mapping $g : (X, \tau_e, \preceq_1) \to (Y, \theta^\star_{\phi(e)}, \preceq_2)$ is I (resp. D, B) β-closed.*

Proposition 4. *Consider τ is not the indiscrete topology on X. If an injective soft mapping $f_\phi : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$ is soft $B\beta$-open or soft $B\beta$-closed, then $\preceq_2$ is not linearly ordered.*

Proposition 5. *Let $f_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta, F, \preceq_2)$ and $g_\lambda : (Y, \theta, F, \preceq_2) \to (Z, \upsilon, K, \preceq_3$) be two soft mappings. Then then following properties hold, for $x \in \{I, D, B\}$.*

(i) *If f_ϕ is a soft $x\beta$-continuous mapping and g_λ is a soft continuous mapping, then $g_\lambda \circ f_\phi$ is a soft x-continuous mapping.*

(ii) *If f_ϕ is a soft open (resp. soft closed) mapping and g_λ is a soft $x\beta$-open (resp. $x\beta$-closed) mapping, then $g_\lambda \circ f_\phi$ is a soft x-open (resp. $x\beta$-closed) mapping.*

(iii) *If $g_\lambda \circ f_\phi$ is a soft x-open mapping and f_ϕ is surjective soft continuous, then g_λ is a soft x-open mapping.*

(iv) *If $g_\lambda \circ f_\phi$ is a soft closed mapping and g_λ is an injective soft x-continuous mapping, then f_ϕ is a soft y-closed mapping, where $(x, y) \in \{(I, D), (D, I), (B, B)\}$.*

5. Soft $I(D,B)\beta$-homeomorphism

The concepts of soft $I(D,B)$-homeomorphism mappings are established and their main properties are discussed. Illustrative examples are provided to show the relationships among them.

Definition 22. *A bijective soft mapping $g_\phi : (X,\tau,E,\preceq_1) \to (Y,\theta,F,\preceq_2)$ is called soft I (resp. soft D, soft B) β-homeomorphism if it is soft Iβ-continuous and soft Iβ-open (resp. soft Dβ-continuous and soft Dβ-open, soft Bβ-continuous and soft Bβ-open).*

Remark 4. *From Definition (22), we can note the following:*

(i) *Every soft I (soft D, soft B) β-homeomorphism mapping is soft β-homeomorphism.*

(ii) *Every soft Bβ-homeomorphism mapping is soft Iβ-homeomorphism or soft Dβ-homeomorphism.*

The two items of the remark above are not conversely as the following examples show.

Example 5. *Let $X = \{u,v,w,x,y,z\}$ be an universe set and $A = \{a_1,a_2\}$ be a parameters set. Consider $\phi : A \to A$ and $f : X \to X$ are both identity mappings. We define two partial order relations on X and Y, respectively, as $\preceq_1 = \triangle \bigcup \{(w,v)\}$ and $\preceq_2 = \triangle \bigcup \{(z,x)\}$ and we define two soft topologies τ and θ on X and Y, respectively, as $\tau = \{\widetilde{\emptyset}, \widetilde{X}, F_A, G_A, H_A\}$ and $\theta = \{\widetilde{\emptyset}, \widetilde{Y}, L_A\}$, where $F_A = \{(a_1,X),(a_2,\{w,z\})\}$, $G_A = \{(a_1,\{u,v\}),(a_2,X)\}$, $H_A = \{(a_1,\{u,v\}),(a_2,\{w,z\})\}$ and $L_A = \{(a_1,\{u,z\}),(a_2,\{v\})\}$. Then one can readily check that a soft mapping $f_\phi : S(X_A) \to S(Y_B)$ is soft β-homeomorphism. On the other hand, $f_\phi(F_A) = F_A$ is not a soft Iβ-open set and $f^{-1}_\phi(L_A) = L_A$ is not a soft Dβ-open set. Hence f_ϕ is not soft I (soft D, soft B) β-homeomorphism.*

Example 6. *In Example above, if we only replace the partial order relation $\preceq_1$ by $\preceq = \triangle \bigcup \{(w,x)\}$, then the soft mapping f_ϕ is soft D-homeomorphism, but is not soft B-homeomorphism. Also, if we only replace the partial order relation $\preceq_2$ by $\preceq = \triangle \bigcup \{(y,z)\}$, then the soft mapping f_ϕ is soft I-homeomorphism, but is not soft B-homeomorphism.*

Theorem 12. *Consider $f_\phi : (X,\tau,E,\preceq_1) \to (Y,\theta,F,\preceq_2)$ is a bijective soft mapping and let $(\gamma,\lambda) \in \{(I\beta, d\beta cl),(D\beta, i\beta cl),(B\beta, b\beta cl)\}$. Then f_ϕ is soft γ-homeomorphism if and only if $(f_\phi(G_E))^\lambda = f_\phi(cl(G_E)) = cl(f_\phi(G_E)) = f_\phi(G_E^\lambda)$, for every $G_E \widetilde{\subseteq} \widetilde{X}$.*

Proof. We make a proof for the theorem in the case of $(\gamma,\lambda) = (I\beta, d\beta cl)$ and the other follow similar line.
Necessity: The property f_ϕ is a soft Iβ-homeomorphism mapping implies that $f_\phi(G_E^{d\beta cl}) \widetilde{\subseteq} cl(f_\phi(G_E))$ and $(f_\phi(G_E))^{d\beta cl} \widetilde{\subseteq} f_\phi(cl(G_E))$, for every $G_E \widetilde{\subseteq} \widetilde{X}$. So $f_\phi(cl(G_E)) \widetilde{\subseteq} f_\phi(G_E^{d\beta cl}) \widetilde{\subseteq} cl(f_\phi(G_E)) \widetilde{\subseteq} (f_\phi(G_E))^{d\beta cl}$ and $cl(f_\phi(G_E)) \widetilde{\subseteq} (f_\phi(G_E))^{d\beta cl} \widetilde{\subseteq} f_\phi(cl(G_E)) \widetilde{\subseteq} f_\phi(G_E^{d\beta cl})$. By the preceding two inclusion relations, we obtain the required equality relation.
Sufficiency: The equality relation $(f_\phi(G_E))^{d\beta cl} = f_\phi(cl(G_E)) = cl(f_\phi(G_E)) = f_\phi(G_E^{d\beta cl})$ implies that $f_\phi(G_E^{d\beta cl}) \widetilde{\subseteq} cl(f_\phi(G_E))$ and $(f_\phi(G_E))^{d\beta cl} \widetilde{\subseteq} f_\phi(cl(G_E))$. So f_ϕ is soft Iβ-continuous and soft Dβ-closed mapping. Hence the desired result is proved.

Theorem 13. *If a bijective soft mapping $f_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta, F, \preceq_2)$ is soft Iβ-continuous (resp. soft Dβ-continuous, soft Bβ-continuous), Then the following three statements are equivalent:*

(i) *f_ϕ is soft Iβ-homeomorphism (resp. soft Dβ-homeomorphism, soft Bβ-homeomorphism);*

(ii) *f_ϕ^{-1} is soft Iβ-continuous (resp. soft Dβ-continuous, soft Bβ-continuous);*

(iii) *f_ϕ is soft Dβ-closed (resp. soft Iβ-closed, soft Bβ-closed).*

Proof. **(i)** ⇒ **(ii)** Since f_ϕ is a soft Iβ-homeomorphism (resp. soft Dβ-homeomorphism , soft Bβ-homeomorphism) mapping, then f_ϕ is soft Iβ-open (resp. soft Dβ-open , soft Bβ-open). It follows from item **(ii)** of Theorem (9), that f_ϕ^{-1} is soft Iβ-continuous (resp. soft Dβ-continuous, soft Bβ-continuous).
(ii) ⇒ **(iii)** The proof follows from item **(iii)** of Theorem (9).
(iii) ⇒ **(i)** It sufficient to prove that f_ϕ is a soft Iβ-open (resp. soft Dβ-open, soft Bβ-open) mapping. This follows from item **(i)** of Theorem (9).

Theorem 14. *Let $\tau^\star$ and $\theta^\star$ be extended soft topologies on X and Y, respectively. Then a soft mapping $g_\phi : (X, \tau^\star, E, \preceq_1) \to (Y, \theta^\star, F, \preceq_2)$ is soft I (resp. soft D, soft B) β-homeomorphism if and only if a mapping $g : (X, \tau^\star_e, \preceq_1) \to (Y, \theta^\star_{\phi(e)}, \preceq_2)$ is I (resp. D, B) β-homeomorphism.*

Proof. The proof is obtained immediately from Theorem (5) and Theorem (10)

Proposition 6. *Let a soft mapping $f_\phi : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$ be soft Bβ-homeomorphism. Then:*

(i) *If $\preceq_1$ and $\preceq_2$ are linearly order, then τ and θ are the soft indiscrete topologies.*

(ii) *If τ and θ are the soft discrete topologies, then $\preceq_1$ and $\preceq_2$ are equality relations.*

Conclusion

In [13], the authors have initiated the concept of soft topological ordered spaces as an extended of the soft topological spaces notion and have defined soft ordered separation axioms. Then they [14] have introduced several types of ordered mappings and have established main features. As a contribution of this, we have utilized a soft β-open set notion to present the concepts of soft $x\beta$-continuous, soft $x\beta$-open, soft $x\beta$-closed and soft $x\beta$-homeomorphism mappings, for $x \in \{I, D, B\}$. We have completely described these concepts and have deduced some results which connect the initiated soft mappings with those mappings via topological ordered spaces. It can be seen that our results are certainly more general than many results in [14]. Finally, hopefully that this study is a good contribution for the further researches on soft ordered spaces.

Acknowledgements

The authors thank the reviewers for their valuable comments.

References

[1] A. M. Abd El-Latif and R. A. Hosny, On soft separation axioms via β-open soft sets, South Asian Journal of Mathematics, 5 (6) (2015) 252-264.

[2] M. E. Abd El-Monsef, S. N. El-Deeb and R. A. Mahmoud, β-open sets and β-continuous mappings, Bulletin of the Faculty of Science Assiut University, 12 (1983) 77-90.

[3] M. Abo-Elhamayel and T. M. Al-shami, Supra homeomorphism in supra topological ordered spaces, Facta Universitatis, Series: Mathematics and Informatics, 31 (5) (2016) 1091-1106.

[4] U. Acar, F. Koyuncu and B. Tanay, Soft sets and soft rings, Computers and Mathematics with Applications, 59 (2010) 3458-3463.

[5] M. Akdag and A. Ozkan, On soft β-open sets and soft β-continuous functions, The Scientific World Journal, Volume 2014, Article ID 843456, 6 pages.

[6] H. Aktas and N. Cagman, Soft sets and soft groups, Information Sciences, 77 (2007) 2726-2735.

[7] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, On some new operations in soft set theory, Computers and Mathematics with Applications, 57 (2009) 1547-1553.

[8] T. M. Al-shami, Supra β-bicontinuous maps via topological ordered spaces, Mathematical Sciences Letters, 6 (3) (2017) 239-247.

[9] T. M. Al-shami, Corrigendum to "Separation axioms on soft topological spaces, Ann. Fuzzy Math. Inform. 11 (4) (2016) 511-525", Annals of Fuzzy Mathematics and Informatics, 15 (3) (2018) 309-312.

[10] T. M. Al-shami, On some maps in supra topological ordered spaces, Journal of New Theory, 20 (2018) 76-92.

[11] T. M. Al-shami, Soft somewhere dense sets on soft topological spaces, Communications of the Korean Mathematical Society, (2018) Accepted.

[12] T. M. Al-shami and M. K. Tahat, I (D, B)-supra pre maps via supra topological ordered spaces, Journal of Progressive Research in Mathematics, 12 (3) (2017) 1989-2001.

[13] T. M. Al-shami, M. E. El-Shafei and M. Abo-Elhamayel, On soft topological ordered spaces, Journal of King Saud University-Science, (2018) https://doi.org/10.1016/j.jksus.2018.06.005.

[14] T. M. Al-shami, M. E. El-Shafei and M. Abo-Elhamayel, On soft ordered maps, Submitted.

[15] T. M. Al-shami, L. D. R. Kočinac, The equivalence between the enriched and extended soft topologies, submitted.

[16] A. Aygünoğlu and H. Aygün, Some notes on soft topological spaces, Neural Computers and Applications, 21 (2012) 113-119.

[17] K. V. Babitha and J.J. Sunil, Soft set relations and functions, Computers and Mathematics with Applications, 60 (2010) 1840-1849.

[18] P. Das, Separation axioms in ordered spaces, Soochow Journal of Mathematics, 30 (4) (2004) 447-454.

[19] S. Das and S. K. Samanta, Soft metric, Annals of Fuzzy Mathematics and Informatics, 1 (2013) 77-94.

[20] M. E. El-Shafei, M. Abo-Elhamayel and T. M. Al-shami, Generating ordered maps via supra topological ordered spaces, International Journal of Modern Mathematical Sciences, 15 (3) (2017) 339-357.

[21] M. E. El-Shafei, M. Abo-Elhamayel and T. M. Al-shami, Strong separation axioms in supra topological ordered spaces, Mathematical Sciences Letters, 6 (3) (2017) 271-277.

[22] M. E. El-Shafei, M. Abo-Elhamayel and T. M. Al-shami, Supra R-homeomorphism in supra topological ordered spaces, International Journal of Algebra and Statistics, 6 (1-2) (2017) 158-167.

[23] M. E. El-Shafei, M. Abo-Elhamayel and T. M. Al-shami, Partial soft separation axioms and soft compac spaces, Filomat 32 (2018) Accepted.

[24] F. Feng and Y. M. Li, Soft subsets and soft product operations, Information Science, 232 (2013) 1468-1470.

[25] M. D. Green, Locally convex topology on a preordered space, Pascific Journal of Mathematics, 26 (1968) 487-491.

[26] T. Hida, A comprasion of two formulations of soft compactness, Annals of Fuzzy Mathematics and Informatics, 8(4) (2014) 511-524.

[27] T. Hida, Soft topological group, Annals of Fuzzy Mathematics and Informatics, 8 (6) (2014) 1001-1025.

[28] D. S. Leela and G. Balasubramanian, New separation axioms in ordered topological spaces, Indian Journal of Pure and Applied Mathematics, 33 (2002) 1011-1016.

[29] F. Li, Notes on the soft operations, ARPN Journal of Systems and Software, 1 (6) (2011) 205-208.

[30] X. Liu and F. Feng, Y. B. Jun, A note on generalized soft equal relations, Computers and Mathematics with Applications, 64 (2012) 572-578.

[31] P. K. Maji, R. Biswas and R. Roy, Soft set theory, Computers and Mathematics with Applications, 45 (2003) 555-562.

[32] S. D. McCartan, Separation axioms for topological ordered spaces, Mathematical Proceedings of the Cambridge Philosophical Society, 64 (1986) 965-973.

[33] S. D. McCartan, Bicontinuous preordered topological spaces, Pacific Journal of Mathematics, 38 (1971) 523-529.

[34] O. Mendez, L. H. Popescu and E. D. Schwab, Inner separation structures for topological spaces, Blakan Journal of Geometry and Its Applications, 13 (2008) 59-65.

[35] W. K. Min, A note on soft topological spaces, Computers and Mathematics with Applications, 62 (2011) 3524-3528.

[36] D. Molodtsov, Soft set theory-first results, Computers and Mathematics with Applications, 37 (1999) 19-31.

[37] L. Nachbin, Topology and ordered, D. Van Nostrand Inc. Princeton, New Jersey, (1965).

[38] S. Nazmul and S. K. Samanta, Neigbourhood properties of soft topological spaces, Annals of Fuzzy Mathematics and Informatics, 1 (2013) 1-15.

[39] D. Pei and D. Miao, From soft sets to information system, In Proceedings of the IEEE International Conference on Granular Computing, 2 (2005) 617-621.

[40] L. Popescu, R-Separated spaces, Blakan Journal of Geometry and Its Applications, 6 (2001) 81-88.

[41] K. Qin and Z. Hong, On soft equality, Journal of Computational and Applied Mathematics, 234 (2010) 1347-1355.

[42] K. K. Rao and R. Chudamani, β-homeomorphism in topological ordered spaces, International Journal of Mathematical and Engineering, 182 (2012) 1734-1755.

[43] W. Rong, The countabilities of soft topological spaces, International Journal of Mathematical, Computational, Physical, Electrical and Computer Engineering, 6 (8) (2012) 952-955.

[44] M. Shabir and M. Naz, On soft topological spaces, Computers and Mathematics with Applications, 61 (2011) 1786-1799.

[45] T. Shah and S. Shaheen, Soft topological groups and rings, Annals of Fuzzy Mathematics and Informatics, 7 (5) (2014) 725-743.

[46] I. Zorlutuna, M. Akdag, W. K. Min and S. K. Samanta, Remarks on soft topological spaces, Annals of Fuzzy Mathematics and Informatics, 2 (2012) 171-185.