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Analyzing the Locus of Soft Spheres: Illustrative Cases and Drawings

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Abstract. In the studies that have been carried out so far, the definition of soft sphere has been made, although it has been accepted very hard to revive it mathematically. In this study, the differences among soft real numbers and soft points are used for the first time to revive the soft sphere mathematically thus the locus of the soft sphere can be analyzed. Applications of soft spheres are supported by suitable examples to discuss the locus of them with scrupulous attention to detail.

2010 Mathematics Subject Classifications: 03F03, 03B10, 03E75, 54E35

Key Words and Phrases: Soft set, Soft real number, Soft point, Soft sphere, Soft topology, Soft metric

1. Introduction

In some theories such as fuzzy set, rough set, vague set, intuitionistic fuzzy set and interval mathematics complicated problems can not be solved because of not being free from the parametrization tool of the theory.
Molodtsov [10] provided a general framework with the involvement of parameters and paved the way for new valuable works such as [1–4, 10–15].
The topological structures of set theories dealing with uncertainities were first defined by Chang [5]. In this study, fuzzy topology is introduced and its properties are studied. The notion of soft topological spaces introduced by Shabir and Naz [9] on an initial universe with a fixed set of parameters.
For a deeper discussion of soft topological spaces, more specialized notions from soft topological space theory were introduced in [6] and [7] as needed. We shall discuss this again at somewhat greater length in the 2. Section. These development studies on soft topological spaces motivated researchers to introduce the notion of soft metric space.

In the early 21th century, attempts to develop appropriate conceptual frameworks for dealing with soft topological structure led Das and Samanta [7] to the definition of soft

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metric spaces. Metric space is widely used in mathematics and soft metric space is a generalization of metric spaces. That is to say, soft metric spaces can be developed in a metric view with soft sets. While these important results caused most mathematicians to think of soft metric spaces as just a rather convenient tool to define and to deal with soft topological spaces, a few began to study soft metric spaces for their own sake as [11]. The development of the theory of soft metric spaces has proceeded in the following main directions: General theory of soft metric spaces and soft topological theory of metric spaces. For each of the cases the simplest and most fruitful method which the soft metric proposed was the introduction of the notion of soft distance and soft spheres. Rather than discuss the soft metric spaces in full generality, let us look at a particular situation of the view of soft spheres.

One of the most important component of soft metric spaces is soft spheres. Spherical structure plays an important role in the study of soft metric space's properties. Generally speaking, the soft spherical structure in soft metric space is equivalent to the soft open sets in a soft topological space.

Up to today, studies of soft metric spaces are made without trying to analyze the meaning of soft spheres. The searchers on soft spheres just mere "presentation" of the underlying soft metric spaces. I suspect that more can be said in specifically about which soft spheres can be obtained from soft metric space, but the researchers explore this matter further here.

The main source of this paper inspiration is to achieve the structure of soft sphere in detail. The motivate is to gain in interest if the locus of the sphere is revived mathematically. Thus it is reasonable to attempt, using the soft real number and soft point, to achieve the structure of soft spheres. In the last section this main result is stated and proved by soft sphere drawings that is shed a light on analyzing the locus of soft spheres.

In one of our applications we describe a more interesting example featuring a soft metric that is not given a soft cyclic sphere. Hence we show different shapes of soft spheres. It is obvious that this result shows a soft sphere supplies an algorithm to effectively recognize soft points in $\tilde{R}^*(E)$.

The aim of this article is to study the relationship between the size of soft spheres, as measured by its defined soft metric, and the extent to which the soft sphere fails to be defined in different soft metric. It seems that the relations between soft open and soft closed spheres emerge most clearly when the setting is quite abstract, and this motivates our approach to the subject.

2. Background in Soft Sets and Soft Metric Spaces

In the course of writing this paper I learned that the authors have simultaneously obtained results similar to each other in certain respects as the proofs are similar in spirit to that of each other. In the end of the literature search, nonetheless, the references that are written below were my main source of inspiration:

I recall some basic notions of soft set theory which may be found in [1–3, 10, 11] for further details. For a deeper discussion of soft point, I refer the reader to [12]. Also, for detailed definitions, propositions and theorems about soft positive and negative real numbers, the reference [6] can be examined. This case has been thoroughly studied; see [7] for more details, theorems and examples.

For a comprehensive treatment and for references to the extensive literature on the related subject in this article one may refer to these references to related subject.

Throughout this work, U refers to an initial universe, E is a set of parameters, $A \subseteq E$ and $P(U)$ is the power set of U.

Definition 1. *([3, 10]) A soft set f_A on the universe U is a set defined by*

$$f_A : E \to \mathcal{P}(U) \quad \text{such that} \quad f_A(x) = \emptyset \quad \text{if} \quad x \notin A.$$

Here f_A is also called an approximate function. A soft set over U can be represented by the set of ordered pairs

$$f_A = \Big\{ \big(x, f_A(x)\big) : x \in E, f_A(x) \in P(U) \Big\}.$$

Note that the set of all soft sets over U will be denoted by $\mathbb{S}$.

Definition 2. *([3]) Let $f_A \in \mathbb{S}$. Then,*
If $f_A(x) = \emptyset$ for all $x \in E$, then f_A is called an empty soft set, denoted by Φ.
If $f_A(x) = U$ for all $x \in E$, then f_A is called universal soft set, denoted by $\tilde{E}$.
If $f_A(x)$ is a singleton set for all $x \in E$, then f_A is called a singleton soft set.

Definition 3. *([6]) Let X be a non-empty set and E be a non-empty parameter set. Then a function $\varepsilon : E \to X$ is said to be a soft element of X. A soft element ε of X is said to belong to a soft set A of X, denoted by $\varepsilon \in A$, if $\varepsilon(e) \in A(e), \forall e \in E$. Thus a soft set A of X with respect to the index set E can be expressed as $A(e) = \{\varepsilon(e), \varepsilon \in A\}, e \in E$.*

This chapter continues to constitute sufficient preparation by giving definitions of soft real number and soft point:

Definition 4. *([6]) Let $\mathbb{R}$ be the set of real numbers, $\mathcal{P}(\mathbb{R})$ be the collection of all non-empty bounded subsets of $\mathbb{R}$, E be a set of parameters and $A \subseteq E$. Then a mapping $f : A \to \mathcal{P}(\mathbb{R})$ is called a soft real set. It is denoted by (f, A) . If in particular (f, A) is a singleton soft set, then identifying (f, A) with the corresponding soft element, it will be called a soft real number.*

Now on, the single parameter set to be used is denoted by E to define soft sets and their operations. Thus, the subscript E can be deleted from the soft sets (f, E), *i.e*, a soft set f_E will be denoted shortly by f unless this will cause confusion. In general, soft sets and their approximate functions are denoted by f, g, h, ..., and soft sets and their approximate functions are used interchangeably.

Definition 5. *([6]) Let us denote the collection of all soft points of f by $SP(f)$, the set of all soft real sets by $R(E)$, the set of all soft real numbers by $\tilde{R}(E)$ and the set of all* *non-negative soft real numbers by $\tilde{R}^*(E)$. If a soft real set is a singleton soft set, it will be called a soft real number and denoted by $\tilde{r}$, $\tilde{s}$ etc.*

The best general reference about aritmetic operations of soft real sets and soft real numbers is [6] for a fuller treatment.

Definition 6. *([2]) The soft set f is called a soft point in $\mathbb{S}$, if for the parameter $e_i \in E$ such that $f(e_i) \neq \emptyset$ and $f(e_j) = \emptyset$, for all $e_j \in E \setminus \{e_i\}$ is denoted by $\left(e_{i_f}\right)_j$ for all $i, j \in \mathbb{N}^+$.*
Note that the set of all soft points of f will be denoted by $SP(f)$.

Definition 7. *([12]) Let $f \in \mathbb{S}$. Then,*
If $f(e) = \emptyset$ for all $e \in E$, then f is called null soft point, denoted by e_Φ.
If $f(e) = U$ for all $e \in E$, then f is called universal soft point, denoted by $e_{\tilde{E}}$.
If there is only one parameter $e \in E$ in f, then f is denoted by e_{f_i}. If there is only one parameter $e \in E$ in f, and $f(e) = \{u\}$ then f is denoted by e_f.

Definition 8. *([12]) The soft point $\left(e_{i_f}\right)_j$ is said to belong to a soft set $g \in \mathbb{S}$, denoted by $\left(e_{i_f}\right)_j \tilde{\in} g$, if for the parameter $e_i \in E$ and $f(e_i) \subseteq g(e_i)$.*

Proposition 1. *([12])*

(i) $f \tilde{\subseteq} g$, *if* $\left(e_{i_f}\right)_j \tilde{\in} f$ *then* $\left(e_{i_f}\right)_j \tilde{\in} g$ *for all* $e_i \tilde{\in} f$.

(ii) $f = g$, *if and only if* $\left(e_{i_f}\right)_j \tilde{\in} f$ *then* $\left(e_{i_f}\right)_j \tilde{\in} g$ *and* $\left(e_{i_f}\right)_j \tilde{\in} g$ *and* $\left(e_{i_f}\right)_j \tilde{\in} f$ *for all* $e_i \tilde{\in} f$.

Definition 9. *([11]) Let $\emptyset \neq X \subseteq E$, $f \in S_X(U)$ and $f : X \to \mathcal{P}(U)$ be one to one function. We denote by $\tilde{R}^*(E)$ the set of all soft real numbers and $f_i, f_j, f_s \in S_X(U)$; $\left(e_{i_f}\right)_j, \left(e_{x_f}\right)_y, \left(e_{s_f}\right)_k \tilde{\in} f$. A mapping*

$$\begin{aligned} d :& E \times E \to \tilde{R}^*(E) \\ & \left(\left(e_{i_f}\right)_j, \left(e_{x_f}\right)_y\right) \to \tilde{d}\left(\left(e_{i_f}\right)_j, \left(e_{x_f}\right)_y\right) \end{aligned}$$

is said to be a soft metric on the soft set f if $\tilde{d}$ satisties the following conditions:

(i) $\tilde{d}\left(\left(e_{i_f}\right)_j, \left(e_{x_f}\right)_y\right) \geq 0$.

(ii) $\tilde{d}\left(\left(e_{i_f}\right)_j, \left(e_{x_f}\right)_y\right) = 0 \Leftrightarrow \left(e_{i_f}\right)_j = \left(e_{x_f}\right)_y$

(iii) $\tilde{d}\left(\left(e_{i_f}\right)_j, \left(e_{x_f}\right)_y\right) = \tilde{d}\left(\left(e_{x_f}\right)_y, \left(e_{i_f}\right)_j\right)$

(iv) $\tilde{d}\left(\left(e_{i_f}\right)_j, \left(e_{x_f}\right)_y\right) \leq \tilde{d}\left(\left(e_{i_f}\right)_j, \left(e_{s_f}\right)_k\right) + \tilde{d}\left(\left(e_{s_f}\right)_k, \left(e_{x_f}\right)_y\right)$

If $\tilde{d}$ is a soft metric on the soft set f then, f is called soft metric space and denoted by $(f, \tilde{d})$.

Definition 10. *([11]) Let $(e_{i_f})_j$ and $(e_{x_f})_y$ be soft points of a soft metric space. The value of $\tilde{d}((e_{i_f})_j, (e_{x_f})_y)$ is called as the soft distance between the soft points $(e_{i_f})_j$ and $(e_{x_f})_y$.*

Definition 11. *([7]) Let $(f, \tilde{d})$ be a soft metric space. $g \neq \Phi$ and $g \tilde{\subseteq} f$. Then the diameter of g is denoted by $\delta(g)$ and is defined by*

$$d((g)(\gamma)) = \sup\left\{\tilde{d}((e_{i_f})_j, (e_{x_f})_y); (e_{i_f})_j, (e_{x_f})_y \tilde{\in} g\right\}, \forall \gamma \tilde{\in} f.$$

Definition 12. *([7]) Let $(f, \tilde{d})$ be a soft metric space, $(e_{x_f})_y$ be a fixed soft point of f and $g \subseteq f$. Then the distance of the soft point $(e_{x_f})_y$ from the soft set g is denoted by $\delta((e_{x_f})_y, g)$ and defined by*

$$\delta((e_{x_f})_y, g)(\gamma) = \inf\left\{\tilde{d}((e_{x_f})_y, (e_{i_f})_j)(\gamma); (e_{i_f})_j)\tilde{\in} g\right\}$$

Definition 13. *([7]) Let $(f, \tilde{d})$ a soft metric space, $\tilde{r}$ be a non-negative soft real number and $(e_{i_f})_j, (e_{x_f})_y \tilde{\in} f$.*

$$\tilde{B}_{\tilde{d}}((e_{x_f})_y, \tilde{r}) = \left\{(e_{i_f})_j \tilde{\in} f : \tilde{d}((e_{i_f})_j, (e_{x_f})_y) \tilde{<} \tilde{r}\right\}$$

$$\tilde{B}_{\tilde{d}}[(e_{x_f})_y, \tilde{r}] = \left\{(e_{i_f})_j \tilde{\in} f : \tilde{d}((e_{i_f})_j, (e_{x_f})_y) \tilde{\leq} \tilde{r}\right\}$$

$$\tilde{K}_{\tilde{d}}((e_{x_f})_y, \tilde{r}) = \left\{(e_{i_f})_j \tilde{\in} f : \tilde{d}((e_{i_f})_j, (e_{x_f})_y) \tilde{=} \tilde{r}\right\}$$

These soft sets are called respectively soft open sphere, soft closed sphere and soft k-sphere with center $(e_{x_f})_y$ and radius $\tilde{r}$.

For the sake of clarity, the notation can be written as $\tilde{B}_{\tilde{d}}((e_{x_f})_y, \tilde{r}) = \tilde{B}((e_{x_f})_y, \tilde{r})$.

Besides, there is a property of soft spheres that is valid for all soft metrics:

For $\tilde{r}_1 < \tilde{r}_2$,

$$\tilde{B}((e_{x_f})_y, \tilde{r}_1) \tilde{\subset} \tilde{B}[(e_{x_f})_y, \tilde{r}_1] \tilde{\subset} \tilde{B}((e_{x_f})_y, \tilde{r}_2) \tilde{\subset} \tilde{B}[(e_{x_f})_y, \tilde{r}_2]$$

It is obvious from the definition of soft metric.

3. More on Soft Spheres with Using Soft Real Numbers and Soft Points

In this section, the most important facts about soft spheres are compiled for the purpose of exposition the structure of soft spheres are studied in [13].

Proposition 2. *([13]) Any soft intersection of soft open spheres need not to be soft open. The following example shows that:*

Example 1. *([13]) Let me show the equation* $\bigcap\limits_{\bar{n}=\bar{1}}^{\infty} \tilde{B}\Big(\big(e_{x_f}\big)_y, \bar{1}+\frac{\bar{1}}{\bar{n}}\Big) = \tilde{B}\Big(\big(e_{x_f}\big)_y, \bar{1}\Big)$.

a) $\bigcap\limits_{\bar{n}=\bar{1}}^{\infty} \tilde{B}\Big(\big(e_{x_f}\big)_y, \bar{1}+\frac{\bar{1}}{\bar{n}}\Big) \tilde{\subset} \tilde{B}\Big(\big(e_{x_f}\big)_y, \bar{1}\Big)$ *(?)*

$$\big(e_{i_f}\big)_j \tilde{\in} \bigcap\limits_{\bar{n}=\bar{1}}^{\infty} \tilde{B}\Big(\big(e_{x_f}\big)_y, \bar{1}+\frac{\bar{1}}{\bar{n}}\Big) \Rightarrow \big(e_{i_f}\big)_j \tilde{\in} B\Big(\big(e_{x_f}\big)_y, \bar{1}+\frac{\bar{1}}{\bar{n}}\Big) \Rightarrow |\bar{i}-\bar{x}| \tilde{<} \bar{1}+\frac{\bar{1}}{\bar{n}}$$

$$\left.\begin{array}{l} (\forall \varepsilon > 0\ ), \bar{i} \tilde{<} \bar{j}+\varepsilon\ ,\ \bar{i} \tilde{\geq} \bar{j} \\ (\forall \varepsilon > 0)\ \ (\frac{\bar{1}}{\bar{n}} \tilde{<} \varepsilon)\ :\ \exists \bar{n} \tilde{\in} \mathbb{N} \end{array}\right\} \Rightarrow |\bar{i}-\bar{x}| \tilde{<} \bar{1}+\frac{\bar{1}}{\bar{n}} \tilde{\leq} \bar{1} \Rightarrow \big(e_{i_f}\big)_j \tilde{\in} \tilde{B}\Big(\big(e_{x_f}\big)_y, \bar{1}\Big).$$

b)

$$\big(e_{i_f}\big)_j \tilde{\in} \tilde{B}\Big(\big(e_{x_f}\big)_y, \bar{1}\Big) \Rightarrow \tilde{\rho}\Big(\big(e_{i_f}\big)_j, \big(e_{x_f}\big)_y\Big) \tilde{\leq} \bar{1} \tilde{<} \bar{1}+\frac{\bar{1}}{\bar{n}}\ , (\exists \bar{n} \tilde{\in} \mathbb{N}^+)$$

$$\tilde{\rho}\Big(\big(e_{i_f}\big)_j, \big(e_{x_f}\big)_y\Big) \tilde{<} \bar{1}+\frac{\bar{1}}{\bar{n}} \Rightarrow \big(e_{i_f}\big)_j \tilde{\in} \bigcap_{\bar{n}=\bar{1}}^{\infty} \tilde{B}\Big(\big(e_{x_f}\big)_y, \bar{1}+\frac{\bar{1}}{\bar{n}}\Big)$$

Proposition 3. *([13]) Any soft union of soft closed spheres need not to be soft closed. The following example shows that:*

Example 2. *([13]) Let me show the equation* $\bigcup\limits_{n=1}^{\infty} \tilde{B}\Big(\big(e_{x_f}\big)_y, \frac{\bar{n}}{n+1}\Big) \tilde{=} B\Big(\big(e_{x_f}\big)_y, \bar{1}\Big)$.

a) $\bigcup\limits_{n=1}^{\infty} \tilde{B}\Big(\big(e_{x_f}\big)_y, \frac{\bar{n}}{n+1}\Big) \tilde{\subset} B\Big(\big(e_{x_f}\big)_y, \bar{1}\Big)$ *(?)*

$\exists \big(e_{x_f}\big)_y \tilde{\in} \tilde{B}\Big(e_{x_f}\big)_y, \frac{\bar{n}}{n+1}\Big) \Rightarrow \tilde{\rho}\Big(\big(e_{x_f}\big)_y, \big(e_{i_f}\big)_j\Big) \tilde{\leq} \frac{\bar{n}}{n+1} \tilde{<} \bar{1}$ $(\forall \bar{n} \tilde{\in} \mathbb{N}^+)$

$\Rightarrow \tilde{\rho}\Big(\big(e_{x_f}\big)_y, \big(e_{i_f}\big)_j\Big) \tilde{<} \bar{1} \Rightarrow \big(e_{i_f}\big)_j \tilde{\in} B_\rho\Big(\big(e_{x_f}\big)_y, \bar{1}\Big)$.

b) $\big(e_{i_f}\big)_j \tilde{\in} \tilde{B}\Big(e_{x_f}\big)_y, \bar{1}\Big) \Rightarrow \big(e_{i_f}\big)_j \tilde{\in} \bigcup\limits_{\bar{n}=\bar{1}}^{\infty} \tilde{B}\Big(\big(e_{x_f}\big)_y, \frac{\bar{n}}{n+1}\Big)$ *(?)*

$\big(e_{i_f}\big)_j \tilde{\in} \tilde{B}\Big(e_{x_f}\big)_y, \bar{1}\Big) \Rightarrow \tilde{\rho}\Big(\big(e_{x_f}\big)_y, \big(e_{i_f}\big)_j\Big) \tilde{<} \bar{1}$

Suppose that, $\big(e_{x_f}\big)_y < \big(e_{i_f}\big)_j < \big(e_{x_f}\big)_y + \frac{\bar{1}}{\bar{n}}$ *and* $(1/\bar{n} = \varepsilon)$.
Then, $\big(e_{x_f}\big)_y < \big(e_{i_f}\big)_j \leq \big(e_{x_f}\big)_y$. *So,* $\big(e_{x_f}\big)_y = \big(e_{i_f}\big)_j$.

This contradicts our assumption and leads to the contradiction that $\big(e_{i_f}\big)_j \tilde{\in} \tilde{B}\Big(e_{x_f}\big)_y, \bar{1}\Big)$. *Hence,*

$\left(e_{x_f}\right)_y + \frac{\bar{1}}{\bar{n}} < \left(e_{i_f}\right)_j$
$\Rightarrow \left(e_{i_f}\right)_j < \frac{\bar{n}}{an+1} \leq \frac{\bar{n}}{n+1}$
$\Rightarrow \left(e_{x_f}\right)_y < \left(e_{i_f}\right)_j \leq \frac{\bar{n}}{n+1}$
$\Rightarrow \left(e_{i_f}\right)_j \tilde{\in} \left(\left(e_{x_f}\right)_y, \frac{\bar{n}}{n+1}\right)$
$\Rightarrow \tilde{\rho}\left(\left(e_{x_f}\right)_y, \left(e_{i_f}\right)_j\right)\tilde{} \leq \frac{\bar{n}}{n+1}.$

The benefit of formulating the notion of 'spheres' as above will become clear with applications in the next section:

4. Analizing The Locus of the Soft Spheres with Examples and Drawings

This section is one of the major steps in analizing the locus of the soft spheres has been accepted very hard to revive until now. For this purpose, different soft metrics are defined and obtained a soft metric that is not given a soft cyclic sphere. Results are proved that a soft sphere supplies an algorithm to effectively recognize soft points in $\tilde{R}^*(E)$.

Example 3. *Let d_1 function is defined as $\tilde{d}_1\left(\left(e_{i_f}\right)_j, \left(e_{x_f}\right)_y\right) = \sqrt{(\bar{i}-\bar{x})^2 + (\bar{j}-\bar{y})^2}$ in f. Then, show the following cases:*

(i) $(f, \tilde{d}_1)$ is a soft metric space.

(ii) Find the soft sets $\tilde{B}_{\tilde{d}_1}\ ((\bar{0},\bar{0}),\bar{2})$, $\tilde{B}_{\tilde{d}_1}\ [(\bar{1},\bar{1}),\bar{3}]$ and $\tilde{K}_{\tilde{d}_1}\ ((\bar{1},\bar{2}),\bar{1})$.

(iii) Draw the soft spheres that are found in (2).

Solution:

$$\tilde{d}_1 : E \times E \to \tilde{R}^*(E)$$
$$\left(\left(e_{i_f}\right)_j, \left(e_{x_f}\right)_y\right) \to \tilde{d}_1\left(\left(e_{i_f}\right)_j, \left(e_{x_f}\right)_y\right) = \sqrt{(\bar{i}-\bar{x})^2 + (\bar{j}-\bar{y})^2}$$

(m_1) *The square root function is positive defined. Hence, this function is positive, too.*

(m_2) $\tilde{d}_1\left(\left(e_{i_f}\right)_j, \left(e_{x_f}\right)_y\right) = \bar{0} \Leftrightarrow \left(e_{i_f}\right)_j = \left(e_{x_f}\right)_y$ *(?)*
$\sqrt{(\bar{i}-\bar{x})^2 + (\bar{j}-\bar{y})^2} = \bar{0} \Rightarrow (\bar{i}-\bar{x})^2 + (\bar{j}-\bar{y})^2 = \bar{0}$. *Then,* $(\bar{i}-\bar{x})^2 = \bar{0}$ *and* $(\bar{j}-\bar{y})^2 = \bar{0}$.

(m_3) $\tilde{d}\left(\left(e_{i_f}\right)_j, \left(e_{x_f}\right)_y\right) = \tilde{d}\left(\left(e_{x_f}\right)_y, \left(e_{i_f}\right)_j\right)$ *(?)*
If square function is applied both of sides, it is obvious.

(m_4) $\forall \left(e_{i_f}\right)_j, \left(e_{x_f}\right)_y, \left(e_{s_f}\right)_k \tilde{\in} f$

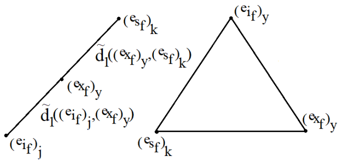

Figure 1

$$\tilde{d}\big((e_{i_f})_j,(e_{x_f})_y\big) \leq \tilde{d}\big((e_{i_f})_j,(e_{s_f})_k\big) + \tilde{d}\big((e_{s_f})_k),(e_{x_f})_y)\big)$$

$$\begin{aligned}\tilde{B}_{\tilde{d}_1}\ ((\bar{0},\bar{0}),\bar{2}) &= \left\{(e_{i_f})_j \,\tilde{\in}\, f|\tilde{d}_1((\bar{0},\bar{0}),(\bar{i},\bar{j})) \,\tilde{<}\, \bar{2}\right\}\\ &= \left\{(e_{i_f})_j \,\tilde{\in}\, f|\sqrt{(\bar{0}-\bar{i})^2+(\bar{0}-\bar{j})^2} \,\tilde{<}\, \bar{2}\right\}\\ &= \left\{(e_{i_f})_j \,\tilde{\in}\, f|\bar{i}^2+\bar{j}^2 \,\tilde{<}\, \bar{4}\right\},\ \ \tilde{r}=\bar{2}\ \textit{and}\ O(\bar{0},\bar{0}).\end{aligned}$$

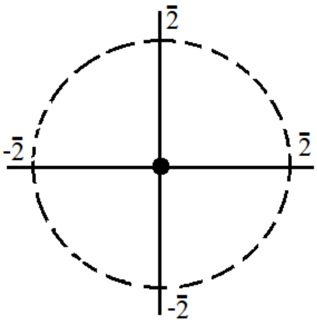

Figure 2

$$\begin{aligned}K_{\tilde{d}_1}\ ((\bar{1},\bar{2}),\bar{1}) &= \left\{(e_{i_f})_j \,\tilde{\in}\, f|\tilde{d}_1((\bar{1},\bar{2}),(\bar{i},\bar{j})) = \bar{1}\right\}\\ &= \left\{(e_{i_f})_j \,\tilde{\in}\, f|\sqrt{(\bar{i}-\bar{1})^2+(\bar{j}-2)^2} = \bar{1}\right\}\\ &= \left\{(e_{i_f})_j \,\tilde{\in}\, f|(\bar{i}-\bar{1})^2+(\bar{j}-\bar{2})^2 = \bar{1}\right\}\ \ \tilde{r}=\bar{1}\ \textit{and}\ O(\bar{1},\bar{2}).\end{aligned}$$

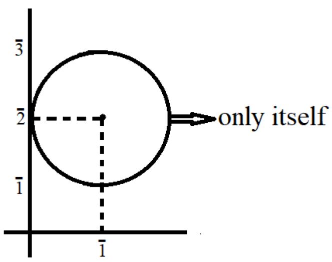

Figure 3

We now describe a more interesting example featuring a metric that is not posed a cyclic sphere:

Example 4. *Let d_2 function is defined as $\tilde{d}_2\big((e_{i_f})_j,(e_{x_f})_y\big)=|\bar{i}-\bar{x}|+|\bar{j}-\bar{y}|$ in f. Then, show the following cases:*

(i) $(f,\tilde{d}_2)$ *is a soft metric space.*

(ii) Find the soft sets $\tilde{B}_{\tilde{d}_2}\ ((\bar{1},\bar{1}),\bar{2})$, $\tilde{B}_{\tilde{d}_2}\ [(\bar{0},\bar{0}),\bar{1}]$ *and* $\tilde{K}_{\tilde{d}_2}\ ((\bar{0},\bar{-2}),\bar{1})$.

(iii) Draw the soft spheres that are found in (2).

Solution:

$$\tilde{d}_2 : E\times E\to \tilde{R}^*(E)$$

$$\big((e_{i_f})_j,(e_{x_f})_y\big)\to \tilde{d}_2\big((e_{i_f})_j,(e_{x_f})_y\big)=|\bar{i}-\bar{x}|+|\bar{j}-\bar{y}|\,\tilde{\geq}\,\bar{0}$$

(m₂) $\tilde{d}_2\big((e_{i_f})_j,(e_{x_f})_y\big)=\bar{0}\Leftrightarrow|\bar{i}-\bar{x}|+|\bar{j}-\bar{y}|=\bar{0}\Leftrightarrow|\bar{i}-\bar{x}|=\bar{0}$ *and* $|\bar{j}-\bar{y}|=\bar{0}$

(m₄) $\forall(e_{i_f})_j,(e_{x_f})_y,(e_{s_f})_k\,\tilde{\in}\,f$

$$\begin{aligned}\tilde{d}_2\big((e_{i_f})_j,(e_{x_f})_y\big)&=|\bar{i}-\bar{x}|+|\bar{j}-\bar{y}|\\&=|\bar{i}-\bar{s}+\bar{s}-\bar{x}|+|\bar{j}-\bar{k}+\bar{k}-\bar{y}|\\&\tilde{\leq}|\bar{i}-\bar{s}|+|\bar{s}-\bar{x}|+|\bar{j}-\bar{k}|+|\bar{k}-\bar{y}|\\&=\tilde{d}_2\big((e_{i_f})_j,(e_{s_f})_k\big)+\tilde{d}_2\big((e_{s_f})_k,(e_{x_f})_y\big)\end{aligned}$$

$$\begin{aligned}\tilde{B}_{\tilde{d}_2}\ ((\bar{1},\bar{1}),\bar{2})&=\Big\{(e_{i_f})_j\,\tilde{\in}\,f|\tilde{d}_2((\bar{1},\bar{1}),(\bar{i},\bar{j}))\,\tilde{<}\,\bar{2}\Big\}\\&=\Big\{(e_{i_f})_j\,\tilde{\in}\,f||\bar{1}-\bar{i}|+|\bar{1}-\bar{j}|\,\tilde{<}\,\bar{2}\Big\}\end{aligned}$$

Critical points : $|\bar{1}-\bar{i}|+|\bar{1}-\bar{j}|\,\tilde{<}\,\bar{2}\Rightarrow\bar{i}=\bar{1}$ *and* $\bar{j}=\bar{1}$
(i) $\bar{i}\,\tilde{<}\,\bar{1}$ *and* $\bar{j}\,\tilde{<}\,\bar{1}\Rightarrow\bar{1}-\bar{i}+\bar{1}-\bar{j}\,\tilde{<}\,\bar{2}\Rightarrow\bar{i}+\bar{j}>\bar{0}$ *or*
(ii) $\bar{i}\,\tilde{<}\,\bar{1}$ *and* $\bar{j}\,\tilde{<}\,\bar{1}\Rightarrow-\bar{1}+\bar{i}-\bar{1}+\bar{j}\,\tilde{<}\,\bar{2}\Rightarrow\bar{i}+\bar{j}\,\tilde{<}\,\bar{4}$ *or*
(iii) $\bar{i}\,\tilde{<}\,\bar{1}$ *and* $\bar{j}\,\tilde{<}\,\bar{1}\Rightarrow-\bar{1}+\bar{i}+\bar{1}-\bar{j}\,\tilde{<}\,\bar{2}\Rightarrow\bar{i}-\bar{j}\,\tilde{<}\,\bar{2}$ *or*

(iv) $\bar{i}\,\tilde{<}\,\bar{1}$ *and* $\bar{j}\,\tilde{<}\,\bar{1} \Rightarrow \bar{1}-\bar{i}-\bar{1}+\bar{j}\,\tilde{<}\,\bar{2} \Rightarrow \bar{j}-\bar{i}\,\tilde{<}\,\bar{2}$ *or*
(v) $\bar{i}=\bar{1} \Rightarrow |\bar{1}-\bar{j}|\,\tilde{<}\,\bar{2} \Rightarrow -\bar{2}\,\tilde{<}\,\bar{1}-\bar{j}\,\tilde{<}\,\bar{2} \Rightarrow -\bar{1}\,\tilde{<}\,\bar{j}\,\tilde{<}\,\bar{3}$ *or*
(vi) $\bar{j}=\bar{1} \Rightarrow |\bar{1}-\bar{i}|\,\tilde{<}\,\bar{2} \Rightarrow -\bar{1}\,\tilde{<}\,\bar{i}\,\tilde{<}\,\bar{3}$.

We shall draw heavily on ideas from these soft real numbers:

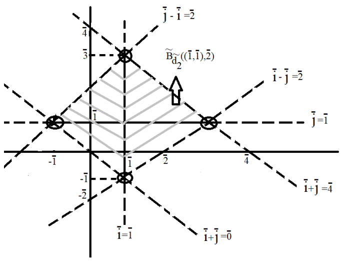

Figure 4

$$\tilde{K}_{\tilde{d}_2}\ ((\bar{0},\bar{-2}),\bar{1}) = \left\{ \left(e_{i_f}\right)_j \tilde{\in} f | \tilde{d}_2((\bar{0},\bar{-2}),(\bar{i},\bar{j})) = \bar{1} \right\}$$
$$= \left\{ \left(e_{i_f}\right)_j \tilde{\in} f ||\bar{i}| + |\bar{j}+\bar{2}| = \bar{1} \right\}$$

It is obvious that the above k-sphere supplies an algorithm to effectively recognize soft points in f that can be seen on the drawing:
(i) $\bar{i}\,\tilde{<}\,0$ and $\bar{j}\,\tilde{<}\,-\bar{2} \Rightarrow \bar{i}+\bar{j}+\bar{2}=\bar{1} \Rightarrow \bar{i}+\bar{j}=-\bar{1}$ or
(ii) $\bar{i}\,\tilde{<}\,\bar{0}$ and $\bar{j}\,\tilde{<}\,-\bar{2} \Rightarrow -\bar{i}-\bar{j}-\bar{2}=\bar{1} \Rightarrow \bar{i}+\bar{j}=-\bar{3}$ or
(iii) $\bar{i}\,\tilde{<}\,\bar{0}$ and $\bar{j}\,\tilde{<}\,-\bar{2} \Rightarrow \bar{i}-\bar{j}-\bar{2}=\bar{1} \Rightarrow \bar{i}-\bar{j}=\bar{3}$ or
(iv) $\bar{i}\,\tilde{<}\,\bar{0}$ and $\bar{j}\,\tilde{<}\,-\bar{2} \Rightarrow -\bar{i}+\bar{j}+\bar{2}=\bar{1} \Rightarrow \bar{j}-\bar{i}=-\bar{1}$ or
(v) $\bar{i}=\bar{0} \Rightarrow |\bar{i}+\bar{2}|=\bar{1} \Rightarrow \bar{j}=-\bar{1},\ \bar{j}=-\bar{3}$ or
(vi) $\bar{j}=-\bar{2} \Rightarrow |\bar{i}|=\bar{1} \Rightarrow \bar{i}=\bar{1},\ \bar{i}=-\bar{1}$.

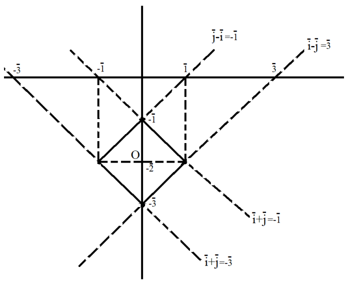

Figure 5

The control of the radius is made and it is equal to 1:

$$\tilde{d}_2\Big((\bar{0},-\bar{2}),\Big(\frac{-\bar{1}}{\bar{2}},\frac{-\bar{5}}{\bar{2}}\Big)\Big) = \Big|\frac{-\bar{1}}{\bar{2}}\Big| + \Big| -\bar{2} - \Big(\frac{-\bar{5}}{\bar{2}}\Big)\Big| = \frac{\bar{1}}{\bar{2}} + \frac{\bar{1}}{\bar{2}} = \bar{1}.$$

The solution shows that the shape of spheres undergoes radical changes as the definition of soft metric changes.

5. Conclusion

This article is intended to revive soft spheres with soft real numbers and soft points in soft metric spaces of curvature bounded below and a survey of applicable results. The locus of the soft sphere is analyzed and the most important results are proved comprehensively. I hope that this article will provide a good beginning point for mathematicians interested in this area and a common reference for future papers on this subject.

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