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Fuzzy Soft Sets over Fully UP-Semigroups*

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Abstract. In this paper, we introduce ten types of fuzzy soft sets over fully UP-semigroups, and investigate the algebraic properties of fuzzy soft sets under the operations of (extended) intersection and (restricted) union. Further, we discuss the relation between some conditions of fuzzy soft sets and fuzzy soft $\mathrm{UP_s}$-subalgebras (resp., fuzzy soft $\mathrm{UP_i}$-subalgebras, fuzzy soft near $\mathrm{UP_s}$-filters, fuzzy soft near $\mathrm{UP_i}$-filters, fuzzy soft $\mathrm{UP_s}$-filters, fuzzy soft $\mathrm{UP_i}$-filters, fuzzy soft $\mathrm{UP_s}$-ideals, fuzzy soft $\mathrm{UP_i}$-ideals, fuzzy soft strongly $\mathrm{UP_s}$-ideals, fuzzy soft strongly $\mathrm{UP_i}$-ideals) of fully UP-semigroups.

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Key Words and Phrases: UP-algebra, fully UP-semigroup, fuzzy soft set.

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1. Introduction and Preliminaries

Several researches introduced a new class of algebras related to logical algebras and semigroups such as: In 1993, Jun et al. [10] introduced the notion of BCI-semigroups. In 1998, Jun et al. [13] renamed the BCI-semigroup as the IS-algebra. In 2006, Kim [14] introduced the notion of KS-semigroups. In 2015, Endam and Vilela [3] introduced the notion of JB-semigroups. In 2018, Iampan [6] introduced the notion of fully UP-semigroups.

A fuzzy subset F of a set X is a function from X to a closed interval [0,1]. The concept of a fuzzy subset of a set was first considered by Zadeh [27] in 1965. The fuzzy set theories developed by Zadeh and others have found many applications in the domain of mathematics and elsewhere. After the introduction of the concept of fuzzy sets by Zadeh [27], several researches were conducted on the generalizations of the notion of fuzzy set and application to many logical algebras such as: In 1998, Jun et al. [9] applied the notion of fuzzy sets to BCI-semigroups (it was renamed as an IS-algebra for the convenience of study), and introduced the concept of fuzzy I-ideals. In 2000, Roh et al. [21] considered the fuzzification of an associative I-ideal of an IS-algebra. They proved that every fuzzy associative I-ideal is a fuzzy I-ideal. By giving an appropriate example, they verified that

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a fuzzy I-ideal may not be a fuzzy associative I-ideal. They gave a condition for a fuzzy I-ideal to be a fuzzy associative I-ideal, and they investigated some related properties. In 2003, Jun and Kondo [11] proved that some concepts of BCK/BCI-algebras expressed by a certain formula can be naturally extended to the fuzzy setting and that many results are obtained immediately with the use of our method. Moreover, they proved that these results can be extended to fuzzy IS-algebras. In 2003, Jianming and Dajing [8] introduced the concept of intuitionistic fuzzy associative I-ideals of IS-algebras and they investigated some related properties. In 2007, Prince Williams and Husain [26] studied fuzzy KS-semigroups. In 2016, Endam and Manahon [2] introduced the notion of fuzzy JB-semigroups and they investigated some of its properties. In 2018, Satirad and Iampan [23] introduced the notion of fuzzy sets in fully UP-semigroups and they investigated some of its properties.

In 1999, to solve complicated problems in economics, engineering, and environment, we cannot successfully use classical methods because of various uncertainties typical for those problems. Uncertainties cannot be handled using traditional mathematical tools but may be dealt with using a wide range of existing theories such as the probability theory, the theory of (intuitionistic) fuzzy sets, the theory of vague sets, the theory of interval mathematics, and the theory of rough sets. However, all of these theories have their own difficulties which are pointed out in [18]. In 2001, Maji et al. [17] introduced the concept of fuzzy soft sets as a generalization of the standard soft sets, and presented an application of fuzzy soft sets in a decision making problem. In 2010, Jun et al. [12] applied fuzzy soft set for dealing with several kinds of theories in BCK/BCI-algebras. The notions of fuzzy soft BCK/BCI-algebras, (closed) fuzzy soft ideals and fuzzy soft p-ideals are introduced, and related properties are investigated.

Before we begin our study, we will introduce the definition of a UP-algebra.

Definition 1. [5] *An algebra* $A = (A, \cdot, 0)$ *of type* $(2, 0)$ *is called a UP-algebra where* A *is a nonempty set,* $\cdot$ *is a binary operation on* A*, and* 0 *is a fixed element of* A (*i.e., a nullary operation*) *if it satisfies the following axioms:*

(UP-1) $(\forall x, y, z \in A)((y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0)$,

(UP-2) $(\forall x \in A)(0 \cdot x = x)$,

(UP-3) $(\forall x \in A)(x \cdot 0 = 0)$, *and*

(UP-4) $(\forall x, y \in A)(x \cdot y = 0, y \cdot x = 0 \Rightarrow x = y)$,

From [5], we know that the notion of UP-algebras is a generalization of KU-algebras (see [19]).

On a UP-algebra $A = (A, \cdot, 0)$, we define a binary relation $\leq$ on A [5] as follows:

$$(\forall x, y \in A)(x \leq y \Leftrightarrow x \cdot y = 0).$$

Example 1. [24] *Let* X *be a universal set and let* $\Omega \in \mathcal{P}(X)$. *Let* $\mathcal{P}_\Omega(X) = \{A \in \mathcal{P}(X) \mid \Omega \subseteq A\}$. *Define a binary operation* $\cdot$ *on* $\mathcal{P}_\Omega(X)$ *by putting* $A \cdot B = B \cap (A' \cup \Omega)$ *for all* $A, B \in \mathcal{P}_\Omega(X)$. *Then* $(\mathcal{P}_\Omega(X), \cdot, \Omega)$ *is a UP-algebra and we shall call it the generalized power UP-algebra of type* 1 *with respect to* Ω.

Example 2. [24] *Let X be a universal set and let $\Omega \in \mathcal{P}(X)$. Let $\mathcal{P}^{\Omega}(X) = \{A \in \mathcal{P}(X) \mid A \subseteq \Omega\}$. Define a binary operation $*$ on $\mathcal{P}^{\Omega}(X)$ by putting $A * B = B \cup (A' \cap \Omega)$ for all $A, B \in \mathcal{P}^{\Omega}(X)$. Then $(\mathcal{P}^{\Omega}(X), *, \Omega)$ is a UP-algebra and we shall call it the generalized power UP-algebra of type 2 with respect to Ω.*

In particular, $(\mathcal{P}(X), \cdot, \emptyset)$ is the power UP-algebra of type 1 and $(\mathcal{P}(X), *, X)$ is the power UP-algebra of type 2.

In a UP-algebra $A = (A, \cdot, 0)$, the following assertions are valid (see [5, 6]).

$$(\forall x \in A)(x \cdot x = 0), \tag{1.1}$$
$$(\forall x, y, z \in A)(x \cdot y = 0, y \cdot z = 0 \Rightarrow x \cdot z = 0), \tag{1.2}$$
$$(\forall x, y, z \in A)(x \cdot y = 0 \Rightarrow (z \cdot x) \cdot (z \cdot y) = 0), \tag{1.3}$$
$$(\forall x, y, z \in A)(x \cdot y = 0 \Rightarrow (y \cdot z) \cdot (x \cdot z) = 0), \tag{1.4}$$
$$(\forall x, y \in A)(x \cdot (y \cdot x) = 0), \tag{1.5}$$
$$(\forall x, y \in A)((y \cdot x) \cdot x = 0 \Leftrightarrow x = y \cdot x), \tag{1.6}$$
$$(\forall x, y \in A)(x \cdot (y \cdot y) = 0), \tag{1.7}$$
$$(\forall a, x, y, z \in A)((x \cdot (y \cdot z)) \cdot (x \cdot ((a \cdot y) \cdot (a \cdot z))) = 0), \tag{1.8}$$
$$(\forall a, x, y, z \in A)((((a \cdot x) \cdot (a \cdot y)) \cdot z) \cdot ((x \cdot y) \cdot z) = 0), \tag{1.9}$$
$$(\forall x, y, z \in A)(((x \cdot y) \cdot z) \cdot (y \cdot z) = 0), \tag{1.10}$$
$$(\forall x, y, z \in A)(x \cdot y = 0 \Rightarrow x \cdot (z \cdot y) = 0), \tag{1.11}$$
$$(\forall x, y, z \in A)(((x \cdot y) \cdot z) \cdot (x \cdot (y \cdot z)) = 0), \text{ and} \tag{1.12}$$
$$(\forall a, x, y, z \in A)(((x \cdot y) \cdot z) \cdot (y \cdot (a \cdot z)) = 0). \tag{1.13}$$

Definition 2. [4, 5, 7, 25] *A nonempty subset S of a UP-algebra $(A, \cdot, 0)$ is called*

(1) *a UP-subalgebra of A if $(\forall x, y \in S)(x \cdot y \in S)$.*

(2) *a near UP-filter of A if it satisfies the following properties:*

   (i) *the constant 0 of A is in S, and*

   (ii) $(\forall x, y \in A)(x \in A, y \in S \Rightarrow x \cdot y \in S)$.

(3) *a UP-filter of A if it satisfies the following properties:*

   (i) *the constant 0 of A is in S, and*

   (ii) $(\forall x, y \in A)(x \cdot y \in S, x \in S \Rightarrow y \in S)$.

(4) *a UP-ideal of A if it satisfies the following properties:*

   (i) *the constant 0 of A is in S, and*

   (ii) $(\forall x, y, z \in A)(x \cdot (y \cdot z) \in S, y \in S \Rightarrow x \cdot z \in S)$.

(5) *a strongly UP-ideal of A if it satisfies the following properties:*

(*i*) *the constant* 0 *of* A *is in* S*, and*

(*ii*) $(\forall x, y, z \in A)((z \cdot y) \cdot (z \cdot x) \in S, y \in S \Rightarrow x \in S)$.

We know that the notion of UP-subalgebras is a generalization of near UP-filters, the notion of near UP-filters is a generalization of UP-filters, the notion of UP-filters is a generalization of UP-ideals, and the notion of UP-ideals is a generalization of strongly UP-ideals. Moreover, they also proved that a UP-algebra A is the only one strongly UP-ideal of itself.

Definition 3. [15] *A nonempty subset* S *of a semigroup* $(A, *)$ *is called*

(1) *a subsemigroup of* A *if* $(\forall x, y \in S)(x * y \in S)$.

(2) *an ideal of* A *if* $(\forall x, y \in A)(x \in A, y \in S \Rightarrow x * y, y * x \in S)$.

Clearly, an ideal is a subsemigroup.

Definition 4. [6] *Let* A *be a nonempty set,* $\cdot$ *and* $*$ *are binary operations on* A*, and* 0 *is a fixed element of* A (*i.e., a nullary operation*)*. An algebra* $A = (A, \cdot, *, 0)$ *of type* $(2, 2, 0)$ *in which* $(A, \cdot, 0)$ *is a UP-algebra and* $(A, *)$ *is a semigroup is called a fully UP-semigroup* (*in short, an f-UP-semigroup*) *if the operation "$*$" is distributive* (*on both sides*) *over the operation "$\cdot$".*

Definition 5. [27] *A fuzzy set* F *in a nonempty set* U *(or a fuzzy subset of* U*) is described by its membership function* f_F*. To every point* $x \in U$*, this function associates a real number* $f_F(x)$ *in the interval* $[0, 1]$*. The number* $f_F(x)$ *is interpreted for the point as a degree of belonging* x *to the fuzzy set* F*, that is,* $F := \{(x, f_F(x)) \mid x \in U\}$*. We say that a fuzzy set* F *in* U *is constant if its membership function* f_F *is constant.*

Definition 6. [16] *Let* F *and* G *be fuzzy sets in a nonempty set* U*. Then* $F \leq G$ *is defined by* $f_F(x) \leq f_G(x)$ *for all* $x \in U$*.*

Definition 7. [15] *Let* F *and* G *be fuzzy sets in a semigroup* $A = (A, *)$*. Then the product of* F *and* G*, denoted by* $F \circ G$*, is described by their membership function* f_F *and* f_G*, respectively which defined as follows:*

$$(\forall x \in A)\left((f_F \circ f_G)(x) = \begin{cases} \sup\{\min\{f_F(y), f_G(z)\}\}_{x=y*z} & \text{if } \exists y, z \in A \text{ such that } x = y * z, \\ 0 & \text{otherwise.} \end{cases}\right)$$

Rosenfeld [22] introduced the notion of fuzzy subsemigroups (resp., fuzzy ideals) of semigroups as follows:

Definition 8. *A fuzzy set* F *in a semigroup* $A = (A, *)$ *is called*

(1) *a fuzzy subsemigroup of* A *if* $(\forall x, y \in A)(f_F(x * y) \geq \min\{f_F(x), f_F(y)\})$.

(2) *a fuzzy ideal of* A *if* $(\forall x, y \in A)(f_F(x * y) \geq \max\{f_F(x), f_F(y)\})$.

Clearly, a fuzzy ideal is a fuzzy subsemigroup.

Definition 9. [15] *The semigroup A itself is a fuzzy set of A, denoted by* A *such that* $\mathrm{f}_{\mathrm{A}}(x) = 1$ *for all* $x \in A$.

Lemma 1. [15] *Let* F *be a fuzzy set in a semigroup* $A = (A, *)$. *Then*

(1) F *is a fuzzy subsemigroup of A if and only if it satisfies the condition*

$$\mathrm{F} \circ \mathrm{F} \leq \mathrm{F}. \tag{1.14}$$

(2) F *is a fuzzy ideal of A if and only if it satisfies the condition*

$$\mathrm{A} \circ \mathrm{F} \leq \mathrm{F} \text{ and } \mathrm{F} \circ \mathrm{A} \leq \mathrm{F}. \tag{1.15}$$

Definition 10. [16] *Let* $\{\mathrm{F}_i\}_{i \in I}$ *be a nonempty family of fuzzy sets in a nonempty set U where I is an arbitrary index set. The intersection of* F_i, *denoted by* $\bigcap_{i \in I} \mathrm{F}_i$, *is described by its membership function* $\mathrm{f}_{\bigcap_{i \in I} \mathrm{F}_i}$ *which defined as follows:*

$$(\forall x \in U)(\mathrm{f}_{\bigcap_{i \in I} \mathrm{F}_i}(x) = \inf\{\mathrm{f}_{\mathrm{F}_i}(x)\}_{i \in I}).$$

The union of F_i, *denoted by* $\bigcup_{i \in I} \mathrm{F}_i$, *is described by its membership function* $\mathrm{f}_{\bigcup_{i \in I} \mathrm{F}_i}$ *which defined as follows:*

$$(\forall x \in U)(\mathrm{f}_{\bigcup_{i \in I} \mathrm{F}_i}(x) = \sup\{\mathrm{f}_{\mathrm{F}_i}(x)\}_{i \in I})$$

Somjanta et al. [25] and Guntasow et al. [4] introduced the notion of fuzzy UP-subalgebras (resp., fuzzy UP-filters, fuzzy UP-ideals, fuzzy strongly UP-ideals) of UP-algebras as follows:

Definition 11. *A fuzzy set* F *in a UP-algebra* $A = (A, \cdot, 0)$ *is called*

(1) *a fuzzy UP-subalgebra of A if* $(\forall x, y \in A)(\mathrm{f}_{\mathrm{F}}(x \cdot y) \geq \min\{\mathrm{f}_{\mathrm{F}}(x), \mathrm{f}_{\mathrm{F}}(y)\})$.

(2) *a fuzzy UP-filter of A if*

  (*i*) $(\forall x \in A)(\mathrm{f}_{\mathrm{F}}(0) \geq \mathrm{f}_{\mathrm{F}}(x))$, *and*

  (*ii*) $(\forall x, y \in A)(\mathrm{f}_{\mathrm{F}}(y) \geq \min\{\mathrm{f}_{\mathrm{F}}(x \cdot y), \mathrm{f}_{\mathrm{F}}(x)\})$.

(3) *a fuzzy UP-ideal of A if*

  (*i*) $(\forall x \in A)(\mathrm{f}_{\mathrm{F}}(0) \geq \mathrm{f}_{\mathrm{F}}(x))$, *and*

  (*ii*) $(\forall x, y, z \in A)(\mathrm{f}_{\mathrm{F}}(x \cdot z) \geq \min\{\mathrm{f}_{\mathrm{F}}(x \cdot (y \cdot z)), \mathrm{f}_{\mathrm{F}}(y)\})$.

(4) *a fuzzy strongly UP-ideal of A if*

  (*i*) $(\forall x \in A)(\mathrm{f}_{\mathrm{F}}(0) \geq \mathrm{f}_{\mathrm{F}}(x))$, *and*

  (*ii*) $(\forall x, y, z \in A)(\mathrm{f}_{\mathrm{F}}(x) \geq \min\{\mathrm{f}_{\mathrm{F}}((z \cdot y) \cdot (z \cdot x)), \mathrm{f}_{\mathrm{F}}(y)\})$.

Now, we introduce the notion of fuzzy near UP-filters of UP-algebras as follows:

Definition 12. *A fuzzy set* F *in a UP-algebra* $A = (A, \cdot, 0)$ *is called a fuzzy near UP-filter of* A *if*

(*i*) $(\forall x \in A)(\mathrm{f_F}(0) \geq \mathrm{f_F}(x))$, *and*

(*ii*) $(\forall x, y \in A)(\mathrm{f_F}(x \cdot y) \geq \mathrm{f_F}(y))$.

We know that the notion of fuzzy UP-subalgebras is a generalization of fuzzy near UP-filters, the notion of fuzzy near UP-filters is a generalization of fuzzy UP-filters, the notion of fuzzy UP-filters is a generalization of fuzzy UP-ideals, and the notion of fuzzy UP-ideals is a generalization of fuzzy strongly UP-ideals. Moreover, fuzzy strongly UP-ideals and constant fuzzy sets coincide in UP-algebras.

Satirad and Iampan [23] introduced the notion of fuzzy $\mathrm{UP_s}$-subalgebras (resp., fuzzy $\mathrm{UP_i}$-subalgebras, fuzzy $\mathrm{UP_s}$-filters, fuzzy $\mathrm{UP_i}$-filters, fuzzy $\mathrm{UP_s}$-ideals, fuzzy $\mathrm{UP_i}$-ideals, fuzzy strongly $\mathrm{UP_s}$-ideals, fuzzy strongly $\mathrm{UP_i}$-ideals) of f-UP-semigroups as follows:

Definition 13. [23] *A fuzzy set* F *in an* f*-UP-semigroup* $A = (A, \cdot, *, 0)$ *is called*

(1) *a fuzzy* $\mathrm{UP_s}$*-subalgebra of* A *if* F *is a fuzzy UP-subalgebra of* $(A, \cdot, 0)$ *and a fuzzy subsemigroup of* $(A, *)$.

(2) *a fuzzy* $\mathrm{UP_i}$*-subalgebra of* A *if* F *is a fuzzy UP-subalgebra of* $(A, \cdot, 0)$ *and a fuzzy ideal of* $(A, *)$.

(3) *a fuzzy* $\mathrm{UP_s}$*-filter of* A *if* F *is a fuzzy UP-filter of* $(A, \cdot, 0)$ *and a fuzzy subsemigroup of* $(A, *)$.

(4) *a fuzzy* $\mathrm{UP_i}$*-filter of* A *if* F *is a fuzzy UP-filter of* $(A, \cdot, 0)$ *and a fuzzy ideal of* $(A, *)$.

(5) *a fuzzy* $\mathrm{UP_s}$*-ideal of* A *if* F *is a fuzzy UP-ideal of* $(A, \cdot, 0)$ *and a fuzzy subsemigroup of* $(A, *)$.

(6) *a fuzzy* $\mathrm{UP_i}$*-ideal of* A *if* F *is a fuzzy UP-ideal of* $(A, \cdot, 0)$ *and a fuzzy ideal of* $(A, *)$.

(7) *a fuzzy strongly* $\mathrm{UP_s}$*-ideal of* A *if* F *is a fuzzy strongly UP-ideal of* $(A, \cdot, 0)$ *and a fuzzy subsemigroup of* $(A, *)$.

(8) *a fuzzy strongly* $\mathrm{UP_i}$*-ideal of* A *if* F *is a fuzzy strongly UP-ideal of* $(A, \cdot, 0)$ *and a fuzzy ideal of* $(A, *)$.

Now, we introduce the notion fuzzy near $\mathrm{UP_s}$-filters of f-UP-semigroups (resp., fuzzy near $\mathrm{UP_i}$-filters) as follows:

Definition 14. *A fuzzy set* F *in an* f*-UP-semigroup* $A = (A, \cdot, *, 0)$ *is called*

(1) *a fuzzy near* $\mathrm{UP_s}$*-filter of* A *if* F *is a fuzzy near UP-filter of* $(A, \cdot, 0)$ *and a fuzzy subsemigroup of* $(A, *)$.

(2) *a fuzzy near* $\mathrm{UP_i}$*-filter of* A *if* F *is a fuzzy near UP-filter of* $(A, \cdot, 0)$ *and a fuzzy ideal of* $(A, *)$.

Clearly, a fuzzy near $\mathrm{UP_i}$-filter is a fuzzy near $\mathrm{UP_s}$-filter.

Example 3. *Let* $A = \{0, 1, 2, 3\}$ *be a set with two binary operations* $\cdot$ *and* $*$ *defined by the following Cayley tables:*

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	2	3
2	0	1	0	3
3	0	1	2	0

$*$	0	1	2	3
0	0	0	0	0
1	0	0	0	0
2	0	0	0	1
3	0	0	1	0

Then $A = (A, \cdot, *, 0)$ *is an* f*-UP-semigroup. We define a membership function* $\mathrm{f_F}$ *as follows:*

$$\mathrm{f_F}(0) = 1, \mathrm{f_F}(1) = 0.4, \mathrm{f_F}(2) = 0.5, \text{ and } \mathrm{f_F}(3) = 0.2.$$

Then F *is a fuzzy near* $\mathrm{UP_s}$*-filter of* A. *Since* $\mathrm{f_F}(2*3) = \mathrm{f_F}(1) = 0.4 \not\geq 0.5 = \max\{0.5, 0.2\} = \max\{\mathrm{f_F}(2), \mathrm{f_F}(3)\}$, *we have* F *is not a fuzzy near* $\mathrm{UP_i}$*-filter of* A.

From [7], we can easily prove Theorems 1, 2, 3, and 4.

Theorem 1. *Every fuzzy near* $\mathrm{UP_s}$*-filter of an* f*-UP-semigroup is a fuzzy* $\mathrm{UP_s}$*-subalgebra.*

The following example shows that the converse of Theorem 1 is not true.

Example 4. *Let* $A = \{0, 1, 2, 3\}$ *be a set with two binary operations* $\cdot$ *and* $*$ *defined by the following Cayley tables:*

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	1	3
2	0	0	0	3
3	0	1	1	0

$*$	0	1	2	3
0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
3	0	0	0	1

Then $A = (A, \cdot, *, 0)$ *is an* f*-UP-semigroup. We define a membership function* $\mathrm{f_F}$ *as follows:*

$$\mathrm{f_F}(0) = 1, \mathrm{f_F}(1) = 0.8, \mathrm{f_F}(2) = 0.9, \text{ and } \mathrm{f_F}(3) = 0.7.$$

Then F *is a fuzzy* $\mathrm{UP_s}$*-subalgebra of* A. *Since* $\mathrm{f_F}(1 \cdot 2) = \mathrm{f_F}(1) = 0.8 \not\geq 0.9 = \mathrm{f_F}(2)$, *we have* F *is not a fuzzy near* $\mathrm{UP_s}$*-filter of* A.

Theorem 2. *Every fuzzy near* $\mathrm{UP_i}$*-filter of an* f*-UP-semigroup is a fuzzy* $\mathrm{UP_i}$*-subalgebra.*

In Example 4, we have F is a fuzzy $\mathrm{UP_i}$-subalgebra of A but F is not a fuzzy near $\mathrm{UP_i}$-filter of A.

Theorem 3. *Every fuzzy* $\mathrm{UP_s}$*-filter of an* f*-UP-semigroup is a fuzzy near* $\mathrm{UP_s}$*-filter.*

The following example shows that the converse of Theorem 3 is not true.

Example 5. *Let $A = \{0, 1, 2, 3\}$ be a set with two binary operations $\cdot$ and $*$ defined by the following Cayley tables:*

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	2	3
2	0	0	0	3
3	0	0	0	0

$*$	0	1	2	3
0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
3	0	0	0	2

Then $A = (A, \cdot, *, 0)$ is an f-UP-semigroup. We define a membership function $\mathrm{f_F}$ *as follows:*

$$\mathrm{f_F}(0) = 1, \mathrm{f_F}(1) = 0.7, \mathrm{f_F}(2) = 0.9, \text{ and } \mathrm{f_F}(3) = 0.8.$$

Then F *is a fuzzy near UP*$_{\mathrm{s}}$*-filter of A. Since $\mathrm{f_F}(1) = 0.7 \ngeq 0.8 = \min\{1, 0.8\} = \min\{\mathrm{f_F}(0), \mathrm{f_F}(3)\} = \min\{\mathrm{f_F}(3 \cdot 1), \mathrm{f_F}(3)\}$, we have* F *is not a fuzzy UP*$_{\mathrm{s}}$*-filter of A.*

Theorem 4. *Every fuzzy UP*$_{\mathrm{i}}$*-filter of an f-UP-semigroup is a fuzzy near UP*$_{\mathrm{i}}$*-filter.*

In Example 5, we have F is a fuzzy near UP$_{\mathrm{i}}$-filter of A but it is not a fuzzy UP$_{\mathrm{i}}$-filter of A.

Hence, we get the diagram of generalization of fuzzy sets in fully UP-semigroups as shown in Figure 1.

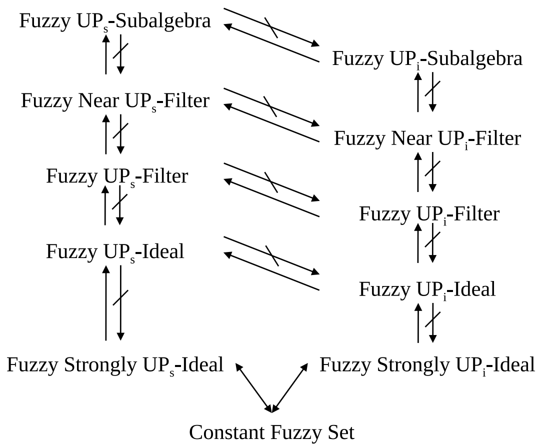

Figure 1: Fuzzy sets in fully UP-semigroups

Theorem 5. [23] *The intersection of any nonempty family of fuzzy UP*$_{\mathrm{s}}$*-subalgebras (resp., fuzzy UP*$_{\mathrm{i}}$*-subalgebras, fuzzy UP*$_{\mathrm{s}}$*-filters, fuzzy UP*$_{\mathrm{i}}$*-filters, fuzzy UP*$_{\mathrm{s}}$*-ideals, fuzzy UP*$_{\mathrm{i}}$*-ideals, fuzzy strongly UP*$_{\mathrm{s}}$*-ideals, fuzzy strongly UP*$_{\mathrm{i}}$*-ideals) of an f-UP-semigroup is also a fuzzy UP*$_{\mathrm{s}}$*-subalgebra (resp., fuzzy UP*$_{\mathrm{i}}$*-subalgebra, fuzzy UP*$_{\mathrm{s}}$*-filter, fuzzy UP*$_{\mathrm{i}}$*-filter, fuzzy UP*$_{\mathrm{s}}$*-ideal, fuzzy UP*$_{\mathrm{i}}$*-ideal, fuzzy strongly UP*$_{\mathrm{s}}$*-ideal, fuzzy strongly UP*$_{\mathrm{i}}$*-ideal).*

Theorem 6. [23] *The union of any nonempty family of fuzzy strongly* $UP_{\rm s}$*-ideals* (*resp., fuzzy strongly* $UP_{\rm i}$*-ideals*) *of an* f*-UP-semigroup is also a fuzzy strongly* $UP_{\rm s}$*-ideal* (*resp., fuzzy strongly* $UP_{\rm i}$*-ideal*).

Theorem 7. *The intersection of any nonempty family of fuzzy near* $UP_{\rm s}$*-filters of an* f*-UP-semigroup* $A = (A, \cdot, *, 0)$ *is also a fuzzy near* $UP_{\rm s}$*-filter.*

Proof. Let F_i be a fuzzy near $\mathrm{UP_s}$-filter of an f-UP-semigroup $A = (A, \cdot, *, 0)$ for all $i \in I$. Then

$$\mathrm{f}_{\bigcap_{i\in I}\mathrm{F}_i}(0) = \inf\{\mathrm{f}_{\mathrm{F}i}(0)\}_{i\in I} \geq \inf\{\mathrm{f}_{\mathrm{F}i}(x)\}_{i\in I} = \mathrm{f}_{\bigcap_{i\in I}\mathrm{F}_i}(x),$$

$$\mathrm{f}_{\bigcap_{i\in I}\mathrm{F}_i}(x \cdot y) = \inf\{\mathrm{f}_{\mathrm{F}i}(x \cdot y)\}_{i\in I} \geq \inf\{\mathrm{f}_{\mathrm{F}i}(y)\}_{i\in I} = \mathrm{f}_{\bigcap_{i\in I}\mathrm{F}_i}(y), \text{ and}$$

$$\begin{aligned}\mathrm{f}_{\bigcap_{i\in I}\mathrm{F}_i}(x * y) &= \inf\{\mathrm{f}_{\mathrm{F}i}(x * y)\}_{i\in I}\\ &\geq \inf\{\min\{\mathrm{f}_{\mathrm{F}i}(x), \mathrm{f}_{\mathrm{F}i}(y)\}\}_{i\in I}\\ &= \min\{\inf\{\mathrm{f}_{\mathrm{F}i}(x)\}_{i\in I}, \inf\{\mathrm{f}_{\mathrm{F}i}(y)\}_{i\in I}\}\\ &= \min\{\mathrm{f}_{\bigcap_{i\in I}\mathrm{F}_i}(x), \mathrm{f}_{\bigcap_{i\in I}\mathrm{F}_i}(y)\}.\end{aligned}$$

Hence, $\bigcap_{i\in I}\mathrm{F}_i$ is a fuzzy near $\mathrm{UP_s}$-filter of A.

The following example shows that the union of two fuzzy near $\mathrm{UP_s}$-filters of an f-UP-semigroup is not a fuzzy near $\mathrm{UP_s}$-filter.

Example 6. *By Cayley tables in Example 3, we know that* $A = (A, \cdot, *, 0)$ *is an* f*-UP-semigroup. We define two membership functions* $\mathrm{f}_{\mathrm{F}_1}$ *and* $\mathrm{f}_{\mathrm{F}_2}$ *as follows:*

A	0	1	2	3
$\mathrm{f}_{\mathrm{F}_1}$	1	0.7	1	0.5
$\mathrm{f}_{\mathrm{F}_2}$	1	0.5	0.3	0.8

Then F_1 *and* F_2 *are fuzzy near* $UP_{\rm s}$*-filters of* A *but* $\mathrm{F}_1 \cup \mathrm{F}_2$ *is not a fuzzy near* $UP_{\rm s}$*-filter of* A*. Indeed,* $\mathrm{f}_{\mathrm{F}_1\cup\mathrm{F}_2}(3*2) = \mathrm{f}_{\mathrm{F}_1\cup\mathrm{F}_2}(1) = 0.7 \not\geq 0.8 = \min\{0.8, 1\} = \min\{\mathrm{f}_{\mathrm{F}_1\cup\mathrm{F}_2}(3), \mathrm{f}_{\mathrm{F}_1\cup\mathrm{F}_2}(2)\}$.

Theorem 8. *The intersection of any nonempty family of fuzzy near* $UP_{\rm i}$*-filters of an* f*-UP-semigroup* $A = (A, \cdot, *, 0)$ *is also a fuzzy near* $UP_{\rm i}$*-filter.*

Proof. Let F_i be a fuzzy near $\mathrm{UP_i}$-filter of an f-UP-semigroup $A = (A, \cdot, *, 0)$ for all $i \in I$. Then, by the proof of Theorem 7, we have $\mathrm{f}_{\bigcap_{i\in I}\mathrm{F}_i}(0) \geq \mathrm{f}_{\bigcap_{i\in I}\mathrm{F}_i}(x)$ and $\mathrm{f}_{\bigcap_{i\in I}\mathrm{F}_i}(x \cdot y) \geq \mathrm{f}_{\bigcap_{i\in I}\mathrm{F}_i}(y)$. Thus

$$\begin{aligned}\mathrm{f}_{\bigcap_{i\in I}\mathrm{F}_i}(x * y) &= \inf\{\mathrm{f}_{\mathrm{F}i}(x * y)\}_{i\in I}\\ &\geq \inf\{\max\{\mathrm{f}_{\mathrm{F}i}(x), \mathrm{f}_{\mathrm{F}i}(y)\}\}_{i\in I}\\ &\geq \max\{\inf\{\mathrm{f}_{\mathrm{F}i}(x)\}_{i\in I}, \inf\{\mathrm{f}_{\mathrm{F}i}(y)\}_{i\in I}\}\\ &= \max\{\mathrm{f}_{\bigcap_{i\in I}\mathrm{F}_i}(x), \mathrm{f}_{\bigcap_{i\in I}\mathrm{F}_i}(y)\}.\end{aligned}$$

Hence, $\bigcap_{i\in I}\mathrm{F}_i$ is a fuzzy near $\mathrm{UP_i}$-filter of A.

Theorem 9. *The union of any nonempty family of fuzzy near* $\mathrm{UP_i}$*-filters of an* f*-UP-semigroup* $A = (A, \cdot, *, 0)$ *is also a fuzzy near* $\mathrm{UP_i}$*-filter.*

Proof. Let F_i be a fuzzy near $\mathrm{UP_i}$-filter of an f-UP-semigroup $A = (A, \cdot, *, 0)$ for all $i \in I$. Then

$$\mathrm{f}_{\bigcup_{i\in I}\mathrm{F}_i}(0) = \sup\{\mathrm{f}_{\mathrm{F}i}(0)\}_{i\in I} \geq \sup\{\mathrm{f}_{\mathrm{F}i}(x)\}_{i\in I} = \mathrm{f}_{\bigcup_{i\in I}\mathrm{F}_i}(x),$$

$$\mathrm{f}_{\bigcup_{i\in I}\mathrm{F}_i}(x \cdot y) = \sup\{\mathrm{f}_{\mathrm{F}i}(x \cdot y)\}_{i\in I} \geq \sup\{\mathrm{f}_{\mathrm{F}i}(y)\}_{i\in I} = \mathrm{f}_{\bigcup_{i\in I}\mathrm{F}_i}(y), \text{ and}$$

$$\begin{aligned}\mathrm{f}_{\bigcup_{i\in I}\mathrm{F}_i}(x * y) &= \sup\{\mathrm{f}_{\mathrm{F}i}(x * y)\}_{i\in I}\\ &\geq \sup\{\max\{\mathrm{f}_{\mathrm{F}i}(x), \mathrm{f}_{\mathrm{F}i}(y)\}\}_{i\in I}\\ &= \max\{\sup\{\mathrm{f}_{\mathrm{F}i}(x)\}_{i\in I}, \sup\{\mathrm{f}_{\mathrm{F}i}(y)\}_{i\in I}\}\\ &= \max\{\mathrm{f}_{\bigcup_{i\in I}\mathrm{F}_i}(x), \mathrm{f}_{\bigcup_{i\in I}\mathrm{F}_i}(y)\}.\end{aligned}$$

Hence, $\bigcup_{i\in I}\mathrm{F}_i$ is a fuzzy near $\mathrm{UP_i}$-filter of A.

2. Properties of Fuzzy Sets in UP-Algebras

In this section, we shall let A be a UP-algebra $A = (A, \cdot, 0)$ and find some properties of fuzzy sets in UP-algebras.

Proposition 1. [25] *If* F *is a fuzzy UP-subalgebra of* A*, then*

$$(\forall x \in A)(\mathrm{f_F}(0) \geq \mathrm{f_F}(x)). \tag{2.1}$$

Proposition 2. [23] *If* F *is a fuzzy UP-filter of* A*, then*

$$(\forall x, y \in A)(x \leq y \Rightarrow \mathrm{f_F}(x) \leq \mathrm{f_F}(y)). \tag{2.2}$$

Proposition 3. *If* F *is a fuzzy set in* A *satisfying the condition*

$$(\forall x, y, z \in A)(z \leq x \Rightarrow \mathrm{f_F}(x \cdot y) \geq \min\{\mathrm{f_F}(z), \mathrm{f_F}(y)\}), \tag{2.3}$$

then F *is a fuzzy UP-subalgebra of* A*.*

Proof. Let $x, y \in A$. By (1.1), we have $x \leq x$. It follows from (2.3) that $\mathrm{f_F}(x \cdot y) \geq \min\{\mathrm{f_F}(x), \mathrm{f_F}(y)\}$. Hence, F is a fuzzy UP-subalgebra of A.

Theorem 10. *If* F *is a fuzzy set in* A *satisfying the condition* (2.3)*, then* F *satisfies the condition* (2.1)*.*

Proof. It is straightforward by Proposition 3.

The following example shows that the converse of Theorem 10 is not true.

Example 7. *Let* $A = \{0, 1, 2, 3\}$ *be a set with a binary operation* $\cdot$ *defined by the following Cayley table:*

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	2	2
2	0	1	0	2
3	0	1	0	0

Then $A = (A, \cdot, 0)$ *is a UP-algebra. We define a membership function* $\mathrm{f_F}$ *as follows:*

$$\mathrm{f_F}(0) = 1, \mathrm{f_F}(1) = 0.6, \mathrm{f_F}(2) = 0.2, \text{ and } \mathrm{f_F}(3) = 0.9.$$

Then F *satisfies the condition* (2.1) *but it does not satisfy the condition* (2.3). *Indeed,* $1 \leq 1$ *but* $\mathrm{f_F}(1 \cdot 3) = \mathrm{f_F}(2) = 0.2 \not\geq 0.6 = \min\{0.6, 0.9\} = \min\{\mathrm{f_F}(1), \mathrm{f_F}(3)\}$.

It is clear that we have the following proposition.

Proposition 4. *If* F *is a fuzzy set in* A *satisfying the condition*

$$(\forall x, y, z \in A)(\mathrm{f_F}(x \cdot y) \geq \min\{\mathrm{f_F}(z), \mathrm{f_F}(y)\}), \tag{2.4}$$

then F *satisfies the condition* (2.3).

The following example shows that the converse of Proposition 4 is not true.

Example 8. *Let* $A = \{0, 1, 2, 3\}$ *be a set with a binary operation* $\cdot$ *defined by the following Cayley table:*

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	3	3
2	0	1	0	0
3	0	1	2	0

Then $A = (A, \cdot, 0)$ *is a UP-algebra. We define a membership function* $\mathrm{f_F}$ *as follows:*

$$\mathrm{f_F}(0) = 1, \mathrm{f_F}(1) = 0.1, \mathrm{f_F}(2) = 0.8, \text{ and } \mathrm{f_F}(3) = 0.2.$$

Then F *satisfies the condition* (2.3) *but it does not satisfy the condition* (2.4). *Indeed,* $\mathrm{f_F}(1 \cdot 2) = \mathrm{f_F}(3) = 0.2 \not\geq 0.8 = \min\{1, 0.8\} = \min\{\mathrm{f_F}(0), \mathrm{f_F}(2)\}$.

Proposition 5. *If* F *is a fuzzy set in* A *satisfying the condition* (2.2), *then* F *is a fuzzy near UP-filter of* A.

Proof. Let $x, y \in A$. By (UP-3), we have $x \leq 0$. It follows from (2.2) that $\mathrm{f_F}(0) \geq \mathrm{f_F}(x)$. By (1.5), we have $y \leq x \cdot y$. It follows from (2.2) that $\mathrm{f_F}(x \cdot y) \geq \mathrm{f_F}(y)$. Hence, F is a fuzzy near UP-filter of A. ∎

Theorem 11. *If* F *is a fuzzy set in* A *satisfying the condition* (2.2), *then* F *satisfies the condition* (2.4).

Proof. Let $x, y, z \in A$. By (1.5), we have $y \leq x \cdot y$. It follows from (2.2) that $\mathrm{f_F}(x \cdot y) \geq \mathrm{f_F}(y) \geq \min\{\mathrm{f_F}(z), \mathrm{f_F}(y)\}$. Hence, F satisfies (2.4).

The following example shows that the converse of Theorem 11 is not true.

Example 9. *Let $A = \{0, 1, 2, 3\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:*

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	2	3
2	0	0	0	3
3	0	0	0	0

Then $A = (A, \cdot, 0)$ is a UP-algebra. We define a membership function $\mathrm{f_F}$ as follows:

$$\mathrm{f_F}(0) = 1, \mathrm{f_F}(1) = 0.1, \mathrm{f_F}(2) = 0.7, \text{ and } \mathrm{f_F}(3) = 0.8.$$

Then F satisfies the condition (2.4) but it does not satisfy the condition (2.2). Indeed, $3 \leq 2$ but $\mathrm{f_F}(2) = \mathrm{f_F}(1) = 0.7 \ngeq 0.8 = \mathrm{f_F}(3)$.

Theorem 12. *If F is a fuzzy UP-subalgebra of A satisfying the condition*

$$(\forall x, y \in A)(x \cdot y \neq 0 \Rightarrow \mathrm{f_F}(x) \geq \mathrm{f_F}(y)), \tag{2.5}$$

then F is a fuzzy near UP-filter of A.

Proof. Let $x, y \in A$. If $x \cdot y = 0$, then by (2.1), we have $\mathrm{f_F}(x \cdot y) = \mathrm{f_F}(0) \geq \mathrm{f_F}(y)$. If $x \cdot y \neq 0$, then by (2.5), we have $\mathrm{f_F}(x \cdot y) \geq \min\{\mathrm{f_F}(x), \mathrm{f_F}(y)\} = \mathrm{f_F}(y)$. Hence, F is a fuzzy near UP-filter of A.

Proposition 6. *A fuzzy set F in A satisfies the condition*

$$(\forall x, y, z \in A)(z \leq x \cdot y \Rightarrow \mathrm{f_F}(y) \geq \min\{\mathrm{f_F}(z), \mathrm{f_F}(x)\}) \tag{2.6}$$

if and only if F is a fuzzy UP-filter of A.

Proof. Let $x \in A$. By (UP-3), we have $x \leq x \cdot 0$. It follows from (2.6) that $\mathrm{f_F}(0) \geq \min\{\mathrm{f_F}(x), \mathrm{f_F}(x)\} = \mathrm{f_F}(x)$. Let $x, y \in A$. By (1.1), we have $x \cdot y \leq x \cdot y$. It follows from (2.6) that $\mathrm{f_F}(y) \geq \min\{\mathrm{f_F}(x \cdot y), \mathrm{f_F}(x)\}$. Hence, F is a fuzzy UP-filter of A.

Conversely, let $x, y, z \in A$ be such that $z \leq x \cdot y$. Then $z \cdot (x \cdot y) = 0$, so

$$\mathrm{f_F}(x \cdot y) \geq \min\{\mathrm{f_F}(z \cdot (x \cdot y)), \mathrm{f_F}(z)\} = \min\{\mathrm{f_F}(0), \mathrm{f_F}(z)\} = \mathrm{f_F}(z).$$

Thus $\mathrm{f_F}(y) \geq \min\{\mathrm{f_F}(x \cdot y), \mathrm{f_F}(x)\} \geq \min\{\mathrm{f_F}(z), \mathrm{f_F}(x)\}$. Hence, F satisfies (2.6).

Theorem 13. *If F is a fuzzy set in A satisfying the condition (2.6), then F satisfies the condition (2.2).*

Proof. Let $x, y \in A$ such that $x \leq y$. By (1.11), we have $x \leq x \cdot y$. It follows from (2.6) that $\mathrm{f_F}(y) \geq \min\{\mathrm{f_F}(x), \mathrm{f_F}(x)\} = \mathrm{f_F}(x)$. Hence, F satisfies (2.2).

The following example shows that the converse of Theorem 13 is not true.

Example 10. *Let $A = \{0, 1, 2, 3\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:*

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	2	2
2	0	1	0	1
3	0	0	0	0

Then $A = (A, \cdot, 0)$ is a UP-algebra. We define a membership function $\mathrm{f_F}$ as follows:

$$\mathrm{f_F}(0) = 0.9, \mathrm{f_F}(1) = 0.3, \mathrm{f_F}(2) = 0.6, \text{ and } \mathrm{f_F}(3) = 0.2.$$

Then F satisfies the condition (2.2) but it does not satisfy the condition (2.6). Indeed, $1 \leq 2 \cdot 3$ but $\mathrm{f_F}(3) = 0.2 \ngeq 0.3 = \min\{0.3, 0.6\} = \min\{\mathrm{f_F}(1), \mathrm{f_F}(2)\}$.

Theorem 14. *If F is a fuzzy near UP-filter of A satisfying the condition*

$$(\forall x, y \in A)(\mathrm{f_F}(x \cdot y) = \mathrm{f_F}(y)), \tag{2.7}$$

then F is a fuzzy UP-filter of A.

Proof. Let $x, y \in A$. By (2.7), we have $\mathrm{f_F}(y) \geq \min\{\mathrm{f_F}(y), \mathrm{f_F}(x)\} = \min\{\mathrm{f_F}(x \cdot y), \mathrm{f_F}(x)\}$. Hence, F is a fuzzy UP-filter of A.

Proposition 7. *A fuzzy set F in A satisfies the condition*

$$(\forall a, x, y, z \in A)(a \leq x \cdot (y \cdot z) \Rightarrow \mathrm{f_F}(x \cdot z) \geq \min\{\mathrm{f_F}(a), \mathrm{f_F}(y)\}) \tag{2.8}$$

if and only if F is a fuzzy UP-ideal of A.

Proof. Let $x \in A$. By (UP-3), we have $x \leq x \cdot (x \cdot 0)$. By (UP-3) and (2.8), we have

$$\mathrm{f_F}(0) = \mathrm{f_F}(x \cdot 0) \geq \min\{\mathrm{f_F}(x), \mathrm{f_F}(x)\} = \mathrm{f_F}(x).$$

Let $x, y, z \in A$. By (1.1), we have $x \cdot (y \cdot z) \leq x \cdot (y \cdot z)$. It follows from (2.8) that

$$\mathrm{f_F}(x \cdot z) \geq \min\{\mathrm{f_F}(x \cdot (y \cdot z)), \mathrm{f_F}(y)\}.$$

Hence, F is a fuzzy UP-ideal of A.

Conversely, let $a, x, y, z \in A$ be such that $a \leq x \cdot (y \cdot z)$. By Proposition 2, we have $\mathrm{f_F}(a) \leq \mathrm{f_F}(x \cdot (y \cdot z))$. Thus

$$\mathrm{f_F}(x \cdot z) \geq \min\{\mathrm{f_F}(x \cdot (y \cdot z)), \mathrm{f_F}(y)\} \geq \min\{\mathrm{f_F}(a), \mathrm{f_F}(y)\}.$$

Hence, F satisfies (2.8).

Proposition 8. *If* F *is a fuzzy UP-ideal of A, then*

$$(\forall a, x, y, z \in A)(a \leq x \cdot (y \cdot z) \Rightarrow \mathrm{f_F}(a \cdot z) \geq \min\{\mathrm{f_F}(x), \mathrm{f_F}(y)\}). \tag{2.9}$$

Proof. Let $a, x, y, z \in A$ be such that $a \leq x \cdot (y \cdot z)$. Then $a \cdot (x \cdot (y \cdot z)) = 0$, so

$$\mathrm{f_F}(a \cdot (y \cdot z)) \geq \min\{\mathrm{f_F}(a \cdot (x \cdot (y \cdot z))), \mathrm{f_F}(x)\} = \min\{\mathrm{f_F}(0), \mathrm{f_F}(x)\} = \mathrm{f_F}(x).$$

Thus

$$\mathrm{f_F}(a \cdot z) \geq \min\{\mathrm{f_F}(a \cdot (y \cdot z)), \mathrm{f_F}(y)\} \geq \min\{\mathrm{f_F}(x), \mathrm{f_F}(y)\}.$$

Corollary 1. *If* F *is a fuzzy set in A satisfying the condition* (2.8)*, then* F *satisfies the condition* (2.9)*.*

Proof. It is straightforward by Propositions 7 and 8.

Theorem 15. *Let A be a UP-algebra satisfying the condition*

$$(\forall x, y, z \in A)(z \cdot (y \cdot x) = y \cdot (z \cdot x)). \tag{2.10}$$

If F *is a fuzzy set in A satisfying the condition* (2.9)*, then* F *satisfies the condition* (2.8)*.*

Proof. Let $a, x, y, z \in A$ such that $a \leq x \cdot (y \cdot z)$. By (2.10), we have $0 = a \cdot (x \cdot (y \cdot z)) = x \cdot (a \cdot (y \cdot z))$, that is, $x \leq a \cdot (y \cdot z)$. It follows from (2.9) that $\mathrm{f_F}(x \cdot z) \geq \min\{\mathrm{f_F}(a), \mathrm{f_F}(y)\}$. Hence, F satisfies (2.8).

Theorem 16. *If* F *is a fuzzy set in A satisfying the condition* (2.9)*, then* F *satisfies the condition* (2.6)*.*

Proof. Let $x, y, z \in A$ be such that $z \leq x \cdot y$. By (1.1) and (1.3), we have $0 = z \cdot z \leq z \cdot (x \cdot y)$. By (UP-2) and (2.9), we have $\mathrm{f_F}(y) = \mathrm{f_F}(0 \cdot y) \geq \min\{\mathrm{f_F}(z), \mathrm{f_F}(x)\}$. Hence, F satisfies (2.6).

Corollary 2. *If* F *is a fuzzy set in A satisfying the condition* (2.8)*, then* F *satisfies the condition* (2.6)*.*

Proof. It is straightforward by Corollary 1 and Theorem 16.

The following example shows that the converse of Theorem 16 is not true.

Example 11. *Let $A = \{0, 1, 2, 3\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:*

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	3	3
2	0	1	0	0
3	0	1	2	0

Then $A = (A, \cdot, 0)$ *is a UP-algebra. We define a membership function* $\mathrm{f_F}$ *as follows:*

$$\mathrm{f_F}(0) = 1, \mathrm{f_F}(1) = 0.9, \mathrm{f_F}(2) = 0.1, \text{ and } \mathrm{f_F}(3) = 0.1.$$

Then F *satisfies the condition* (2.6) *but it does not satisfy the condition* (2.9)*. Indeed,* $3 \leq 1 \cdot (1 \cdot 2)$ *but* $\mathrm{f_F}(3 \cdot 2) = \mathrm{f_F}(2) = 0.1 \ngeq 0.9 = \mathrm{f_F}(1) = \min\{\mathrm{f_F}(1), \mathrm{f_F}(1)\}$.

The following example shows that fuzzy set in a UP-algebra which satisfies the condition (2.8) is not constant.

Example 12. *Let* $A = \{0, 1, 2, 3\}$ *be a set with a binary operation* $\cdot$ *defined by the following Cayley table:*

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	2	3
2	0	1	0	3
3	0	1	2	0

Then $A = (A, \cdot, 0)$ *is a UP-algebra. We define a membership function* $\mathrm{f_F}$ *as follows:*

$$\mathrm{f_F}(0) = 0.7, \mathrm{f_F}(1) = 0.5, \mathrm{f_F}(2) = 0.4, \text{ and } \mathrm{f_F}(3) = 0.4.$$

Then F *satisfies the condition* (2.8) *but it is not constant.*

Theorem 17. *If* F *is a fuzzy UP-filter of* A *satisfying the condition*

$$(\forall x, y, z \in A)(\mathrm{f_F}(y \cdot (x \cdot z)) = \mathrm{f_F}(x \cdot (y \cdot z))), \tag{2.11}$$

then F *is a fuzzy UP-ideal of* A.

Proof. Let $x, y, z \in A$. By (2.11), we have

$$\mathrm{f_F}(x \cdot z) \geq \min\{\mathrm{f_F}(y \cdot (x \cdot z)), \mathrm{f_F}(y)\} = \min\{\mathrm{f_F}(x \cdot (y \cdot z)), \mathrm{f_F}(y)\}.$$

Hence, F is a fuzzy UP-ideal of A.

Proposition 9. *A fuzzy set* F *in* A *satisfies the condition*

$$(\forall a, x, y, z \in A)(a \leq (z \cdot y) \cdot (z \cdot x) \Rightarrow \mathrm{f_F}(x) \geq \min\{\mathrm{f_F}(a), \mathrm{f_F}(y)\}) \tag{2.12}$$

if and only if F *is a fuzzy strongly UP-ideal of* A.

Proof. Let $x \in A$. By (UP-3), we have $x \leq 0 = x \cdot 0 = (0 \cdot x) \cdot (0 \cdot 0)$. By (2.12), we have $\mathrm{f_F}(0) \geq \min\{\mathrm{f_F}(x), \mathrm{f_F}(x)\} = \mathrm{f_F}(x)$. Let $x, y, z \in A$. By (1.1), we have $(z \cdot y) \cdot (z \cdot x) \leq (z \cdot y) \cdot (z \cdot x)$. By (2.12), we have $\mathrm{f_F}(x) \geq \min\{\mathrm{f_F}((z \cdot y) \cdot (z \cdot x)), \mathrm{f_F}(y)\}$. Hence, F is a fuzzy strongly UP-ideal of A.

The converse is obvious because F is constant.

Theorem 18. *If* F *is a fuzzy set in* A *satisfying the condition*

$$(\forall x, y, z \in A)(z \leq x \cdot y \Rightarrow \mathrm{f_F}(z) \geq \min\{\mathrm{f_F}(x), \mathrm{f_F}(y)\}), \tag{2.13}$$

then F *satisfies the condition* (2.3)*.*

Proof. Let $x, y, z \in A$ be such that $z \leq x$. By (1.4), we have $x \cdot y \leq z \cdot y$. By (2.13), we have $\mathrm{f_F}(x \cdot y) \geq \min\{\mathrm{f_F}(z), \mathrm{f_F}(y)\}$. Hence, F satisfies (2.3).

Proposition 10. *A fuzzy set* F *in* A *satisfies the condition* (2.13) *if and only if* F *is a fuzzy strongly UP-ideal of* A*.*

Proof. Let $x \in A$. By (UP-3), we have $x \leq 0 = 0 \cdot 0$. By (2.13), we have $\mathrm{f_F}(x) \geq \min\{\mathrm{f_F}(0), \mathrm{f_F}(0)\} = \mathrm{f_F}(0)$. By Theorem 18 and Proposition 3, we have $\mathrm{f_F}(0) \geq \mathrm{f_F}(x)$. Thus $\mathrm{f_F}(x) = \mathrm{f_F}(0)$ for all $x \in A$, so F is constant. Hence, F a fuzzy strongly UP-ideal of A.

The converse is obvious because F is constant.

Theorem 19. *If* F *is a fuzzy set in* A *satisfying the condition*

$$(\forall x, y, z \in A)(z \leq x \cdot y \Rightarrow \mathrm{f_F}(z) \geq \mathrm{f_F}(y)), \tag{2.14}$$

then F *satisfies the condition* (2.3)*.*

Proof. Let $x, y, z \in A$ be such that $z \leq x$. By (1.4), we have $x \cdot y \leq z \cdot y$. It follows from (2.14) that $\mathrm{f_F}(x \cdot y) \geq \mathrm{f_F}(y) \geq \min\{\mathrm{f_F}(z), \mathrm{f_F}(y)\}$. Hence, F satisfies (2.3).

Proposition 11. *A fuzzy set* F *in* A *satisfies the condition* (2.14) *if and only if* F *is a fuzzy strongly UP-ideal of* A*.*

Proof. Let $x \in A$. By (UP-3), we have $x \leq 0 = 0 \cdot 0$. By (2.14), we have $\mathrm{f_F}(x) \geq \mathrm{f_F}(0)$. By Theorem 19 and Proposition 3, we have $\mathrm{f_F}(0) \geq \mathrm{f_F}(x)$. Thus $\mathrm{f_F}(x) = \mathrm{f_F}(0)$ for all $x \in A$, so F is constant. Hence, F is a fuzzy strongly UP-ideal of A.

The converse is obvious because F is constant.

We have provided various important properties of fuzzy sets in various types in UP-algebras which will be used in the next section. We get the diagram of the properties of fuzzy sets in UP-algebras as shown in Figure 2.

3. Fuzzy Soft Sets over Fully UP-Semigroups

From now on, we shall let A be an f-UP-semigroup $A = (A, \cdot, *, 0)$ and P be a set of parameters. Let $\mathcal{F}(A)$ denotes the set of all fuzzy sets in A. A subset E of P is called a *set of statistics*.

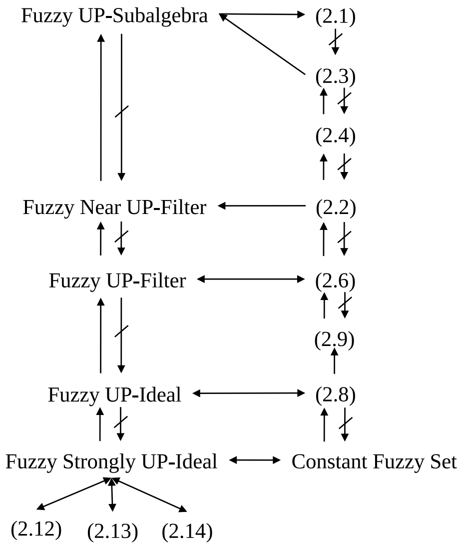

Figure 2: Properties of fuzzy sets in UP-algebras

Definition 15. *Let $E \subseteq P$. A pair $(\widetilde{\mathrm{F}}, E)$ is called a fuzzy soft set over A if $\widetilde{\mathrm{F}}$ is a mapping given by $\widetilde{\mathrm{F}} \colon E \to \mathcal{F}(A)$, that is, a fuzzy soft set is a statistic family of fuzzy sets in A. In general, for every $e \in E$, $\widetilde{\mathrm{F}}[e] := \{(x, \mathrm{f}_{\widetilde{\mathrm{F}}[e]}(x)) \mid x \in A\}$ is a fuzzy set in A and it is called a fuzzy value set of statistic e.*

Definition 16. *Let $(\widetilde{\mathrm{F}}, E_1)$ and $(\widetilde{\mathrm{G}}, E_2)$ be two fuzzy soft sets over a common universe U. The union* [17] *of $(\widetilde{\mathrm{F}}, E_1)$ and $(\widetilde{\mathrm{G}}, E_2)$ is defined to be the fuzzy soft set $(\widetilde{\mathrm{F}}, E_1) \cup (\widetilde{\mathrm{G}}, E_2) = (\widetilde{\mathrm{H}}, E)$ satisfying the following conditions:*

(i) $E = E_1 \cup E_2$ *and*

(ii) for all $e \in E$,

$$\widetilde{\mathrm{H}}[e] = \begin{cases} \widetilde{\mathrm{F}}[e] & \text{if } e \in E_1 \setminus E_2 \\ \widetilde{\mathrm{G}}[e] & \text{if } e \in E_2 \setminus E_1 \\ \widetilde{\mathrm{F}}[e] \cup \widetilde{\mathrm{G}}[e] & \text{if } e \in E_1 \cap E_2. \end{cases}$$

The restricted union [20] *of $(\widetilde{\mathrm{F}}, E_1)$ and $(\widetilde{\mathrm{G}}, E_2)$ is defined to be the fuzzy soft set $(\widetilde{\mathrm{F}}, E_1) \Cup (\widetilde{\mathrm{G}}, E_2) = (\widetilde{\mathrm{H}}, E)$ satisfying the following conditions:*

(i) $E = E_1 \cap E_2 \neq \emptyset$ *and*

(ii) $\widetilde{\mathrm{H}}[e] = \widetilde{\mathrm{F}}[e] \cup \widetilde{\mathrm{G}}[e]$ *for all* $e \in E$.

Definition 17. [20] *Let $(\widetilde{\mathrm{F}}, E_1)$ and $(\widetilde{\mathrm{G}}, E_2)$ be two fuzzy soft sets over a common universe U. The extended intersection of $(\widetilde{\mathrm{F}}, E_1)$ and $(\widetilde{\mathrm{G}}, E_2)$ is defined to be the fuzzy soft set $(\widetilde{\mathrm{F}}, E_1) \cap (\widetilde{\mathrm{G}}, E_2) = (\widetilde{\mathrm{H}}, E)$ satisfying the following conditions:*

(i) $E = E_1 \cup E_2$ *and*

(*ii*) *for all* $e \in E$,

$$\widetilde{\mathrm{H}}[e] = \begin{cases} \widetilde{\mathrm{F}}[e] & \text{if } e \in E_1 \setminus E_2 \\ \widetilde{\mathrm{G}}[e] & \text{if } e \in E_2 \setminus E_1 \\ \widetilde{\mathrm{F}}[e] \cap \widetilde{\mathrm{G}}[e] & \text{if } e \in E_1 \cap E_2. \end{cases}$$

The intersection [1] *of* $(\widetilde{\mathrm{F}}, E_1)$ *and* $(\widetilde{\mathrm{G}}, E_2)$ *is defined to be the fuzzy soft set* $(\widetilde{\mathrm{F}}, E_1) \Cap (\widetilde{\mathrm{G}}, E_2) = (\widetilde{\mathrm{H}}, E)$ *satisfying the following conditions:*

(*i*) $E = E_1 \cap E_2 \neq \emptyset$ *and*

(*ii*) $\widetilde{\mathrm{H}}[e] = \widetilde{\mathrm{F}}[e] \cap \widetilde{\mathrm{G}}[e]$ *for all* $e \in E$.

3.1. Fuzzy Soft UP$_\mathrm{s}$-Subalgebras

Definition 18. *A fuzzy soft set* $(\widetilde{\mathrm{F}}, E)$ *over* A *is called a fuzzy soft UP*$_\mathrm{s}$*-subalgebra based on* $e \in E$ (*we shortly call an e-fuzzy soft UP*$_\mathrm{s}$*-subalgebra*) *of* A *if a fuzzy set* $\widetilde{\mathrm{F}}[e]$ *in* A *is a fuzzy UP*$_\mathrm{s}$*-subalgebra of* A. *If* $(\widetilde{\mathrm{F}}, E)$ *is an e-fuzzy soft UP*$_\mathrm{s}$*-subalgebra of* A *for all* $e \in E$, *we say that* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft UP*$_\mathrm{s}$*-subalgebra of* A.

In the next theorem, we give necessary condition for fuzzy soft UP$_\mathrm{s}$-subalgebras of f-UP-semigroups.

Theorem 20. *If* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft set over* A *such that for all* $e \in E$, *a fuzzy set* $\widetilde{\mathrm{F}}[e]$ *in* A *satisfies the conditions* (2.3) *and* (1.14), *then* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft UP*$_\mathrm{s}$*-subalgebra of* A.

Proof. It is straightforward by Proposition 3 and Lemma 1 (1).

The proof of the following theorem can be verified easily.

Theorem 21. *If* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft UP*$_\mathrm{s}$*-subalgebra of* A *and* $\emptyset \neq E^* \subseteq E$, *then* $(\widetilde{\mathrm{F}}|_{E^*}, E^*)$ *is a fuzzy soft UP*$_\mathrm{s}$*-subalgebra of* A.

The following example shows that there exists a nonempty subset E^* of E such that $(\widetilde{\mathrm{F}}|_{E^*}, E^*)$ is a fuzzy soft UP$_\mathrm{s}$-subalgebra of A, but $(\widetilde{\mathrm{F}}, E)$ is not a fuzzy soft UP$_\mathrm{s}$-subalgebra of A.

Example 13. *Let* A *be the set of four series of the iPhone, that is,*

$$A = \{5, 6, 7, X\}.$$

Define two binary operations $\cdot$ *and* $*$ *on* A *as the following Cayley tables:*

$\cdot$	X	7	6	5
X	X	7	6	5
7	X	X	6	5
6	X	7	X	5
5	X	7	6	X

$*$	X	7	6	5
X	X	X	X	X
7	X	X	X	X
6	X	X	X	7
5	X	X	7	X

Then $A = (A, \cdot, *, X)$ is an f-UP-semigroup. Let $(\widetilde{\mathrm{F}}, E)$ be a fuzzy soft set over A where

$$E := \{price,\ beauty,\ specifications,\ stability\}$$

with $\widetilde{\mathrm{F}}[price], \widetilde{\mathrm{F}}[beauty], \widetilde{\mathrm{F}}[specifications]$, and $\widetilde{\mathrm{F}}[stability]$ are fuzzy sets in A defined as follows:

$\widetilde{\mathrm{F}}$	X	7	6	5
price	0.8	0.3	0.7	0.1
beauty	0.5	0.3	0.2	0.4
specifications	0.9	0.8	0.5	0.6
stability	1	0.4	0.7	0.6

Then $\widetilde{\mathrm{F}}[stability]$ is not a fuzzy UP$_{\mathrm{s}}$-subalgebra of A. Indeed,

$$\mathrm{f}_{\widetilde{\mathrm{F}}[stability]}(5 * 6) = \mathrm{f}_{\widetilde{\mathrm{F}}[stability]}(7) = 0.4 \ngeq 0.6 = \min\{0.6, 0.7\} = \min\{\mathrm{f}_{\widetilde{\mathrm{F}}[stability]}(5), \mathrm{f}_{\widetilde{\mathrm{F}}[stability]}(6)\}.$$

Hence, $(\widetilde{\mathrm{F}}, E)$ is not a fuzzy soft UP$_{\mathrm{s}}$-subalgebra of A. We take

$$E^* := \{price,\ beauty,\ specifications\}.$$

Thus $(\widetilde{\mathrm{F}}|_{E^*}, E^*)$ is a fuzzy soft UP$_{\mathrm{s}}$-subalgebra of A.

Theorem 22. *The extended intersection of two fuzzy soft UP$_{\mathrm{s}}$-subalgebras of A is also a fuzzy soft UP$_{\mathrm{s}}$-subalgebra. Moreover, the intersection of two fuzzy soft UP$_{\mathrm{s}}$-subalgebras of A is also a fuzzy soft UP$_{\mathrm{s}}$-subalgebra.*

Proof. Let $(\widetilde{\mathrm{F}}, E_1)$ and $(\widetilde{\mathrm{G}}, E_2)$ be two fuzzy soft UP$_{\mathrm{s}}$-subalgebras of A. Assume that $(\widetilde{\mathrm{F}}, E_1) \cap (\widetilde{\mathrm{G}}, E_2) = (\widetilde{\mathrm{H}}, E)$ with $E = E_1 \cup E_2$. Let $e \in E$.

Case 1: $e \in E_1 \setminus E_2$ (resp., $e \in E_2 \setminus E_1$). Then $\widetilde{\mathrm{H}}[e] = \widetilde{\mathrm{F}}[e]$ (resp., $\widetilde{\mathrm{H}}[e] = \widetilde{\mathrm{G}}[e]$) is a fuzzy soft UP$_{\mathrm{s}}$-subalgebra of A.

Case 2: $e \in E_1 \cap E_2$. By Theorem 5, we have $\widetilde{\mathrm{H}}[e] = \widetilde{\mathrm{F}}[e] \cap \widetilde{\mathrm{G}}[e]$ is a fuzzy soft UP$_{\mathrm{s}}$-subalgebra.

Thus $(\widetilde{\mathrm{H}}, E)$ is an e-fuzzy soft UP$_{\mathrm{s}}$-subalgebra of A for all $e \in E$. Hence, $(\widetilde{\mathrm{H}}, E)$ is a fuzzy soft UP$_{\mathrm{s}}$-subalgebra of A.

Theorem 23. *The union of two fuzzy soft UP$_{\mathrm{s}}$-subalgebras of A is also a fuzzy soft UP$_{\mathrm{s}}$-subalgebra if sets of statistics of two fuzzy soft UP$_{\mathrm{s}}$-subalgebras are disjoint.*

Proof. Let $(\widetilde{\mathrm{F}}, E_1)$ and $(\widetilde{\mathrm{G}}, E_2)$ be two fuzzy soft UP$_{\mathrm{s}}$-subalgebras of A such that $E_1 \cap E_2 = \emptyset$. Assume that $(\widetilde{\mathrm{F}}, E_1) \cup (\widetilde{\mathrm{G}}, E_2) = (\widetilde{\mathrm{H}}, E)$ with $E = E_1 \cup E_2$. Let $e \in E$. Since $E_1 \cap E_2 = \emptyset$, we have $e \in E_1 \setminus E_2$ or $e \in E_2 \setminus E_1$.

Case 1: $e \in E_1 \setminus E_2$. Then $\widetilde{\mathrm{H}}[e] = \widetilde{\mathrm{F}}[e]$ is a fuzzy soft UP$_{\mathrm{s}}$-subalgebra of A.

Case 2: $e \in E_2 \setminus E_1$. Then $\widetilde{\mathrm{H}}[e] = \widetilde{\mathrm{G}}[e]$ is a fuzzy soft UP$_{\mathrm{s}}$-subalgebra of A.

Thus $(\widetilde{\mathrm{H}}, E)$ is an e-fuzzy soft UP$_{\mathrm{s}}$-subalgebra of A for all $e \in E$. Hence, $(\widetilde{\mathrm{H}}, E)$ is a fuzzy soft UP$_{\mathrm{s}}$-subalgebra of A.

The following example shows that Theorem 23 is not valid if sets of statistics of two fuzzy soft UP$_{\mathrm{s}}$-subalgebras are not disjoint.

Example 14. *By Cayley tables in Example 13, we know that $A = (A, \cdot, *, X)$ is an f-UP-semigroup. Let $(\widetilde{\mathrm{G}}_1, E_1)$ and $(\widetilde{\mathrm{G}}_2, E_2)$ be two fuzzy soft sets over A where*

$$E_1 := \{price,\ beauty,\ specifications\} \text{ and } E_2 := \{price,\ stability\}$$

with $\widetilde{\mathrm{G}}_1[price], \widetilde{\mathrm{G}}_1[beauty], \widetilde{\mathrm{G}}_1[specifications], \widetilde{\mathrm{G}}_2[price]$, and $\widetilde{\mathrm{G}}_2[stability]$ are fuzzy sets in A defined as follows:

$\widetilde{\mathrm{G}}_1$	X	7	6	5
price	0.9	0.7	0.9	0.2
beauty	1	0.8	0.3	0.2
specifications	0.6	0.5	0.3	0.4

$\widetilde{\mathrm{G}}_2$	X	7	6	5
price	0.9	0.3	0.2	0.8
stability	0.7	0.2	0.5	0.2

Then $(\widetilde{\mathrm{G}}_1, E_1)$ and $(\widetilde{\mathrm{G}}_2, E_2)$ are two fuzzy soft UP$_{\mathrm{s}}$-subalgebras of A. Since $price \in E_1 \cap E_2$, we have

$$\begin{aligned}
(\mathrm{f}_{\widetilde{\mathrm{G}}_1[price] \cup \widetilde{\mathrm{G}}_2[price]})(6 * 5) &= (\mathrm{f}_{\widetilde{\mathrm{G}}_1[price] \cup \widetilde{\mathrm{G}}_2[price]})(7) \\
&= 0.7 \\
&\ngeq 0.8 \\
&= \min\{0.9, 0.8\} \\
&= \min\{(\mathrm{f}_{\widetilde{\mathrm{G}}_1[price] \cup \widetilde{\mathrm{G}}_2[price]})(6), (\mathrm{f}_{\widetilde{\mathrm{G}}_1[price] \cup \widetilde{\mathrm{G}}_2[price]})(5)\}.
\end{aligned}$$

Thus $\widetilde{\mathrm{G}}_1[price] \cup \widetilde{\mathrm{G}}_2[price]$ is not a fuzzy UP$_{\mathrm{s}}$-subalgebra of A, that is, $(\widetilde{\mathrm{G}}_1, E_1) \cup (\widetilde{\mathrm{G}}_2, E_2)$ is not a price-fuzzy soft UP$_{\mathrm{s}}$-subalgebra of A. Hence, $(\widetilde{\mathrm{G}}_1, E_1) \cup (\widetilde{\mathrm{G}}_2, E_2)$ is not a fuzzy soft UP$_{\mathrm{s}}$-subalgebra of A. Moreover, $(\widetilde{\mathrm{G}}_1, E_1) \Cup (\widetilde{\mathrm{G}}_2, E_2)$ is not a fuzzy soft UP$_{\mathrm{s}}$-subalgebra of A.

3.2. Fuzzy Soft UP$_{\mathrm{i}}$-Subalgebras

Definition 19. *A fuzzy soft set $(\widetilde{\mathrm{F}}, E)$ over A is called a fuzzy soft UP$_{\mathrm{i}}$-subalgebra based on $e \in E$ (we shortly call an e-fuzzy soft UP$_{\mathrm{i}}$-subalgebra) of A if a fuzzy set $\widetilde{\mathrm{F}}[e]$ in A is a fuzzy UP$_{\mathrm{i}}$-subalgebra of A. If $(\widetilde{\mathrm{F}}, E)$ is an e-fuzzy soft UP$_{\mathrm{i}}$-subalgebra of A for all $e \in E$, we say that $(\widetilde{\mathrm{F}}, E)$ is a fuzzy soft UP$_{\mathrm{i}}$-subalgebra of A.*

In the next theorem, we give necessary condition for fuzzy soft UP$_{\mathrm{i}}$-subalgebras of f-UP-semigroups.

Theorem 24. *If $(\widetilde{\mathrm{F}}, E)$ is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\widetilde{\mathrm{F}}[e]$ in A satisfies the conditions (2.3) and (1.15), then $(\widetilde{\mathrm{F}}, E)$ is a fuzzy soft UP$_{\mathrm{i}}$-subalgebra of A.*

Proof. It is straightforward by Proposition 3 and Lemma 1 (2).

From Figure 1, we have the following theorem.

Theorem 25. *Every e-fuzzy soft* UP_{i}*-subalgebra of* A *is an e-fuzzy soft* UP_{s}*-subalgebra. Moreover, every fuzzy soft* UP_{i}*-subalgebra of* A *is a fuzzy soft* UP_{s}*-subalgebra.*

The following example shows that the converse of Theorem 25 is not true.

Example 15. *In Example 13, we know that* $(\widetilde{\mathrm{F}}, E)$ *is a price-fuzzy soft* UP_{s}*-subalgebra of* A *but* $\widetilde{\mathrm{F}}[price]$ *is not a fuzzy* UP_{i}*-subalgebra of* A*. Indeed,*

$$\mathrm{f}_{\widetilde{\mathrm{F}}[price]}(6 * 5) = \mathrm{f}_{\widetilde{\mathrm{F}}[price]}(7) = 0.3 \ngeq 0.7 = \max\{0.7, 0.1\} = \max\{\mathrm{f}_{\widetilde{\mathrm{F}}[price]}(6), \mathrm{f}_{\widetilde{\mathrm{F}}[price]}(5)\}.$$

Hence, $(\widetilde{\mathrm{F}}, E)$ *is not a price-fuzzy soft* UP_{i}*-subalgebra of* A*.*

The proof of the following theorem can be verified easily.

Theorem 26. *If* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft* UP_{i}*-subalgebra of* A *and* $\emptyset \neq E^* \subseteq E$*, then* $(\widetilde{\mathrm{F}}|_{E^*}, E^*)$ *is a fuzzy soft* UP_{i}*-subalgebra of* A*.*

The following two theorems can be deduced in the same way as Theorems 22 and 23.

Theorem 27. *The extended intersection of two fuzzy soft* UP_{i}*-subalgebras of* A *is also a fuzzy soft* UP_{i}*-subalgebra. Moreover, the intersection of two fuzzy soft* UP_{i}*-subalgebras of* A *is also a fuzzy soft* UP_{i}*-subalgebra.*

Theorem 28. *The union of two fuzzy soft* UP_{i}*-subalgebras of* A *is also a fuzzy soft* UP_{i}*-subalgebra if sets of statistics of two fuzzy soft* UP_{i}*-subalgebras are disjoint.*

The following example shows that Theorem 28 is not valid if sets of statistics of two fuzzy soft $\mathrm{UP_i}$-subalgebras are not disjoint.

Example 16. *Let* A *be the set of four types of a music, that is,*

$$A = \{pop,\ rock,\ classic,\ disco\}.$$

Define two binary operations $\cdot$ *and* $*$ *on* A *as the following Cayley tables:*

$\cdot$	*pop*	*rock*	*disco*	*classic*
pop	*pop*	*rock*	*disco*	*classic*
rock	*pop*	*pop*	*disco*	*disco*
disco	*pop*	*rock*	*pop*	*disco*
classic	*pop*	*rock*	*pop*	*pop*

$*$	*pop*	*rock*	*disco*	*classic*
pop	*pop*	*pop*	*pop*	*pop*
rock	*pop*	*pop*	*pop*	*pop*
disco	*pop*	*pop*	*pop*	*pop*
classic	*pop*	*pop*	*pop*	*pop*

Then $A = (A, \cdot, *, pop)$ *is an* f*-UP-semigroup. Let* $(\widetilde{\mathrm{G}}_1, E_1)$ *and* $(\widetilde{\mathrm{G}}_2, E_2)$ *be two fuzzy soft sets over* A *where*

$$E_1 := \{sorrow,\ modernity\} \text{ and } E_2 := \{modernity,\ enjoyment\}$$

with $\widetilde{\mathrm{G}}_1[sorrow], \widetilde{\mathrm{G}}_1[modernity], \widetilde{\mathrm{G}}_2[modernity]$*, and* $\widetilde{\mathrm{G}}_2[enjoyment]$ *are fuzzy sets in* A *defined as follows:*

$\widetilde{\mathrm{G}}_1$	*pop*	*rock*	*disco*	*classic*
sorrow	0.7	0.7	0.5	0.5
modernity	0.9	0.8	0.3	0.3

$\widetilde{\mathrm{G}}_2$	*pop*	*rock*	*disco*	*classic*
modernity	0.8	0.3	0.4	0.5
enjoyment	1	0.9	0.1	0.1

Then $(\widetilde{\mathrm{G}}_1, E_1)$ *and* $(\widetilde{\mathrm{G}}_2, E_2)$ *are two fuzzy soft UP*$_\mathrm{i}$*-subalgebras of* A*. Since modernity* $\in E_1 \cap E_2$*, we have*

$$\begin{aligned}
&(\mathrm{f}_{\widetilde{\mathrm{G}}_1[modernity]\cup\widetilde{\mathrm{G}}_2[modernity]})(rock \cdot classic)\\
&\quad = (\mathrm{f}_{\widetilde{\mathrm{G}}_1[modernity]\cup\widetilde{\mathrm{G}}_2[modernity]})(disco)\\
&\quad = 0.4\\
&\quad \not\geq 0.5\\
&\quad = \min\{0.8, 0.5\}\\
&\quad = \min\{(\mathrm{f}_{\widetilde{\mathrm{G}}_1[modernity]\cup\widetilde{\mathrm{G}}_2[modernity]})(rock), (\mathrm{f}_{\widetilde{\mathrm{G}}_1[modernity]\cup\widetilde{\mathrm{G}}_2[modernity]})(classic)\}.
\end{aligned}$$

Thus $\widetilde{\mathrm{G}}_1[modernity] \cup \widetilde{\mathrm{G}}_2[modernity]$ *is not a fuzzy UP*$_\mathrm{i}$*-subalgebra of* A*, that is,* $(\widetilde{\mathrm{G}}_1, E_1) \cup (\widetilde{\mathrm{G}}_2, E_2)$ *is not a modernity-fuzzy soft UP*$_\mathrm{i}$*-subalgebra of* A*. Hence,* $(\widetilde{\mathrm{G}}_1, E_1) \cup (\widetilde{\mathrm{G}}_2, E_2)$ *is not a fuzzy soft UP*$_\mathrm{i}$*-subalgebra of* A*. Moreover,* $(\widetilde{\mathrm{G}}_1, E_1) \Cup (\widetilde{\mathrm{G}}_2, E_2)$ *is not a fuzzy soft UP*$_\mathrm{i}$*-subalgebra of* A*.*

3.3. Fuzzy Soft Near UP$_\mathrm{s}$-Filters

Definition 20. *A fuzzy soft set* $(\widetilde{\mathrm{F}}, E)$ *over* A *is called a fuzzy soft near UP*$_\mathrm{s}$*-filter based on* $e \in E$ *(we shortly call an e-fuzzy soft near UP*$_\mathrm{s}$*-filter) of* A *if a fuzzy set* $\widetilde{\mathrm{F}}[e]$ *in* A *is a fuzzy near UP*$_\mathrm{s}$*-filter of* A*. If* $(\widetilde{\mathrm{F}}, E)$ *is an e-fuzzy soft near UP*$_\mathrm{s}$*-filter of* A *for all* $e \in E$*, we say that* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft near UP*$_\mathrm{s}$*-filter of* A*.*

In the next theorem, we give necessary condition for fuzzy soft near UP$_\mathrm{s}$-filters of f-UP-semigroups.

Theorem 29. *If* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft set over* A *such that for all* $e \in E$*, a fuzzy set* $\widetilde{\mathrm{F}}[e]$ *in* A *satisfies the conditions* (2.2) *and* (1.14)*, then* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft near UP*$_\mathrm{s}$*-filter of* A*.*

Proof. It is straightforward by Proposition 5 and Lemma 1 (1).

From Figure 1, we have the following theorem.

Theorem 30. *Every e-fuzzy soft near UP*$_\mathrm{s}$*-filter of* A *is an e-fuzzy soft UP*$_\mathrm{s}$*-subalgebra. Moreover, every fuzzy soft near UP*$_\mathrm{s}$*-filter of* A *is a fuzzy soft UP*$_\mathrm{s}$*-subalgebra.*

The following example shows that the converse of Theorem 30 is not true.

Example 17. *Let A be a set of four foods, that is,*

$$A = \{apple,\ banana,\ meat,\ rice\}.$$

Define two binary operations $\cdot$ and $*$ on A as the following Cayley tables:

$\cdot$	*rice*	*apple*	*banana*	*meat*
rice	*rice*	*apple*	*banana*	*meat*
apple	*rice*	*rice*	*apple*	*meat*
banana	*rice*	*rice*	*rice*	*meat*
meat	*rice*	*apple*	*apple*	*rice*

$*$	*rice*	*apple*	*banana*	*meat*
rice	*rice*	*rice*	*rice*	*rice*
apple	*rice*	*rice*	*rice*	*rice*
banana	*rice*	*rice*	*rice*	*rice*
meat	*rice*	*rice*	*rice*	*apple*

Then $A = (A, \cdot, *, rice)$ is an f-UP-semigroup. Let $(\widetilde{\mathrm{F}}, E)$ be a fuzzy soft set over A where

$$E := \{pig,\ monkey,\ chicken\}$$

with $\widetilde{\mathrm{F}}[pig], \widetilde{\mathrm{F}}[monkey]$, and $\widetilde{\mathrm{F}}[chicken]$ are fuzzy sets in A defined as follows:

$\widetilde{\mathrm{F}}$	*rice*	*apple*	*banana*	*meat*
pig	1	0.8	0.9	0.3
monkey	0.8	0.4	0.8	0.3
chicken	0.7	0.4	0.3	0.2

Then $(\widetilde{\mathrm{F}}, E)$ is a pig-fuzzy soft UP$_{\mathrm{s}}$-subalgebra of A. But $(\widetilde{\mathrm{F}}, E)$ is not a pig-fuzzy soft near UP$_{\mathrm{s}}$-filter of A since

$$\begin{aligned} \mathrm{f}_{\widetilde{\mathrm{F}}[pig]}(meat \cdot banana) &= \mathrm{f}_{\widetilde{\mathrm{F}}[pig]}(apple) \\ &= 0.8 \\ &\not\geq 0.9 \\ &= \mathrm{f}_{\widetilde{\mathrm{F}}[pig]}(banana), \end{aligned}$$

that is, $\widetilde{\mathrm{F}}[pig]$ is not a fuzzy near UP$_{\mathrm{s}}$-filter of A.

In the next theorem, we give necessary condition for fuzzy soft UP$_{\mathrm{s}}$-subalgebras as fuzzy soft near UP$_{\mathrm{s}}$-filters of f-UP-semigroups.

Theorem 31. *If $(\widetilde{\mathrm{F}}, E)$ is a fuzzy soft UP$_{\mathrm{s}}$-subalgebra of A such that for all $e \in E$, a fuzzy set $\widetilde{\mathrm{F}}[e]$ in A satisfies the condition* (2.5)*, then $(\widetilde{\mathrm{F}}, E)$ is a fuzzy soft near UP$_{\mathrm{s}}$-filter of A.*

Proof. It is straightforward by Theorem 12.

The proof of the following theorem can be verified easily.

Theorem 32. *If $(\widetilde{\mathrm{F}}, E)$ is a fuzzy soft near UP$_{\mathrm{s}}$-filter of A and $\emptyset \neq E^* \subseteq E$, then $(\widetilde{\mathrm{F}}|_{E^*}, E^*)$ is a fuzzy soft near UP$_{\mathrm{s}}$-filter of A.*

The following two theorems can be deduced in the same way as Theorems 22 and 23.

Theorem 33. *The extended intersection of two fuzzy soft near UP*$_{\text{s}}$*-filters of* A *is also a fuzzy soft near UP*$_{\text{s}}$*-filter. Moreover, the intersection of two fuzzy soft near UP*$_{\text{s}}$*-filters of* A *is also a fuzzy soft near UP*$_{\text{s}}$*-filter.*

Theorem 34. *The union of two fuzzy soft near UP*$_{\text{s}}$*-filters of* A *is also a fuzzy soft near UP*$_{\text{s}}$*-filter if sets of statistics of two fuzzy soft near UP*$_{\text{s}}$*-filters are disjoint.*

The following example shows that Theorem 34 is not valid if sets of statistics of two fuzzy soft near UP$_{\text{s}}$-filters are not disjoint.

Example 18. *In Example 14, we have* $(\widetilde{\mathrm{G}}_1, E_1)$ *and* $(\widetilde{\mathrm{G}}_2, E_2)$ *are two fuzzy soft near UP*$_{\text{s}}$*-filters of* A*. Since* $price \in E_1 \cap E_2$*, we have*

$$\begin{aligned}(\mathrm{f}_{\widetilde{\mathrm{G}}_1[price]\cup\widetilde{\mathrm{G}}_2[price]})(6*5) &= (\mathrm{f}_{\widetilde{\mathrm{G}}_1[price]\cup\widetilde{\mathrm{G}}_2[price]})(7)\\ &= 0.7\\ &\ngeq 0.8\\ &= \min\{0.9, 0.8\}\\ &= \min\{(\mathrm{f}_{\widetilde{\mathrm{G}}_1[price]\cup\widetilde{\mathrm{G}}_2[price]})(6), (\mathrm{f}_{\widetilde{\mathrm{G}}_1[price]\cup\widetilde{\mathrm{G}}_2[price]})(5)\}.\end{aligned}$$

Thus $\widetilde{\mathrm{G}}_1[price] \cup \widetilde{\mathrm{G}}_2[price]$ *is not a fuzzy near UP*$_{\text{s}}$*-filter of* A*, that is,* $(\widetilde{\mathrm{G}}_1, E_1) \cup (\widetilde{\mathrm{G}}_2, E_2)$ *is not a price-fuzzy soft near UP*$_{\text{s}}$*-filter of* A*. Hence,* $(\widetilde{\mathrm{G}}_1, E_1) \cup (\widetilde{\mathrm{G}}_2, E_2)$ *is not a fuzzy soft near UP*$_{\text{s}}$*-filter of* A*. Moreover,* $(\widetilde{\mathrm{G}}_1, E_1) \Cup (\widetilde{\mathrm{G}}_2, E_2)$ *is not a fuzzy soft near UP*$_{\text{s}}$*-filter of* A*.*

3.4. Fuzzy Soft Near UP$_{\text{i}}$-Filters

Definition 21. *A fuzzy soft set* $(\widetilde{\mathrm{F}}, E)$ *over* A *is called a fuzzy soft near UP*$_{\text{i}}$*-filter based on* $e \in E$ *(we shortly call an* e*-fuzzy soft near UP*$_{\text{i}}$*-filter) of* A *if a fuzzy set* $\widetilde{\mathrm{F}}[e]$ *in* A *is a fuzzy near UP*$_{\text{i}}$*-filter of* A*. If* $(\widetilde{\mathrm{F}}, E)$ *is an* e*-fuzzy soft near UP*$_{\text{i}}$*-filter of* A *for all* $e \in E$*, we say that* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft near UP*$_{\text{i}}$*-filter of* A*.*

In the next theorem, we give necessary condition for fuzzy soft near UP$_{\text{i}}$-filters of f-UP-semigroups.

Theorem 35. *If* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft set over* A *such that for all* $e \in E$*, a fuzzy set* $\widetilde{\mathrm{F}}[e]$ *in* A *satisfies the conditions* (2.2) *and* (1.15)*, then* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft near UP*$_{\text{i}}$*-filter of* A*.*

Proof. It is straightforward by Proposition 5 and Lemma 1 (2).

From Figure 1, we have the following two theorems.

Theorem 36. *Every* e*-fuzzy soft near UP*$_{\text{i}}$*-filter of* A *is an* e*-fuzzy soft near UP*$_{\text{s}}$*-filter. Moreover, every fuzzy soft near UP*$_{\text{i}}$*-filter of* A *is a fuzzy soft near UP*$_{\text{s}}$*-filter.*

Theorem 37. *Every e-fuzzy soft near UP$_{\rm i}$-filter of A is an e-fuzzy soft UP$_{\rm i}$-subalgebra. Moreover, every fuzzy soft near UP$_{\rm i}$-filter of A is a fuzzy soft UP$_{\rm i}$-subalgebra.*

The following two examples show that the converse of Theorems 36 and 37 is not true.

Example 19. *In Example 13, we know that $(\widetilde{\rm F}, E)$ is a price-fuzzy soft near UP$_{\rm s}$-filter of A but $\widetilde{\rm F}[price]$ is not a fuzzy near UP$_{\rm i}$-filter of A. Indeed,*

$$\mathrm{f}_{\widetilde{\rm F}[price]}(6 * 5) = \mathrm{f}_{\widetilde{\rm F}[price]}(7) = 0.3 \ngeq 0.7 = \max\{0.7, 0.1\} = \max\{\mathrm{f}_{\widetilde{\rm F}[price]}(6), \mathrm{f}_{\widetilde{\rm F}[price]}(5)\}.$$

Hence, $(\widetilde{\rm F}, E)$ is not a price-fuzzy soft near UP$_{\rm i}$-filter of A.

Example 20. *In Example 17, we know that $(\widetilde{\rm F}, E)$ is a monkey-fuzzy soft UP$_{\rm i}$-subalgebra of A but $\widetilde{\rm F}[monkey]$ is not a fuzzy near UP$_{\rm i}$-filter of A. Indeed,*

$$\mathrm{f}_{\widetilde{\rm F}[monkey]}(apple \cdot banana) = \mathrm{f}_{\widetilde{\rm F}[monkey]}(apple) = 0.4 \ngeq 0.8 = \mathrm{f}_{\widetilde{\rm F}[monkey]}(banana).$$

Hence, $(\widetilde{\rm F}, E)$ is not a monkey-fuzzy soft near UP$_{\rm i}$-filter of A.

In the next theorem, we give necessary condition for fuzzy soft UP$_{\rm i}$-subalgebras as fuzzy soft near UP$_{\rm i}$-filters of f-UP-semigroups.

Theorem 38. *If $(\widetilde{\rm F}, E)$ is a fuzzy soft UP$_{\rm i}$-subalgebra of A such that for all $e \in E$, a fuzzy set $\widetilde{\rm F}[e]$ in A satisfies the condition (2.5), then $(\widetilde{\rm F}, E)$ is a fuzzy soft near UP$_{\rm i}$-filter of A.*

Proof. It is straightforward by Theorem 12.

The proof of the following theorem can be verified easily.

Theorem 39. *If $(\widetilde{\rm F}, E)$ is a fuzzy soft near UP$_{\rm i}$-filter of A and $\emptyset \neq E^* \subseteq E$, then $(\widetilde{\rm F}|_{E^*}, E^*)$ is a fuzzy soft near UP$_{\rm i}$-filter of A.*

By using Theorem 9, we can obtain the following two theorems in the same way as Theorems 22 and 23.

Theorem 40. *The extended intersection of two fuzzy soft near UP$_{\rm i}$-filters of A is also a fuzzy soft near UP$_{\rm i}$-filter. Moreover, the intersection of two fuzzy soft near UP$_{\rm i}$-filters of A is also a fuzzy soft near UP$_{\rm i}$-filter.*

Theorem 41. *The union of two fuzzy soft near UP$_{\rm i}$-filters of A is also a fuzzy soft near UP$_{\rm i}$-filter. Moreover, the restricted union of two fuzzy soft near UP$_{\rm i}$-filters of A is also a fuzzy soft near UP$_{\rm i}$-filter.*

3.5. Fuzzy Soft UP_s-Filters

Definition 22. *A fuzzy soft set* $(\widetilde{F}, E)$ *over* A *is called a fuzzy soft* UP_s*-filter based on* $e \in E$ *(we shortly call an e-fuzzy soft* UP_s*-filter) of* A *if a fuzzy set* $\widetilde{F}[e]$ *in* A *is a fuzzy* UP_s*-filter of* A*. If* $(\widetilde{F}, E)$ *is an e-fuzzy soft* UP_s*-filter of* A *for all* $e \in E$*, we say that* $(\widetilde{F}, E)$ *is a fuzzy soft* UP_s*-filter of* A*.*

In the next theorem, we give necessary condition for fuzzy soft UP_s-filters of f-UP-semigroups.

Theorem 42. *If* $(\widetilde{F}, E)$ *is a fuzzy soft set over* A *such that for all* $e \in E$*, a fuzzy set* $\widetilde{F}[e]$ *in* A *satisfies the conditions* (2.6) *and* (1.14)*, then* $(\widetilde{F}, E)$ *is a fuzzy soft* UP_s*-filter of* A*.*

Proof. It is straightforward by Proposition 6 and Lemma 1 (1).

From Figure 1, we have the following theorem.

Theorem 43. *Every e-fuzzy soft* UP_s*-filter of* A *is an e-fuzzy soft near* UP_s*-filter. Moreover, every fuzzy soft* UP_s*-filter of* A *is a fuzzy soft near* UP_s*-filter.*

The following example shows that the converse of Theorem 43 is not true.

Example 21. *Let* A *be a set of four coffees, that is,*

$$A = \{Mocha(M),\ Americano(A),\ Cappuccino(C),\ Latte(L)\}.$$

Define two binary operations $\cdot$ *and* $*$ *on* A *as the following Cayley tables:*

$\cdot$	L	A	M	C
L	L	A	M	C
A	L	L	M	C
M	L	L	L	C
C	L	L	L	L

$\cdot$	L	A	M	C
L	L	L	L	L
A	L	L	L	L
M	L	L	L	L
C	L	L	L	M

Then $A = (A, \cdot, *, Latte)$ *is an* f*-UP-semigroup. Let* $(\widetilde{F}, E)$ *be a fuzzy soft set over* A *where*

$$E := \{sweetness,\ strong,\ aroma\}$$

with $\widetilde{F}[sweetness], \widetilde{F}[strong]$*, and* $\widetilde{F}[aroma]$ *are fuzzy sets in* A *defined as follows:*

$\widetilde{F}$	L	A	M	C
sweetness	0.8	0.1	0.6	0.6
strong	0.7	0.7	0.6	0.5
aroma	0.5	0.3	0.4	0.1

Then $(\widetilde{F}, E)$ *is a sweetness-fuzzy soft near* UP_s*-filter of* A *but* $\widetilde{F}[sweetness]$ *is not a fuzzy* UP_s*-filter of* A*. Indeed,*

$$f_{\widetilde{F}[sweetness]}(A) = 0.1 \not\geq 0.6 = \min\{0.8, 0.6\} = \min\{f_{\widetilde{F}[sweetness]}(L), f_{\widetilde{F}[sweetness]}(M)\} = \min\{f_{\widetilde{F}[sweetness]}(M \cdot A), f_{\widetilde{F}[sweetness]}(M)\}$$

Hence, $(\widetilde{F}, E)$ *is not a sweetness-fuzzy soft* UP_s*-filter of* A.

In the next theorem, we give necessary condition for fuzzy soft near UP_s-filters as fuzzy soft UP_s-filters of f-UP-semigroups.

Theorem 44. *If* $(\widetilde{F}, E)$ *is a fuzzy soft near* UP_s*-filter of* A *such that for all* $e \in E$*, a fuzzy set* $\widetilde{F}[e]$ *in* A *satisfies the condition* (2.7)*, then* $(\widetilde{F}, E)$ *is a fuzzy soft* UP_s*-filter of* A.

Proof. It is straightforward by Theorem 14.

The proof of the following theorem can be verified easily.

Theorem 45. *If* $(\widetilde{F}, E)$ *is a fuzzy soft* UP_s*-filter of* A *and* $\emptyset \neq E^* \subseteq E$*, then* $(\widetilde{F}|_{E^*}, E^*)$ *is a fuzzy soft* UP_s*-filter of* A.

The following two theorems can be deduced in the same way as Theorems 22 and 23.

Theorem 46. *The extended intersection of two fuzzy soft* UP_s*-filters of* A *is also a fuzzy soft* UP_s*-filter. Moreover, the intersection of two fuzzy soft* UP_s*-filters of* A *is also a fuzzy soft* UP_s*-filter.*

Theorem 47. *The union of two fuzzy soft* UP_s*-filters of* A *is also a fuzzy soft* UP_s*-filter if sets of statistics of two fuzzy soft* UP_s*-filters are disjoint.*

The following example shows that Theorem 47 is not valid if sets of statistics of two fuzzy soft UP_s-filters are not disjoint.

Example 22. *In Example 14, we have* $(\widetilde{G}_1, E_1)$ *and* $(\widetilde{G}_2, E_2)$ *are two fuzzy soft* UP_s*-filters of* A*. Since price* $\in E_1 \cap E_2$*, we have*

$$(f_{\widetilde{G}_1[price] \cup \widetilde{G}_2[price]})(6 * 5) = (f_{\widetilde{G}_1[price] \cup \widetilde{G}_2[price]})(7) = 0.7 \not\geq 0.8 = \min\{0.9, 0.8\} = \min\{(f_{\widetilde{G}_1[price] \cup \widetilde{G}_2[price]})(6), (f_{\widetilde{G}_1[price] \cup \widetilde{G}_2[price]})(5)\}.$$

Thus $\widetilde{G}_1[price] \cup \widetilde{G}_2[price]$ *is not a fuzzy* UP_s*-filter of* A*, that is,* $(\widetilde{G}_1, E_1) \cup (\widetilde{G}_2, E_2)$ *is not a price-fuzzy soft* UP_s*-filter of* A*. Hence,* $(\widetilde{G}_1, E_1) \cup (\widetilde{G}_2, E_2)$ *is not a fuzzy soft* UP_s*-filter of* A*. Moreover,* $(\widetilde{G}_1, E_1) \Cup (\widetilde{G}_2, E_2)$ *is not a fuzzy soft* UP_s*-filter of* A*.*

3.6. Fuzzy Soft UP_i-Filters

Definition 23. *A fuzzy soft set* $(\widetilde{F}, E)$ *over* A *is called a fuzzy soft* UP_i*-filter based on* $e \in E$ *(we shortly call an* e*-fuzzy soft* UP_i*-filter) of* A *if a fuzzy set* $\widetilde{F}[e]$ *in* A *is a fuzzy* UP_i*-filter of* A*. If* $(\widetilde{F}, E)$ *is an* e*-fuzzy soft* UP_i*-filter of* A *for all* $e \in E$*, we say that* $(\widetilde{F}, E)$ *is a fuzzy soft* UP_i*-filter of* A*.*

In the next theorem, we give necessary condition for fuzzy soft UP_i-filters of f-UP-semigroups.

Theorem 48. *If* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft set over* A *such that for all* $e \in E$*, a fuzzy set* $\widetilde{\mathrm{F}}[e]$ *in* A *satisfies the conditions* (2.6) *and* (1.15)*, then* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft UP*$_{\mathrm{i}}$*-filter of* A*.*

Proof. It is straightforward by Proposition 6 and Lemma 1 (2).

From Figure 1, we have the following two theorems.

Theorem 49. *Every e-fuzzy soft UP*$_{\mathrm{i}}$*-filter of* A *is an e-fuzzy soft UP*$_{\mathrm{s}}$*-filter. Moreover, every fuzzy soft UP*$_{\mathrm{i}}$*-filter of* A *is a fuzzy soft UP*$_{\mathrm{s}}$*-filter.*

Theorem 50. *Every e-fuzzy soft UP*$_{\mathrm{i}}$*-filter of* A *is an e-fuzzy soft near UP*$_{\mathrm{i}}$*-filter. Moreover, every fuzzy soft UP*$_{\mathrm{i}}$*-filter of* A *is a fuzzy soft near UP*$_{\mathrm{i}}$*-filter.*

The following two examples show that the converse of Theorems 49 and 50 is not true.

Example 23. *In Example 13, we know that* $(\widetilde{\mathrm{F}}, E)$ *is a beauty-fuzzy soft UP*$_{\mathrm{s}}$*-filter of* A *but* $\widetilde{\mathrm{F}}[beauty]$ *is not a fuzzy UP*$_{\mathrm{i}}$*-filter of* A*. Indeed,*

$$\mathrm{f}_{\widetilde{\mathrm{F}}[beauty]}(6 * 5) = \mathrm{f}_{\widetilde{\mathrm{F}}[beauty]}(7) = 0.3 \not\geq 0.4 = \max\{0.2, 0.4\} = \max\{\mathrm{f}_{\widetilde{\mathrm{F}}[beauty]}(6), \mathrm{f}_{\widetilde{\mathrm{F}}[beauty]}(5)\}.$$

Hence, $(\widetilde{\mathrm{F}}, E)$ *is not a beauty-fuzzy soft UP*$_{\mathrm{i}}$*-filter of* A*.*

Example 24. *In Example 21, we know that* $(\widetilde{\mathrm{F}}, E)$ *is a aroma-fuzzy soft near UP*$_{\mathrm{i}}$*-filter of* A *but* $\widetilde{\mathrm{F}}[aroma]$ *is not a fuzzy UP*$_{\mathrm{i}}$*-filter of* A*. Indeed,*

$$\begin{aligned}\mathrm{f}_{\widetilde{\mathrm{F}}[aroma]}(A) = 0.3 \not\geq 0.4 = \min\{0.5, 0.4\} = \min\{\mathrm{f}_{\widetilde{\mathrm{F}}[aroma]}(L), \mathrm{f}_{\widetilde{\mathrm{F}}[aroma]}(M)\} = \\ \min\{\mathrm{f}_{\widetilde{\mathrm{F}}[aroma]}(M \cdot A), \mathrm{f}_{\widetilde{\mathrm{F}}[aroma]}(M)\}.\end{aligned}$$

Hence, $(\widetilde{\mathrm{F}}, E)$ *is not a aroma-fuzzy soft UP*$_{\mathrm{i}}$*-filter of* A*.*

In the next theorem, we give necessary condition for fuzzy soft near UP$_{\mathrm{i}}$-filters as fuzzy soft UP$_{\mathrm{i}}$-filters of f-UP-semigroups.

Theorem 51. *If* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft near UP*$_{\mathrm{i}}$*-filter of* A *such that for all* $e \in E$*, a fuzzy set* $\widetilde{\mathrm{F}}[e]$ *in* A *satisfies the condition* (2.7)*, then* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft UP*$_{\mathrm{i}}$*-filter of* A*.*

Proof. It is straightforward by Theorem 14.

The proof of the following theorem can be verified easily.

Theorem 52. *If* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft UP*$_{\mathrm{i}}$*-filter of* A *and* $\emptyset \neq E^* \subseteq E$*, then* $(\widetilde{\mathrm{F}}|_{E^*}, E^*)$ *is a fuzzy soft UP*$_{\mathrm{i}}$*-filter of* A*.*

The following two theorems can be deduced in the same way as Theorems 22 and 23.

Theorem 53. *The extended intersection of two fuzzy soft UP*$_{\mathrm{i}}$*-filters of* A *is also a fuzzy soft UP*$_{\mathrm{i}}$*-filter. Moreover, the intersection of two fuzzy soft UP*$_{\mathrm{i}}$*-filters of* A *is also a fuzzy soft UP*$_{\mathrm{i}}$*-filter.*

Theorem 54. *The union of two fuzzy soft* $\mathrm{UP_i}$*-filters of* A *is also a fuzzy soft* $\mathrm{UP_i}$*-filter if sets of statistics of two fuzzy soft* $\mathrm{UP_i}$*-filters are disjoint.*

The following example shows that Theorem 54 is not valid if sets of statistics of two fuzzy soft $\mathrm{UP_i}$-filters are not disjoint.

Example 25. *Let* A *be a set of four colors, that is,*

$$A = \{blue, green, cyan, black\}.$$

Define two binary operations $\cdot$ *and* $*$ *on* A *as the following Cayley tables:*

$\cdot$	*black*	*cyan*	*blue*	*green*
black	*black*	*cyan*	*blue*	*green*
cyan	*black*	*black*	*blue*	*blue*
blue	*black*	*cyan*	*black*	*cyan*
green	*black*	*black*	*black*	*black*

$*$	*black*	*cyan*	*blue*	*green*
black	*black*	*black*	*black*	*black*
cyan	*black*	*black*	*black*	*black*
blue	*black*	*black*	*black*	*black*
green	*black*	*black*	*black*	*black*

Then $A = (A, \cdot, *, black)$ *is an* f*-UP-semigroup. Let* $(\widetilde{\mathrm{G}}_1, E_1)$ *and* $(\widetilde{\mathrm{G}}_2, E_2)$ *be two fuzzy soft sets over* A *where*

$$E_1 := \{endurance, beauty\} \text{ and } E_2 := \{endurance, warmth\}$$

with $\widetilde{\mathrm{G}}_1[endurance], \widetilde{\mathrm{G}}_1[beauty], \widetilde{\mathrm{G}}_2[endurance]$*, and* $\widetilde{\mathrm{G}}_2[warmth]$ *are fuzzy sets in* A *defined as follows:*

$\widetilde{\mathrm{G}}_1$	*black*	*cyan*	*blue*	*green*
endurance	1	0.5	0.7	0.5
beauty	0.4	0.3	0.2	0.2

$\widetilde{\mathrm{G}}_2$	*black*	*cyan*	*blue*	*green*
endurance	1	0.6	0.5	0.5
warmth	0.9	0.4	0.5	0.4

Then $(\widetilde{\mathrm{G}}_1, E_1)$ *and* $(\widetilde{\mathrm{G}}_2, E_2)$ *are two fuzzy soft* $\mathrm{UP_i}$*-filters of* A*. Since endurance* $\in E_1 \cap E_2$*, we have*

$$\begin{gathered}(\mathrm{f}_{\widetilde{\mathrm{G}}_1[endurance] \cup \widetilde{\mathrm{G}}_2[endurance]})(green) = 0.5 \ngeq 0.6 = \min\{0.6, 0.7\} = \\ \min\{(\mathrm{f}_{\widetilde{\mathrm{G}}_1[endurance] \cup \widetilde{\mathrm{G}}_2[endurance]})(cyan), (\mathrm{f}_{\widetilde{\mathrm{G}}_1[endurance] \cup \widetilde{\mathrm{G}}_2[endurance]})(blue)\} = \\ \min\{(\mathrm{f}_{\widetilde{\mathrm{G}}_1[endurance] \cup \widetilde{\mathrm{G}}_2[endurance]})(blue \cdot green), (\mathrm{f}_{\widetilde{\mathrm{G}}_1[endurance] \cup \widetilde{\mathrm{G}}_2[endurance]})(blue)\}.\end{gathered}$$

Thus $\widetilde{\mathrm{G}}_1[endurance] \cup \widetilde{\mathrm{G}}_2[endurance]$ *is not a fuzzy* $\mathrm{UP_i}$*-filter of* A*, that is,* $(\widetilde{\mathrm{G}}_1, E_1) \cup (\widetilde{\mathrm{G}}_2, E_2)$ *is not a endurance-fuzzy soft* $\mathrm{UP_i}$*-filter of* A*. Hence,* $(\widetilde{\mathrm{G}}_1, E_1) \cup (\widetilde{\mathrm{G}}_2, E_2)$ *is not a fuzzy soft* $\mathrm{UP_i}$*-filter of* A*. Moreover,* $(\widetilde{\mathrm{G}}_1, E_1) \Cup (\widetilde{\mathrm{G}}_2, E_2)$ *is not a fuzzy soft* $\mathrm{UP_i}$*-filter of* A*.*

3.7. Fuzzy Soft $\mathrm{UP_s}$-Ideals

Definition 24. *A fuzzy soft set* $(\widetilde{\mathrm{F}}, E)$ *over* A *is called a fuzzy soft* UP_{s}*-ideal based on* $e \in E$ (*we shortly call an* e*-fuzzy soft* UP_{s}*-ideal*) *of* A *if a fuzzy set* $\widetilde{\mathrm{F}}[e]$ *in* A *is a fuzzy* UP_{s}*-ideal of* A. *If* $(\widetilde{\mathrm{F}}, E)$ *is an* e*-fuzzy soft* UP_{s}*-ideal of* A *for all* $e \in E$*, we say that* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft* UP_{s}*-ideal of* A.

In the next theorem and corollary, we give necessary condition for fuzzy soft $\mathrm{UP_s}$-ideals of f-UP-semigroups.

Theorem 55. *If* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft set over* A *such that for all* $e \in E$*, a fuzzy set* $\widetilde{\mathrm{F}}[e]$ *in* A *satisfies the conditions* (2.8) *and* (1.14)*, then* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft* UP_{s}*-ideal of* A.

Proof. It is straightforward by Proposition 7 and Lemma 1 (1).

Corollary 3. *Let* A *be an* f*-UP-semigroup satisfying the condition* (2.10)*. If* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft set over* A *such that for all* $e \in E$*, a fuzzy set* $\widetilde{\mathrm{F}}[e]$ *in* A *satisfies the conditions* (2.9) *and* (1.14)*, then* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft* UP_{s}*-ideal of* A.

Proof. It is straightforward by Theorems 55 and 15.

From Figure 1, we have the following theorem.

Theorem 56. *Every* e*-fuzzy soft* UP_{s}*-ideal of* A *is an* e*-fuzzy soft* UP_{s}*-filter. Moreover, every fuzzy soft* UP_{s}*-ideal of* A *is a fuzzy soft* UP_{s}*-filter.*

The following example shows that the converse of Theorem 56 is not true.

Example 26. *By Cayley tables in Example 16, we know that* $A = (A, \cdot, *, pop)$ *is an* f*-UP-semigroup. Let* $(\widetilde{\mathrm{F}}, E)$ *be a fuzzy soft set over* A *where*

$$E := \{sorrow,\ relaxation,\ enjoyment\}$$

with $\widetilde{\mathrm{F}}[sorrow], \widetilde{\mathrm{F}}[modernity]$*, and* $\widetilde{\mathrm{F}}[enjoyment]$ *are fuzzy sets in* A *defined as follows:*

$\widetilde{\mathrm{F}}$	*pop*	*rock*	*disco*	*classic*
sorrow	0.6	0.2	0.1	0.1
modernity	1	0.5	0.5	0.5
enjoyment	0.7	0.5	0.2	0.2

Then $(\widetilde{\mathrm{F}}, E)$ *is a sorrow-fuzzy soft* UP_{s}*-filter of* A *but* $\widetilde{\mathrm{F}}[sorrow]$ *is not a fuzzy* UP_{s}*-ideal of* A. *Indeed,*

$$\mathrm{f}_{\widetilde{\mathrm{F}}[sorrow]}(disco \cdot classic) = \mathrm{f}_{\widetilde{\mathrm{F}}[sorrow]}(disco) = 0.1 \ngeq 0.2 = \min\{0.6, 0.2\} = \min\{\mathrm{f}_{\widetilde{\mathrm{F}}[sorrow]}(pop), \mathrm{f}_{\widetilde{\mathrm{F}}[sorrow]}(rock)\} = \min\{\mathrm{f}_{\widetilde{\mathrm{F}}[sorrow]}(disco \cdot (rock \cdot classic)), \mathrm{f}_{\widetilde{\mathrm{F}}[sorrow]}(rock)\}.$$

Hence, $(\widetilde{\mathrm{F}}, E)$ *is not a sorrow-fuzzy soft* UP_{s}*-ideal of* A.

In the next theorem, we give necessary condition for fuzzy soft $\mathrm{UP_s}$-filters as fuzzy soft $\mathrm{UP_s}$-ideals of f-UP-semigroups.

Theorem 57. *If $(\widetilde{\mathrm{F}}, E)$ is a fuzzy soft UP_{s}-filter of A such that for all $e \in E$, a fuzzy set $\widetilde{\mathrm{F}}[e]$ in A satisfies the condition (2.11), then $(\widetilde{\mathrm{F}}, E)$ is a fuzzy soft UP_{s}-ideal of A.*

Proof. It is straightforward by Theorem 17.

The proof of the following theorem can be verified easily.

Theorem 58. *If $(\widetilde{\mathrm{F}}, E)$ is a fuzzy soft UP_{s}-ideal of A and $\emptyset \neq E^* \subseteq E$, then $(\widetilde{\mathrm{F}}|_{E^*}, E^*)$ is a fuzzy soft UP_{s}-ideal of A.*

The following two theorems can be deduced in the same way as Theorems 22 and 23.

Theorem 59. *The extended intersection of two fuzzy soft UP_{s}-ideals of A is also a fuzzy soft UP_{s}-ideal. Moreover, the intersection of two fuzzy soft UP_{s}-ideals of A is also a fuzzy soft UP_{s}-ideal.*

Theorem 60. *The union of two fuzzy soft UP_{s}-ideals of A is also a fuzzy soft UP_{s}-ideal if sets of statistics of two fuzzy soft UP_{s}-ideals are disjoint.*

The following example shows that Theorem 60 is not valid if sets of statistics of two fuzzy soft $\mathrm{UP_s}$-ideals are not disjoint.

Example 27. *In Example 14, we have $(\widetilde{\mathrm{G}}_1, E_1)$ and $(\widetilde{\mathrm{G}}_2, E_2)$ are two fuzzy soft UP_{s}-ideals of A. Since price $\in E_1 \cap E_2$, we have*

$$(\mathrm{f}_{\widetilde{\mathrm{G}}_1[price] \cup \widetilde{\mathrm{G}}_2[price]})(6 * 5) = (\mathrm{f}_{\widetilde{\mathrm{G}}_1[price] \cup \widetilde{\mathrm{G}}_2[price]})(7) = 0.7 \not\geq 0.8 = \min\{0.9, 0.8\} = \min\{(\mathrm{f}_{\widetilde{\mathrm{G}}_1[price] \cup \widetilde{\mathrm{G}}_2[price]})(6), (\mathrm{f}_{\widetilde{\mathrm{G}}_1[price] \cup \widetilde{\mathrm{G}}_2[price]})(5)\}.$$

Thus $\widetilde{\mathrm{G}}_1[price] \cup \widetilde{\mathrm{G}}_2[price]$ is not a fuzzy UP_{s}-ideal of A, that is, $(\widetilde{\mathrm{G}}_1, E_1) \cup (\widetilde{\mathrm{G}}_2, E_2)$ is not a price-fuzzy soft UP_{s}-ideal of A. Hence, $(\widetilde{\mathrm{G}}_1, E_1) \cup (\widetilde{\mathrm{G}}_2, E_2)$ is not a fuzzy soft UP_{s}-ideal of A. Moreover, $(\widetilde{\mathrm{G}}_1, E_1) \Cup (\widetilde{\mathrm{G}}_2, E_2)$ is not a fuzzy soft UP_{s}-ideal of A.

3.8. Fuzzy Soft $\mathrm{UP_i}$-Ideals

Definition 25. *A fuzzy soft set $(\widetilde{\mathrm{F}}, E)$ over A is called a fuzzy soft UP_{i}-ideal based on $e \in E$ (we shortly call an e-fuzzy soft UP_{i}-ideal) of A if a fuzzy set $\widetilde{\mathrm{F}}[e]$ in A is a fuzzy UP_{i}-ideal of A. If $(\widetilde{\mathrm{F}}, E)$ is an e-fuzzy soft UP_{i}-ideal of A for all $e \in E$, we say that $(\widetilde{\mathrm{F}}, E)$ is a fuzzy soft UP_{i}-ideal of A.*

In the next theorem and corollary, we give necessary condition for fuzzy soft $\mathrm{UP_i}$-ideals of f-UP-semigroups.

Theorem 61. *If $(\widetilde{\mathrm{F}}, E)$ is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\widetilde{\mathrm{F}}[e]$ in A satisfies the conditions (2.8) and (1.15), then $(\widetilde{\mathrm{F}}, E)$ is a fuzzy soft UP_{i}-ideal of A.*

Proof. It is straightforward by Proposition 7 and Lemma 1 (2).

Corollary 4. *Let A be an f-UP-semigroup satisfying the condition* (2.10)*. If $(\widetilde{\mathrm{F}}, E)$ is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\widetilde{\mathrm{F}}[e]$ in A satisfies the conditions* (2.9) *and* (1.15)*, then $(\widetilde{\mathrm{F}}, E)$ is a fuzzy soft UP$_{\mathrm{i}}$-ideal of A.*

Proof. It is straightforward by Theorems 61 and 15.

From Figure 1, we have the following two theorems.

Theorem 62. *Every e-fuzzy soft UP$_{\mathrm{i}}$-ideal of A is an e-fuzzy soft UP$_{\mathrm{s}}$-ideal. Moreover, every fuzzy soft UP$_{\mathrm{i}}$-ideal of A is a fuzzy soft UP$_{\mathrm{s}}$-ideal.*

Theorem 63. *Every e-fuzzy soft UP$_{\mathrm{i}}$-ideal of A is an e-fuzzy soft UP$_{\mathrm{i}}$-filter. Moreover, every fuzzy soft UP$_{\mathrm{i}}$-ideal of A is a fuzzy soft UP$_{\mathrm{i}}$-filter.*

The following two examples show that the converse of Theorems 62 and 63 is not true.

Example 28. *In Example 13, we know that $(\widetilde{\mathrm{F}}, E)$ is a price-fuzzy soft UP$_{\mathrm{s}}$-ideal of A but $\widetilde{\mathrm{F}}[price]$ is not a fuzzy UP$_{\mathrm{i}}$-ideal of A. Indeed,*

$$\mathrm{f}_{\widetilde{\mathrm{F}}[price]}(5 * 6) = \mathrm{f}_{\widetilde{\mathrm{F}}[price]}(7) = 0.3 \not\geq 0.7 = \max\{0.1, 0.7\} = \max\{\mathrm{f}_{\widetilde{\mathrm{F}}[price]}(5), \mathrm{f}_{\widetilde{\mathrm{F}}[price]}(6)\}.$$

Hence, $(\widetilde{\mathrm{F}}, E)$ is not a price-fuzzy soft UP$_{\mathrm{i}}$-ideal of A.

Example 29. *In Example 26, we know that $(\widetilde{\mathrm{F}}, E)$ is a enjoyment-fuzzy soft UP$_{\mathrm{i}}$-filter of A but $\widetilde{\mathrm{F}}[enjoyment]$ is not a fuzzy UP$_{\mathrm{i}}$-ideal of A. Indeed,*

$$\begin{gathered}\mathrm{f}_{\widetilde{\mathrm{F}}[enjoyment]}(disco \cdot classic) = \mathrm{f}_{\widetilde{\mathrm{F}}[enjoyment]}(disco) = 0.2 \not\geq 0.5 = \min\{0.7, 0.5\} = \\ \min\{\mathrm{f}_{\widetilde{\mathrm{F}}[enjoyment]}(pop), \mathrm{f}_{\widetilde{\mathrm{F}}[enjoyment]}(rock)\} = \\ \min\{\mathrm{f}_{\widetilde{\mathrm{F}}[enjoyment]}(disco \cdot (rock \cdot classic)), \mathrm{f}_{\widetilde{\mathrm{F}}[enjoyment]}(rock)\}.\end{gathered}$$

Hence, $(\widetilde{\mathrm{F}}, E)$ is not a enjoyment-fuzzy soft UP$_{\mathrm{i}}$-ideal of A.

In the next theorem, we give necessary condition for fuzzy soft UP$_{\mathrm{i}}$-filters as fuzzy soft UP$_{\mathrm{i}}$-ideals of f-UP-semigroups.

Theorem 64. *If $(\widetilde{\mathrm{F}}, E)$ is a fuzzy soft UP$_{\mathrm{i}}$-filter of A such that for all $e \in E$, a fuzzy set $\widetilde{\mathrm{F}}[e]$ in A satisfies the condition* (2.11)*, then $(\widetilde{\mathrm{F}}, E)$ is a fuzzy soft UP$_{\mathrm{i}}$-ideal of A.*

Proof. It is straightforward by Theorem 17.

The proof of the following theorem can be verified easily.

Theorem 65. *If $(\widetilde{\mathrm{F}}, E)$ is a fuzzy soft UP$_{\mathrm{i}}$-ideal of A and $\emptyset \neq E^* \subseteq E$, then $(\widetilde{\mathrm{F}}|_{E^*}, E^*)$ is a fuzzy soft UP$_{\mathrm{i}}$-ideal of A.*

The following two theorems can be deduced in the same way as Theorems 22 and 23.

Theorem 66. *The extended intersection of two fuzzy soft* UP_{i}*-ideals of* A *is also a fuzzy soft* UP_{i}*-ideal. Moreover, the intersection of two fuzzy soft* UP_{i}*-ideals of* A *is also a fuzzy soft* UP_{i}*-ideal.*

Theorem 67. *The union of two fuzzy soft* UP_{i}*-ideals of* A *is also a fuzzy soft* UP_{i}*-ideal if sets of statistics of two fuzzy soft* UP_{i}*-ideals are disjoint.*

The following example shows that the converse of Theorem 67 is not true.

Example 30. *In Example 25, we have* $(\widetilde{\mathrm{G}}_1, E_1)$ *and* $(\widetilde{\mathrm{G}}_2, E_2)$ *are two fuzzy soft* UP_{i}*-ideals of* A*. Since endurance* $\in E_1 \cap E_2$*, we have*

$$(\mathrm{f}_{\widetilde{\mathrm{G}}_1[endurance] \cup \widetilde{\mathrm{G}}_2[endurance]})(black \cdot green) = (\mathrm{f}_{\widetilde{\mathrm{G}}_1[endurance] \cup \widetilde{\mathrm{G}}_2[endurance]})(green) = 0.5 \ngeq$$
$$0.6 = \min\{0.6, 0.7\} =$$
$$\min\{(\mathrm{f}_{\widetilde{\mathrm{G}}_1[endurance] \cup \widetilde{\mathrm{G}}_2[endurance]})(cyan), (\mathrm{f}_{\widetilde{\mathrm{G}}_1[endurance] \cup \widetilde{\mathrm{G}}_2[endurance]})(blue)\} =$$
$$\min\{(\mathrm{f}_{\widetilde{\mathrm{G}}_1[endurance] \cup \widetilde{\mathrm{G}}_2[endurance]})(black \cdot (blue \cdot green)), (\mathrm{f}_{\widetilde{\mathrm{G}}_1[endurance] \cup \widetilde{\mathrm{G}}_2[endurance]})(blue)\}.$$

Thus $\widetilde{\mathrm{G}}_1[endurance] \cup \widetilde{\mathrm{G}}_2[endurance]$ *is not a fuzzy* UP_{i}*-ideal of* A*, that is,* $(\widetilde{\mathrm{G}}_1, E_1) \cup (\widetilde{\mathrm{G}}_2, E_2)$ *is not a endurance-fuzzy soft* UP_{i}*-ideal of* A*. Hence,* $(\widetilde{\mathrm{G}}_1, E_1) \cup (\widetilde{\mathrm{G}}_2, E_2)$ *is not a fuzzy soft* UP_{i}*-ideal of* A*. Moreover,* $(\widetilde{\mathrm{G}}_1, E_1) \Cup (\widetilde{\mathrm{G}}_2, E_2)$ *is not a fuzzy soft* UP_{i}*-ideal of* A*.*

3.9. Fuzzy Soft Strongly $\mathrm{UP_s}$-Ideals

Definition 26. *A fuzzy soft set* $(\widetilde{\mathrm{F}}, E)$ *over* A *is called a fuzzy soft strongly* UP_{s}*-ideal based on* $e \in E$ *(we shortly call an* e*-fuzzy soft strongly* UP_{s}*-ideal) of* A *if a fuzzy set* $\widetilde{\mathrm{F}}[e]$ *in* A *is a fuzzy strongly* UP_{s}*-ideal of* A*. If* $(\widetilde{\mathrm{F}}, E)$ *is an* e*-fuzzy soft strongly* UP_{s}*-ideal of* A *for all* $e \in E$*, we say that* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft strongly* UP_{s}*-ideal of* A*.*

Definition 27. *A fuzzy soft set* $(\widetilde{\mathrm{F}}, E)$ *over* A *is called a constant fuzzy soft set based on* $e \in E$ *(we shortly call an* e*-constant fuzzy soft set) of* A *if a fuzzy set* $\widetilde{\mathrm{F}}[e]$ *in* A *is constant. If* $(\widetilde{\mathrm{F}}, E)$ *is an* e*-constant fuzzy soft set over* A *for all* $e \in E$*, we say that* $(\widetilde{\mathrm{F}}, E)$ *is a constant fuzzy soft set over* A*.*

From Figure 1, we have the following two theorem.

Theorem 68. *Every* e*-fuzzy soft strongly* UP_{s}*-ideal of* A *is an* e*-fuzzy soft* UP_{s}*-ideal. Moreover, every fuzzy soft strongly* UP_{s}*-ideal of* A *is a fuzzy soft* UP_{s}*-ideal.*

Theorem 69. e*-fuzzy soft strongly* UP_{s}*-ideals and* e*-constant fuzzy soft sets coincide in* A*. Moreover, fuzzy soft strongly* UP_{s}*-ideals and constant fuzzy soft sets coincide in* A*.*

In the next theorem, we give necessary condition for fuzzy soft strongly $\mathrm{UP_s}$-ideals of f-UP-semigroups.

Theorem 70. *If* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft set over* A *such that for all* $e \in E$*, a fuzzy set* $\widetilde{\mathrm{F}}[e]$ *in* A *satisfies the conditions* (2.12) *(or* (2.13) *or* (2.14)*) and* (1.14)*, then* $(\widetilde{\mathrm{F}}, E)$ *is a fuzzy soft strongly* UP_{s}*-ideal of* A*.*

Proof. It is straightforward by Propositions 9 (or 10 or 11) and Lemma 1 (1).

The following example shows that the converse of Theorem 68 is not true.

Example 31. *Let A be a set of four brands of a pick-up truck, that is,*

$A = \{$*Toyota Hilux*(TH), *Mitsubishi Triton*(MT), *Ford Ranger*(FR), *Isuzu D-Max*$(ID)\}$.

Define two binary operations $\cdot$ and $*$ on A as the following Cayley tables:

$\cdot$	MT	FR	ID	TH
MT	MT	FR	ID	TH
FR	MT	MT	ID	TH
ID	MT	FR	MT	TH
TH	MT	FR	ID	MT

$*$	MT	FR	ID	TH
MT	MT	MT	MT	MT
FR	MT	FR	MT	MT
ID	MT	MT	ID	MT
TH	MT	TH	MT	MT

Then $A = (A, \cdot, *, \textit{Mitsubishi Triton})$ is an f-UP-semigroup. Let $(\widetilde{\mathrm{F}}, E)$ be a fuzzy soft set over A where

$$E := \{\textit{displacement}, \textit{horse power}, \textit{torque}\}$$

with $\widetilde{\mathrm{F}}[\textit{displacement}], \widetilde{\mathrm{F}}[\textit{horse power}]$, and $\widetilde{\mathrm{F}}[\textit{torque}]$ are fuzzy sets in A defined as follows:

$\widetilde{\mathrm{F}}$	MT	FR	ID	TH
displacement	1	0.6	0.4	0.7
horse power	0.9	0.6	0.5	0.5
torque	0.9	0.7	0.6	0.5

Then $(\widetilde{\mathrm{F}}, E)$ is a torque-fuzzy soft UP$_{\mathrm{s}}$-ideal of A but $\widetilde{\mathrm{F}}[\textit{torque}]$ is not a fuzzy strongly UP$_{\mathrm{s}}$-ideal of A. Indeed,

$$\mathrm{f}_{\widetilde{\mathrm{F}}[\textit{torque}]}(ID) = 0.6 \not\geq 0.7 = \min\{0.9, 0.7\} = \min\{\mathrm{f}_{\widetilde{\mathrm{F}}[\textit{torque}]}(MT), \mathrm{f}_{\widetilde{\mathrm{F}}[\textit{torque}]}(FR)\} = \min\{\mathrm{f}_{\widetilde{\mathrm{F}}[\textit{torque}]}((ID \cdot FR) \cdot (ID \cdot ID)), \mathrm{f}_{\widetilde{\mathrm{F}}[\textit{torque}]}(FR)\}.$$

Hence, $(\widetilde{\mathrm{F}}, E)$ is not a torque-fuzzy soft strongly UP$_{\mathrm{s}}$-ideal of A.

The proof of the following theorem can be verified easily.

Theorem 71. *If $(\widetilde{\mathrm{F}}, E)$ is a fuzzy soft strongly UP$_{\mathrm{s}}$-ideal of A and $\emptyset \neq E^* \subseteq E$, then $(\widetilde{\mathrm{F}}|_{E^*}, E^*)$ is a fuzzy soft strongly UP$_{\mathrm{s}}$-ideal of A.*

By using Theorem 6, we can obtain the following two theorems in the same way as Theorems 22 and 23.

Theorem 72. *The extended intersection of two fuzzy soft strongly UP$_{\mathrm{s}}$-ideals of A is also a fuzzy soft strongly UP$_{\mathrm{s}}$-ideal. Moreover, the intersection of two fuzzy soft strongly UP$_{\mathrm{s}}$-ideals of A is also a fuzzy soft strongly UP$_{\mathrm{s}}$-ideal.*

Theorem 73. *The union of two fuzzy soft strongly UP$_{\mathrm{s}}$-ideals is also a fuzzy soft strongly UP$_{\mathrm{s}}$-ideal. Moreover, the restricted union of two fuzzy soft strongly UP$_{\mathrm{s}}$-ideals of A is also a fuzzy soft strongly UP$_{\mathrm{s}}$-ideal.*

3.10. Fuzzy Soft Strongly $\mathrm{UP_i}$-Ideals

Definition 28. *A fuzzy soft set $(\widetilde{\mathrm{F}}, E)$ over A is called a fuzzy soft strongly UP_{i}-ideal based on $e \in E$ (we shortly call an e-fuzzy soft strongly UP_{i}-ideal) of A if a fuzzy set $\widetilde{\mathrm{F}}[e]$ in A is a fuzzy strongly UP_{i}-ideal of A. If $(\widetilde{\mathrm{F}}, E)$ is an e-fuzzy soft strongly UP_{i}-ideal of A for all $e \in E$, we say that $(\widetilde{\mathrm{F}}, E)$ is a fuzzy soft strongly UP_{i}-ideal of A.*

From Figure 1, we have the following two theorem.

Theorem 74. *Every e-fuzzy soft strongly UP_{i}-ideal of A is an e-fuzzy soft UP_{i}-ideal. Moreover, every fuzzy soft strongly UP_{i}-ideal of A is a fuzzy soft UP_{i}-ideal.*

Theorem 75. *e-fuzzy soft strongly UP_{i}-ideals and e-constant fuzzy soft sets coincide in A. Moreover, fuzzy soft strongly UP_{i}-ideals and constant fuzzy soft sets coincide in A.*

Corollary 5. *e-fuzzy soft strongly UP_{s}-ideals, e-fuzzy soft strongly UP_{i}-ideals, and e-constant fuzzy soft sets coincide in A. Moreover, fuzzy soft strongly UP_{s}-ideals, fuzzy soft strongly UP_{i}-ideals and constant fuzzy soft sets coincide in A.*

Proof. It is straightforward by Theorems 69 and 75.

In the next theorem, we give necessary condition for fuzzy soft strongly $\mathrm{UP_i}$-ideals of f-UP-semigroups.

Theorem 76. *If $(\widetilde{\mathrm{F}}, E)$ is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\widetilde{\mathrm{F}}[e]$ in A satisfies the conditions* (2.12) (*or* (2.13) *or* (2.14)) *and* (1.15)*, then $(\widetilde{\mathrm{F}}, E)$ is a fuzzy soft strongly UP_{i}-ideal of A.*

Proof. It is straightforward by Proposition 9 (or 10 or 11) and Lemma 1 (2).

The following example shows that the converse of Theorem 74 is not true.

Example 32. *In Example 31, we know that $(\widetilde{\mathrm{F}}, E)$ is a displacement-fuzzy soft UP_{i}-ideal of A but $\widetilde{\mathrm{F}}[displacement]$ is not a fuzzy strongly UP_{i}-ideal of A. Indeed,*

$$\begin{gathered}\mathrm{f}_{\widetilde{\mathrm{F}}[displacement]}(ID) = 0.4 \ngeq 0.6 = \min\{1, 0.6\} = \\ \min\{\mathrm{f}_{\widetilde{\mathrm{F}}[displacement]}(MT), \mathrm{f}_{\widetilde{\mathrm{F}}[displacement]}(FR)\} = \\ \min\{\mathrm{f}_{\widetilde{\mathrm{F}}[displacement]}((ID \cdot FR) \cdot (ID \cdot ID)), \mathrm{f}_{\widetilde{\mathrm{F}}[displacement]}(FR)\}.\end{gathered}$$

Hence, $(\widetilde{\mathrm{F}}, E)$ is not a displacement-fuzzy soft strongly UP_{i}-ideal of A.

The proof of the following theorem can be verified easily.

Theorem 77. *If $(\widetilde{\mathrm{F}}, E)$ is a fuzzy soft strongly UP_{i}-ideal of A and $\emptyset \neq E^* \subseteq E$, then $(\widetilde{\mathrm{F}}|_{E^*}, E^*)$ is a fuzzy soft strongly UP_{i}-ideal of A.*

By using Theorem 6, we can obtain the following two theorems in the same way as Theorems 22 and 23.

Theorem 78. *The extended intersection of two fuzzy soft strongly* UP_{i}*-ideals of* A *is also a fuzzy soft strongly* UP_{i}*-ideal. Moreover, the intersection of two fuzzy soft strongly* UP_{i}*-ideals of* A *is also a fuzzy soft strongly* UP_{i}*-ideal.*

Theorem 79. *The union of two fuzzy soft strongly* UP_{i}*-ideals of* A *is also a fuzzy soft strongly* UP_{s}*-ideal. Moreover, the restricted union of two fuzzy soft strongly* UP_{i}*-ideals of* A *is also a fuzzy soft strongly* UP_{i}*-ideal.*

4. Conclusions

In this paper, we have introduced the notions of fuzzy soft $\mathrm{UP_s}$-subalgebras, fuzzy soft $\mathrm{UP_i}$-subalgebras, fuzzy soft near $\mathrm{UP_s}$-filters, fuzzy soft near $\mathrm{UP_i}$-filters, fuzzy soft $\mathrm{UP_s}$-filters, fuzzy soft $\mathrm{UP_i}$-filters, fuzzy soft $\mathrm{UP_s}$-ideals, fuzzy soft $\mathrm{UP_i}$-ideals, fuzzy soft strongly $\mathrm{UP_s}$-ideals, and fuzzy soft strongly $\mathrm{UP_i}$-ideals of fully UP-semigroups and the conditions for fuzzy soft sets over fully UP-semigroups, proved its generalizations and investigated some of its important properties. Then, we get the diagram of generalization of fuzzy soft sets over fully UP-semigroups as shown in Figure 3.

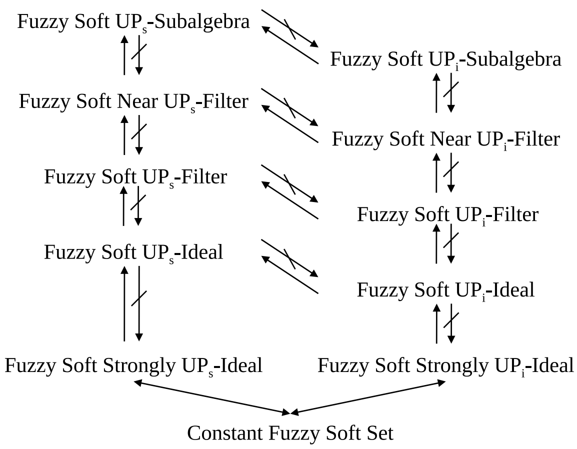

Figure 3: Fuzzy soft sets over fully UP-semigroups

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