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Some Results on Fuzzy Implicative Hyper GR-ideals

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Abstract. The implicative hyper GR-ideals, the fuzzy implicative hyper GR-ideals of type 1 and the fuzzy implicative hyper GR-ideals of type 2 are introduced, and several properties are investigated. Characterizations of fuzzy implicative hyper GR-ideals of type 1 are established using level subsets of fuzzy sets.

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1. Introduction

Hyperstructure theory was introduced in 1934 by F. Marty [11] at the $8th$ Congress of Scandinavian Mathematics. It is studied from the theoritical point of view and for their applications to many areas of pure and applied mathematics. Y.B. Jun et al. applied this concept to BCK-algebras [8] and X.X. Long introduced hyper BCI-algebras [10] as a generalization of BCI-algebras. Different types of hyper BCI-ideals are also defined in [10]. After the introduction of the concept of hyper BCI-algebras, several researches were conducted. Among these studies are fuzzy hyper BCK-ideals of hyper BCK-algebras [6], fuzzy ideals in hyper BCI-algebras [13], fuzzy implicative hyper BCK-ideals of hyper BCK-algebras [7], bi-polar-valued fuzzy hyper subalgebras of a hyper BCI-algebra [12], intuitionistic fuzzy hyper BCK-ideals of hyper BCK-algebras [2], and intuitionistic fuzzy ideals in hyper BCI-algebras [14] where fuzzy sets are applied to hyper BCK-algebras and hyper BCI-algebras. By following these hyperstructures, Indangan et al. introduced hyper GR-algebras [4]. They established some results on hyper GR-ideals and hyper homomorphic properties on hyper GR-algebras [5]. Fuzzy set was introduced by L.A. Zadeh [17]. This concept of fuzzy sets are extremely useful for many people involved in research and

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development including engineers, mathematicians, computer software developers and researchers, natural scientists, medical researchers, social scientists, public policy analysts, business analysts, and jurists [15]. With this, several researches investigated on the generalization of the notion of fuzzy sets. Other studies where fuzzy sets are applied are hyper K-subalgebras based on fuzzy points [9] and on fuzzy hyper B-ideals of hyper B-algebras [16].

In this paper, we introduce an implicative hyper GR-ideal and apply fuzzy sets on this notion. The definition of implicative hyper GR-ideal is somewhat alike with the definition of weak implicative hyper BCK-ideal [1]. However, some examples in section 3 are not hyper BCI-algebras. These examples show that implicative hyper GR-ideals and weak implicative hyper BCK-ideals are not equivalent. Other than implicative hyper GR-ideals, two types of fuzzy implicative hyper GR-ideals are introduced and investigated. Using the notion of level subsets of a fuzzy set, we give a characterization of a fuzzy implicative hyper GR-ideal of type 1.

2. Preliminaries

Let $P(H)$ be the power set of H. Consider $P^*(H) = P(H) \setminus \{\phi\}$. A *hyperoperation* on a nonempty set H is a function $\circledast : H \times H \to P^*(H)$. The image of $(x, y) \in H \times H$ under $\circledast$ is denoted by $x \circledast y$. If $x \in H$ and A, B are nonempty subsets of H, then we define $A \circledast B = \bigcup_{a \in A, b \in B} a \circledast b$; $A \circledast x = A \circledast \{x\}$; and $x \circledast B = \{x\} \circledast B$. Moreover, $x \ll y$ if and only if $0 \in x \circledast y$; and $A \ll B$ if and only if for any $a \in A$, there exists $b \in B$ such that $a \ll b$. We call " $\ll$ " a *hyperorder* on H.

Definition 2.1. [4] Let H be a nonempty set and $\circledast$ be a hyperoperation on H. Then $(H; \circledast, 0)$ is called a *hyper GR-algebra* if it contains a constant $0 \in H$ and it satisfies the following conditions, for all $x, y, z \in H$:

(HGR1) $(x \circledast z) \circledast (y \circledast z) \ll x \circledast y$;

(HGR2) $(x \circledast y) \circledast z = (x \circledast z) \circledast y$;

(HGR3) $x \ll x$;

(HGR4) $0 \circledast (0 \circledast x) \ll x, x \neq 0$; and

(HGR5) $(x \circledast y) \circledast z \ll y \circledast z$.

For the sake of simplicity, we also call H a hyper GR-algebra.

Example 2.2. Consider a set $H = \{0, 1, 2, 3\}$ with the Cayley table below.

$\circledast$	0	1	2	3
0	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$
2	$\{0,2\}$	$\{0,2\}$	$\{0,1,2\}$	$\{0,1,2\}$
3	$\{0,3\}$	$\{0,3\}$	$\{0,3\}$	$\{0,1,3\}$

It can be verified that H is a hyper GR-algebra.

Definition 2.3. [4] Let H be a hyper GR-algebra and S be a subset of H containing 0. If S is a hyper GR-algebra with respect to the hyperoperation $\circledast$ on H, then we say that S is a *hyper subGR-algebra* on H.

Theorem 2.4. [4](Hyper SubGR-algebra Criterion) Let H be a hyper GR-algebra and S be a non-empty subset of H. Then S is a hyper subGR-algebra of H if and only if $(x \circledast y) \subseteq S$ for all $x, y \in S$.

Definition 2.5. [4] Let I be a subset of a hyper GR-algebra H. Then I is said to be a *hyper GR-ideal* of H if

i) $0 \in I$; and

ii) for all $x, y \in H$, $x \circledast y \subseteq I$ and $y \in I$ imply that $x \in I$.

Definition 2.6. [8] By a *hyper BCK-algebra* we mean a non-empty set H endowed with a hyperoperation "$\circ$" and a constant 0 satisfying the following axioms:

(HK1) $(x \circ z) \circledast (y \circ z) \ll x \circ y$,

(HK2) $(x \circ y) \circ z = (x \circ z) \circ y$,

(HK3) $x \circ H \ll \{x\}$,

(HK4) $x \ll y$ and $y \ll x$ imply $x = y$,

for all $x, y, z \in H$ where $x \ll y$ is defined by $0 \in x \circ y$ and for every $A, B \subseteq H$, $A \ll B$ is defined by for all $a \in A$, there exist $b \in B$ such that $a \ll b$.

Definition 2.7. [1] Let I be a non-empty subset of H and $0 \in H$. Then I is called a weak implicative hyper BCK-ideal of H if, $(x \circ z) \circ (y \circ x) \subset I$ and $z \in I$ imply $x \in I$, for all $x, y, z \in H$.

Definition 2.8. [17] Let M be a nonempty set. A *fuzzy set* μ in M is a function $\mu : M \to [0, 1]$.

Definition 2.9. [3] Let μ be a fuzzy set in M. For a fixed $t \in [0, 1]$, the set $\mu_t = \{x \in M | \mu(x) \geq t\}$ is a subset of M, called a *level subset* of μ.

3. Implicative Hyper GR-Ideals

Definition 3.1. A nonempty subset I of a hyper GR-algebra H is called an *implicative hyper GR-ideal* of H if for any $x, y, z \in H$

(IH1) $0 \in I$, and

(IH2) $(x \circledast z) \circledast (y \circledast x) \subseteq I$ and $z \in I$ imply $x \in I$.

Example 3.2. Consider a set $H = \{0, 1, 2, 3\}$ with the Cayley table below.

$\circledast$	0	1	2	3
0	$\{0, 1, 3\}$	$\{0, 1, 3\}$	$\{0, 1, 3\}$	$\{0, 1, 3\}$
1	$\{0, 1, 3\}$	$\{0, 1, 3\}$	$\{0, 1, 3\}$	$\{0, 1, 3\}$
2	$\{0, 1, 3\}$	$\{0, 1\}$	$\{0, 1\}$	$\{3\}$
3	$\{0, 1, 3\}$	$\{0, 1, 3\}$	$\{0, 1, 3\}$	$\{0\}$

It can be shown that H is a hyper GR-algebra. By routine calculations, we can see that the following are true:

(i) $H, \{0\}, \{0, 1\}, \{0, 2\}, \{0, 3\}, \{0, 2, 3\}$, and $\{0, 1, 2\}$ are the only implicative hyper GR-ideals of H; and

(ii) $H, \{0\}, \{0, 1, 2\}$, and $\{0, 2\}$ are the only hyper GR-ideals of H.

Moreover, since $2 \circledast 1 = \{0, 1\}$ and $1 \in \{0, 1\}$ but $2 \notin \{0, 1\}$, $\{0, 1\}$ is not a hyper GR-ideal of H. Furthermore, note that $(2 \circledast 0) \circledast (0 \circledast 2) = \{0, 1, 3\}$ and $0 \in \{0, 1, 3\}$ but $2 \notin \{0, 1, 3\}$ and so $\{0, 1, 3\}$ is not an implicative hyper GR-ideal of H.

Remark 3.3. Example 3.2 shows that not all implicative hyper GR-ideals are hyper GR-ideals. Moreover, H in Example 3.2 is not a hyper BCI-algebra since $2 \ll 1$ and $1 \ll 2$ but $2 \neq 1$. This shows that implicative hyper GR-ideals and weak implicative hyper BCK-ideals are not equivalent.

Example 3.4. Consider a set $H = \{0, 1, 2\}$ with the Cayley table below.

$\circledast$	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{0\}$	$\{0\}$	$\{1\}$
2	$\{0, 2\}$	$\{2\}$	$\{0\}$

It can be verified that H is a hyper GR-algebra. Since $(1 \circledast 0) \circledast (0 \circledast 1) = \{0\}$ and $0 \in \{0\}$ but $1 \neq \{0\}$, $\{0\}$ is not an implicative hyper GR-ideal of H. Furthermore, note that $1 \circledast 0 = \{0\}$ and $0 \in \{0\}$ but $1 \notin \{0\}$. This implies that $\{0\}$ is not a hyper GR-ideal of H.

Remark 3.5. Example 3.4 shows that $\{0\}$ is not always an implicative hyper GR-ideal and is not always a hyper GR-ideal of a hyper GR-algebra H.

In Example 3.4, we can see that $1 \notin 1 \circledast 0$ and not all subset I of H containing 0 is an implicative hyper GR-ideal of H.

Example 3.6. Consider a set $H = \{0, 1, 2\}$ with the Cayley table below.

$\circledast$	0	1	2
0	$\{0, 1, 2\}$	$\{0, 1, 2\}$	$\{0, 1, 2\}$
1	$\{1\}$	$\{0, 1, 2\}$	$\{1, 2\}$
2	$\{1, 2\}$	$\{0, 1, 2\}$	$\{0, 1, 2\}$

It can be verified that H is a hyper GR-algebra. Clearly, $x \in x \circledast y$ for all $x, y \in H$. By routine calculations, we can show that any subset of I of H containing 0 is an implicative hyper GR-ideal of H.

The next proposition will give a generalization when $x \in x \circledast y$ for all x and y in a hyper GR-algebra H.

Proposition 3.7. Let H be a hyper GR-algebra such that $x \in x \circledast y$ for any $x, y \in H$. Then any subset I of H containing 0 is an implicative hyper GR-ideal of H.

Proof. Let $I \subseteq H$ and $0 \in I$. Let $x, y, z \in H$ such that $(x \circledast z) \circledast (y \circledast x) \subseteq I$ and $z \in I$. By hypothesis, $x \in x \circledast z$. Then $x \circledast w \subseteq I$ for any $w \in y \circledast x$. It follows from the hypothesis that $x \in x \circledast w \subseteq I$. Hence, I is an implicative hyper GR-ideal of H.

Lemma 3.8. If I is a hyper GR-ideal of a hyper GR-algebra H, then $A \circledast B \subseteq I$ and $B \subseteq I$ imply $A \subseteq I$.

Proof. Suppose $A \circledast B \subseteq I$ and $B \subseteq I$. Let $a \in A$. Then $a \circledast b \subseteq I$ for any $b \in B$. Since I is a hyper GR-ideal of H, $a \in I$. Hence, $A \subseteq I$.

Example 3.9. Consider a set $H = \{0, 1, 2\}$ with the Cayley table below.

$\circledast$	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{0,1\}$	$\{0,2\}$	$\{0\}$
2	$\{0,2\}$	$\{2\}$	$\{0\}$

It can be verified that H is a hyper GR-algebra and $\{0\}$ is a hyper GR-ideal of H. Note that $1 \circledast (2 \circledast 1) = \{0\}$ and $1 \notin \{0\}$. Since $(1 \circledast 0) \circledast (2 \circledast 1) = \{0\}$ and $0 \in \{0\}$ but $1 \notin \{0\}$, $\{0\}$ is not an implicative hyper GR-ideal of H.

Remark 3.10. Example 3.9 shows that not all hyper GR-ideals are implicative hyper GR-ideals.

Example 3.11. Consider the hyper GR-algebra H in Example 2.2. By routine calculations, it can be shown that the following are true:

- $\{0, 1, 2\}$ is a hyper GR-ideal of H.
- For any $x, y, \in H$ such that $x \circledast (y \circledast x) \subseteq \{0, 1, 2\}$ implies that $x \in \{0, 1, 2\}$.

Moreover, $\{0, 1, 2\}$ can be verified to be an implicative hyper GR-ideal of H.

The preceding example is generalized in the following proposition.

Proposition 3.12. Let I be a hyper GR-ideal of H. If $x \circledast (y \circledast x) \subseteq I$ implies $x \in I$, then I is implicative hyper GR-ideal of H.

Proof. Let $x, y, z \in H$ such that $(x \circledast z) \circledast (y \circledast x) \subseteq I$ and $z \in I$. By HGR2, $[x \circledast (y \circledast x)] \circledast z = (x \circledast z) \circledast (y \circledast x) \subseteq I$. By Lemma 3.8, $x \circledast (y \circledast x) \subseteq I$. Thus, $x \in I$ and so I is implicative hyper GR-ideal of H.

The converse of Proposition 3.12 does not hold in general as shown in the following example.

Example 3.13. Consider the hyper GR-algebra H in Example 3.2. $I = \{0, 1, 2\}$ can be verified to be both hyper GR-ideal and implicative hyper GR-ideal of H. But $3 \circledast (2 \circledast 3) = \{0\} \subseteq \{0, 1, 2\}$ and $3 \notin \{0, 1, 2\} = I$.

Proposition 3.14. Let I be a hyper subGR-algebra of a hyper GR-algebra H. If I is an implicative hyper GR-ideal, then $x \circledast (y \circledast x) \subseteq I$ implies $x \in I$.

Proof. Let $x, y, z \in H$ such that $x \circledast (y \circledast x) \subseteq I$ and let $z \in I$. It follows from Theorem 2.4, $[x \circledast (y \circledast x)] \circledast z \in I$. By HGR2, $(x \circledast z) \circledast (y \circledast x) = [x \circledast (y \circledast x)] \subseteq I$. Since I is an implicative hyper GR-ideals, $x \in I$.

Proposition 3.15. Let A, B and C be subsets of a hyper GR-algebra H. If I is implicative hyper GR-ideal of H, then $(A \circledast C) \circledast (B \circledast A) \subseteq I$ and $C \subseteq I$ imply $A \subseteq I$.

Proof. Suppose $(A \circledast C) \circledast (B \circledast A) \subseteq I$ and $C \subseteq I$. Let $a \in A$. Then, $(a \circledast c) \circledast (b \circledast a) \subseteq I$ for $c \in C \subseteq I$ and $b \in B$. Since I is implicative hyper GR-ideal of H, $x \in I$. Hence, $A \subseteq I$.

4. Fuzzy Implicative Hyper GR-ideals

Definition 4.1. A fuzzy set μ in a hyper GR-algebra H is called a *fuzzy implicative hyper GR-ideal of type 1* in H if

(FIM1) $\mu(0) \geq \mu(x) \geq min\left\{\inf_{u \in (x \circledast z) \circledast (y \circledast x)} \mu(u), \mu(z)\right\}$ for $x, y, z \in H$.

Example 4.2. Consider the hyper GR-algebra $H = [0, 1]$ such that for any $x, y \in [0, 1]$,

$$x \circledast y = \begin{cases} [0, 0.3], & \text{if } y \neq 0 \text{ or } x = 0 = y, \\ \{x\}, & \text{if } x \neq 0 \text{ and } y = 0. \end{cases}$$

Define a fuzzy set μ in H by

$$\mu(x) = \begin{cases} 1, & \text{if } x = 0, \\ 0.3 + x, & \text{if } x \in (0, 0.3], \\ 0.7, & \text{if } x \in (0.3, 1]. \end{cases}$$

It can be shown that μ is a fuzzy implicative hyper GR-ideal of type 1 in H.

Definition 4.3. A fuzzy set μ in H is called a *fuzzy implicative hyper GR-ideal of type 2* in H if FIM1 holds and

(FIM2) for any $x, y, z \in H$ such that $x \ll y$, we have $\mu(y) \leq \mu(x)$.

Example 4.4. Consider the hyper GR-algebra $H = [0,1]$ in Example 4.2. Note that from FIM2 $0 \neq x \ll y \neq 0$ and $0 \neq y \ll x \neq 0$ for $x, y \in H$ such that $x \neq y$ should imply to $\mu(x) = \mu(y)$. On the contrary, the μ in Example 4.2 does not give $\mu(x) = \mu(y)$ whenever $x \neq y$ for $x, y \in H$. Hence, μ is not a fuzzy implicative hyper GR-ideal of type 2. However, redefining fuzzy set μ in H by

$$\mu(x) = \begin{cases} n, & \text{if } x = 0, \\ m, & \text{if } x \neq 0, \end{cases}$$

where $m, n \in [0,1]$ and $m < n$ will provide a fuzzy implicative hyper GR-ideal of type 2 μ in H . It can be seen that this is true by routine calculation.

Example 4.5. Consider the hyper GR-algebra H in Example 3.6. Define a fuzzy set μ in H by $\mu(1) = 0.1, \mu(2) = 0.2$ and $\mu(0) = 0.3$. By routine calculations, we see that μ is a fuzzy implicative hyper GR-ideal of type 2 in H.

Remark 4.6. From Definitions 4.1 and 4.3, it shows that every fuzzy implicative hyper GR-ideal of type 2 is a fuzzy implicative hyper GR-ideal of type 1. Example 4.4 shows that not all fuzzy implicative hyper GR-ideal of type 1 is a fuzzy implicative hyper GR-ideal of type 2.

Theorem 4.7. A fuzzy set μ in a hyper GR-algebra H is a fuzzy implicative hyper GR-ideal of type 1 if and only if μ_t is an implicative hyper GR-ideal of H whenever $\mu_t \neq \phi$ and $t \in [0,1]$.

Proof. Suppose μ is a fuzzy implicative hyper GR-ideal of type 1 and $\mu_t \neq \phi$. Let $t \in [0,1]$. Since $\mu_t \neq \phi$, there exist $x \in \mu_t$. By FIM1, $\mu(0) \geq \mu(x) \geq t$. It follows that $0 \in \mu_t$. Let $x, y, z \in H$ such that $(x \circledast z) \circledast (y \circledast x) \subseteq \mu_t$ and $z \in \mu_t$. Then $\mu(z) \geq t$ and $\mu(u) \geq t$ for any $u \in (x \circledast z) \circledast (y \circledast x)$. This implies that t is a lowerbound for the set $\{\mu(u) | u \in (x \circledast z) \circledast (y \circledast x)\}$. Then, $\inf\limits_{u \in (x \circledast z) \circledast (y \circledast x)} \mu(u) \geq t$. By FIM1,

$$\mu(x) \geq min \left\{ \inf_{u \in (x \circledast z) \circledast (y \circledast x)} \mu(u), \mu(z) \right\} \geq min\{t, t\} = t.$$

Hence, $x \in \mu_t$ and so μ_t is implicative hyper GR-ideal of H. Conversely, suppose μ_t is an implicative hyper GR-ideal of H. Let $x \in H$ and let $t \in [0,1]$ such that $t = \mu(x)$. Since $0 \in \mu_t$, $\mu(0) \geq t = \mu(x)$. Moreover, let $x, y, z \in H$ and let $t \in [0,1]$ such that $t = min \left\{ \inf\limits_{u \in (x \circledast z) \circledast (y \circledast x)} \mu(u), \mu(z) \right\}$. Since $\mu(z) \geq min \left\{ \inf\limits_{u \in (x \circledast z) \circledast (y \circledast x)} \mu(u), \mu(z) \right\} = t$, $z \in \mu_t$. Let $v \in (x \circledast z) \circledast (y \circledast x)$. Then $\mu(v) \geq \inf\limits_{u \in (x \circledast z) \circledast (y \circledast x)} \mu(u) \geq min \left\{ \inf\limits_{u \in (x \circledast z) \circledast (y \circledast x)} \mu(u), \mu(z) \right\} = t$. Then $v \in \mu_t$ and hence $(x \circledast z) \circledast (y \circledast x) \subset \mu_t$. Since μ_t is implicative hyper GR-ideal, $x \in \mu_t$. Thus $\mu(x) \geq t = min \left\{ \inf\limits_{u \in (x \circledast z) \circledast (y \circledast x)} \mu(u), \mu(z) \right\}$.

Theorem 4.8. For any nonempty subset I of H, let μ_t be a fuzzy set in H defined by

$$\mu_I(x) = \begin{cases} k, & \text{if } x \in I \\ l, & \text{otherwise,} \end{cases}$$

for all $x \in H$, where $k, l \in [0,1]$ with $k > l$. Then I is an implicative hyper GR-ideal of H if and only if μ_I is a fuzzy implicative hyper GR-ideal of type 1 in H.

Proof. Note that

$$(\mu_I)_t = \begin{cases} \phi, & \text{if } k < t \leq 1, \\ I, & \text{if } l < t \leq k, \\ H, & \text{if } 0 \leq t \leq l. \end{cases} \qquad (1)$$

Assume that I is an implicative hyper GR-ideal of H. It follows from (1) that a nonempty $(\mu_I)_t$ is an implicative hyper GR-ideal of H for all $t \in [0,1]$. By Theorem 4.7, μ_I is a fuzzy implicative hyper GR-ideal of type 1 in H. Conversely, suppose that μ_I is a fuzzy implicative hyper GR-ideal of type 1 in H. By Theorem 4.7, we can see in (1) that a nonempty $(\mu_I)_t$ is an implicative hyper GR-ideal for all $t \in [0,1]$ and so I is an implicative hyper GR-ideal of H.

Theorem 4.9. If a fuzzy set μ in a hyper GR-algebra H is a fuzzy implicative hyper GR-ideal of type 2, then μ_t is an implicative hyper GR-ideal of H whenever $\mu_t \neq \phi$ and $t \in [0,1]$.

Proof. Suppose μ is a fuzzy implicative hyper GR-ideal of type 2 and $\mu_t \neq \phi$. Let $t \in [0,1]$. Since $\mu_t \neq \phi$, there exist $x \in \mu_t$. By FIM1, $\mu(0) \geq \mu(x) \geq t$. It follows that $0 \in \mu_t$. Let $x, y, z \in H$ such that $(x \circledast z) \circledast (y \circledast x) \subseteq \mu_t$ and $z \in \mu_t$. Then for any $u \in (x \circledast z) \circledast (y \circledast x)$ there exist $a \in \mu_t$ such that $u \ll a$. By FIM2, $\mu(u) \geq \mu(a) \geq t$. It follows from FIM1 that

$$\mu(x) \geq min\left\{ \inf_{u \in (x \circledast z) \circledast (y \circledast x)} \mu(u), \mu(z) \right\} \geq min\{t, t\} = t.$$

Hence, $x \in \mu_t$ and so μ_t is implicative hyper GR-ideal of H.

Theorem 4.10. If μ is a fuzzy implicative hyper GR-ideal of type 1 (type 2) in H, then the set

$$I = \{x \in H | \mu(x) = \mu(0)\}$$

is an implicative hyper GR-ideal of H.

Proof. Let $x, y, z \in H$ such that $(x \circledast z) \circledast (y \circledast x) \subset I$ and $z \in I$. Then $\mu(z) = \mu(0)$ and $\mu(u) = \mu(0)$ for each $u \in (x \circledast z) \circledast (y \circledast z)$. By $FIM2$, $\mu(0) \geq \mu(x) \geq min\left\{ \inf_{u \in (x \circledast z) \circledast (y \circledast x)} \mu(u), \mu(z) \right\} = \mu(0)$. Then, $\mu(x) = \mu(0)$ and so $x \in I$. Thus, I is fuzzy implicative hyper GR-ideal of type 1 (type 2) in H.

References

[1] R Borzooei and M Bakhsi. (Weak) Implicative Hyper BCK-ideals. *Quasigroups and Related Systems*, 12:13–28, 2004.

[2] R Borzooei and Y Jun. Intuitionistic Fuzzy Hyper BCK-ideals of Hyper BCK-algebras. *Iranian Journal of Fuzzy Systems*, 1(1):61–73, 2004.

[3] P Das. Fuzzy Groups and Level Subgroups. *J. Math. Anal. Appl.*, 67:549–564, 1979.

[4] R Indangan and G Petalcorin. Some Results on Hyper GR-ideals of a Hyper GR-algebra. *Journal of Algebra and Applied Mathematics*, 14:101–119, 2016.

[5] R Indangan, G Petalcorin, and A Villa. Some Hyper Homomorphic Properties on Hyper GR-algebras. *Journal of Algebra and Applied Mathematics*, 15:100–121, 2017.

[6] Y Jun and X Long. Fuzzy Hyper BCK-ideals of Hyper BCK-algebras. *Scientiae Mathematics Japonicae Online*, 4:415–422, 2001.

[7] Y Jun and W Shim. Fuzzy Implicative Hyper BCK-ideals of Hyper BCK-algebras. *International Journal of Mathematics and Mathematical Sciences*, 29(2):63–70, 2002.

[8] Y Jun, M Zahedi, X Xin, and R Borzoei. On Hyper BCK-algebras. *Italian J. Pure Appl. Math.*, 8:127–136, 2000.

[9] M Kang. Hyper K-subalgebras Based on Fuzzy Points. *Communications of the Korean Mathematical Society*, 26(3):385–403, 2011.

[10] X Long. Hyper BCI-algebras. *Discuss Math. Soc.*, 26:5–19, 2006.

[11] F Marty. Sur une generalization de la notion de group. *8th Congress Math. Scandenaves (Stockholm)*, pages 45–49, 1934.

[12] F Nisar, R Tariq, and S Bhatti. Bi-polar-valued Fuzzy Hyper Subalgebras of a Hyper BCI-algebra. *World Applied Sciences Journal*, 9(1):25–33, 2010.

[13] F Nisar, R Tariq, and S Bhatti. Fuzzy Ideals in Hyper BCI-algebras. *World Applied Sciences Journal*, 16(12):1771–1777, 2012.

[14] N Palaniappan, P Veerappan, and R Devi. Intuitionistic Fuzzy Ideals in Hyper BCI-algebras. *International Journal of Computational Science and Mathematics*, 4(3):271–285, 2012.

[15] H Singh, M Gupta, T Meitzler, Z Hou, K Garg, A Solo, and L Zadeh. Real-life Applications of Fuzzy Logic. *Hindawi Publishing Corporation Advances in Fuzzy Systems Volume*, 2013:271–285, 2013.

[16] G Tabaranza and J Vilela. On Fuzzy Hyper B-ideals of Hyper B-algebras. *International Journal of Algebra*, 12(5):197–209, 2018.

[17] L Zadeh. Fuzzy Sets. *Information and Control*, 8:338–353, 1965.