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Voting method based on an average gap assessment

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Abstract. The theory of social choice is the study of voting methods. In the literature many studies have been conducted for the development of a fair voting system, that is to say a voting method that allows to aggregate the individual preferences in a collective preference representing in the most possible faithfull way individual preferences. Yet some voting methods do not allow to obtain a consensus. So there are a lot of paradoxes in electoral systems and related results in the theory of social choice are also paradoxical. This is the case, for example, with Arrow's theorem showing that no voting method can simultaneously verify a restricted list of properties that are desirable in a democratic political system. That is to mean that the search for a system that makes it possible to reach a consensus remains a concern in the theory of social choice. In this article we have combined various voting methods based on grading, scoring or approving to contribute to literature with a new voting system filling fair properties.

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1. Introduction

Votes rank prominently in all countries. Indeed to elect a president of the republic, deputies and mayors are proceeded by votes. According to [9] in many countries (communities, groups, committees), the translation of the democratic ideal takes place by resorting to one version or another of a "majority" method, and that this voting system can lead to surprising results. In the literature, this system is challenged by many theorists of the theory of social choice. This is the case for example in [3] where Michel Balinski and Rida

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Laraki presented some setbacks of the majority vote.

Similarly, according to [9], the one or two-round majority voting system contains a large number of known and proven defects. Like the majority vote, many other voting methods abound in the literature, but many of them have shortcomings, and therefore do not allow to determine the real will of the voters.

Indeed in [14], it is shown that all systems of voting by classification, rating voting systems also generate defects. Sometimes the results of votes lead to huge problems such as popular uprisings, wars and so on. In [14], the way of voting and the current voting system in France are extremely simple but have many flaws that can pervert the expression of the popular will.

According to [13] the electoral procedures are very often formalized and studied in an ordinal context where every voter is supposed to be able to classify in order of preference, all the options subject to the collective decision, and that this method is dominant in social choice theory. Long dominated by the votes by classification, one of the major concerns turns to the votes by notes. This is the case in [13] where authors question classification voting systems and propose as an alternative aggregation methods based on the evaluation principle.

Similarly in [5], some authors reject the ordinal approach of individual preferences. Efforts seem to be focusing more and more on searching for a voting system whose result of aggregation operated reflects at best the preferences of the electors. The search for a voting method that best reflects individual preferences is timely.

According to [4], approval voting does not require preferences ordinal. In this system, each voter is free to vote for (approve) the number of candidates he wishes. In addition to its interesting theoretical properties, approval voting offers electors great flexibility in expressing their choices [5] and in [13]. Approval voting, introduced notably by Brams and Fishburn [7, 8] and by Weber [16] , can be regarded as the simplest example of these systems of vote by note. In order to mitigate all these difficulties related to the electoral system, in this work, we have combined several methods in order to achieve an adequate voting method. The main purpose of this article is to improve the compensation found when using the arithmetic mean. This method uses both the median and the arithmetic mean and the mean deviation. It also raises “cumulative” or “approval voting”.

2. State of the art

2.1. Definition of Approval Voting

Approval voting is a method with good properties according to the literature. It allows each voter to approve one, two, all candidates or none of them. The elector is not restricted to voting for a single candidate. He may choose or, approve, several candidates, all or none if he wishes but he can not vote several times for the same candidate. The winner is the candidate who has received the most approvals. This type of ballot, though simple, verifies very interesting properties and shows itself in many points superior to the majority vote [6].

2.2. Specific voting properties

According to [15], the desirable properties for a social choice function are:

- Neutrality (N): If each voter reverses his preference, the selected candidate changes too.
- Anonymity (A): If one permutes voters (or even with other voters), the elected candidate does not change.
- Pareto-consistency (P): If all the voters prefer candidate x to candidate y, then x must be the election winner.
- Monotoncity (M): If one or more voters reclassify a candidate x better, then x does not have ultimately to be less better placed in this election.
- Independence of Irrelevant Alternatives (IIA): To classify two candidates among several others, it must be enough to know preferences of each voter for these two candidates. Their choices for others do not influence the classification between these candidates as well as the addition or the withdrawal of a candidate.
- Absence of dictator (ND): The rule should not simply reflect the views of a single and even judge, whatever the circumstances.
- Condorcet Criterion (CC): If a competitor is ranked in front of each of the others competitors by a majority of judges in a one-to-one comparison, it must be placed alone at the top of the final standings.

2.3. Description of the Majority Judgment

This description comes from [11, 12]. Balinski and Laraki [1] adopted, in their 2007 experience in the French presidential elections, the following common language:

{*Excellent (E), Very Good (VG), Good (G), Acceptable (A), Poor (P), To Reject (R)*}

We call common language a set $\mathcal{L} = \{g_1, g_2, \ldots, g_k\}$ strictly ordered by ">" such as $g_1 > g_2 > \ldots > g_k$ ($g_i \geqslant g_j := g_i > g_j$ or $g_i = g_j$). Note that we can also have a common language be an infinite set such as the interval $[0, 1]$ real numbers with its natural order. Note the possibility for a voter to assign the same assessment to more than two candidates.

As such, a voter may award a candidate x the **VG** score, a candidate y the **VG** note and another candidate z the note **G**. In the context of Arrow, we will say: "x is at least as good as y", "y is at least as good as x", "x is at least as good as z", "y is at least as good as z", "x is preferred to z", "y is preferred to z", "x is indifferent to y".

A function F is a it ranking method if it associates to any profile a single rank [in the same language] for any candidate. So, $F : L^{m\times n} \longrightarrow L^m$. Where m is the number of candidates, and n the number of judges or voters.

Let A_i be a candidate or competitor with grades $g_{i1}, g_{i2}, \ldots, g_{in}$ where $g_{i1} \geqslant g_{i2} \geqslant \ldots g_{in}$. Then the majority or majority grade $f^{maj}[A_i]$ is by definition:

$$f^{maj}(A_i) = \begin{cases} f^{\frac{n+1}{2}}(g_{i1}, g_{i2}, \ldots, g_{in}), & \text{if } n \text{ is odd;} \\ f^{\frac{n+2}{2}}(g_{i1}, g_{i2}, \ldots, g_{in}), & \text{if si } n \text{ is even.} \end{cases}$$

For example, if 5 judges award grades 4, 8, 7, 9, 5 to A_i,

$$f^{maj}(A_i) = f^3(9, 8, 7, 5, 4) = 7.$$

And if 8 judges award grades 9, 7, 3, 6, 5, 4, 5, 8 to A_i,

$$f^{maj}(A_i) = f^5(9, 8, 7, 6, 5, 5, 4, 3) = 5$$

Tie-Breaking (See [1, 2])

When the majority grades of two candidates are different, the one with the highest rank is ranked before the other. The majority ranking $>_{maj}$ between two candidates evaluated by the same jury is determined by a repeated application of the majority rank:

- If $f^{maj}(A) > f^{maj}(B)$ so $A >_{maj} B$
- If $f^{maj}(A) = f^{maj}(B)$ one majority-grade is dropped from the grades of each of the contestants, and the procedure is repeated.

Balinski and Laraki [2] give this example to illustrate their definition:
Suppose A and B are evaluated by a 7 voting jury:

A	85	73	78	90	69	70	73
B	77	70	95	81	73	73	66

The ordered profile is:

A	90	85	78	**73**	73	70	69
B	95	81	77	**73**	73	70	69

$f_1^{maj}(A) = f_1^{maj}(B) = 73$. By definition, we reject 7 from both lists and we get:

90	85	78	**73**	70	69
95	81	77	**73**	70	66

$f_2^{maj}(A) = f_2^{maj}(B) = 73$. By definition, we reject 7 from both lists and we get:

A	90	85	**78**	70	69
B	95	81	**77**	70	6

$f_3^{maj}(A) = 78 > f_3^{maj}(B) = 77$. Since then , $A >_{maj} B$.
It is clear that the majority position always ranks one candidate before the other unless the judges give them the same rank. In case there are several judges or voters (presidential elections for example), Balinski and Laraki present a way of dealing with *tie-break*. The majority of a candidate with $f^{maj}(A) = \alpha$ is a triplet (p_A, α^*, q_A) where p is the number or percentage of the candidate's ranks that are greater than the majority rank, q is the number or percentage of the candidate's ranks that are lower than the rank of majority, and $\alpha^* = \alpha^+$ if $p > q$ and $\alpha^* = \alpha^-$ if $p \leqslant q$. α^* is called the modified majority rank of the candidate.

By definition, $\alpha^* > \beta^*$ if and only if $\alpha > \beta$ or ($\alpha = \beta$ and $\alpha^* = \alpha^+$ and $\beta^* = \alpha^-$). Balinski and Laraki use majority gauge to define the majority-gauge-ranking $\succ_{mg}$.
Let A and B two candidates with respective majority gauges (p_A, α_A^*, q_A) and (p_B, α_B^*, q_B). So $A >_{mg} B$ or $(p_A, \alpha_A^*, q_A) >_{mg} (p_B, \alpha_B^*, q_B)$ if and only if $\alpha^* > \beta^*$ or ($\alpha_A^* = \alpha_A^* = \alpha^+$ and $p_A > p_B$) or ($\alpha_A^* = \alpha_A^* = \alpha^-$ and $p_A < p_B$).
Manzoor Ahmed Zahid [17, 18] shows that the ranking by majority rule may not decide between candidates in certain cases. A theorem uttered by Balinski and Laraki (Theorem 14.1 in [2]) shows that:

$$A >_{mg} B \Rightarrow A >_{maj} B.$$

Zahid then takes an example that illustrates a case where $A >_{mg} B$, but neither $B >_{maj} A$ nor $B >_{mg} A$.

Candidate	p	*Excellent*	*Very good*	*Good*	*Pretty good*	*Passable*	*To Reject*	q	*Total*
A	**5**	2	3	**3**	1	3	3	7	15
B	**6**	3	3	**2**	0	2	5	7	15

The majority of A is $(5, Good^-, 7)$ and that of B is $(6, Good^-, 7)$. As $q_A = q_B = 7$, the majority rule makes no decision and yet it is easy to check that $A >_{maj} B$.

2.4. Vote by rating

This example was designed by ourselves.
Consider a vote of 11 voters and 4 candidates:
Candidate 1: 1 1 1 1 1 1 1 5 5 5 5

Candidate 2: 1 3 3 3 3 3 3 3 3 3 3
Candidate 3: 1 1 1 1 1 5 5 5 5 5 5
Candidate 4: 1 2 2 2 2 3 3 3 3 3 3
Candidate 5:1 1 1 1 1 1 5 5 5 5 5
Candidate 6:1 1 1 2 4 4 4 4 4 5 5

Using the arithmetic mean we find: On the one hand candidates 1 and candidate 4 each have 27 points; But most of the majority does not want the candidate 1 to be elected. On the other hand candidates 2 and candidate 5 each have 31 points. It is also noted that candidate 2 is better liked than candidate 5. Candidates 3 and 6 also have the same number of points, but candidate 6 seems more than candidate 3. It is summarized in the following table:

	c_1	c_2	c_3	c_4	c_5	c_6
x_1	1	1	1	1	1	1
x_2	1	3	1	2	1	1
x_3	1	3	1	2	1	1
x_4	1	3	1	2	1	2
x_5	1	3	1	2	1	4
x_6	1	3	5	3	1	4
x_7	1	3	5	3	5	4
x_8	5	3	5	3	5	4
x_9	5	3	5	3	5	4
x_{10}	5	3	5	3	5	5
x_{11}	5	3	5	3	5	5
$\overline{x}$	2.45	2.81	3.18	2.45	2.81	3.18

2.5. Ranking vote

These examples are taken from [9].

Example 1. *Let { a, b, c ..., z } be an example of 26 candidates for which there are 100 voters whose preferences are as follows:*
51 voters have preference $a \succ b \succ c \succ \ldots \succ y \succ z$
49 voters have preference $z \succ b \succ c \succ \ldots \succ y \succ a$.

Example 2. *Let { a,b,c} be the set of all candidates in an election involving 21 voters whose preferences are as follows: 10 voters have preferences* $a \succ b \succ c$
6 voters have preferences $b \succ c \succ a$
5 voters have preferences $c \succ b \succ a$.

Example 3. *Let { a,b,c,d} be all candidates in an election involving 21 voters whose preferences are as follows: 10 voters have preferences* $b \succ a \succ c \succ d$

6 voters have preferences $c \succ a \succ d \succ b$
5 voters have preferences $a \succ d \succ b \succ c$.

In example 1, under the assumption of the sincerity of the voters, the candidate will receive 51 votes against 49 for the candidate z. The other candidates do not receive any votes; the candidate is elected by absolute majority. But the elected candidate is badly perceived by a large proportion of the voters whereas the candidate b could constitute "a good compromise".

In example 2, under the hypothesis of the sincerity of voters the candidate will receive 10 votes against 6 and 5 respectively for candidates b and c. The candidate is elected with 10 votes out of 21. This reflects little the wishes of the majority of voters. It is noted, however, that an absolute majority prefers all other candidates to the one elected (11 voters out of 21 prefer b and c to a).

In example 3, in the French system, only candidates b and c remain in the running for the second round, and b win with 15/21, although an absolute majority (11/21) of voters prefers both candidate a and candidate d.

2.6. Problematic

The conclusion that emerges is that at the general level, many voting methods generate paradoxes. According to [11], Majority Judgment may produce controversial results in some cases. According to [2], majority voting is a poor measure of opinion. Why ? He forces the elector to vote for a single candidate, while he has much more nuanced opinions on all. Some might support several; others do not like the candidate for whom they voted; still others make the strategic choice of the least bad among those whom they consider to have a chance. Nevertheless, each vote is interpreted as a membership and is worth "1". So, the sum of votes that are totally different in sense determines the result. It is not surprising that the induced political weight is far from reality.

According to [5], the two-round majority vote suffers from several defaults. It does not meet Condorcet's criterion that a candidate who beats all his competitors in majority duels must be elected. It encourages strategic voting and does not encourage participation in the sense that some voters may have an interest in not to vote. According to [13] classification voting systems use ordinal preferences voters and that this ordinal context does not make it possible to judge (or appreciate) the different options independently. According to [14], the one or two-round majority voting system has a large number of widely known and proven defects.

We also found through our example on vote by grading that the aggregation using the arithmetic mean gives controversial results. It is in this context that we will try through this article to find a new method, with the main concern that this one result in results with little contrast. A method that avoids Borda's paradox that a candidate can be elected while he has against him a majority of voters.

3. Voting system based on average deviation assessments

3.1. Description

The method consists of classifying candidates into five classes in order of preference. A candidate ranked in the best class gets 5 points. If he is ranked in the one following the first, he gets 4, and so on until the last of the classes where he gets 1 point. The next phase is to order a candidate's notes in ascending order. The median of the series is then determined. If the median score of a candidate is 1 or 1.5 depending on whether the number of voters is even or odd, then it is automatically eliminated.

In other cases, the arithmetic mean and the mean deviation are calculated. Of course, the candidate with the best average and the smallest average difference is elected. In case of a tie, the process is repeated.

So let us consider a E set of m candidates for an election, with $m \geq 2$, that is to say $E = \{c_1, c_2, ..., c_m\}$ and F a set of voters with $s \geq 2$, that is to say $F = \{v_1, v_2, ..., v_s\}$. So the method is as follows:

- Each of the s voters uses $P(E)$ items that are disjoint and whose meeting gives E, according to the following order: choice 1, choice 2, choice 3, choice 4, choice 5.
- It assigns each element of each of its subsets respectively the note 1, 2, 3, 4, 5. Example : Let $E = \{c_1, c_2, c_3, c_4, c_5\}$ be a set of 5 candidates. The following table represents the choice of a voter:

$\{c_2\}$	$\{c_1\}$	$\{c_4\}$	$\{c_5\}$	$\{c_3\}$

Let us now calculate the absolute average difference between the scores of the different candidates. Note that in this table, we also calculated the average deviation of candidate 1, this is optional because the median score of the latter is 1. And according to the method the candidate 1 is automatically eliminated.

Candidate 1												
x_1	1	1	1	1	1	1	1	5	5	5	5	E_m
$\|x_i - \overline{x}\|$	1.45	1.45	1.45	1.45	1.45	1.45	1.45	2.55	2.55	2.55	2.55	**1.85**

Candidate 2												
x_i	1	3	3	3	3	3	3	3	3	3	3	E_m
$\|x_i - \overline{x}\|$	1.81	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	**0.33**

Candidate 3												
x_i	1	1	1	1	1	5	5	5	5	5	5	E_m
$\lvert x_i - \overline{x}\rvert$	2.18	2.18	2.18	2.18	2.18	1.82	1.82	1.82	1.82	1.82	1.82	**1.98**

Candidate 4												
x_i	1	2	2	2	2	3	3	3	3	3	3	E_m
$\lvert x_i - \overline{x}\rvert$	1.45	0.45	0.45	0.45	0.45	0.55	0.55	0.55	0.55	0.55	0.55	**0.57**

Candidate 5												
x_i	1	1	1	1	1	1	5	5	5	5	5	E_m
$\lvert x_i - \overline{x}\rvert$	1.81	1.81	1.81	1.81	1.81	1.81	2.19	2.19	2.19	2.19	2.19	**1.98**

Candidate 5												
x_i	1	1	1	2	4	4	4	4	4	5	5	E_m
$\lvert x_i - \overline{x}\rvert$	2.18	2.18	2.18	1.18	0.82	0.82	0.82	0.82	0.82	1.82	1.82	**1.47**

3.2. Theorem

The new voting method has the following properties:

(i) It verifies the condorcet criterion.

(ii) The binary actions are independent: To classify for example 2 candidates among several others, it is enough to know the preferences of each voter for these two candidates; their choices for the others do not change the classification between these two candidates.

(iii) Monotony; if one or more voters better reclassify a candidate x, then x must not, ultimately be less placed in this election. In other words, if x is elected in a first election and in a second, a voter who has voted against x changes his mind in favor of x, then x is still elected.

(iv) Unanimity or pareto.

3.3. Proof

(i) It checks the condorcet criterion: If a candidate beats everyone else at all duels, then x is the winner.

(ii) The binary actions are independent: to classify for example two candidates among several others, it is enough to know the preferences of each voter for these two candidates; their choices for the others do not change the classification between these two candidates.
The calculation of the average and the average difference of these two candidates does not depend on the marks obtained by the other candidates.

(iii) It is monotonous: in a second election, if an elector who was not for the x candidate first elected, changes his position, the average of x increases and x is still elected

(iv) Pareto is trivial.

4. Discussion and Perspectives

Voting methods play a vital role in any society. They seem to be involved in the stability of a country. Indeed a voting system leading to a consensus generates less post-election contestation. Votes by ratings that seem better than ratings by theory of choice theorists, present some difficulties. In this article, we found that by only being satisfied with the cumulative points, different candidates, you can elect a candidate who does not reflect a consensus.

We have also, combined the assent voting at the median, the average and the average deviation to determine the new voting mode. Voting by assent is well appreciated in the literature because it fulfills some good properties [6].

Using the median eliminates candidates who are not at all liked by at least (50%) of voters.

Which is an advantage in ballot counting because we do not waste time calculating the mean and the average deviation of the candidates downgraded.

In addition to the combination used gave birth to a voting method with good properties and to have a good compromise, despite some limitations it could generate. Because according to [10], we know since the 1970s, thanks to the work of Gibbard and Satterthwaite that any democratic voting procedure is manipulable: thus, one can never be sure that a voter has revealed his sincere preference, or that he has revealed a false preference, in order to favor the election of a candidate who is better placed strategically.

Our method fulfills many good properties certainly, but is it practicable in terms of optimization of calculations and monetary, because it seems that the average difference is easy to calculate and simple to interpret that unfortunately, it is not provided by most statistical software.

In addition to this, does the method allow to reduce conflicts with those who still refuse to lose votes despite the results provided by the ballot boxes ?

Will future research allow us to study the complexity of the new voting system and apply it to the group decision, to solve a multi-criteria with several decision makers problem ?

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