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A note on "On β-Open Sets and Ideals in Topological Spaces"[European Journal of Pure and Applied Mathematics 6 (2019) 893–903]

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Abstract. Let X be a non-empty set. $\mathcal{I} \neq \phi$, $\mathcal{I} \in P(X)$ is an ideal on X, if $A \in \mathcal{I}$ and $B \in \mathcal{I} \Rightarrow A \cup B \in \mathcal{I}$ & $A \in \mathcal{I}$ and $B \subseteq A \Rightarrow B \in \mathcal{I}$ [7]. Let (X, τ) be a topological space. $A \subseteq X$ is called β-open, if $A \subseteq cl(int(cl(A)))$ [1]. Let $(X, \tau, \mathcal{I})$ be an ideal topological space. $A \subseteq X$ is called $\beta_{\mathcal{I}}$-open, if $\exists\ U \in \tau$ such that $(U - A) \in \mathcal{I}$ and $(A - cl(int(cl(U)))) \in \mathcal{I}$ [4]. This note shows that the main results of the paper [4] [European Journal of Pure and Applied Mathematics 6 (2019) 893–903] are incorrect in general, by giving counter examples. The correct form of the incorrect results in [4] is presented.

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1. Introduction

General topology has been considered the entrance to understand topology science, moreover the base of general topology is the topological space, which has been considered a representation of universal space in general, and geometric shape in special, also the mathematical analysis concepts. The concept of topology shows up naturally in almost every branch of mathematics [8, 14, 17, 18]. This has made topology one of the great unifying ideas of mathematics. Ordinary topology now has been used in many subfields of artificial intelligence, such as knowledge representation, spatial reasoning etc.

The main component of a topological space is the open sets, and overtime there have been so many generalizations of it. Stone [19] introduced the concept of regular open sets.

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The concept of semi open sets was presented in [10]. Meantime, the concept of α-open sets was proposed by Najastad [15]. Mashhour et al. [11] introduced the concept of pre-open sets. In 1983, Abd El-Monsef et al. [1] discussed the concept of β-open sets.

Ideal is a fundamental concept in studying the topological problems. The notion of ideal topological spaces was first studied by Kuratowski [9] and Vaidyanathaswamy [20] which is one of the important areas of research in the branch of mathematics. After them different mathematicians applied the concept of ideals in topological spaces (see: [3, 5–7, 13]). The interest in the idealized version of many general topological properties has grown drastically in the past 20 years.

In [2, 12], the concept of semi-open sets with respect to an ideal was investigated. Nasef et al. [16] presented and studied the concept of α-open sets with respect to an ideal. Recently, in [4], the concept of β-open sets with respect to an ideal ($\beta_{\mathcal{I}}$-open) was introduced.

2. Counter examples

In this section, i point out where the errors occur in [4] and then give counter examples to confirm my claim. Eventually, the correct form of the incorrect results is introduced.

In [[4], Lemma 1, p. 895], the authors proved that in the topological space (X, τ), and $A \subseteq X$. Then $int(A) = int(cl(A)) = int(cl(int(A)))$.

The following examples show that [Lemma 1, p. 895] is not true in general.

Example 2.1. *Let* $X = \{1, 2, 3, 4\}$ *and* $\tau = \{X, \phi, \{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}\}$. *Take* $A = \{1, 2\}$. *Then,* $cl(A) = X, int(A) = A, int(cl(A)) = X, cl(int(A)) = X, int(cl(int(A))) = X$, *but* $int(A) = A \neq X = int(cl(A))$ *and* $int(A) = A \neq X = int(cl(int(A)))$.

Example 2.2. *Let* $X = \{1, 2, 3, 4\}$ *and* $\tau = \{X, \phi, \{1\}, \{2\}, \{1, 2\}, \{3, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\}$. *Take* $A = \{1, 2, 3\}$. *Then,* $cl(A) = X, int(A) = \{1, 2\}, int(cl(A)) = X, cl(int(A)) = \{1, 2\}$, $int(cl(int(A))) = \{1, 2\}$, *but* $int(cl(A)) = X \neq \{1, 2\} = int(cl(int(A)))$.

In [[4], Lemma 2, p. 895], the authors proved that in an ideal topological space $(X, \tau, \mathcal{I})$. A subset A of X is β-open if and only if there exists an open set U such that $U \subseteq A \subseteq cl(int(cl(U)))$.

The following example shows that the necessary condition (A is β-open, then there exists an open set U such that $U \subseteq A \subseteq cl(int(cl(U)))$) in [Lemma 2, p. 895] is not true in general.

Example 2.3. *Let* $X = \{1, 2, 3, 4\}$ *and* $\tau = \{X, \phi, \{1, 2, 3\}\}$. *Take* $A = \{3\}$. *Then,* A *is* β*-open, but* $\nexists\, U \in \tau$ *such that* $U \subseteq A \subseteq cl(int(cl(U)))$.

It should be noted that, in the head of [Lemma 2, p. 895] the authors supposed that "$(X, \tau, \mathcal{I})$" is an ideal topological space, but they didn't use it in the proof as this lemma studied only the properties of β-open sets. So, it must be replaced by "(X, τ)" is a topological space and the correct form of this lemma is:

Lemma 2.1. *Let (X, τ) be a topological space and $A \subseteq X$. If there exists an open set U such that $U \subseteq A \subseteq cl(int(cl(U)))$, then A is β-open set.*

In [[4], Lemma 3, p. 895], the authors proved in part (iii) that in an ideal topological space $(X, \tau, \mathcal{I})$. If A is β-open, then A is $\beta_{\mathcal{I}}$-open.

In Example 2.3, put $\mathcal{I} = \{\phi, \{3\}\}$. Then it shows part (iii) in [Lemma 3, p. 895] is not true in general. Take $A = \{1\}$. Then, A is β-open, but A is not $\beta_{\mathcal{I}}$-open.

In [[4], Lemma 4, p. 896], the authors proved that in an ideal topological space $(X, \tau, \mathcal{I})$. If $\mathcal{I}$ is not countably additive, then the following statements are equivalent.

(i) If $\mathcal{I} = \{\phi\}$.

(ii) A is a β-open set if and only if A is a $\beta_{\mathcal{I}}$-open set.

1. In Example 2.3, $\mathcal{I} = \{\phi\}$. Then it shows for $[(i) \rightarrow (ii)]$ in [Lemma 4, p. 896] is not true in general. Take $A = \{3\}$. Then, A is β-open, but A is not $\beta_{\mathcal{I}}$-open.

2. The following example shows that for $[(ii) \rightarrow (i)]$ in [Lemma 4, p. 896] is not true in general.

Example 2.4. *Let $X = \{1, 2, 3, 4\}, \mathcal{I} = P(X)$ and $\tau = \{X, \phi\}$. Then, the family of all β-open sets is $P(X)$ which is precisely the family of all $\beta_{\mathcal{I}}$-open sets, but $\mathcal{I} \neq \{\phi\}$.*

The following lemma is the correct form of [Lemma 4, p. 896].

Lemma 2.2. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $A \subseteq X$. If $\mathcal{I} = \{\phi\}$ and A is a $\beta_{\mathcal{I}}$-open set, then A is a β-open set.*

It should be noted that, i can add examples in the same manner, to show that [Theorem 1, p. 896], [Theorem 8 for part (ii), Claim 1, Claim 2 p. 899] and [Theorem 9, p. 900], are also not true in general.

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