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Generalized Nörlund and Nörlund-type Means of Sequences of Fuzzy Numbers

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Abstract. In this article we study some properties of generalized Nörlund and Nörlund-type means of sequences of fuzzy real numbers. We establish necessary and sufficient conditions for our purposed methods to transform convergent sequences of fuzzy real numbers into convergent sequences of fuzzy real numbers which also preserve the limit. Finally, we establish some results showing the connection between the generalized Nörlund and Nörlund-type limits and the usual limits under slow oscillation of sequences of fuzzy real numbers.

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Key Words and Phrases: Generalized Nörlund mean, Generalized Riesz mean, Fuzzy real numbers, Slow oscillation

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1. Introduction

Let D be the set of all closed and bounded intervals on the real line $\mathbb{R}$. For $X, Y \in D$, we define

$$d(X, Y) = \max(|a_1 - b_1|, |a_2 - b_2|),$$

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where $X = [a_1, a_2]$, $Y = [b_1, b_2]$. It is known that (D, d) is a metric space which is also complete.

A fuzzy real number X is a fuzzy set on $\mathbb{R}$, and is a mapping $X : \mathbb{R} \to I\ (= [0, 1])$ associating each real number r with its grade of membership $X(r)$.
Recalling some basic terminologies, a fuzzy real number X is called *convex* if, $X(r) \geq X(s) \wedge X(t) = \min(X(s), X(t))$, where $s < r < t$. A fuzzy real number X is called *normal* if, there exists $r_0 \in \mathbb{R}$ such that $X(r_0) = 1$. Further, if for every $\epsilon > 0$, $X^{-1}([0, a + \epsilon])$, for all $a \in I$ (is open in the usual topology of $\mathbb{R}$) then X is called *upper semi-continuous.*

Let $\mathbb{R}(I)$ denotes the set of all *convex, upper semi continuous and normal* fuzzy numbers, and let X^α $(0 < \alpha \leq 1)$ be the α level set of X, which is defined by $X^\alpha = \{r \in R : X(r) \geq \alpha\}$. Also, for $\alpha = 0$, it is closure of the strong 0-cut.

Note that, the set of all numbers $\mathbb{R}$ can be embedded in $\mathbb{R}(I)$. For each $t \in \mathbb{R}$, $\bar{t} \in \mathbb{R}(I)$ is defined by

$$\bar{t}(r) = \begin{cases} 1, & if \quad r = t, \\ \\ 0, & if \quad r \neq t. \end{cases}$$

Let $\bar{d} : \mathbb{R}(I) \times \mathbb{R}(I) \to \mathbb{R}$ be defined by

$$\overline{d}(X, Y) = \sup_{0 \leq \alpha \leq 1} d(X^\alpha, Y^\alpha).$$

Also, $\overline{d}$ defines a metric on $\mathbb{R}(I)$. It is trivial that $(\mathbb{R}(I), \overline{d})$ is a metric space, and is complete. Here, $\overline{0}$ and $\overline{1}$ are the additive identity and multiplicative identity respectively.

The preliminary idea of fuzzy set theory was introduced and studied by Zadeh [18] in the year 1965. Gradually this theory has entered into many diversified areas of science and technology. In particular, mathematicians and researchers working on sequence spaces preferred to use fuzzy sequences because of its wide applications. The scope of such theory has been studied in the different areas of (for instance) fuzzy logic, fuzzy graph theory, fuzzy topological spaces, fuzzy differential equations, fuzzy mathematical programming, and so on. In this article we study the characterization of generalized Nörlund and Nörlund-type (Riesz) means of sequences of fuzzy real numbers.

2. Preliminaries and Definitions

Let (p_n) and (q_n) be two sequences of non-negative real numbers which are not all zero, that is,

$$P_n = \sum_{u=1}^{n} p_u, \quad n \in \mathbb{N}$$

and

$$Q_n = \sum_{u=0}^{n} q_u, \quad n \in \mathbb{N}.$$

Let us consider

$$R_n = \sum_{u=0}^{n} p_u q_u \quad \text{and} \quad R_n' = \sum_{u=0}^{n} p_{n-u} q_u$$

Definition 1. *A sequence (X_n) of fuzzy real numbers is generalized Nörlund (N, p_n, q_n) summable to l if,*

$$\overline{d}\left(\frac{1}{R_n'}\sum_{u=0}^{n} p_{n-u} q_u X_u, l\right) \to 0 \ \text{as } n \to \infty.$$

Definition 2. *A sequence (X_n) of real numbers is generalized Nörlund-type $(\overline{N}, p_n, q_n)$ summable or generalized Riesz (R, p_n, q_n) summable to l if,*

$$\overline{d}\left(\frac{1}{R_n}\sum_{u=1}^{n} p_u q_u X_u, l\right) \to 0 \quad \text{as} \quad n \to \infty.$$

Definition 3. *A sequence (X_n) of fuzzy real numbers is slowly oscillating if,*

$$\overline{d}(X_n, X_m) \to 0 \quad \text{as} \quad n, m \to \infty \ \text{with } 1 \leq \frac{n}{m} \to 1.$$

Equivalently, we can say that; a sequence of fuzzy real numbers (X_n) is oscillating slowly iff for every $\epsilon > 0$, there exists $\delta = \delta(\epsilon) > 0$ and $n_0(\epsilon) \in \mathbb{N}$ such that $\overline{d}(X_n, X_m) < \epsilon$ whenever $1 \leq \left(\frac{n}{m}\right) < 1 + \delta$ and $m, n \geq n_0(\epsilon)$.

Several summability methods have been defined for different fuzzy numbers valued sequences. The Cesàro summability of order one for sequences of fuzzy real numbers was studied by Altın *et al.* [1] in the year 2010. In the year 2017, Yavuz [17] defined a Euler summability method of sequences of fuzzy numbers and a proved Tauberian theorem. Dealing with statistical summability of sequences of fuzzy numbers, in 2016, Talo and Bal [13] studied some results based on Nörlund-type means. Recently, some works on Nörlund and Riesz means have been studied by Srivastava *et al.* [9], [10], [11], and [12] based on statistical convergence. Very recently, Jena *et al.* [6] studied the Cesàro summability of double sequences of fuzzy real numbers and proved Tauberian theorems on that basis. Also, Das *et al.* [2] used statistical $(C, 1)(E, \mu)$ product summability mean for sequences of fuzzy numbers to prove a fuzzy Korovkin-type approximation theorem. For more studies in this direction one may refer to [3], [4], [5], [7], [8], [14], [15] and [16]. Motivated essentially by the above mentioned works, we investigate here the characterization of generalized Nörlund and Nörlund-type (Riesz) means of sequences of fuzzy real numbers. We establish necessary and sufficient conditions for our purposed methods to transform convergent sequences of fuzzy real numbers into convergent sequences of fuzzy real numbers which also preserve the limit. Moreover, we establish some results demonstrating the connection between the generalized Nörlund and Nörlund-type limit and the usual limit under slow oscillation of sequences of fuzzy real numbers.

3. Main Theorem

The objective of this paper to prove the following theorem.

Theorem 1. *The method* (N, p_n, q_n) *is regular if and only if* $\frac{p_{n-k}q_k}{R'_n} \to 0$ *as* $n \to \infty$.

Proof. Let (X_n) be any sequence of fuzzy real numbers which is convergent to L. That is, $\lim\limits_{n\to\infty} X_n = L$; then for given $\epsilon > 0$ there exists a positive integer n_0 for which $\overline{d}(X_n, L) < \epsilon$ for $n \geq n_0$ and $\overline{d}(X_n, L) < H$, for all $n \in \mathbb{N}$. Let $\frac{p_{n-k}q_k}{R'_n} \to 0$ as $n \to \infty$, then for $\epsilon > 0$ there exists $n_1 \in \mathbb{N}$ such that $\frac{p_{n-k}q_k}{R'_n} < \left(\frac{\epsilon}{2H\max(n_0,n_1)}\right)$ for $n > n_1$. Let $n_2 = \max(n_0, n_1)$. Then for all $n \geq n_2$, we have

$$\begin{aligned}
\overline{d}\left(\frac{1}{R'_n}\sum_{i=1}^{n} p_{n-i+1}q_iX_i, L\right) &\leq \overline{d}\left(\frac{1}{R'_n}\sum_{i=1}^{n_2} p_{n-i+1}q_iX_i, L\right) + \overline{d}\left(\frac{1}{R'_n}\sum_{i=n_2+1}^{n} p_{n-i+1}q_iX_i, L\right) \\
&= \overline{d}\left(\frac{1}{R'_n}(p_nq_0X_0 + p_{n-1}q_1X_1 + ... + p_{n-n_2+1}q_{n_2-1}X_{n_2-1}), L\right) \\
&\quad + \overline{d}\left(\frac{1}{R'_n}(p_{n-n_2}q_{n_2}X_{n_2} + ... + p_0q_nX_n), L\right) \\
&\leq \frac{p_nq_0}{R'_n}\overline{d}(X_0, L) + \frac{p_{n-1}q_1}{R'_n}\overline{d}(X_1, L) + ... + \frac{p_{n-n_2+1}q_{n_2-1}}{R'_n}\overline{d}(X_{n_2-1}, L) \\
&\quad + \frac{p_{n-n_2}q_{n_2}}{R'_n}\overline{d}(X_{n_2}, L) + ... + \frac{p_0q_n}{R'_n}\overline{d}(X_n, L) \\
&\leq \frac{\epsilon}{2Hn_2}H + \frac{\epsilon}{2Hn_2}H + ... + \frac{\epsilon}{2Hn_2}H + \frac{p_{n-n_2}q_{n_2}}{R'_n}\frac{\epsilon}{2} \\
&\quad + ... + \frac{p_0q_n}{R'_n}\frac{\epsilon}{2} \\
&\leq \frac{\epsilon}{2n_2} + \frac{\epsilon}{2n_2} + ... + \frac{\epsilon}{2n_2} + \frac{p_{n-n_2}q_{n_2} + ... + p_0q_n}{R'_n}\frac{\epsilon}{2} \\
&< \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon.
\end{aligned}$$

Conversely, let (N, p_n, q_n) be a regular method. Consider the sequence

$$\overline{e}_k = (\overline{0}, \overline{0}, ..., \overline{1}, \overline{0}, \overline{0}...) = X_n$$

where $\overline{1}$ appears at the k^{th} place. Also, we have $X_n \to \overline{0}$ as $n \to \infty$. Thus,

$$\overline{d}\left(\sum_{k=1}^{n}\frac{p_{n-k+1}q_k}{R'_n}\bar{e_k}, \overline{0}\right) = \frac{p_{n-k}q_n}{R'_n} \to 0 \quad (n \to \infty).$$

Theorem 2. *The method* (R, p_n, q_n) *is regular if and only if* $\frac{p_nq_n}{R_n} \to 0$ *as* $n \to \infty$.

Proof. Let (X_n) be any sequence of fuzzy real numbers which is convergent to L. That is, $\lim_{n\to\infty} X_n = L$; then for given $\epsilon > 0$ there exists a positive integer n_0 for which $\overline{d}(X_n, L) < \epsilon$, for all $n \geq n_0$ and $\overline{d}(X_n, L) < H$ for all $n \in \mathbb{N}$. Let $\frac{p_n q_n}{R_n} \to 0$ as $n \to \infty$, then there exists $n_1 \in \mathbb{N}$ such that $\frac{p_n q_n}{R_n} < \frac{\epsilon}{2H max(n_0,n_1)}$ for all $n > n_1$. Let $n_2 = \max(n_0, n_1)$. Then for all $n \geq n_2$, we have

$$\begin{aligned}
\overline{d}\left(\frac{1}{R_n}\sum_{i=0}^{n} p_i q_i X_i, L\right) &\leq \overline{d}\left(\frac{1}{R_n}\sum_{i=0}^{n_2} p_i q_i X_i, L\right) + \overline{d}\left(\frac{1}{R_n}\sum_{n_2+1}^{n} p_i q_i X_i, L\right)\\
&\leq \overline{d}\left(\frac{1}{R_n}(p_0q_0X_0 + p_1q_1X_1 + ... + p_{n_2}q_{n_2}X_{n_2}), L\right)\\
&\quad + \overline{d}\left(\frac{1}{R_n}(p_{n_2+1}q_{n_2+1}X_{n_2+1} + ... + p_nq_nX_n), L\right)\\
&\leq \frac{p_0q_0}{R_n}\overline{d}(X_0, L) + \frac{p_1q_1}{R_n}\overline{d}(X_1, L) + ... + \frac{p_{n_2}q_{n_2}}{R_n}\overline{d}(X_{n_2}, L)\\
&\quad + \frac{p_{n_2+1}q_{n_2+1}}{R_n}\overline{d}(X_{n_2+1}, L) + ... + \frac{p_nq_n}{R_n}\overline{d}(X_n, L)\\
&\leq \frac{p_0q_0}{R_n}H + \frac{p_1q_1}{R_n}H + ... + \frac{p_{n_2}q_{n_2}}{R_n}H\\
&\quad + \frac{p_{n_2+1}q_{n_2+1}}{R_n}\frac{\epsilon}{2} + ... + \frac{p_nq_n}{R_n}\frac{\epsilon}{2}\\
&\leq \frac{\epsilon}{2Hn_2}H + \frac{\epsilon}{2Hn_2}H + ... + \frac{\epsilon}{2Hn_2}H\\
&\quad + \frac{\frac{p_{n_2}}{q_{n-n_2}} + ... + p_nq_0}{R_n}\frac{\epsilon}{2}\\
&< \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon.
\end{aligned}$$

Conversely, let (R, p_n, q_n) be regular. Consider the sequence $\overline{e_k} = (\overline{0}, \overline{0}, ..., \overline{1}, \overline{0}, ...) = X_n$, where $\overline{1}$ appears at the k^{th} place. Also, we have $X_n \to \overline{0}$ as $n \to \infty$. Thus,

$$\overline{d}\left(\sum_{k=1}^{n}\frac{p_nq_n}{R_n}\bar{e}_k, \overline{0}\right) = \frac{p_nq_n}{R_n} \to 0 \quad as \quad n \to \infty.$$

Theorem 3. *If (X_n) is (N, p_n, q_n) summable to L in $\mathbb{R}(I)$ and slowly oscillating then it is convergent to L in $\mathbb{R}(I)$.*

Proof. Without loss of generality we may assume that $L = 0$. Suppose that $\lim_{n\to\infty} \overline{d}(X_n, \overline{0}) > 0$. Then there exists $\alpha > 0$ and a subsequence X_{n_i} of (X_n) such that

$$\overline{d}(X_{n_i}, \overline{0}) \geq \alpha \text{ for all } i \in \mathbb{N}.$$

Since (X_n) is slowly oscillating, so (X_{n_i}) as a subsequence of (X_n) is also slowly oscillating. Then for a given $\delta > 0$, there exists $g_0 \in \mathbb{N}$ such that $g_0 \leq n \leq m < (1 + \delta)n$ and

$$\overline{d}(X_n, X_m) < \frac{\alpha}{2}.$$

Moreover, (X_n) being (N,p_n,q_n)- summable to $\overline{0}$, that means, (σ_n) is convergent to $\overline{0}$ in $(\mathbb{R}(I),\overline{d})$ with

$$\sigma_n = \frac{1}{R_n'}\sum_{k=1}^{n} p_{n-k}q_kX_k;$$

thus for all $m_i \geq n_i$,

$$\begin{aligned}\sigma_{m_i} - \frac{R_{n_i}'}{R_{m_i}'}\sigma_{n_i} &= \frac{1}{R_{m_i}'}\sum_{k=1}^{m_i} p_{m_i-k}q_kX_k - \frac{R_{n_i}'}{R_{m_i}'}\frac{1}{R_{n_i}'}\sum_{k=1}^{n_i} p_{n_i-k}q_kX_k\\ &= \frac{1}{R_{m_i}'}\sum_{k=n_i+1}^{m_i} p_{m_i-k}q_kX_k.\end{aligned}$$

Clearly, $n_i \geq g_1$ and $n_i \leq m \leq m_i = [(1+\delta)n_i]$, where $[x]$ denote the integral part of x, we have

$$\overline{d}(\overline{0},X_m) \geq \overline{d}(\overline{0},X_{n_i}) - \overline{d}(X_{n_i},X_m) \geq \alpha - \frac{\alpha}{2}.$$

Again,

$$\begin{aligned}\overline{d}(\sigma_{m_i},\sigma_{n_i}) + \overline{d}\left(\sigma_{n_i},\frac{R_{n_i}'}{R_{m_i}'}\sigma_{n_i}\right) &\geq \overline{d}\left(\sigma_{m_i},\frac{R_{n_i}'}{R_{m_i}'}\sigma_{n_i}\right)\\ &\geq \overline{d}\left(\frac{1}{R_{m_i}'}\sum_{k=n_i+1}^{m_i} p_{m_i-k}q_kX_k,\overline{0}\right)\\ &\geq \overline{d}\left(\frac{p_{m_i-k}q_{m_i} - p_{n_i-k}q_{n_i}}{R_{m_i}'}X_{n_i},\overline{0}\right)\\ &\quad - \overline{d}\left(\sum_{k=n_i+1}^{m_i}\frac{p_{m_i-k}q_kX_k - p_{n_i-k}q_{n_i}X_{n_i}}{R_{m_i}'},\overline{0}\right)\\ &\geq \frac{p_{m_i-k}q_{m_i} - p_{n_i-k}q_{n_i}}{R_{m_i}'}\overline{d}(X_{n_i},\overline{0})\\ &\quad - \sum_{k=n_i+1}^{m_i}\overline{d}\left(\frac{p_{m_i-k}q_kX_k - p_{n_i-k}q_{n_i}X_{n_i}}{R_{m_i}'},\overline{0}\right)\\ &= \frac{p_{m_i-k}q_{m_i} - p_{n_i-k}q_{n_i}}{R_{m_i}'}\overline{d}(X_{n_i},0)\\ &\quad - \sum_{k=n_i+1}^{m_i}\frac{1}{R_{m_i}'}\overline{d}\left(p_{m_i-k}q_kX_k, p_{n_i-k}q_{n_i}X_{n_i}\right)\\ &\geq \frac{p_{m_i-k}q_{m_i} - p_{n_i-k}q_{n_i}}{R_{m_i}'}\overline{d}(X_{n_i},\overline{0})\\ &\quad - \frac{p_{m_i-k}q_{m_i} - p_{n_i-k}q_{n_i}}{R_{m_i}'}\overline{d}(X_k,X_{n_i})\end{aligned}$$

$$\geq \frac{p_{m_i-k}q_{m_i} - p_{n_i-k}q_{n_i}}{R'_{m_i}}\alpha - \frac{p_{m_i-k}q_{m_i} - p_{n_i-k}q_{n_i}}{R'_{m_i}}\frac{\alpha}{2}$$
$$= \frac{p_{m_i-k}q_{m_i} - p_{n_i-k}q_{n_i}}{R'_{m_i}}\left(\alpha - \frac{\alpha}{2}\right)$$
$$\geq \frac{p_{m_i-k}q_{m_i} - p_{n_i-k}q_{n_i}}{R'_{m_i}}\left(\frac{\delta}{1+\delta}\right) \geq 0.$$

Thus, for all $m_i \geq n_i \geq g_i$,

$$\overline{d}(\sigma_{m_i}, \sigma_{n_i}) + \overline{d}\left(\sigma_{n_i}, \frac{R'_{n_i}}{R'_{m_i}}\sigma_{n_i}\right) \geq \overline{d}\left(\sigma_{m_i}, \frac{R'_{n_i}}{R'_{m_i}}\sigma_{n_i}\right)\frac{\alpha}{2}\left(\frac{\delta}{1+\delta}\right).$$

Consequently,

$$0 = \lim \overline{d}\left(\sigma_{n_i}, \frac{R'_{n_i}}{R'_{m_i}}\sigma_{n_i}\right) \geq \frac{\alpha}{2}\left(\frac{\delta}{1+\delta}\right) > 0$$

which contradicts that (X_n) converges in $\mathbb{R}(I)$. Therefore, (X_n) is convergent to L in $\mathbb{R}(I)$. This completes the proof of the theorem.

Theorem 4. *If (X_n) is $(\bar{N}, p_n, q_n)$ summable to L in $\mathbb{R}(I)$ and slowly oscillating then it is convergent to L in $\mathbb{R}(I)$.*

Proof. The proof can be followed in the similar lines from the proof of Theorem 3.

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