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# Generalized Iterative Decreasing Dimension Method

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**Abstract.** In this study, we have given a generalization of the iterative decreasing dimension method given in [3] and a generalization of the iterative decreasing dimension algorithm based on this method. The algorithm is suited for implementation using computer algebra systems such as Maple and MATLAB. So we also have given symbolic and numerical examples using this algorithm and a Maple procedure for the algorithm.

**2000 Mathematics Subject Classifications**: 65F10

**Key Words and Phrases**: Iterative Decreasing Dimension Method, Iterative Decreasing Dimension Algorithm, Linear Algebraic Equations

## 1. Introduction

Studying on solution of the systems of linear algebraic equation

$$AX = f \tag{1}$$

is a classical problem which is important not only in linear algebra but also in other branches of science, engineering, economics. A decreasing dimension method (DDM) has been proposed in [4] to solve the system (1) where $A$ is $N \times N$ -regular matrix, $X$ and $f$ are $N$-vectors. In [5] (therein [1, 2]), it has been said that the proposed DDM in [4] is same as the well known domain decomposition technique based on a Schur complement type method. Also it has been said that this method costs more than the standard Schur complement method and does not decrease the dimension of the linear systems. So in [3], the authors improved DDM and gave iterative decreasing dimension method (IDDM) which decreases the dimension of the linear systems, one order in every step without any pre-process.

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In this study; we have given a generalization of IDDM in [3] for the solution of the linear algebraic system (1) taking $A$ is any $M \times N$ - matrix instead of a $N \times N$ - regular matrix.

In section 2; we have given symbols and we have summarized IDDM, then we have given generalized iterative decreasing dimension method (GIDDM) improving the method in [3]. In section 3, we have given generalized iterative decreasing dimension algorithm (GIDDA) based on GIDDM and some symbolic and numerical examples. We have also given a Maple procedure for GIDDA in section 4.

## 2. Generalized Iterative Decreasing Dimension Method (GIDDM)

In this section, after introduce symbols used in this study and IDDM given in [3] we are going to give GIDDM which is the generalization of IDDM. The symbols will be used similar as in [3] in this study.

### 2.1. Symbols

Let us give some symbols and explanations used in procedure.

| | | |
|---|---|---|
| $n$ | : | $n = \min\{M, N\}$ |
| $k$ | : | $k = 1(1)n,\ (k = 1,\ 2, \ldots,\ n)$ - iteration step |
| $A^{(k)}$ | : | $M_k \times N_k$ - reduced coefficient matrix |
| $a_{ps}^{(k)}$ | : | $a_{ij}^{(k)} \neq 0$ which is the first non-zero element of matrix $A^{(k)}$ |
| $p_k$ | : | $p$ which is the number in $a_{ps}^{(k)} \neq 0$ |
| $M_k$ | : | $M_k = M - \sum_{i=1}^{k-1} p_i \;\; ; \;\; \sum_{i=1}^{0} p_i = 0$ |
| $N_k$ | : | $N_k = N - k + 1$ |
| $X^{(k)}$ | : | $N_k$ - solution vector of reduced system |
| $f^{(k)}$ | : | $M_k$ - right side vector of reduced system |
| $a_{ij}^{(k)}$ | : | $(i, j)$ element of matrix $A^{(k)}$ |
| $x_i^{(k)}$ | : | $i^{th}$ element of vector $X^{(k)}$ |
| $f_i^{(k)}$ | : | $i^{th}$ element of vector $f^{(k)}$ |
| $u^{(k)}$ | : | vector composed of $f_p^{(k)}$ element of vector $f^{(k)}$ |
| $v^{(k)}$ | : | vector composed of $f_i^{(k)}$, $i = p + 1(1)M_k$ element of vector $f^{(k)}$ |
| $A_1^{(k)}$ | : | matrix composed of first non-zero row vector of matrix $A^{(k)}$ |
| $A_2^{(k)}$ | : | matrix composed remain line vector of matrix $A^{(k)}$ |
| $X_0^{(k)}$ | : | special solution vector of $A_1^{(k)} X^{(k)} = u^{(k)}$ |
| $R^{(k)}$ | : | base matrix of solution space of $A_1^{(k)} X^{(k)} = 0$ |

**Note:** If $A$ is a $N \times N$-regular matrix, then it is clear that $M_k = N_k = N - k + 1$ for $k = 1(1)n$.

## 2.2. IDDM

Let us summarize IDDM given in [3]. Consider a system of linear algebraic equation

$$AX = f\,;\, A\text{-regular} \tag{2}$$

where $A$ is a $N \times N$-matrix, $X$ and $f$ are $N$-vectors. Suppose that $k = 1(1)n$ is the iteration step, $A^{(k)}$ is a $N_k \times N_k$ - coefficient matrix of reduced system and $f^{(k)}$ is a right side vector of reduced system as following

$$A^{(k)} = \begin{cases} A & k = 1, \\ A_2^{(k-1)} R^{(k-1)} & k = 2(1)n \end{cases} ; f^{(k)} = \begin{cases} f & k = 1, \\ v^{(k-1)} - A_2^{(k-1)} X_0^{(k-1)} & k = 2(1)n \end{cases}.$$

$X_0^{(k)}$ is a special solution as $X_0^{(k)} = \begin{pmatrix} 0 & \dots & 0 & \frac{f_1^{(k)}}{a_{1s}^{(k)}} & 0 & \dots & 0 \end{pmatrix}^T$ where $a_{1s}^{(k)} \neq 0, (1 \leq s \leq N_k)$ which is the first non-zero element of matrix $A_1^{(k)}$ and

$$R^{(k)} = \begin{cases} \begin{pmatrix} r_{1\times(N_k-1)}^{(k)} \\ I_{(N_k-1)\times(N_k-1)} \end{pmatrix} & s = 1, \\ \begin{pmatrix} I_{(s-1)\times(s-1)} & 0_{(s-1)\times(N_k-s)} \\ 0_{1\times(s-1)} & r_{1\times(N_k-s)}^{(k)} \\ 0_{(N_k-s)\times(s-1)} & I_{(N_k-s)\times(N_k-s)} \end{pmatrix} & s = 2(1)N_k - 1,. \\ \begin{pmatrix} I_{(N_k-1)\times(N_k-1)} \\ 0_{1\times(N_k-1)} \end{pmatrix} & s = N_k \end{cases} \tag{3}$$

Then the solution of linear system (2) is given to be

$$X = X^{(1)} = \sum_{i=1}^{n} \left( \prod_{j=1}^{i-1} R^{(j)} \right) X_0^{(i)};\ \prod_{j=1}^{i-1} R^{(j)} = \begin{cases} R^{(1)} R^{(2)} \dots R^{(i-1)} & i > 1, \\ I & i = 1 \end{cases}. \tag{4}$$

## 2.3. Generalized Iterative Decreasing Dimension Method (GIDDM)

Let us consider a system of linear algebraic equation

$$AX = A^{(1)} X^{(1)} = f^{(1)} = f \tag{5}$$

where $A$ is a $M \times N$ -matrix, $X$ is a $N$-vector and $f$ is a $M$-vector and examine the solution of the linear system (5) according to situations of $M$ and $N$.

Now, we divide the given system into two systems such that

$$A_1^{(1)} X^{(1)} = u^{(1)}; A_1^{(1)} = \left( a_{pj} \right)^{j=1(1)N}, u^{(1)} = \left( f_p \right) \tag{6}$$

$$A_2^{(1)} X^{(1)} = v^{(1)}\ ;\ A_2^{(1)} = \left( a_{ij} \right)_{i=p+1(1)M}^{j=1(1)N},\ v^{(1)} = \left( f_i \right)_{i=p+1(1)M} \tag{7}$$

where $p$ is the number of first non-zero row of matrix $A^{(1)}$. If $p > 1$, for the equation (5) to have a solution, $f_i^{(1)} = 0$, $i = 1(1)p-1$ must be satisfied.

$X_0^{(1)}$ is chosen to be

$$X_0^{(1)} = \begin{pmatrix} 0 & \dots & 0 & \frac{f_p^{(1)}}{a_{ps}^{(1)}} & 0 & \dots & 0 \end{pmatrix}^T \tag{8}$$

which is a special solution of (6). In (8), $a_{ps}^{(1)} \neq 0$ $(1 \leq s \leq N,\ 1 \leq p \leq M)$ is the first non-zero element of matrix $A^{(1)}$. Then, $X_h^{(1)}$ -homogeneous solution of (6) is obtained to be

$$X_h^{(1)} = R^{(1)} X^{(2)}$$

where $X^{(2)}$ is a $N_2$-vector composed of $x_i$-parametric variables for $i = 1(1)M_2$, $i \neq s$ and $R^{(1)}$ is a matrix composed of the base vector of this solution space as

$$R^{(1)} = \begin{cases} \begin{pmatrix} r^{(1)}_{1\times(N-1)} \\ I_{(N-1)\times(N-1)} \end{pmatrix} & s = 1, \\ \begin{pmatrix} I_{(s-1)\times(s-1)} & 0_{(s-1)\times(N-s)} \\ 0_{1\times(s-1)} & r^{(1)}_{1\times(N-s)} \\ 0_{(N-s)\times(s-1)} & I_{(N-s)\times(N-s)} \end{pmatrix} & s = 2(1)N-1, \\ \begin{pmatrix} I_{(N-1)\times(N-1)} \\ 0_{1\times(N-1)} \end{pmatrix} & s = N \end{cases}$$

where

$$r^{(1)}_{1\times(N-s)} = \begin{pmatrix} r^{(1)}_{1(s+1)} & r^{(1)}_{1(s+2)} & \dots & r^{(1)}_{1N} \end{pmatrix};\ r^{(1)}_{1j} = -\frac{a^{(1)}_{pj}}{a^{(1)}_{ps}}, j = s+1(1)N.$$

The general solution of (6) is achieved as $X^{(1)} = X_0^{(1)} + R^{(1)} X^{(2)}$, where $X_0^{(1)}$ is a $N$-vector and $R^{(1)}$ is a $N \times (N-1)$-matrix. By substituting solution $X^{(1)}$ into system (7), we have a new linear algebraic system as following

$$A^{(2)} X^{(2)} = f^{(2)}, \tag{9}$$

where $A^{(2)} = A_2^{(1)} R^{(1)}$ and $f^{(2)} = v^{(1)} - A_2^{(1)} X_0^{(1)}$.

Applying the steps given above to the system (9), we can write the systems followed by each other as

$$A^{(k)} X^{(k)} = f^{(k)}\ ;\ \ k = 2(1)n \tag{10}$$

where $A^{(k)} = A_2^{(k-1)} R^{(k-1)}$, $f^{(k)} = v^{(k-1)} - A_2^{(k-1)} X_0^{(k-1)}$. It is known that the general solutions of the system (10) are $X^{(k)} = X_0^{(k)} + R^{(k)} X^{(k+1)}$ if the solutions exist. Here $R^{(k)}$ is a matrix as given in (3).

Now, we are going to examine the situations for solution of linear system (5).

Case 1. Suppose that $A^{(n)} \neq 0$.

We have three situations according to $M$ and $N$.

a) If $M = N$, the system (5) is same as in IDDM, i.e. $A$ is a regular matrix and its solution has been given by equation (4).

b) If $M < N$, then the solution of (5) is expressed by substituting $X^{(k+1)}$ solution into $X^{(k)}$ solution for $k = n(-1)1$ as follows

$$X = \sum_{i=1}^{n} \left( \prod_{j=1}^{i-1} R^{(j)} \right) X_0^{(i)} + \left( \prod_{j=1}^{n} R^{(j)} \right) X^{(n+1)},$$

where $X^{(n+1)} = \begin{pmatrix} x_1^{(n+1)} & x_2^{(n+1)} & \ldots & x_{N_{n+1}}^{(n+1)} \end{pmatrix}^T$ and $x_j^{(n+1)}$ $(j = 1(1)N_{n+1})$ are the arbitrary parameters.

c) If $M > N$, then the system

$$A^{(n)} X^{(n)} = f^{(n)}$$

is obtained, where $A^{(n)}$ is a $M_n \times 1$ - matrix, $f^{(n)}$ is a $M_n$ -vector and $X^{(n)}$ is a 1 -vector given as

$$A^{(n)} = \begin{pmatrix} a_{11}^{(n)} \\ a_{21}^{(n)} \\ \vdots \\ a_{M_n 1}^{(n)} \end{pmatrix}, f^{(n)} = \begin{pmatrix} f_1^{(n)} \\ f_2^{(n)} \\ \vdots \\ f_{M_n}^{(n)} \end{pmatrix}, X^{(n)} = \begin{pmatrix} x_1^{(n)} \end{pmatrix}.$$

Here, if $A^{(n)} = \frac{a_{11}^{(n)}}{f_1^{(n)}} f^{(n)}$ $(f^{(n)} \neq 0)$, $X^{(n)} = X_0^{(n)} = \left( \frac{f_1^{(n)}}{a_{11}^{(n)}} \right)$ and if $f^{(n)} = 0$, $X^{(n)} = X_0^{(n)} = 0$. Therefore, the solution of (5) is

$$X = \sum_{i=1}^{n} \left( \prod_{j=1}^{i-1} R^{(j)} \right) X_0^{(i)}.$$

But, if $A^{(n)} \neq \frac{a_{11}^{(n)}}{f_1^{(n)}} f^{(n)}$ $(f^{(n)} \neq 0)$, the equation (5) has no solution.

Case 2. Suppose that $A^{(k)} \neq 0$ $(k < n)$ and $M_k = 1$ or $p_k$.

In this case; for all of the situations of $M$ and $N$, the solution of the system (5) is obtained as follows

$$X = \sum_{i=1}^{k} \left( \prod_{j=1}^{i-1} R^{(j)} \right) X_0^{(i)} + \left( \prod_{j=1}^{k} R^{(j)} \right) X^{(k+1)},$$

where $X^{(k+1)} = \begin{pmatrix} x_1^{(k+1)} & x_2^{(k+1)} & \dots & x_{N_{k+1}}^{(k+1)} \end{pmatrix}^T$; $x_j^{(k+1)}$ $(j = 1(1)N_{k+1})$ are the arbitrary parameters and $f_i^{(k)} = 0, i = 1(1)p-1$ for $p > 1$.

Case 3. Suppose that $A^{(k)} = 0$ $(k \leq n)$.

a) If $f^{(k)} = 0$, then the solution of the system (5) is obtained as follows

$$X = \sum_{i=1}^{k-1} \left( \prod_{j=1}^{i-1} R^{(j)} \right) X_0^{(i)} + \left( \prod_{j=1}^{k-1} R^{(j)} \right) X^{(k)},$$

where $X^{(k)} = \begin{pmatrix} x_1^{(k)} & x_2^{(k)} & \dots & x_{N_k}^{(k)} \end{pmatrix}^T$; $x_j^{(k)}$ $(j = 1(1)N_k)$ are the arbitrary parameters and $f_i^{(k)} = 0,\ i = 1(1)p-1$for $p > 1$.

b) If $f^{(k)} \neq 0$, then (5) is an inconsistent system and has no solution.

## 3. Generalized Iterative Decreasing Dimension Algorithm (GIDDA)

Here, we are going to give an algorithm based on GIDDM. GIDDA is the modification of the algorithm IDDA given in [3].

**Input.** $A$ - $M \times N$ matrix, $f$ - $M$-vector.

Step 1. Get $n = \min\{M, N\}$, $A^{(1)} = A, f^{(1)} = f$.

Step 2. $k = 1(1)n-1$,

2.1. Calculate $A^{(k)}$, $f^{(k)}$, $M_k$ and $N_k$.

2.2. Control if $a_{ij}^{(k)} \neq 0$ for $i = 1(1)M_k$, $j = 1(1)N_k$; let $a_{ps}^{(k)} \neq 0$ is first element and take $p = p_k$. Otherwise go Step 4.

2.3. If $p > 1$, control if $f_i^{(k)} = 0$ for $i = 1(1)p-1$. If $\exists i \ni f_i^{(k)} \neq 0$ then go Output 2.

2.4. If $M_k = 1$ or $p_k$, calculate $X_0^{(k)}$ , $R^{(k)}$, $m = k$ and go Output 1.

2.5. Determine $A_1^{(k)}$, $A_2^{(k)}$, $u^{(k)}$, $v^{(k)}$.

2.6. Calculate $X_0^{(k)}$ and $R^{(k)}$.

Step 3. For $k = n$ calculate $A^{(k)}$, $f^{(k)}$, $M_k$ and $N_k$.

3.1. Control if $a_{ij}^{(k)} \neq 0$ for $i = 1(1)M_k$, $j = 1(1)N_k$; let $a_{ps}^{(k)} \neq 0$ is first element and take $p = p_k$. Otherwise go Step 4.

3.2. If $M < N$ , calculate $X_0^{(k)}$, $R^{(k)}$, take $m = k$ and go Output 1

3.3. If $A^{(k)} = \frac{a_{11}^{(k)}}{f_1^{(k)}} f^{(k)}$ or $f^{(k)} = 0$, calculate $X_0^{(k)}$, take $m = k$ and go Output 1.

3.4. If $A^{(k)} \neq \frac{a_{11}^{(k)}}{f_1^{(k)}} f^{(k)}$, go Output 2.

Step 4. Control if $f_i^{(k)} = 0$ for $i = 1(1)M_k$. If $\exists i \ni f_i^{(k)} \neq 0$ then go Output 2.

4.1. Take $m = k - 1$ and go Output 1.

**Output 1.** $X = \sum_{i=1}^{m} \left( \prod_{j=1}^{i-1} R^{(j)} \right) X_0^{(i)} + \left( \prod_{j=1}^{m} R^{(j)} \right) X^{(m+1)}$.

**Output 2.** No Solution.

**Note:** Vector $X^{(m+1)}$ in Output 1 is a parametric vector in $N_{m+1}$-dimension, i.e. $x_j^{(m+1)}$ $(j = 1(1)N_m)$ are the arbitrary parameters, if $N_{m+1} \neq 0$. If $N_{m+1} = 0$, $X^{(m+1)} = 0$.

Now, we are going to give some examples solved using algorithm GIDDA.

**Example 1.** ***Input***: $A = \begin{pmatrix} 1 & -2 & 2 & 3 \\ 2 & 1 & 1 & -1 \\ 3 & -1 & 3 & 2 \\ 5 & 0 & 4 & 1 \end{pmatrix}$, $f = \begin{pmatrix} 1 \\ -1 \\ 0 \\ -1 \end{pmatrix}$.

*Step 1.* $n = \min\{4, 4\} = 4$.

*Step 2.* $X_0^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, R^{(1)} = \begin{pmatrix} 2 & -2 & -3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$X_0^{(2)} = \begin{pmatrix} \frac{-3}{5} \\ 0 \\ 0 \end{pmatrix}, R^{(2)} = \begin{pmatrix} \frac{3}{5} & \frac{7}{5} \\ 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

$$A^{(3)} = 0,\ f^{(3)} = 0 \textit{ and } m = 2 \Rightarrow X^{(3)} = \begin{pmatrix} x_1^{(3)} \\ x_2^{(3)} \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}, a, b \in \mathbb{R}$$

***Output.*** *Solution* $X = \begin{pmatrix} -\frac{1}{5} - \frac{4}{5}a - \frac{1}{5}b \\ -\frac{3}{5} + \frac{3}{5}a + \frac{4}{5}b \\ a \\ b \end{pmatrix}$.

**Example 2.** ***Input***: $A = \begin{pmatrix} 1 & 2 \\ 1 & -2 \\ 2 & 3 \\ 1 & 1 \end{pmatrix}$, $f = \begin{pmatrix} 3 \\ 4 \\ \frac{25}{4} \\ \frac{13}{4} \end{pmatrix}$.

*Step 1.* $n = \min\{4, 2\} = 2$.

*Step 2.* $X_0^{(1)} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, R^{(1)} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

$$A^{(2)} = \frac{a_{11}^{(2)}}{f_1^{(2)}} f^{(2)} \text{ and } m = 2 \Rightarrow X_0^{(2)} = \begin{pmatrix} -\frac{1}{4} \end{pmatrix}$$

***Output.*** *Solution* $X = \begin{pmatrix} \frac{7}{2} \\ -\frac{1}{4} \end{pmatrix}$.

**Example 3.** ***Input****:* $A = \begin{pmatrix} 1 & 2 \\ 1 & -2 \\ 2 & 3 \\ 1 & 1 \end{pmatrix}, f = \begin{pmatrix} 3 \\ 1 \\ 5 \\ 2 \end{pmatrix}$.

*Step 1.* $n = \min\{4, 2\} = 2$.

*Step 2.* $X_0^{(1)} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, R^{(1)} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

$$A^{(2)} = \begin{pmatrix} -4 \\ -1 \\ -1 \end{pmatrix}, f^{(2)} = \begin{pmatrix} -2 \\ -1 \\ -1 \end{pmatrix}, A^{(2)} \neq \frac{a_{11}^{(2)}}{f_1^{(2)}} f^{(2)}$$

***Output.*** *No Solution.*

## 4. Maple Procedure for GIDDA

```
>#A Maple Procedure: To compute the solution of the given linear system.
>restart;
>with(LinearAlgebra, Multiply);
>with(linalg, coldim, rowdim, blockmatrix, vectdim);
>gidda:=proc(A::Matrix, f::Vector)

global n,B,M,N,u,v,A1,A2,X0,RR,X,S,XS;
local g,m,z,i,j,p,s,k,t,r,R,H1,H2,B1,B2,B3,B4,B5,B6, Output1, Output2, Step4,
   CalculateX0, CalculateR, Find\_ps, bul;
Output1:= proc()
RR[0]:= Matrix(N[1], N[1], shape = identity); X:= X0[1];
for i from 1 to m-1 do RR[i]:= R[i]; RR[i]:= Multiply(RR[i-1],RR[i]);
S[i]:= Multiply(RR[i], X0[i+1]); X:= X + S[i]; end do:
for i from 1 to m do RR[i]:= R[i]; RR[i]:= Multiply(RR[i-1], RR[i]); end do;
if N[m+1]=0 then XS:= Vector(1 ..coldim(RR[m]), 0);
else XS:= Vector(1 .. coldim(RR[m]), symbol = a); end if:
X:= X + Multiply(RR[m], XS); print(X); break;
end proc:
```

```
Output2:=proc()
printf("No solution"); break;
end proc:

Step4:=proc()
if verify(g[k],Vector(1..vectdim(g[k]),0),Vector)=false then Output2();
   end if:
m:=k-1; Output1();
end proc:

CalculateX0:=proc()
X0[k]:=Vector(1..N[k]); X0[k][s[k]]:=(g[k][p[k]])/(B[k][p[k],s[k]]); }
end proc:

Calculate R:=proc() r[k]:=Matrix(1,1..N[k]-s[k]);
for z from 1 to N[k]-s[k]
   do r[k][1,z]:=-((B[k][p[k],s[k]+z])/(B[k][p[k],s[k]])); end do:
H1:=Matrix(N[k]-1,N[k]-1,shape=identity); H2:=Matrix(1,N[k]-1,0);
B1:=Matrix(s[k]-1,s[k]-1,shape=identity); B2:=Matrix(s[k]-1,N[k]-s[k],0);
B3:=Matrix(1,s[k]-1,0); B4:=r[k]; B5:=Matrix(N[k]-s[k],s[k]-1,0);
B6:=Matrix(N[k]-s[k],N[k]-s[k],shape=identity);
if s[k]=1 then R[k]:=convert(blockmatrix(2,1,[r[k],H1]),Matrix);
   end if:
if s[k]=N[k] then R[k]:=convert(blockmatrix(2,1,[H1,H2]),Matrix);
   end if:
if (s[k]\texttt{>}=2 and s[k]\texttt{<}=N[k]-1) then
R[k]:=convert(blockmatrix(3,2,[B1,B2,B3,B4,B5,B6]),Matrix); end if:
end proc:

Find\_ps:=proc()
bul:=0; for i from 1 to M[k] do for j from 1 to N[k] do
if bul=0 and B[k][i,j]\texttt{<>}0 then p[k]:=i;s[k]:=j; bul:=1;
   end if:
end do: end do: if bul=0 then Step4(); end if:
end proc:

>#Main Procedure
M[1]:=rowdim(A); N[1]:=coldim(A); g[1]:=Vector(1..M[1]);
B[1]:=Matrix(1..M[1],1..N[1]); n:=min(M[1],N[1]); B[1]:=A; g[1]:=f;
for k from 1 to n-1 do
if k<>1 then M[k]:=rowdim(A)-sum(p[t],t=1..k-1); N[k]:=coldim(A)-k+1;
   end if:
```

```
Find\_ps();
if p[k]>1 then for i from 1 to p[k]-1 do if g[k][i]<>0 then Output2();
   end if: end do:
end if:
if M[k]=1 or M[k]=p[k] then CalculateX0(); CalculateR(); m:=k;
   Output1(); end if:
u[k]:=Vector(1..1); u[k]:=g[k][p[1]]; v[k]:=Vector(1..M[k]-p[k]);
for i from 1 to M[k]-p[k] do v[k][i]:=g[k][p[k]+i]; end do:
   A1[k]:=Matrix(1,1..N[k]);
for j from 1 to N[k] do A1[k][1,j]:=B[k][p[k],j]; end do:
A2[k]:=Matrix(1..(M[k]-p[k]),1..N[k]);
for i from 1 to M[k]-p[k] do for j from 1 to N[k] do
   A2[k][i,j]:=B[k][p[k]+i,j]; end do:
end do:
CalculateX0(); CalculateR(); B[k+1]:=Multiply(A2[k],R[k]);
g[k+1]:=v[k]-Multiply(A2[k],X0[k]);
end do:
k:=n; M[k]:=rowdim(A)-sum(p[t],t=1..k-1); N[k]:=coldim(A)-k+1;Find_ps();
if M[1]<N[1] then CalculateX0(); CalculateR(); m:=k;Output1(); end if:
if verify(convert(B[k],Vector),(B[k][1,1])/(g[k][1])*g[k],Vector) or
verify(g[k],Vector(1..vectdim(g[k]),0)) then CalculateX0(); m:=k;
   Output1(); else Output2(); end if:
end proc:
```

**Example 4.** `>A:=Matrix([[1,-2,2,3],[2,1,1,-1],[3,-1,3,2],[5,0,4,1]]);`

$$A = \begin{pmatrix} 1 & -2 & 2 & 3 \\ 2 & 1 & 1 & -1 \\ 3 & -1 & 3 & 2 \\ 5 & 0 & 4 & 1 \end{pmatrix}$$

`>f:=Vector([1,-1,0,-1]);`

$$f = \begin{pmatrix} 1 \\ -1 \\ 0 \\ -1 \end{pmatrix}$$

`>gidda(A,f);`

$$\begin{pmatrix} -\frac{1}{5} - \frac{4}{5}a_1 - \frac{1}{5}a_2 \\ -\frac{3}{5} + \frac{3}{5}a_1 + \frac{4}{5}a_2 \\ a_1 \\ a_2 \end{pmatrix}$$

**Example 5.** `>A:=Matrix([[1,2,-3,1,-1,-2,4],[2,4,-6,2,-2,-4,8],[3,6,-9,3,-3,-6,12],[1,-1,3,-2,0,1,2]]);`

$$A = \begin{pmatrix} 1 & 2 & -3 & 1 & -1 & -2 & 4 \\ 2 & 4 & -6 & 2 & -2 & -4 & 8 \\ 3 & 6 & -9 & 3 & -3 & -6 & 12 \\ 1 & -1 & 3 & -2 & 0 & 1 & 2 \end{pmatrix}$$

`>f:=Vector([4,8,12,1]);`

$$f = \begin{pmatrix} 4 \\ 8 \\ 12 \\ 1 \end{pmatrix}$$

`>gidda(A,f);`

$$\begin{pmatrix} -2 - a_1 + a_2 + \frac{1}{3}a_3 - \frac{8}{3}a_5 \\ 1 + 2a_1 - a_2 + \frac{1}{3}a_3 + a_4 - \frac{2}{3}a_5 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix}$$

## 5. Conclusion

GIDDM produces a special $X_0^{(k)}$ solutions and $R^{(k)}$ matrices by reducing the dimension of a given system of linear algebraic equation. It obtains the solution depending on $X_0^{(k)}$ and $R^{(k)}$. GIDDA is suited for implementation using computer algebra systems such as Maple and MATLAB.

## References

[1] G.H. Golub and J.M. Ortega. *Scientific Computing : An Introduction with Parallel Computing.* Academic Press, Boston, MA, 1993.

[2] G.H. Golub and C.F. van Loan. *Matrix Computations.* The Johns Hopkins University Press, Baltimore, MD, 1983.

[3] T. Keskin and K. Aydın. Iterative decreasing dimension algorithm. *Comp. and Math. with Appl.*, 53:1153–1158, 2007.

[4] H. Wang and J. Jiang. Solution of the system of linear algebraic equations by decreasing dimension. *App. Math. and Comput.*, 109:51–57, 2000.

[5] J. Zhang. Comments on Solution of the system of linear algebraic equations by decreasing dimension. *Appl. Math. and Comput.*, 128:95–98, 2002.