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# Almost Bi-Γ-Ideals and Fuzzy Almost Bi-Γ-Ideals of Γ-Semigroups

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**Abstract.** In this paper, we introduce the notions of almost bi-Γ-ideals and fuzzy almost bi-Γ-ideals of Γ-semigroups and give properties of them. Moreover, we investigate relationships between almost bi-Γ-ideals and fuzzy almost bi-Γ-ideals.

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**Key Words and Phrases**: bi-Γ-ideals, almost bi-Γ-ideals, fuzzy almost bi-Γ-ideals

## 1. Introduction and Preliminaries

Ideal theory in semigroups, like all other algebraic structures, plays an important role in studying them. Good and Hughes [8] introduced the notion of bi-ideals of semigroups in 1952. An introductory definition of left, right, two-sided almost ideals of semigroups was launched by Grosek and Satko [9] in 1980. They gave the characterization of these ideals when a semigroup $S$ contains no proper left, right, two-sided almost ideals in [9], and afterwards, they discovered the minimal almost ideals and the smallest almost ideals of semigroups in [10] and [11], respectively. In 1981, Bogdanovic [3] introduced the definition of almost bi-ideals in semigroups by using the definitions of almost ideals and bi-ideals in semigroups. In [5], Wattanatripop, Chinram and Changphas gave the properties of quasi-almost-ideals and first defined the concept of fuzzy almost ideals in semigroups. Moreover, they provided the relationships between almost ideals and their fuzzification. Furthermore, they investigated fuzzification of almost bi-ideals in semigroups in [4]. Almost $(m, n)$-ideals and their fuzzification in semigroups were studied by Suebsung, Wattanatripop and Chinram in [23]. Moreover, the idea of almost ideals and their fuzzification were extended to $n$-ary semigroups in [21].

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The notion of Γ-semigroups has been first studied by Sen [18] in 1981. In 1986, Sen and Saha [19] improved more general definition as follows:

**Definition 1.** ([19]) Let $M$ and $\Gamma$ be non-empty sets. $(M, \Gamma)$ is called a Γ-*semigroup* if it satisfies the following laws.

(1) $a\alpha b \in M$ for all $a, b \in M$ and $\alpha \in \Gamma$.

(2) $M$ is associative under $\Gamma$, that is

$$(a\alpha b)\beta c = a\alpha(b\beta c)$$

for all $a, b, c \in M$ and all $\alpha, \beta \in \Gamma$.

Every semigroup $(S, \cdot)$ can be considered as a Γ-semigroup $S$ by choosing $\Gamma = \{\cdot\}$. Then a Γ-semigroup is one of the generalizations of semigroups. The investigation on Γ-semigroups was done by certain mathematicians which are parallel to some results of semigroups, for example, one may see [6, 7, 17–19]. Similar to semigroups, ideal theory in Γ-semigroups plays an important role (for example, we can see in [1, 6, 7, 12–14, 20]). Let $M$ be a Γ-semigroup. For nonempty subsets $A$ and $B$ of $M$, let

$$A\Gamma B = \{a\alpha b \mid a \in A, b \in B, \alpha \in \Gamma\}.$$

If $m \in M$, we let $A\Gamma m = A\Gamma\{m\}$ and $m\Gamma A = \{m\}\Gamma A$. If $\alpha \in \Gamma$, we let

$$A\alpha B = \{a\alpha b \mid a \in A, b \in B\}.$$

**Definition 2.** (see [7]) Let $M$ be a Γ-semigroup.

(1) A nonempty subset $T$ of $M$ is called a *sub* Γ-*semigroup* of $M$ if $T\Gamma T \subseteq T$.

(2) A sub Γ-semigroup $B$ of $M$ is called a *bi*-Γ-*ideal* of $M$ if $B\Gamma M\Gamma B \subseteq B$.

A bi-Γ-ideal in Γ-semigroups was sometimes called a bi-ideal (see [14]). Some generalizations of this ideal were studied in [2] and [16]. Recently, Wattanatripop and Changphas first studied the concept of almost ideals in Γ-semigroups. In [22], they defined the definitions of left [right] almost ideals in Γ-semigroups. Moreover, a Γ-semigroup containing no proper left [right] almost ideals was characterized.

In 1965, Zadeh [24] introduced the concept of fundamental fuzzy sets. Since then, fuzzy sets have been studied in various fields. A function from a set $M$ into the closed unit interval $[0, 1]$ is called a fuzzy subset of $M$. Let $f$ and $g$ be any two fuzzy subsets of a set $M$.

(1) A fuzzy subset $f \cap g$ of $M$ is defined by

$$(f \cap g)(m) = \min\{f(m), g(m)\}$$

for all $m \in M$.

(2) A fuzzy subset $f \cup g$ of $M$ is defined by

$$(f \cup g)(m) = \max\{f(m), g(m)\}$$

for all $m \in M$.

(3) If $f(m) \leq g(m)$ for all $m \in M$, we say that $f$ is a subset of $g$, and use the notation $f \subseteq g$ and sometimes we will say that $f$ is contained in $g$.

For a fuzzy subset $f$ of any set $M$, the support of $f$ is the set of points in $M$ defined by

$$supp(f) = \{m \in M \mid f(m) \neq 0\}.$$

For a subset $A$ of any set $M$, the characteristic function $\chi_A$ of $A$ is a fuzzy subset of $M$ defined by

$$\chi_A(m) = \begin{cases} 1 & m \in A, \\ 0 & m \notin A. \end{cases}$$

For any element $m$ of any set $M$ and $t \in (0, 1]$, a fuzzy point $m_t$ of $M$ is a fuzzy subset of $M$ defined by

$$m_t(x) = \begin{cases} t & x = m, \\ 0 & x \neq m \end{cases}$$

(see [15]).

## 2. Almost bi-Γ-ideals

First, we define almost bi-Γ-ideals of Γ-semigroups as follows:

**Definition 3.** A non-empty subset $B$ of a Γ-semigroup $M$ is called an *almost bi-*Γ*-ideal* of $S$ if

$$B\Gamma m\Gamma B \cap B \neq \emptyset$$

for all $m \in M$.

**Example 1.** Let $B$ be any bi-Γ-ideal of a Γ-semigroup $M$. Then $B\Gamma M\Gamma B \subseteq B$. This implies that for any $m \in M, B\Gamma m\Gamma B \subseteq B\Gamma M\Gamma B \subseteq B$. So $B\Gamma m\Gamma B \cap B = B\Gamma m\Gamma B \neq \emptyset$ for all $m \in M$. Then $B$ is an almost bi-Γ-ideal of $M$.

By Example 1, we conclude that every bi-Γ-ideal of a Γ-semigroup $M$ is an almost bi-Γ-ideal of $M$.

**Example 2.** Consider the Γ-semigroup $\mathbb{Z}_8$ with $\Gamma = \left\{\overline{0}, \overline{1}, \overline{2}\right\}$ under the usual addition. Let $B = \{\overline{4}, \overline{6}\}$. We see that

$$\begin{aligned}
(B+\Gamma+\overline{0}+\Gamma+B)\cap B &= \mathbb{Z}_8\cap\{\overline{4},\overline{6}\}\neq\emptyset,\\
(B+\Gamma+\overline{1}+\Gamma+B)\cap B &= \mathbb{Z}_8\cap\{\overline{4},\overline{6}\}\neq\emptyset,\\
(B+\Gamma+\overline{2}+\Gamma+B)\cap B &= \mathbb{Z}_8\cap\{\overline{4},\overline{6}\}\neq\emptyset,\\
(B+\Gamma+\overline{3}+\Gamma+B)\cap B &= \mathbb{Z}_8\cap\{\overline{4},\overline{6}\}\neq\emptyset,\\
(B+\Gamma+\overline{4}+\Gamma+B)\cap B &= \mathbb{Z}_8\cap\{\overline{4},\overline{6}\}\neq\emptyset,\\
(B+\Gamma+\overline{5}+\Gamma+B)\cap B &= \mathbb{Z}_8\cap\{\overline{4},\overline{6}\}\neq\emptyset,\\
(B+\Gamma+\overline{6}+\Gamma+B)\cap B &= \mathbb{Z}_8\cap\{\overline{4},\overline{6}\}\neq\emptyset,\\
(B+\Gamma+\overline{7}+\Gamma+B)\cap B &= \mathbb{Z}_8\cap\{\overline{4},\overline{6}\}\neq\emptyset.
\end{aligned}$$

Therefore, $B$ is an almost bi-$\Gamma$-ideal of $\mathbb{Z}_8$. However, $B$ is not a bi-$\Gamma$-ideal of $\mathbb{Z}_8$ because $B+\Gamma+\mathbb{Z}_8+\Gamma+B=\mathbb{Z}_8\nsubseteq B$.

From Example 2, we see that an almost bi-$\Gamma$-ideal of $\Gamma$-semigroup $S$ need not be a bi-$\Gamma$-ideal of $S$.

**Example 3.** Consider the $\Gamma$-semigroup $M=\{a,b,c,d\}$ with $\Gamma=\{\alpha,\beta\}$ and the multiplication table:

| $\alpha$ | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|
| $a$ | $a$ | $c$ | $c$ | $a$ |
| $b$ | $c$ | $a$ | $a$ | $c$ |
| $c$ | $c$ | $a$ | $a$ | $c$ |
| $d$ | $a$ | $c$ | $c$ | $a$ |

| $\beta$ | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|
| $a$ | $c$ | $a$ | $a$ | $c$ |
| $b$ | $a$ | $c$ | $c$ | $a$ |
| $c$ | $a$ | $c$ | $c$ | $a$ |
| $d$ | $c$ | $a$ | $a$ | $c$ |

Let $B=\{a,c\}$. Then

$$\begin{aligned}
B\Gamma a\Gamma B\cap B &= \{a,c\}\cap\{a,c\}=\{a,c\}\neq\emptyset,\\
B\Gamma b\Gamma B\cap B &= \{a,c\}\cap\{a,c\}=\{a,c\}\neq\emptyset,\\
B\Gamma c\Gamma B\cap B &= \{a,c\}\cap\{a,c\}=\{a,c\}\neq\emptyset,\\
B\Gamma d\Gamma B\cap B &= \{a,c\}\cap\{a,c\}=\{a,c\}\neq\emptyset.
\end{aligned}$$

Therefore, $B$ is an almost bi-$\Gamma$-ideal of $M$.

**Theorem 1.** *Assume that $B$ is an almost bi-$\Gamma$-ideal of a $\Gamma$-semigroup $M$. If $A$ is any subset of $M$ containing $B$, then $A$ is also an almost bi-$\Gamma$-ideal of $M$.*

*Proof.* Since $B$ is an almost bi-$\Gamma$-ideal of $M$ and $B\subseteq A$, we have $B\Gamma m\Gamma B\cap B\neq\emptyset$ and $B\Gamma m\Gamma B\cap B\subseteq A\Gamma m\Gamma A\cap A$ for all $m\in M$, respectively. This implies that $A\Gamma m\Gamma A\cap A\neq\emptyset$ for all $m\in M$. Therefore, $A$ is an almost bi-$\Gamma$-ideal of $M$.

**Corollary 1.** *The union of any two almost bi-$\Gamma$-ideals of a $\Gamma$-semigroup $M$ is also an almost bi-$\Gamma$-ideal of $M$.*

*Proof.* Let $A$ and $B$ be any two almost bi-$\Gamma$-ideals of $M$. Since $A\subseteq A\cup B\subseteq M$, it follows from Theorem 1 that $A\cup B$ is an almost bi-$\Gamma$-ideal of $M$.

**Example 4.** Consider the Γ-semigroup $\mathbb{Z}_8$ with $\Gamma = \{\overline{0}, \overline{1}, \overline{2}\}$ under the usual addition. Let $A = \{\overline{2}, \overline{3}\}$ and $B = \{\overline{4}, \overline{6}\}$. Clearly, $A$ and $B$ are almost bi-Γ-ideals of $\mathbb{Z}_8$ but $A \cap B = \emptyset$, so it is not an almost bi-Γ-ideal of $\mathbb{Z}_8$.

By Example 4, we have the following remark.

**Remark 1.** *The intersection of any two almost bi-Γ-ideals of a Γ-semigroup $M$ need not be an almost bi-Γ-ideal of $M$.*

**Theorem 2.** *A Γ-semigroup $M$ contains a proper almost bi-Γ-ideal if and only if there exists an element $m$ of $M$ such that $M \setminus \{m\}$ is an almost bi-Γ-ideal of $M$.*

*Proof.* Assume that a Γ-semigroup $M$ contains a proper almost bi-Γ-ideal $B$ and let $m \in M \setminus B$. Then $B \subseteq M \setminus \{m\} \subset M$. By Theorem 1, $M \setminus \{m\}$ is an almost bi-Γ-ideal of $M$.

Conversely, let $m \in M$ be such that $M \setminus \{m\}$ is an almost bi-Γ-ideal of $M$. Since $M \setminus \{m\} \subsetneq M$, we get $M \setminus \{m\}$ is a proper almost bi-Γ-ideal of $M$.

**Theorem 3.** *Let $M$ be a Γ-semigroup such that $|M| > 1$. Then $M$ has no proper almost bi-Γ-ideals if and only if for all $m \in M$ there exists $a \in M$ such that*

$$(M \setminus \{m\})\Gamma a\Gamma(M \setminus \{m\}) = \{m\}.$$

*Proof.* Assume that $M$ has no proper almost bi-Γ-ideals and let $m \in M$. By Theorem 2, $M \setminus \{m\}$ is not an almost bi-Γ-ideal of $M$. Thus there exists an element $a$ of $M$ such that $(M \setminus \{m\})\Gamma a\Gamma(M \setminus \{m\}) \cap (M \setminus \{m\}) = \emptyset$. Hence, $(M \setminus \{m\})\Gamma a\Gamma(M \setminus \{m\}) = \{m\}$.

Conversely, suppose $M$ contains a proper almost bi-Γ-ideal $B$. Let $m \in M \setminus B$. By assumption, we have $(M \setminus \{m\})\Gamma a\Gamma(M \setminus \{m\}) = \{m\}$ for some element $a$ in $M$. Since $B \subseteq M \setminus \{m\} \subset M$, we get $M \setminus \{m\}$ is an almost bi-Γ-ideal of $M$ by Theorem 1. This implies that $\emptyset = \{m\} \cap (M \setminus \{m\}) = (M \setminus \{m\})\Gamma a\Gamma(M \setminus \{m\}) \cap (M \setminus \{m\}) \neq \emptyset$, which is a contradiction. Therefore, $M$ has no proper almost bi-Γ-ideals.

## 3. Fuzzy almost bi-Γ-ideals

For a Γ-semigroup $M$, let $\mathcal{F}(M)$ be the set of all fuzzy subsets of $M$. For each $\alpha \in \Gamma$, define a binary operation $\circ_\alpha$ on $\mathcal{F}(M)$ by

$$(f \circ_\alpha g)(m) = \begin{cases} \sup\limits_{m = a\alpha b} \{\min\{f(a), g(b)\}\} & \text{if } m \in M\alpha M, \\ 0 & \text{otherwise.} \end{cases}$$

Let $\Gamma^\star := \{\circ_\alpha \mid \alpha \in \Gamma\}$. Then $(\mathcal{F}(M), \Gamma^\star)$ is a Γ-semigroup.

**Proposition 1.** *For fuzzy subsets $f$ and $g$ of a Γ-semigroup $M$ such that $f \subseteq g$ and $\alpha \in \Gamma$, if $h$ is any fuzzy subset of $M$, then $h \circ_\alpha f \subseteq h \circ_\alpha g$ and $f \circ_\alpha h \subseteq g \circ_\alpha h$.*

We define fuzzification of almost bi-Γ-ideals in Γ-semigroups as follows:

**Definition 4.** A fuzzy subset $f$ of a Γ-semigroup $M$ is called a *fuzzy almost bi-Γ-ideal* of $M$ if for all fuzzy points $m_t$ of $M$, there exist $\alpha, \beta \in \Gamma$ such that $(f \circ_\alpha m_t \circ_\beta f) \cap f \neq 0$.

**Theorem 4.** *Assume that $f$ and $g$ are fuzzy subsets of a Γ-semigroup $M$ such that $f \subseteq g$. If $f$ is a fuzzy almost bi-Γ-ideal of $M$, then $g$ is also a fuzzy almost bi-Γ-ideal of $M$.*

*Proof.* Since $f$ is a fuzzy almost bi-Γ-ideal of $M$, for each fuzzy point $m_t$ of $M$, there exist $\alpha, \beta \in \Gamma$ such that $(f \circ_\alpha m_t \circ_\beta f) \cap f \neq 0$. We have that $(f \circ_\alpha m_t \circ_\beta f) \cap f \subseteq (g \circ_\alpha m_t \circ_\beta g) \cap g$, this implies that $(g \circ_\alpha m_t \circ_\beta g) \cap g \neq 0$. Hence, $g$ is also a fuzzy almost bi-Γ-ideal of $M$.

**Corollary 2.** *If $f$ and $g$ are fuzzy almost bi-Γ-ideals of a Γ-semigroup $M$, then $f \cup g$ is also a fuzzy almost bi-Γ-ideal of $M$.*

*Proof.* It follows by Theorem 4 because of $f \subseteq f \cup g$.

**Example 5.** Consider the Γ-semigroup $\mathbb{Z}_5$ where $\Gamma = \{\overline{0}\}$ and $\overline{a}\gamma\overline{b} := \overline{a} + \gamma + \overline{b}$. Let $f$ and $g$ be fuzzy subsets of $\mathbb{Z}_5$ defined by

$$f(\overline{0}) = 0, f(\overline{1}) = 0.5, f(\overline{2}) = 0, f(\overline{3}) = 0.1, f(\overline{4}) = 0.4$$

and

$$g(\overline{0}) = 0, g(\overline{1}) = 0.3, g(\overline{2}) = 0.7, g(\overline{3}) = 0, g(\overline{4}) = 0.2.$$

It is easy to check that $[(f \circ_\alpha m_t \circ_\beta f) \cap f](\overline{4}) \neq 0$ and $[(g \circ_\alpha m_t \circ_\beta g) \cap g](\overline{4}) \neq 0$ for all $\alpha, \beta \in \Gamma, m \in \mathbb{Z}_5$ and $t \in (0, 1]$. So $f$ and $g$ are fuzzy almost bi-Γ-ideals of $\mathbb{Z}_5$.

From the definition of the intersection of two fuzzy subsets, we have

$$(f \cap g)(\overline{0}) = 0, (f \cap g)(\overline{1}) = 0.3, (f \cap g)(\overline{2}) = 0, (f \cap g)(\overline{3}) = 0, (f \cap g)(\overline{4}) = 0.2.$$

We can easily to check that $[((f \cap g) \circ_\alpha \overline{0}_t \circ_\beta (f \cap g)) \cap (f \cap g)](a) = 0$ for all $\alpha, \beta \in \Gamma, t \in (0, 1]$ and $a \in \mathbb{Z}_5$, so $f \cap g$ is not a fuzzy almost bi-Γ-ideal of $\mathbb{Z}_5$.

The following remark follows from Example 5.

**Remark 2.** *The intersection of two fuzzy almost bi-Γ-ideals of a Γ-semigroup $M$ need not be a fuzzy almost bi-Γ-ideal of $M$.*

## 4. Relationships between almost bi-Γ-ideals and their fuzzification

**Theorem 5.** *A non-empty subset $B$ of a Γ-semigroup $M$ is an almost bi-Γ-ideal of $M$ if and only if $\chi_B$ is a fuzzy almost bi-Γ-ideal of $M$.*

*Proof.* Assume that $B$ is an almost bi-$\Gamma$-ideal of a $\Gamma$-semigroup $M$ and let $m_t$ be any fuzzy point of $M$. Then $B\Gamma m\Gamma B \cap B \neq \emptyset$. Thus there exists $b \in B$ such that $b \in B\alpha m\beta B$ for some $\alpha, \beta \in \Gamma$. This implies that $(\chi_B \circ_\alpha m_t \circ_\beta \chi_B)(b) \neq 0$ and $\chi_B(b) \neq 0$. Hence, $(\chi_B \circ_\alpha m_t \circ_\beta \chi_B) \cap \chi_B \neq 0$. Therefore, $\chi_B$ is a fuzzy almost bi-$\Gamma$-ideal of $M$.

To prove the converse, we assume that $\chi_B$ is a fuzzy almost bi-$\Gamma$-ideal of $M$ and let $m \in M$. Then there exist $\alpha, \beta \in \Gamma$ such that $(\chi_B \circ_\alpha m_t \circ_\beta \chi_B) \cap \chi_B \neq 0$, so $[(\chi_B \circ_\alpha m_t \circ_\beta \chi_B) \cap \chi_B](y) \neq 0$ for some $y \in M$. Hence, $y \in B$ and $y = a\alpha m\beta b$ for some $a, b \in B$ and $\alpha, \beta \in \Gamma$. Therefore, $y \in B\Gamma m\Gamma B \cap B$. So $B\Gamma m\Gamma B \cap B \neq \emptyset$. Consequently, $B$ is an almost bi-$\Gamma$-ideal of $M$

**Theorem 6.** *A fuzzy subset $f$ of a $\Gamma$-semigroup $M$ is a fuzzy almost bi-$\Gamma$-ideal of $M$ if and only if supp$(f)$ is an almost bi-$\Gamma$-ideal of $M$.*

*Proof.* Assume that $f$ is a fuzzy almost bi-$\Gamma$-ideal of a $\Gamma$-semigroup $M$ and let $m \in M$ and $t \in (0, 1]$. Then there exist $\alpha, \beta \in \Gamma$ such that $(f \circ_\alpha m_t \circ_\beta f) \cap f \neq 0$. Hence, $[(f \circ_\alpha m_t \circ_\beta f) \cap f](x) \neq 0$ for some $x \in M$. So there exist $y_1, y_2 \in S$ such that $x = y_1\alpha m\beta y_2, f(x) \neq 0, f(y_1) \neq 0$ and $f(y_2) \neq 0$. That is $x, y_1, y_2 \in supp(f)$. Thus $[\chi_{supp(f)} \circ_\alpha s_t \circ_\beta \chi_{supp(f)}](x) \neq 0$ and $\chi_{supp(f)}(x) \neq 0$. Therefore, $(\chi_{supp(f)} \circ_\alpha m_t \circ_\beta \chi_{supp(f)}) \cap \chi_{supp(f)} \neq 0$. Hence, $\chi_{supp(f)}$ is a fuzzy almost bi-$\Gamma$-ideal of $M$. By Theorem 5, $supp(f)$ is an almost bi-$\Gamma$-ideal of $M$.

On the other hand, we assume that $supp(f)$ is an almost bi-$\Gamma$-ideal of $M$. It follows from Theorem 5 that $\chi_{supp(f)}$ is a fuzzy almost bi-$\Gamma$-ideal of $M$. Let $m_t$ be any fuzzy point of $M$. Thus, $(\chi_{supp(f)} \circ_\alpha m_t \circ_\beta \chi_{supp(f)}) \cap \chi_{supp(f)} \neq 0$ for some $\alpha, \beta \in \Gamma$. Then there exists an element $x$ in $M$ such that $[(\chi_{supp(f)} \circ_\alpha m_t \circ_\beta \chi_{supp(f)}) \cap \chi_{supp(f)}](x) \neq 0$. Therefore, $(\chi_{supp(f)} \circ_\alpha m_t \circ_\beta \chi_{supp(f)})(x) \neq 0$ and $\chi_{supp(f)}(x) \neq 0$. Then there exist $y_1, y_2 \in M$ such that $x = y_1\alpha m\beta y_2, f(x) \neq 0, f(y_1) \neq 0$ and $f(y_2) \neq 0$. This means that $(f \circ_\alpha m_t \circ_\beta f) \cap f \neq 0$. We conclude that $f$ is a fuzzy almost bi-$\Gamma$-ideal of $M$.

Next, we will study the minimality of fuzzy almost bi-$\Gamma$-ideals.

**Definition 5.** A fuzzy almost bi-$\Gamma$-ideal $f$ of a $\Gamma$-semigroup $M$ is called *minimal* if for all fuzzy almost bi-$\Gamma$-ideal $g$ of $M$ contained in $f$, we must have $supp(g) = supp(f)$.

Now, we provide the relationship between minimal almost bi-$\Gamma$-ideals and their fuzzification.

**Theorem 7.** *A non-empty subset $A$ of a $\Gamma$-semigroup $M$ is a minimal almost bi-$\Gamma$-ideal of $M$ if and only if $\chi_A$ is a minimal fuzzy almost bi-$\Gamma$-ideal of $M$.*

*Proof.* Let $A$ be a minimal almost bi-$\Gamma$-ideal of a $\Gamma$-semigroup $M$. By Theorem 5, we have that $\chi_A$ is a fuzzy almost bi-$\Gamma$-ideal of $M$. Assume that $g$ is a fuzzy almost bi-$\Gamma$-ideal of $M$ contained in $\chi_A$. Thus, $supp(g) \subseteq supp(\chi_A) = A$. Because of $g \subseteq \chi_{supp(g)}$, we have $(g \circ_\alpha m_t \circ_\beta g) \cap g \subseteq (\chi_{supp(g)} \circ_\alpha m_t \circ_\beta \chi_{supp(g)}) \cap \chi_{supp(g)}$ for all fuzzy points $m_t$ of $M$. Thus $\chi_{supp(g)}$ is a fuzzy almost bi-$\Gamma$-ideal of $M$. By Theorem 5, $supp(g)$ is an almost bi-$\Gamma$-ideal of $M$. Because of $A$ is a minimal, then $supp(g) = A = supp(\chi_A)$. Therefore, $\chi_A$ is minimal.

To prove the converse, assume that $\chi_A$ is a minimal fuzzy almost bi-$\Gamma$-ideal of $M$ and $B$ is an almost bi-$\Gamma$-ideal of $M$ contained in $A$. Then $\chi_B$ is a fuzzy almost bi-$\Gamma$-ideal of $M$ and $\chi_B \subseteq \chi_A$. Thus, $B = supp(\chi_B) = supp(\chi_A) = A$. We conclude that $A$ is minimal.

**Corollary 3.** *A* $\Gamma$*-semigroup* $M$ *has no proper almost bi-*$\Gamma$*-ideals if and only if for all fuzzy almost bi-*$\Gamma$*-ideal* $f$ *of* $M$*,* $supp(f) = M$*.*

*Proof.* Assume that $M$ has no proper almost bi-$\Gamma$-ideals and let $f$ be a fuzzy almost bi-$\Gamma$-ideal of $M$. By Theorem 6, we have $supp(f)$ is almost bi-$\Gamma$-ideal of $M$. Thus $supp(f) = M$.

To prove the converse, we let $B$ be any almost bi-$\Gamma$-ideal of $M$. Follow by Theorem 5, we have that $\chi_B$ is a fuzzy almost bi-$\Gamma$-ideal of $M$. By assumption, we get $B = supp(\chi_B) = M$. This implies that $M$ has no proper almost bi-$\Gamma$-ideals.

**Definition 6.** Let $M$ be a $\Gamma$-semigroup and $\alpha \in \Gamma$.

(1) An almost bi-$\Gamma$-ideal $B$ of $M$ is called *$\alpha$-prime* if

$$x\alpha y \in B \Rightarrow x \in B \text{ or } y \in B$$

for any $x, y \in M$.

(2) A fuzzy almost bi-$\Gamma$-ideal $f$ of $M$ is called *$\alpha$-prime* if

$$f(x\alpha y) \leq \max\{f(x), f(y)\}$$

for any $x, y \in M$.

Next, we investigate relationship between $\alpha$-prime almost bi-$\Gamma$-ideals and their fuzzificaion.

**Theorem 8.** *A nonempty subset* $A$ *of a* $\Gamma$*-semigroup* $M$ *is an* $\alpha$*-prime almost bi-*$\Gamma$*-ideal of* $M$ *if and only if* $\chi_A$ *is an* $\alpha$*-prime fuzzy almost bi-*$\Gamma$*-ideal of* $M$*.*

*Proof.* Let $A$ be any $\alpha$-prime almost bi-$\Gamma$-ideal of $M$. Then $\chi_A$ is a fuzzy almost bi-$\Gamma$-ideal of $M$ by Theorem 5. Let $x$ and $y$ be elements in $M$. If $x\alpha y \in A$, then $x \in A$ or $y \in A$. This implies that

$$\chi_A(x\alpha y) = 1 \leq \max\{\chi_A(x), \chi_A(y)\}.$$

If $x\alpha y \notin A$, then

$$\chi_A(x\alpha y) = 0 \leq \max\{\chi_A(x), \chi_A(y)\}.$$

We conclude that $\chi_A(x\alpha y) \leq \max\{\chi_A(x), \chi_A(y)\}$ for all $x, y \in M$. Therefore, $\chi_A$ is an $\alpha$-prime fuzzy almost bi-$\Gamma$-ideal of $M$.

To prove the converse, suppose that $\chi_A$ is an $\alpha$-prime fuzzy almost bi-$\Gamma$-ideal of $M$. By Theorem 5, we have that $A$ is an almost bi-$\Gamma$-ideal of $M$. Let $x$ and $y$ be elements in $M$ such that $x\alpha y \in A$. Thus, $\chi_A(x\alpha y) = 1$. By assumption, we have that $\chi_A(x\alpha y) \leq \max\{\chi_A(x), \chi_A(y)\}$. Therefore, $\max\{\chi_A(x), \chi_A(y)\} = 1$. We can conclude that $x \in A$ or $y \in A$. Hence, $A$ is an $\alpha$-prime almost bi-$\Gamma$-ideal of $M$.

**Definition 7.** Let $M$ be a $\Gamma$-semigroup and $\alpha \in \Gamma$.

(1) An almost bi-$\Gamma$-ideal $A$ of $M$ is called *$\alpha$-semiprime* if

$$m\alpha m \in A \Rightarrow m \in A$$

for all $m \in M$.

(2) A fuzzy almost bi-$\Gamma$-ideal $f$ of $M$ is called *$\alpha$-semiprime* if

$$f(m\alpha m) \leq f(m)$$

for all $m \in M$.

Finally, we give relationship between $\alpha$-semiprime almost bi-$\Gamma$-ideals and their fuzzification.

**Theorem 9.** *A nonempty subset $A$ of a $\Gamma$-semigroup $M$ is an $\alpha$-semiprime almost bi-$\Gamma$-ideal of $M$ if and only if $\chi_A$ is an $\alpha$-semiprime fuzzy almost bi-$\Gamma$-ideal of $M$.*

*Proof.* Let $A$ be an $\alpha$-semiprime almost bi-$\Gamma$-ideal of $M$. By Theorem 5, $\chi_A$ is a fuzzy almost bi-$\Gamma$-ideal of $M$. Let $m \in M$. If $m\alpha m \in A$, then $m \in A$. So, $\chi_A(m) = 1$. Hence, $\chi_A(m\alpha m) \leq \chi_A(m)$. If $m\alpha m \notin A$, then $\chi_A(m\alpha m) = 0 \leq \chi_A(m)$. By both cases, we conclude that $\chi_A(m\alpha m) \leq \chi_A(m)$ for all $m \in M$. Thus, $\chi_A$ is an $\alpha$-semiprime fuzzy almost bi-$\Gamma$-ideal of $M$.

Conversely, assume that $\chi_A$ is an $\alpha$-semiprime fuzzy almost bi-$\Gamma$-ideal of $M$. By Theorem 5, we have that $A$ is an almost bi-$\Gamma$-ideal of $M$. Let $m \in M$ be such that $m\alpha m \in A$. Thus $\chi_A(m\alpha m) = 1$. By assumption, we have that $\chi_A(m\alpha m) \leq \chi_A(m)$. Since $\chi_A(m\alpha m) = 1$, it follows that $\chi_A(m) = 1$. Therefore, $m \in A$. Consequently, $A$ is an $\alpha$-semiprime almost bi-$\Gamma$-ideal of $M$.

## 5. Conclusion

In this paper, we define almost bi-$\Gamma$-ideals and their fuzzification of $\Gamma$-semigroups. Every bi-$\Gamma$-ideal is an almost bi-$\Gamma$-ideal but the converse is not true in general. We show that the union of two almost bi-$\Gamma$-ideals is also an almost bi-$\Gamma$-ideal. However, it is not generally true in case the intersection. Similarly, we have that the union of two fuzzy almost bi-$\Gamma$-ideals is also a fuzzy almost bi-$\Gamma$-ideal but it is not generally true in case the intersection. Moreover, the relationships between almost bi-$\Gamma$-ideals and their fuzzification were shown in Section 4.

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