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Use of some Topological Concepts in the study of some COVID−19 Symptoms

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Abstract. We apply some topological concepts on topological spaces generated from both equality and similarity relation for our information system, which examine 10 hypothetical patients who suffer from some COVID−19 symptoms. This is based on the degree of accuracy generated from the cardinality of the lower and upper approximation. This method is clarified by application.

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Key Words and Phrases: Rough sets, topology, similarity, lower and upper approximation, accuracy

1. Introduction

The present time is characterized by an abundance of computers that can collect a lot of information on any subject. This information allows to make decision. We need to create mathematical models for this information that help in analyzing and extracting knowledge from it [5],[7]. In addition to foreign schools in Germany and America, the 2016 Nobel Prize in Physics was awarded for topological uses in the theory of material transformation using topological applications in science and engineering [2]. One of the most important mathematical models is the Rough Set Theory based on topological concepts [4]. The notion of rough sets was introduced by Pawlak [7]. From the outset, rough set theory has been a methodology of database mining or knowledge discovery in relation databases [6], [3]. The rough set methodology is based on the premise that lowering the degree of precision in the data makes the data pattern more visible, whereas the central premise of the rough set philosophy is that the knowledge consists in the ability of classification. Previously induction set introduced by Pawlak [8], using equivalence relation which was considered a major constraint, an therefore, research tended to use unequal relation. In this paper, we use similarity relationships to find neighborhoods of objects and use them in approximations: we calculate the degree of correlation attributes with total information and use this correlation to deduce the effective attributes [1]. We introduce in this paper some symptoms of inflammation of the respiratory system in which

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the inflammation cases extend from easy treatment for chronic disease that the patient suffers throughout his life, such as asthma and pneumonia. However, there are diseases of the respiratory system that are difficult to treat and may lead to death, such as Covid-19. In this paper, we use the topological concepts interior and closure to know the degree of correlation between patients and symptoms of inflammation of the respiratory system, and any other symptom that are more related with respiratory system diseases.

2. Preliminaries

Motivation to use rough set theory has come from the requirement to represent subsets of a universe set in terms of equivalence classes of a partition of that universe set. We consider the partition of equivalence classes is a topological space.
We are given an information system $S = (U, A, C)$ where U, A and C are finite, non-empty sets. We called U is a universe set , A is attributes and C is cases of attributes. Also, we denote of equivalence classes by $R_p,\ p \in U$ and R_X is denoted of equivalence classes in accordance with C of attribute A.

2.1 Topological spase

Definition 1 [4] Let B be a subset of a topological space (X, τ), the union of all open sets contained in B is called the interior of a set B and denoted by $int(B)$ or B°. The interior of a set B is the largest open set contained in B; i.e., $B^\circ = \bigcup\{U \subseteq X : U \subseteq B, U \in \tau\}$, U is open set. The intersection of all closed sets containing B is called the closure of a set B and denoted by $cl(B)$ or $\overline{B}$, i.e. $\overline{B} = \bigcap\{F \subseteq X : B \subseteq F, F \in \tau^c\}$, F is the closed set.

2.2 Approximation spase

Definition 2 [7] Suppose that we are given knowledge base $I = (U, R)$, with each subset $X \subseteq U$ and an equivalence relation $R \in IND(I)$, we associate two subsets:

$$\underline{P(X)} = \bigcup\{p \in U : R_p \subseteq R_X\},$$
and
$$\overline{P(X)} = \bigcup\{p \in U : R_p \cap R_X \neq \phi\}.$$
Two approximations $\underline{P(X)}$, and $\overline{P(X)}$ called the lower approximation and upper approximation of X respectively.
Also, the accuracy of the approximation is defined by

$$\alpha(X) = \frac{|\underline{P(X)}|}{|\overline{P(X)}|}\ ,\quad 0 \leq \alpha(X) \leq 1.$$
If a set X with accuracy equal to 1 is crisp, otherwise X is rough.
Definition 3 [1] For each $B \subseteq A$, the relation $R_B \subseteq U \times U$ defined

$$x \ \mathrm{R}_B \ y = \frac{|\sum_{i=1}^{4}(a(x_i) = b(y_i))|}{4} \qquad \text{where } |.| \text{ is the cardinality of } B.$$

In this paper $U = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}\}$ is patients, $A = \{X, Y, Z, W\}$ is symptoms of inflammation of respiratory system and C is classification of symptoms as in Application down.

3. Topological spaces generated from relations

Application : We consider data of ten hypothetical patients have respiratory disease and suffer from the following symptoms: Temperature,breathing difficulty, cough and muscle pain. We denote the ten patients by $\{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}\}$, and we symbolize for symptoms by the following symbols:

1. Symptom the temperature by the symbol X, where it is classified into high temperature H, very high temperature V and simple temperature S.
2. Symptom breathing difficulty by the symbol Y, where it is classified into a strong situation T, medium situation M and simple situation N.
3. Denote the symptom cough by Z, where it is classified into dry cough C, cough with phlegm L and an attic cough G.
4. Symptom muscle pain by the symbol W, where it is classified into strong pain R, medium pain D and simple pain E.

Table 1: Information system

	X	Y	Z	W
1	V	M	L	D
2	S	N	L	D
3	V	N	L	R
4	V	M	C	R
5	V	T	C	D
6	H	M	C	R
7	H	M	C	E
8	H	M	G	E
9	S	T	G	E
10	S	T	G	E

Part 1 : According to the Pawlak′s model, we constitute patients′s classes R_p depending on the previous symptoms, whereas patients′s classes establish according to the equal in the all symptoms and it produces the following class:
$R_p = \{\{p_1\}, \{p_2\}, \{p_3\}, \{p_4\}, \{p_5\}, \{p_6\}, \{p_7\}, \{p_8\}, \{p_9, p_{10}\}\}$.
We discuss the above symptoms through a table 1 in the following cases:

case1 :
From the table 1 appears the class R_X of symptom temperature that contains three sets of patients according their pain:
$X_V = \{p_1, p_3, p_4, p_5\}$, $X_S = \{p_2, p_9, p_{10}\}$, $X_H = \{p_6, p_7, p_8\}$.
Likewise, from the table 1 appears the classes of the other symptoms such as :difficulty breathing, cough ,and muscle pain. It shows the following patients′s classes R_Y, R_Z, R_W contain the following sets according to symptom situation:

$$Y_M = \{p_1, p_4, p_6, p_7, p_8\}, \ Y_T = \{p_5, p_9, p_{10}\}, \ Y_N = \{p_2, p_3\}.$$

$$Z_C = \{p_4, p_5, p_6, p_7\}, \ Z_L = \{p_1, p_2, p_3\}, \ Z_G = \{p_8, p_9, p_{10}\}.$$

$$W_R = \{p_3, p_4, p_6\}, \ W_D = \{p_1, p_2, p_5\}, \ \ W_E = \{p_7, p_8, p_9, p_{10}\}.$$

We find the accuracy, mean the degree of correlation between the overall information and both previous symptoms. So we find the lower and upper of patients′s classes as to temperature X :

$$\underline{X_V} = \{p_1, p_3, p_4, p_5\}, \ \underline{X_S} = \{p_2, p_9, p_{10}\}, \ \underline{X_H} = \{p_6, p_7, p_8\}.$$

We make union of these classes and we have:
$\underline{P(X)} = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}\}$.

Also in upper case, we get

$$\overline{X_V} = \{p_1, p_3, p_4, p_5\}, \ \overline{X_S} = \{p_2, p_9, p_{10}\}, \ \overline{X_H} = \{p_6, p_7, p_8\}.$$

We make union of these classes and we have:
$\overline{P(X)} = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}\}$.
The degree of correlation between the overall information and temperature is :
$\dfrac{|\underline{P(X)}|}{|\overline{P(X)}|} = \dfrac{10}{10} = 1.$
This mean that accreditation ratio is 100%.
Similarly, we find the lower and upper approximation of patients′s classes with other symptoms breathing difficulty Y, cough Z and muscle pain W,we will find that accreditation ratio between the overall information and any of other symptoms is 100%.

case2 :
From the table 1 appears the class R_{XY} of symptom temperature X and breathing difficulty Y, it shows equivalence class R_{XY} of patients :
$R_{XY} = \{\{p_1, p_4\}, \{p_2\}, \{p_3\}, \{p_5\}, \{p_6, p_7, p_8\}, \{p_9, p_{10}\}\}$.
When we find the lower and upper approximation, we will have the degree of correlation between the overall information on XY is 100%.

Similarly, we find the lower and upper approximation of patients′s classes with XZ, XW, YZ, YW, ZW, XYZ, XYW, YZW, all of them we have the degree of correlation is 100%.

Part 2 :
In this part we do a similarity matrix which represents the degree of similarity between patients. We create the elements of this similarity matrix by sum of the number of similar symptoms at each two patients and divide it by the total number of symptoms , this elements of this similarity matrix are degree of similarity. i.e.

The degree of similarity $= \dfrac{|\sum_{i=1}^{4}(a(x_i) = b(y_i))|}{4}$ where $a, b \in U$ and x_i, y_i denoted to the similar symptoms. See Table 2

We formed the classes of patients at the beginning based on the equal between them, now we form patients classes based on degree of similarity $\frac{1}{4}$ between them.
we have the following topological approximation space depending on degree of similarity $\frac{1}{4}$, this approximation classes is:
$R_{p_{sim(1/4)}} = \{\{p_6, p_7, p_8\}, \{p_5, p_9, p_{10}\}, \{p_5, p_6\}, \{p_8\}, \{p_2, p_3, p_6, p_7, p_9, p_{10}\}, \{p_1, p_3, p_5\}, \{p_1, p_5, p_9, p_{10}\}, \{p_1, p_4\}, \{p_2, p_5, p_7\}\}$.
We discuss these classes through table 1 in the following cases:

case1 :
When we looked at table 1 from through symptom temperature X, it showed us the classes R_X contain three sets of patients according to their pain:
$X_V = \{p_1, p_3, p_4, p_5\}$, $X_S = \{p_2, p_9, p_{10}\}$, $X_H = \{p_6, p_7, p_8\}$.

Now, we find the lower and upper approximation of patient classes $R_{p_{sim(1/4)}}$ with classes R_X, we get:
$\underline{X_V} = \{p_1, p_3, p_4, p_5\}$, $\underline{X_S} = \emptyset$, $\underline{X_H} = \{p_6, p_7, p_8\}$.

$$\overline{X_V} = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}\},$$

$$\overline{X_S} = \{p_1, p_2, p_3, p_5, p_6, p_7, p_9, p_{10}\},$$

$$\overline{X_H} = \{p_2, p_3, p_5, p_6, p_7, p_8, p_9, p_{10}\}.$$

Then the correlation degree between the total information and symptom X depending on the degree of similarity $\frac{1}{4}$ is

$$\frac{|\underline{\underline{P(X)}}|}{|\overline{P(X)}|} = \frac{7}{10}.$$

Table 2: Similarity matrix

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
p_1	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0
p_2	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{4}$	0	0	0	$\frac{1}{4}$	$\frac{1}{4}$
p_3	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	0	0
p_4	$\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0	0
p_5	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	1	$\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{4}$
p_6	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	1	$\frac{3}{4}$	$\frac{1}{2}$	0	0
p_7	$\frac{1}{4}$	0	0	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	1	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
p_8	$\frac{1}{4}$	0	0	$\frac{1}{4}$	0	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$
p_9	0	$\frac{1}{4}$	0	0	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$	1	1
p_{10}	0	$\frac{1}{4}$	0	0	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$	1	1

Where $\underline{P(X)} = \underline{X_V} \cup \underline{X_S} \cup \underline{X_H}$ and $\overline{P(X)} = \overline{X_V} \cup \overline{X_S} \cup \overline{X_H}$

This mean that accreditation ratio is 70%.

Similarly, we will find that accreditation ratio for the symptom Y is 80%. While accreditation ratio of symptoms Z is 30% , and W is 10%

case2 :
From table 1 appears the classes R_{XY} of symptoms XY , as the following :
$R_{XY} = \{\{p_1, p_4\}, \{p_2\}, \{p_3\}, \{p_5\}, \{p_6, p_7, p_8\}, \{p_9, p_{10}\}\}$.

Now, we find the lower and upper approximation of patient classes $R_{p_{sim(1/4)}}$ with classes R_{XY}, we get:
$\underline{P(XY)} = \{p_1, p_4, p_6, p_7, p_8\}$

$$\overline{P(XY)} = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}\}$$

$$\frac{|\underline{P(XY)}|}{|\overline{P(XY)}|} = \frac{5}{10}.$$

This means that the accreditation ratio is 50%.

Also, from table 1 appears the following classes:
R_{XZ} of symptom XZ:
$R_{XZ} = \{\{p_1, p_3\}, \{p_2\}, \{p_4, p_5\}, \{p_6, p_7\}, \{p_8\}, \{p_9, p_{10}\}\}$.

R_{XW} of symptom XW :
$R_{XW} = \{\{p_1, p_5\}, \{p_2\}, \{p_3, p_4\}, \{p_6\}, \{p_7, p_8\}, \{p_9, p_{10}\}\}$.

R_{YZ} of symptom YZ :
$R_{YZ} = \{\{p_1\}, \{p_2, p_3\}, \{p_4, p_6, p_7\}, \{p_5\}, \{p_8\}, \{p_9, p_{10}\}\}$.

R_{YW} of symptom YW:
$R_{YW} = \{\{p_1\}, \{p_2\}, \{p_3\}, \{p_4, p_6\}, \{p_5\},\{p_7, p_8\}, \{p_9, p_{10}\}\}$.

R_{ZW} of symptom ZW:
$R_{ZW} = \{\{p_1, p_2\}, \{p_3\}, \{p_4, p_6\}, \{p_5\}, \{p_7\}, \{p_8, p_9, p_{10}\}\}$.
We have accreditation ratio in all of above with classes which degree of similarity $\frac{1}{4}$ is 10%.

case3 :
From table 1 appears the following classes R_{XYZ} of symptoms XYZ :
$R_{XYZ} = \{\{p_1\}, \{p_2\}, \{p_3\}, \{p_4\}, \{p_5\}, \{p_6, p_7\}, \{p_8\},\{p_9, p_{10}\}\}$.
We find the lower and upper approximation of patient classes $R_{p_{sim(1/4)}}$, we get

$$|\underline{P(XYZ)}| = 1.$$

$$|\overline{P(XYZ)}| = 10$$

$$\frac{|\underline{P(XYZ)}|}{|\overline{P(XYZ)}|} = \frac{1}{10}.$$

This mean that accreditation ratio between the overall information and symptom XYZ is 10% and it is the same ratio with symptoms XYW, YZW .
We do a summary of the above in table 3.

Table 3: Summary the ratio in part 2

Symptoms	The ratio	Symptoms	The ratio
X	70%	YZ	10%
Y	80%	YW	10%
Z	30%	ZW	10%
W	10%	XYZ	10%
XY	50%	XYW	10%
XZ	10%	YZW	10%
XW	10%		

4. Conclusion

We find that accreditation ratio of information on symptom breathing difficulty is the highest, this means that the most symptoms indicate of inflammation of the respiratory system is breathing difficulty. By this we can determine the most important tests to preform saving time, effort, and money from doing tests that have no effect on inflammation of the respiratory system.

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